Assignment 2: Model Predictive Control (MPC)

Contributors:

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Introduction

Defining variables

(x_0, y_0) -> start coordinate

(x_g, y_g) -> goal coordinate

(x_j, y_j) -> j^{th} waypoint coordinates

n -> number of waypoints

(x_ob, y_ob) -> obstracle center coordinate

R -> obstracle radius

Kinematics

$x_{i+1} = x_i + \dot{x_i}dt$

$y_{i+1} = y_i + \dot{y_i}dt$

$x_j = x_0 + \sum_{i=0}^{j-1} \dot{x_i}dt \quad \forall j \in [1, n]$

Optimization problem

$$\begin{aligned} &\min && (x_n - x_g)^2 + (y_n - y_g)^2 \\ &\text{Subject to} && 0 \le \dot{x_i} \le v_{max} \end{aligned}$$

$$
\begin{aligned}
&0 \leq \dot{y}_i \leq v_{max} \quad \forall i \in [0, n-1] \\
\text{if obstracles} \quad &(x_j - x_{ob})^2 + (y_j - y_{ob}) \geq R^2 \quad \forall j \in [1, n]
\end{aligned}
$$

MPC with Obstracles

initial point x_0, y_0

Goal locn x_g, y_g

Circular obstracle center x_{ob}, y_{ob}

radius R

no. of way points n

velocity at waypoints $\dot{x}_0, \dot{x}_1, \ldots, \dot{x}_n$

$\dot{y}_0, \dot{y}_1, \ldots, \dot{y}_n$

Objective
$$\left(x_0 + \sum_{i=0}^{n-1} \dot{x}_i \, dt - x_g\right)^2 + \left(y_0 + \sum_{i=1}^{n-1} \dot{y}_i \, dt - y_g\right)^2$$

such that $v_{min} \leq \dot{x}_i, \dot{y}_i \leq v_{max}$

$$\left(x_0 + \sum_{i=1}^{j-1} \dot{x}_i \, dt - x_{ob}\right)^2 + \left(y_0 + \sum_{i=1}^{j-1} \dot{y}_i \, dt - y_{ob}\right)^2 \geq R^2 \quad \forall j \in [1,n]$$

Linearising the non-linear constraints

$$\left[\left(x_0 + \sum_{i=1}^{j-1} \dot{x}_i \, dt - x_{ob}\right)^2 + \left(y_0 - \sum_{i=0}^{j-1} \dot{y}_i \, dt - y_{ob}\right)^2 - R^2\right] \geq 0 \quad \forall j \in [1,n]$$

$$\underbrace{-\left[\left(x_0 + \sum_{i=1}^{j-1} \dot{x}_i \, dt - x_{ob}\right)^2 + \left(y_0 - \sum_{i=0}^{j-1} \dot{y}_i \, dt - y_{ob}\right)^2 - R^2\right]}_{f(x,y)} \leq 0$$

Taylor series Expansion for Multivariate

$$f(x)\Big| = f(a) + \nabla f(a)\,(x-a)$$

$\big|_{x=a}$

we want to find cvx approximation, hence we consider terms only upto first order.

$$+ \frac{1}{2} (x-a)^T \nabla^2 f(a) (x-a) + \ldots$$

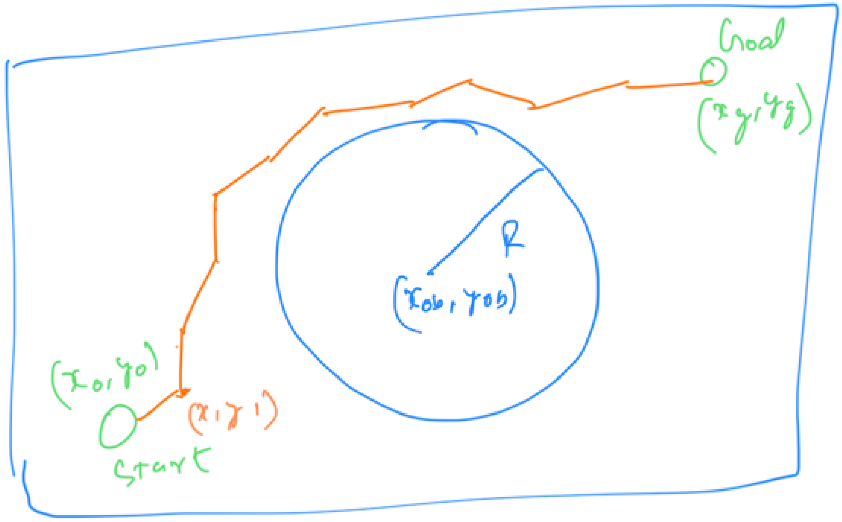
combining above two.

$$f(x)\big|_{x=a} = f(a) + \nabla f(a) \, (x-a)$$

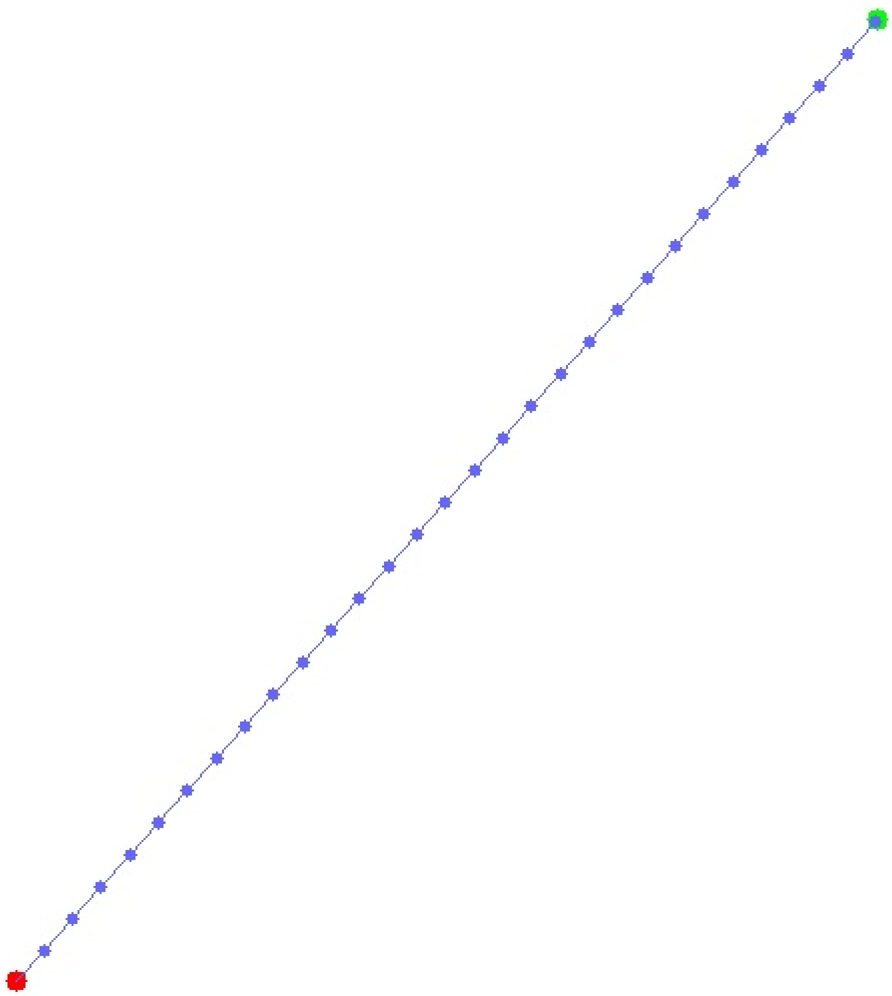
Here $x = \begin{pmatrix} \dot{x}_0 \\ \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \\ \dot{y}_0 \\ \dot{y}_1 \\ \vdots \\ \dot{y}_n \end{pmatrix}$

$$\begin{bmatrix} \frac{\partial f(x)}{\partial \dot{x}_0} \\ \frac{\partial f(x)}{\partial \dot{x}_1} \\ \vdots \\ \frac{\partial f(x)}{\partial \dot{x}_{j-1}} \\ \frac{\partial f(x)}{\partial \dot{x}_j} \\ \vdots \\ \frac{\partial f(x)}{\partial \dot{x}_{n-1}} \\ \frac{\partial f(x)}{\partial \dot{y}_0} \\ \frac{\partial f(x)}{\partial \dot{y}_1} \\ \vdots \\ \frac{\partial f(x)}{\partial \dot{y}_{j-1}} \\ \frac{\partial f(x)}{\partial \dot{y}_j} \\ \vdots \\ \frac{\partial f(x)}{\partial \dot{y}_{n-1}} \end{bmatrix} = \begin{bmatrix} -2\left(x_0 - x_{ob} + \sum_{i=0}^{j-1} \dot{x}_i \, dt\right) dt \\ -2\left(x_0 - x_{ob} + \sum_{i=0}^{j-1} \dot{x}_i \, dt\right) dt \\ \vdots \\ -2\left(x_0 - x_{ob} + \sum_{i=0}^{j-1} \dot{x}_i \, dt\right) dt \\ 0 \\ \vdots \\ 0 \\ -2\left(y_0 - y_{ob} + \sum_{i=0}^{j-1} \dot{y}_i \, dt\right) dt \\ \vdots \\ \vdots \\ -2\left(y_0 - y_{ob} + \sum_{i=0}^{j-1} \dot{y}_i \, d\tau\right) dt \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

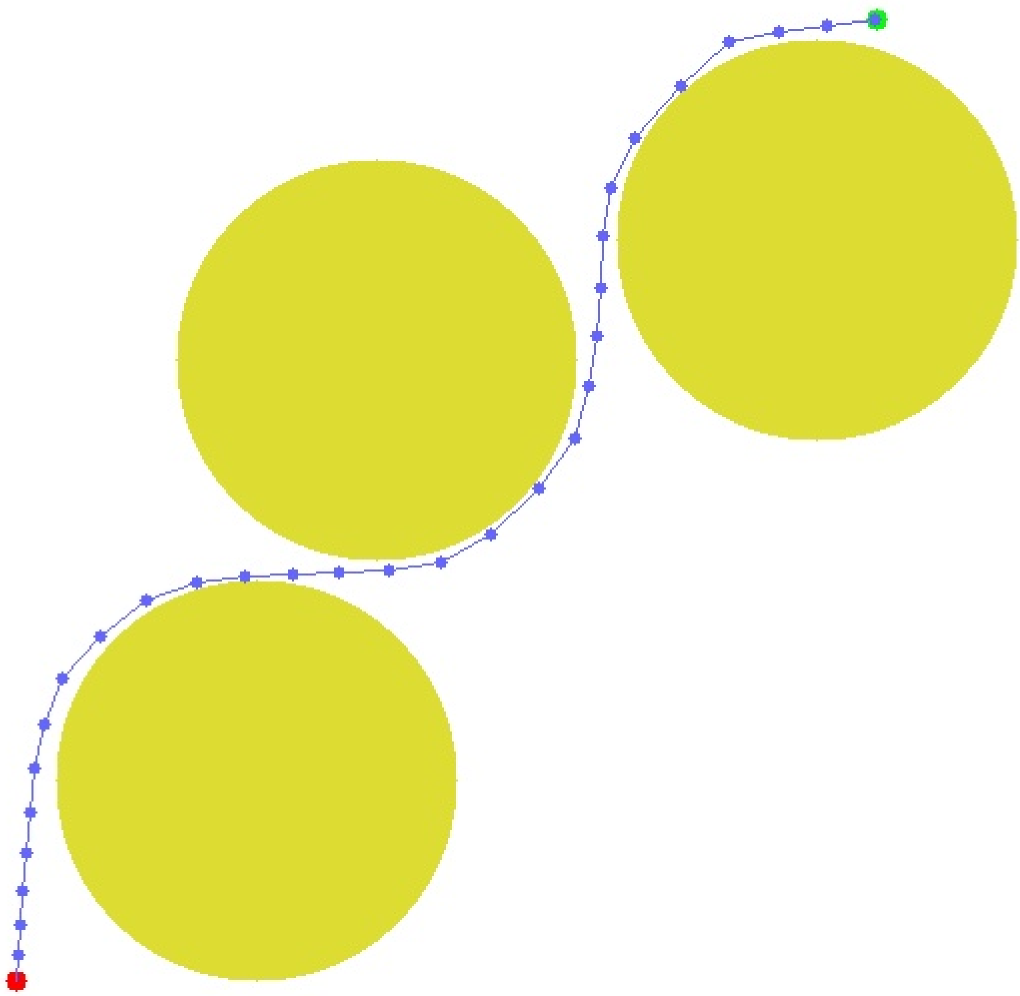
→ The above is ∇f for j^{th} waypoint.



MPC Without Obstracles

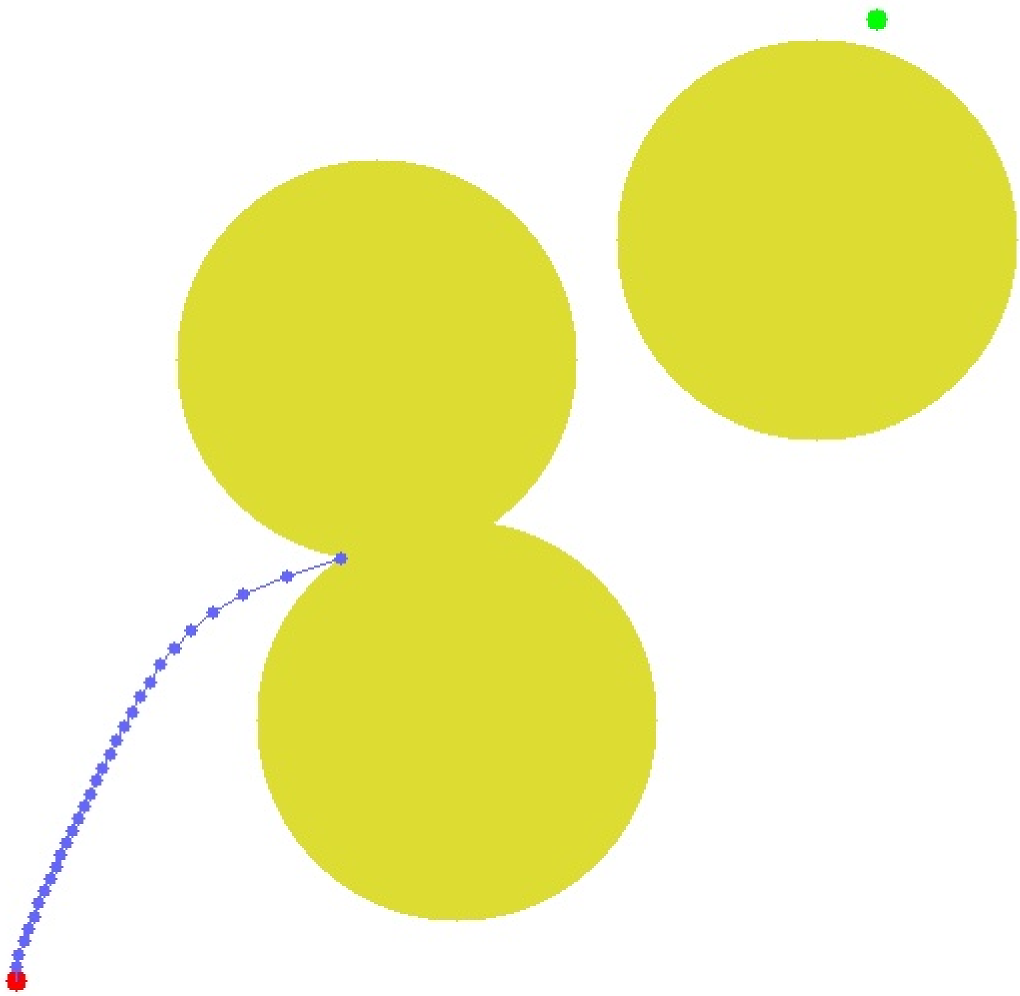


MPC With Obstracles



Failure Case

- Here we observe failure case.
- This is because we linearise the obstracle constraints pointwise which leads to local minima convergence.



Work Distribution

- Mohd Omama: Without Obstracles
- Kinal Mehta: With Obstracles