Homework 2 - Pratik Mistry : Spring DSA 2020 (pdm79)

Solution 1:

Results:

1. Time Complexity for Shell Sort Phase and Insertion Sort Phase

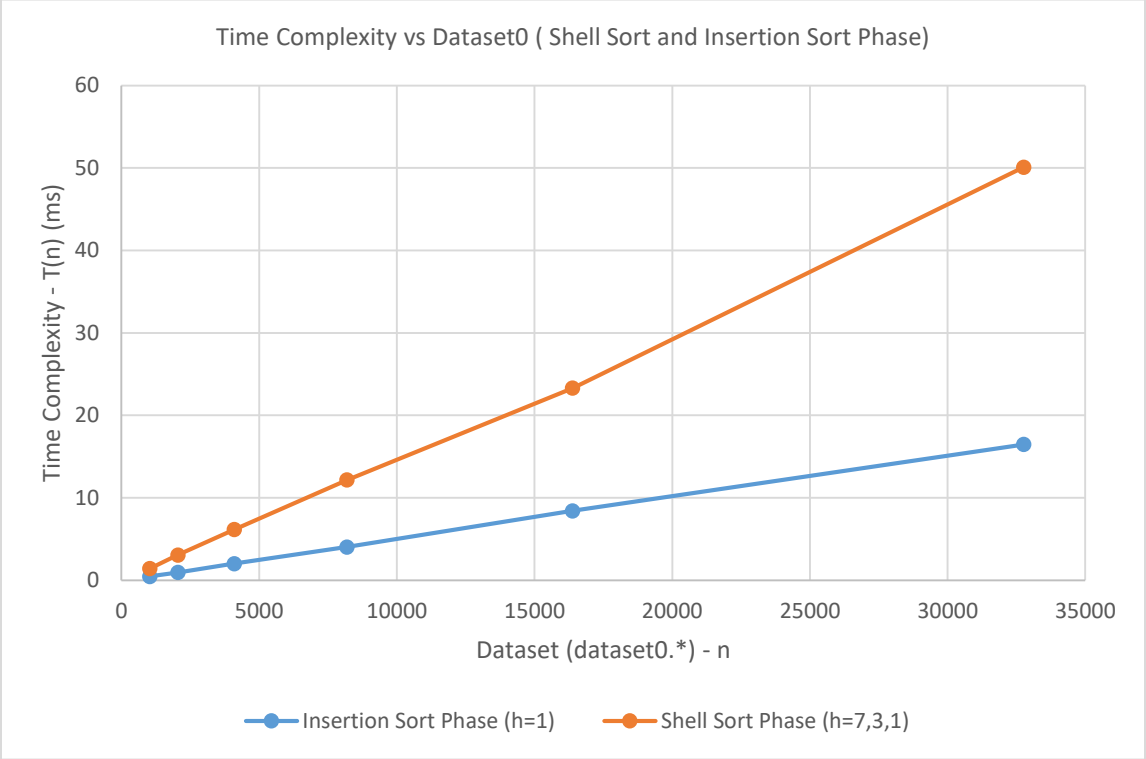
Shell Sort All The Way	Insertion Sort (h=1)	Data Set	
Time (ms)	Time (ms)		
1.41	0.46	1024	dataset0.*
3.05	0.95	2048	
6.14	2	4096	
12.15	4.03	8192	
23.3	8.42	16384	
50.09	16.46	32768	
45.28	332.6	1024	dataset1.*
177.46	1177.15	2048	
710.59	4528.88	4096	
3209.29	19738.74	8192	
13721.08	78297.7	16384	
42303.87	295337.98	32768	

2. Complexity Count for Shell Sort Phase and Insertion Sort Phase

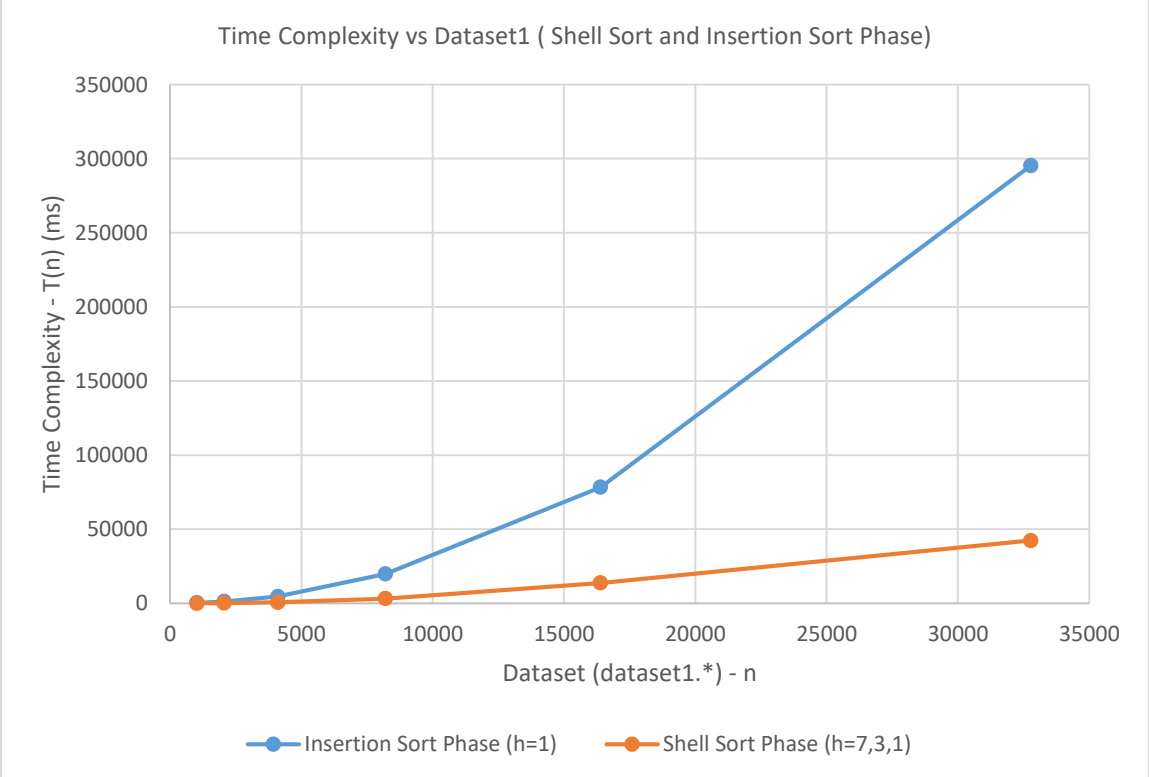
Shell Sort All The Way	Insertion Sort (h=1)	Data Set	
Complexity Count	Complexity Count		
3061	1023	1024	dataset0.*
6133	2047	2048	
12277	4095	4096	
24565	8191	8192	
49141	16383	16384	
98293	32767	32768	
46768	265564	1024	dataset1.*
169081	1029283	2048	
660673	4187899	4096	
2576322	16936958	8192	
9950984	66657566	16384	
39442505	267966675	32768	

Graphs:

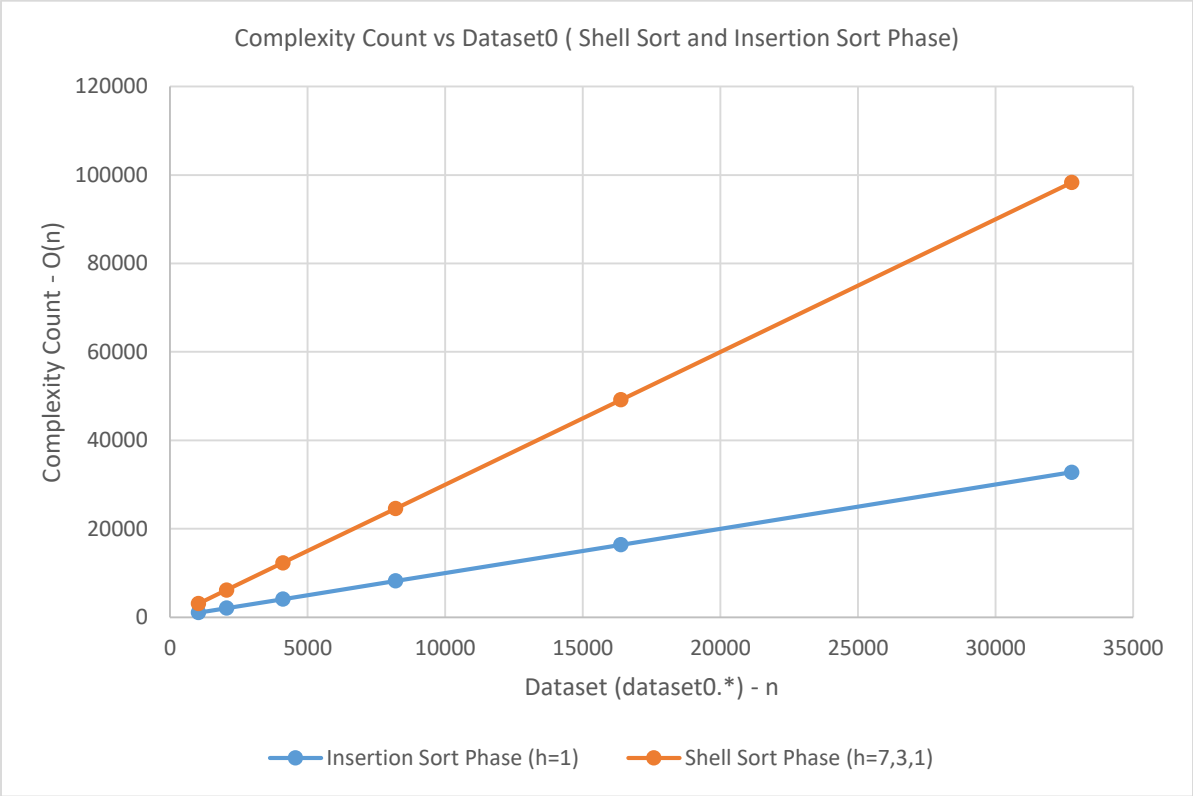
1. Time Complexity v/s Dataset0



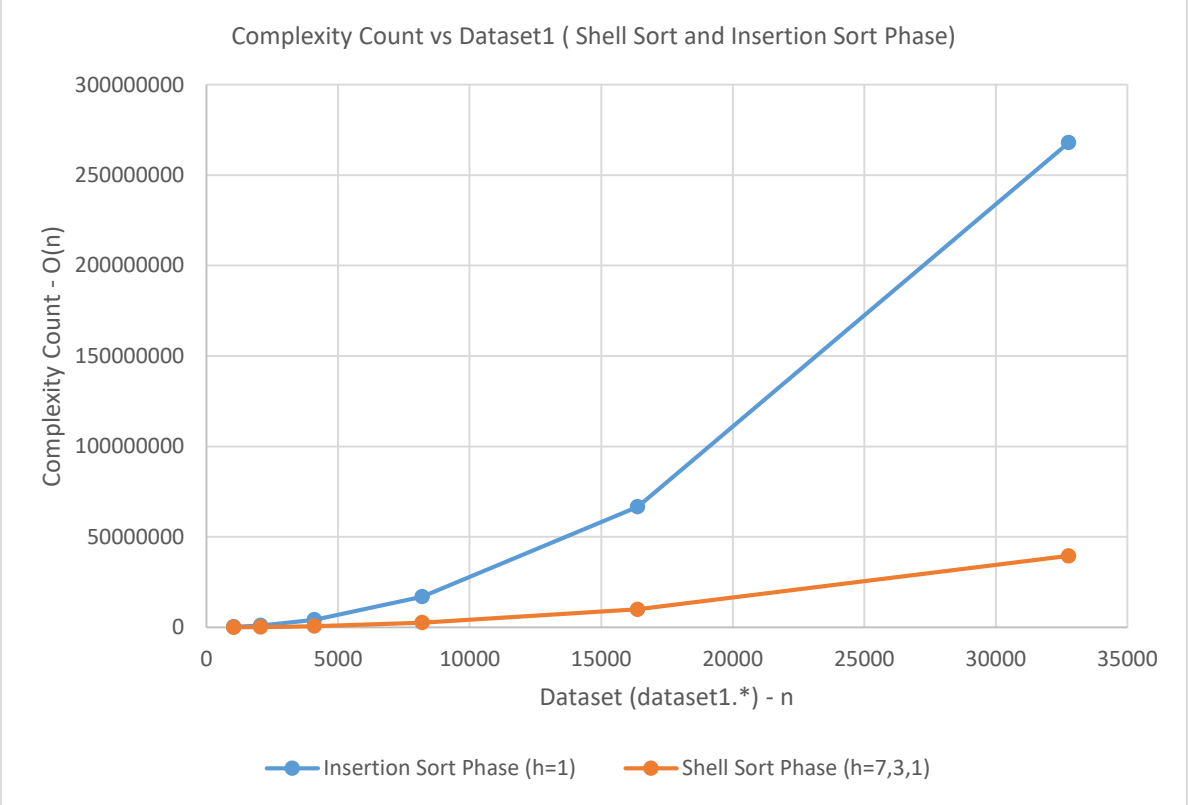
2. Time Complexity v/s Dataset1



3. Complexity Count v/s Dataset0



4. Complexity Count v/s Dataset1



Explanation:

From the results and graphs of Time Complexity (T (n)) or Complexity Count (O (n)), below are the inferences clearly drawn:

- In **dataset0.*** since all the elements are already sorted, the time complexity or complexity count for Shell Sort (Insertion Sort Phase) i.e. when h=1 behaves very well because when h=1 it is Insertion Sort and thus complexity in case of sorted elements is **linear** i.e. **O (n)**.
- While the complexity for the shell sort all the way for h=7, 3, 1 is not linear. It is almost **O (n) = nlog(n)** i.e. **linearithmic** since the data is sorted
- In **dataset1.*** since elements are not sorted, the time complexity or complexity count for Shell Sort (Insertion Sort Phase) i.e. when h=1 does not behave good as it is Insertion sort and the complexity would be nearly **O (n) = n^2** i.e. **quadratic** in worst case.
- While the complexity for the shell sort all the way for h=7, 3, 1 behaves well compared to Insertion Sort Phase with complexity of **O (n) = nlog(n)** i.e. **linearithmic** in average case.
- Thus, for sorted datasets Insertion Sort Phase behaves well compared to Shell Sort all the way while its reverse in case for unsorted datasets.

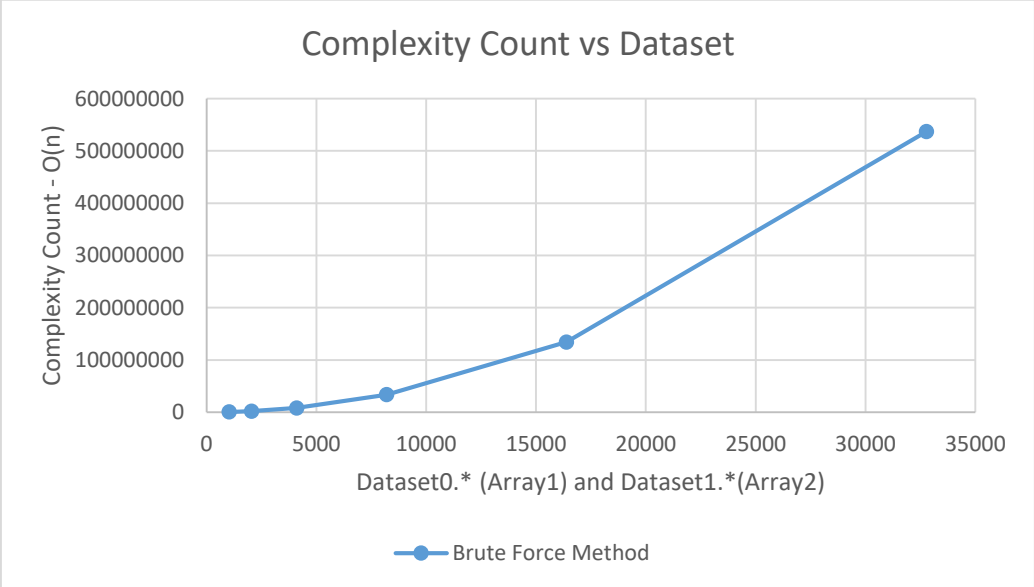
Solution 2:

Results: Complexity Count for Kendall Tau Distance between Sophisticated and Brute Force Methods

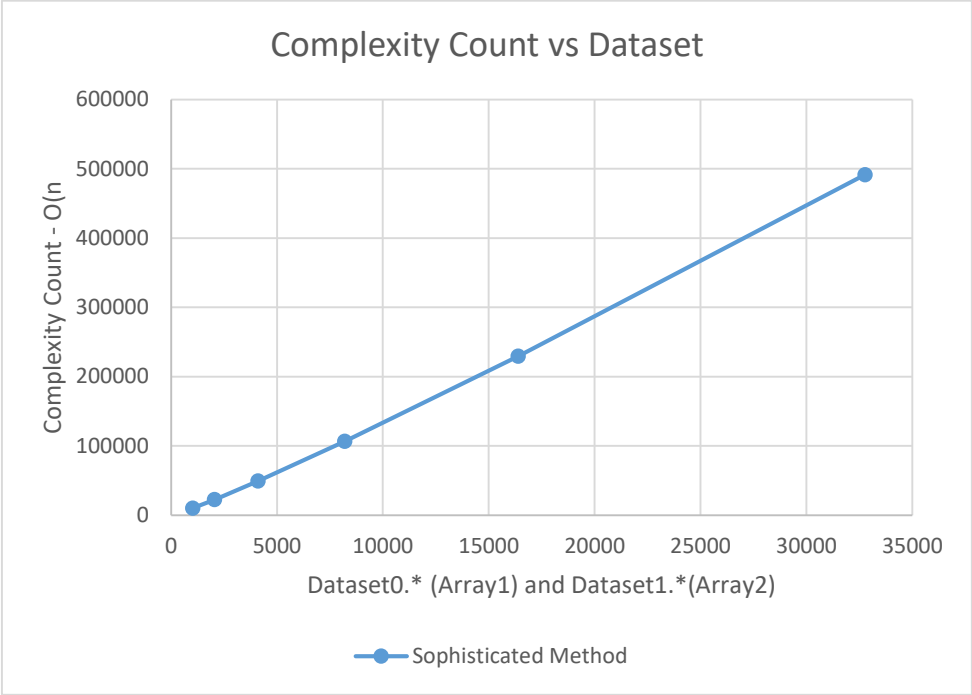
Complexity Count (Sophisticated)	Complexity Count (Brute Force)	Kendall Tau Distance (Sophisticated)	Kendall Tau Distance (Brute Force)	Dataset
10240	523776	264541	264541	1024
22528	2096128	1027236	1027236	2048
49152	8386560	4183804	4183804	4096
106496	33550336	16928767	16928767	8192
229376	134209536	66641183	66641183	16384
491520	536854528	267933908	267933908	32768

Graphs:

1. Complexity Count v/s Dataset : Brute Force Method



2. Complexity Count v/s Dataset : Sophisticated Method



Explanation:

- Kendall Tau Distance between two arrays has some conditions: 1. Array sizes must be same. 2. Array elements must be same.
- Since, we are using array1 as dataset0.* and array2 as dataset1.*, and dataset0.* is already **sorted** then Kendall Tau Distance is number of inversions in dataset1.* i.e. array2
- To find the inversions as Kendall Tau Distance, I tried using Brute Force Method, but the complexity is **O (n) = n^2** i.e. **quadratic** as seen in graphs and results above.
- Thus, to find the inversions using sophisticated method, I have used **Merge Sort** for array2 as sophisticated method and the complexity is nearly **O (n) = nlogn** i.e. **linearithmic**.
- Thus sophisticated method by using merge sort to find inversions i.e. Kendall Tau distance behaves well and can be seen in the graphs and results above.

Solution 3:

Results:

1. Time Complexity for Merge Sort (Top Down) and Merge Sort (Bottom Up)

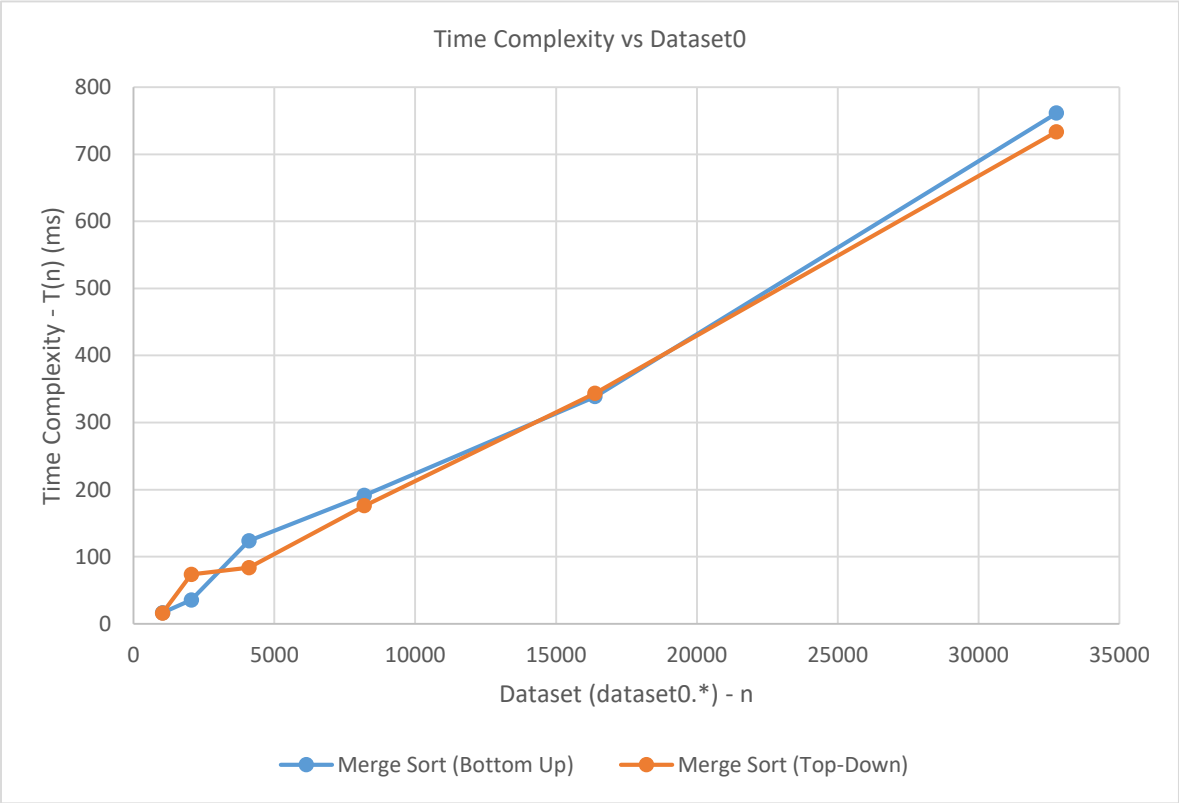
Top Down Time (ms)	Bottom Up Time (ms)	Data Set	
15.87	16.36	1024	dataset0.*
73.72	35.45	2048	
83.58	123.78	4096	
175.96	191.42	8192	
343.58	338.64	16384	
733.3	761.29	32768	
21.02	17.56	1024	dataset1.*
38.02	37.55	2048	
102.44	130.77	4096	
178.35	191.62	8192	
382.42	386.32	16384	
809.21	939.9	32768	

2. Complexity Count for Merge Sort (Top Down) and Merge Sort (Bottom Up)

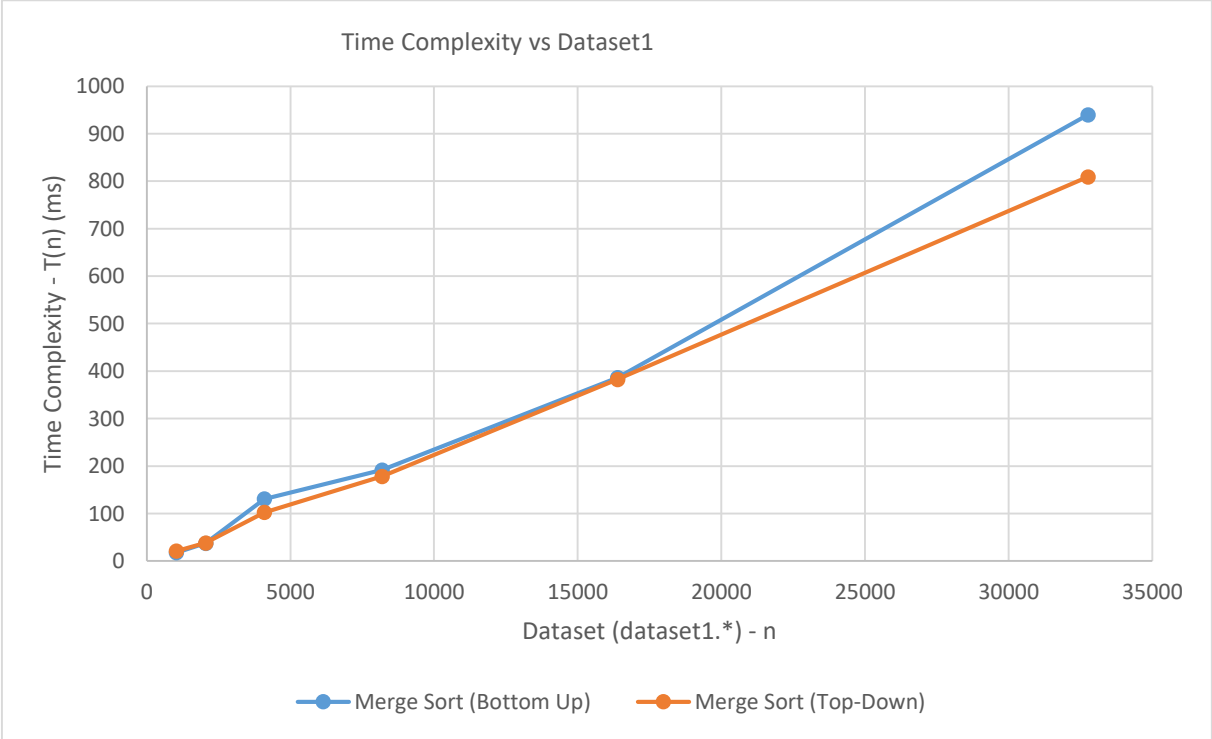
Top Down Complexity Count	Bottom Up Complexity Count	Data Set	
10240	10240	1024	dataset0.*
22528	22528	2048	
49152	49152	4096	
106496	106496	8192	
229376	229376	16384	
491520	491520	32768	
10240	10240	1024	dataset1.*
22528	22528	2048	
49152	49152	4096	
106496	106496	8192	
229376	229376	16384	
491520	491520	32768	

Graphs:

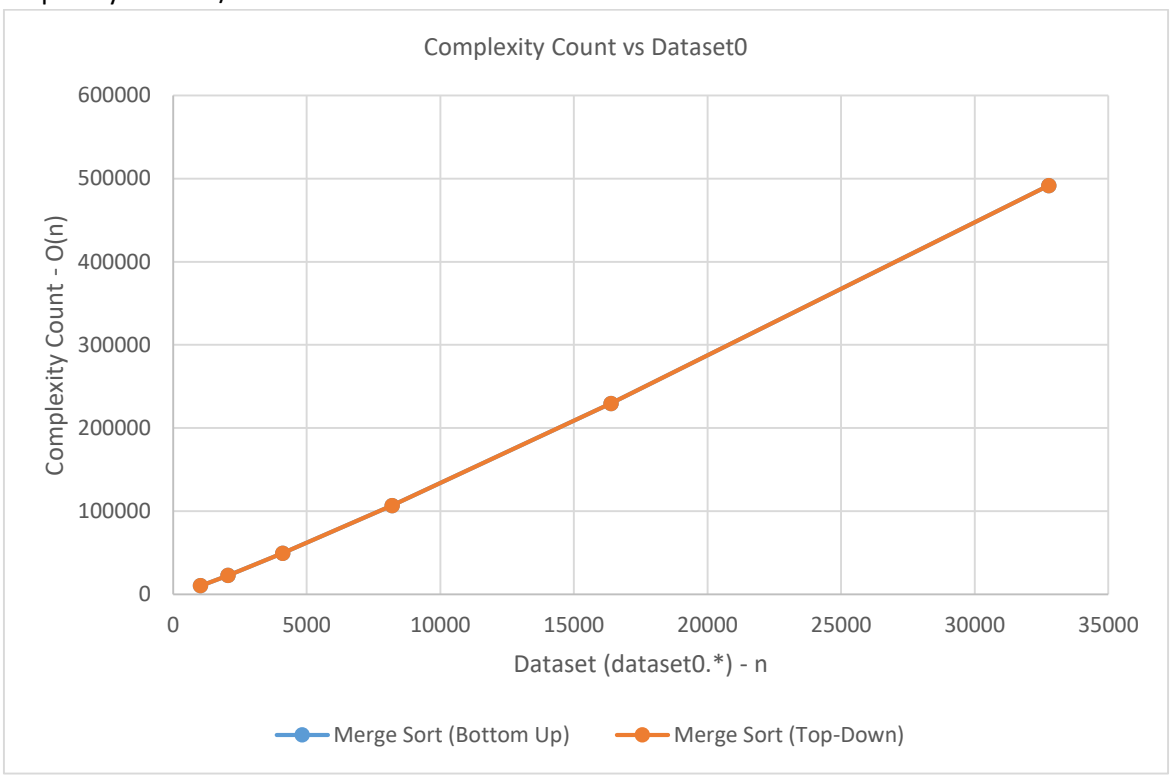
1. Time Complexity v/s Dataset0



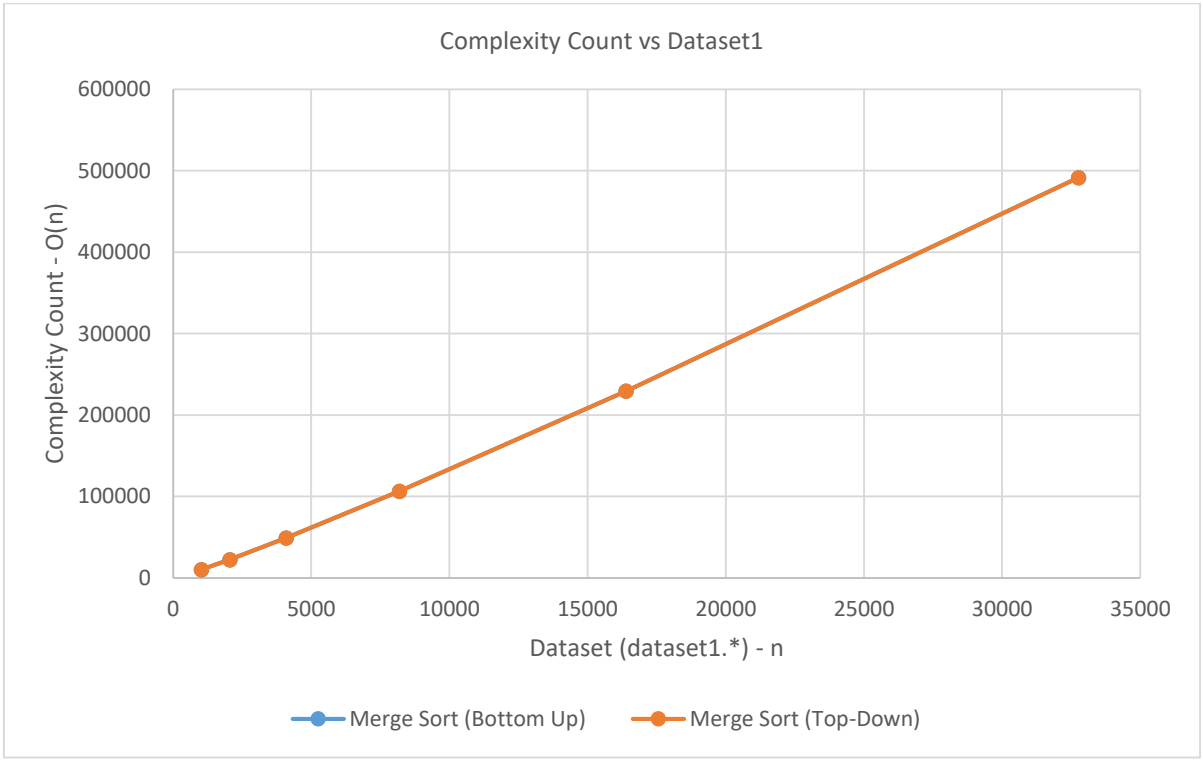
2. Time Complexity v/s Dataset1



3. Complexity Count v/s Dataset0



4. Complexity Count v/s Dataset1



Explanation:

- As seen from the results and graphs above, we can say that the Complexity Count i.e. number of comparisons for both type of Merge Sort – Top Down and Bottom Up are equal since we are counting comparisons in the "merge" operations.
- But, we can see that the time complexity in case of Merge Sort – Top Down approach is better than Bottom Up approach. The only reason is "recursion" which helps in caching the data in the memory while it creates virtual tree in it. Thus, cache locality helps improve time for Top Down approach as seen in graph and results of time complexity
- The only problem in Merge Sort – Top Down method is that the memory may get exhausted in case of higher values of N (i.e. millions of records) due to recursive calls and thus Bottom Up approach would help.
- The complexity for both types of Merge Sort is nearly same and it is **O (n) = nlogn i.e. linearithmic**.

Solution 4:

Results: Complexity Count for the Algorithm

Complexity Count	Dataset
8191	8192

Explanation:

- Since the dataset to be generated of 8192 elements is consecutive orders of 1, 11, 111 and 1111 which is already sorted, there are many sorting algorithms that have linear time complexity. I have implemented **Insertion Sort** which has complexity **O (n) = n** i.e. **linear** since dataset is already sorted.
- We can even implement Merge Sort and Bubble Sort which would have linear complexity but would need some improvements/addition to their standard algorithms.
- For Merge Sort, we can check condition of first/lowest element in second array with highest/largest element in first array before merging which would infer that the two sub-arrays are already sorted.
- For Bubble sort, we can add a flag which will be set to True or 1 every time the data is swapped and flag to False or 0 before entering the inner loop for comparison.
- Since, there is no modifications required for existing standard insertion sort algorithm, it can be used effectively to solve this given problem as it have **O (n) = n** i.e. **linear** complexity as seen in table above.

Solution 5:

Part 1: Performance Comparison between Merge Sort (both types), Quick Sort (Median of 3) and Quick Sort (Cutoff to Insertion Sort with N=7) for the datasets given

Results:

1. Complexity Count for Comparison between Merge Sort (both types), Quick Sort (Median of 3) and Quick Sort (Cutoff to Insertion Sort with N=7)

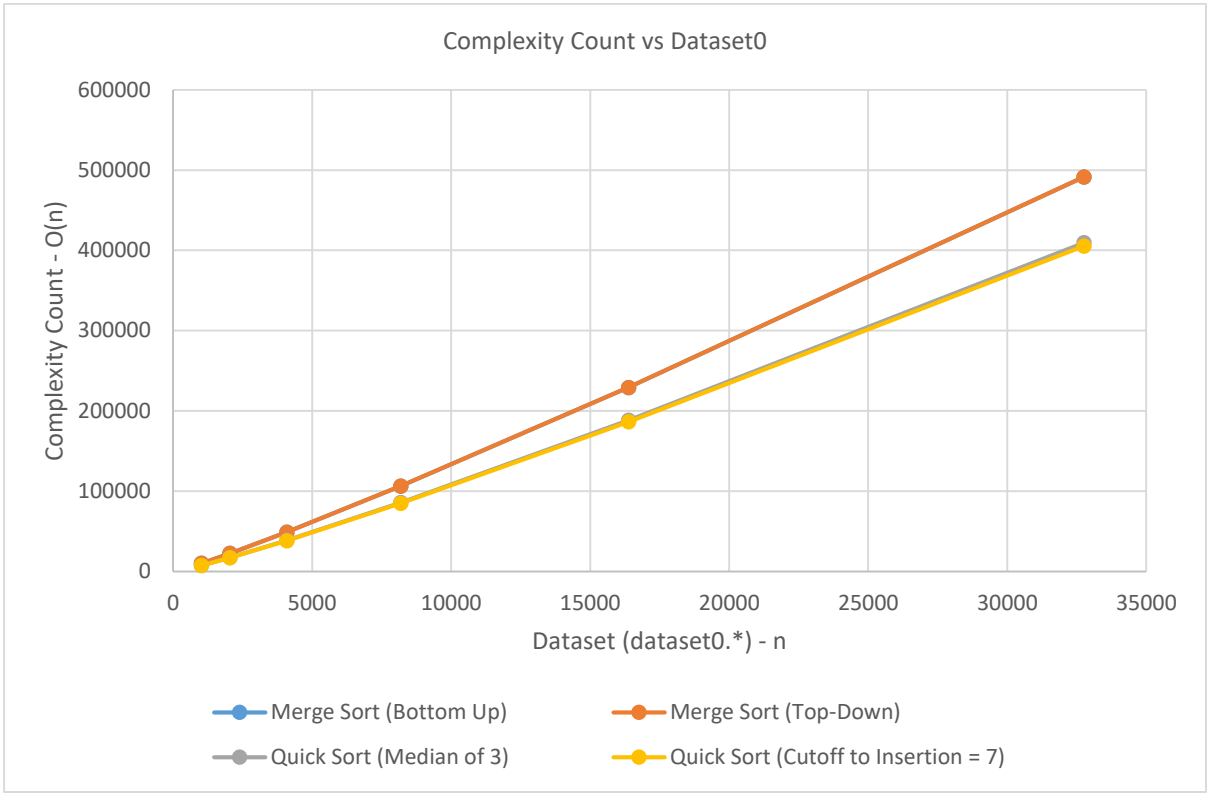
Quick Sort (Cutoff: N=7)	Quick Sort (Median of 3)	Merge Sort (Top Down)	Merge Sort (Bottom Up)	Data Set	
Complexity Count	Complexity Count	Complexity Count	Complexity Count		
7567	7693	10240	10240	1024	dataset0.*
17168	17422	22528	22528	2048	
38417	38927	49152	49152	4096	
85010	86032	106496	106496	8192	
186387	188433	229376	229376	16384	
405524	409618	491520	491520	32768	
6747	5686	10240	10240	1024	dataset1.*
14023	11903	22528	22528	2048	
31125	26978	49152	49152	4096	
67132	58748	106496	106496	8192	
151849	134924	229376	229376	16384	
320151	286896	491520	491520	32768	

2. Time Complexity for Comparison between Merge Sort(both types), Quick Sort(Median of 3) and Quick Sort(Cutoff to Insertion Sort with N=7)

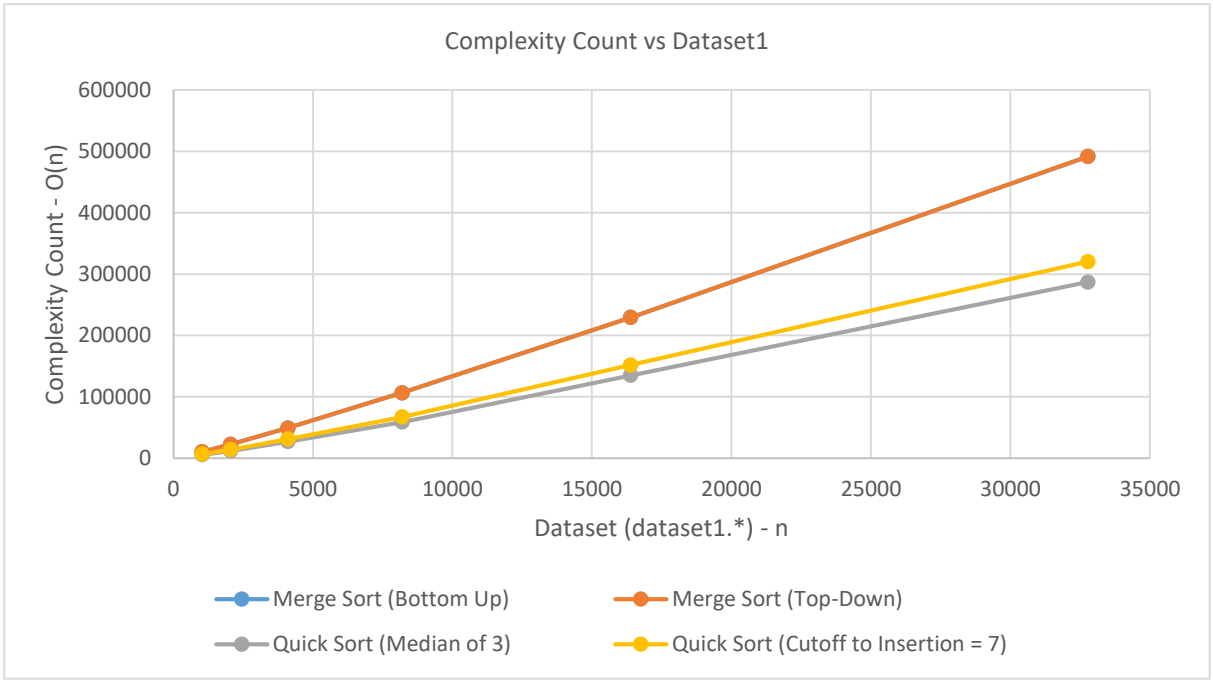
Quick Sort (Cutoff: N=7)	Quick Sort (Median of 3)	Merge Sort (Top Down)	Merge Sort (Bottom Up)	Data Set	
Time (ms)	Time (ms)	Time (ms)	Time (ms)		
4.2	5.59	15.87	16.36	1024	dataset0.*
10.89	12.4	73.72	35.45	2048	
43.12	48.62	83.58	123.78	4096	
72.45	91.73	175.96	191.42	8192	
144.65	183.23	343.58	338.64	16384	
227.61	258.18	733.3	761.29	32768	
7.14	7.11	21.02	17.56	1024	dataset1.*
26.78	16.07	38.02	37.55	2048	
49.17	33.91	102.44	130.77	4096	
119.31	83.67	178.35	191.62	8192	
203.89	177.65	382.42	386.32	16384	
362.87	348.84	809.21	939.9	32768	

Graphs:

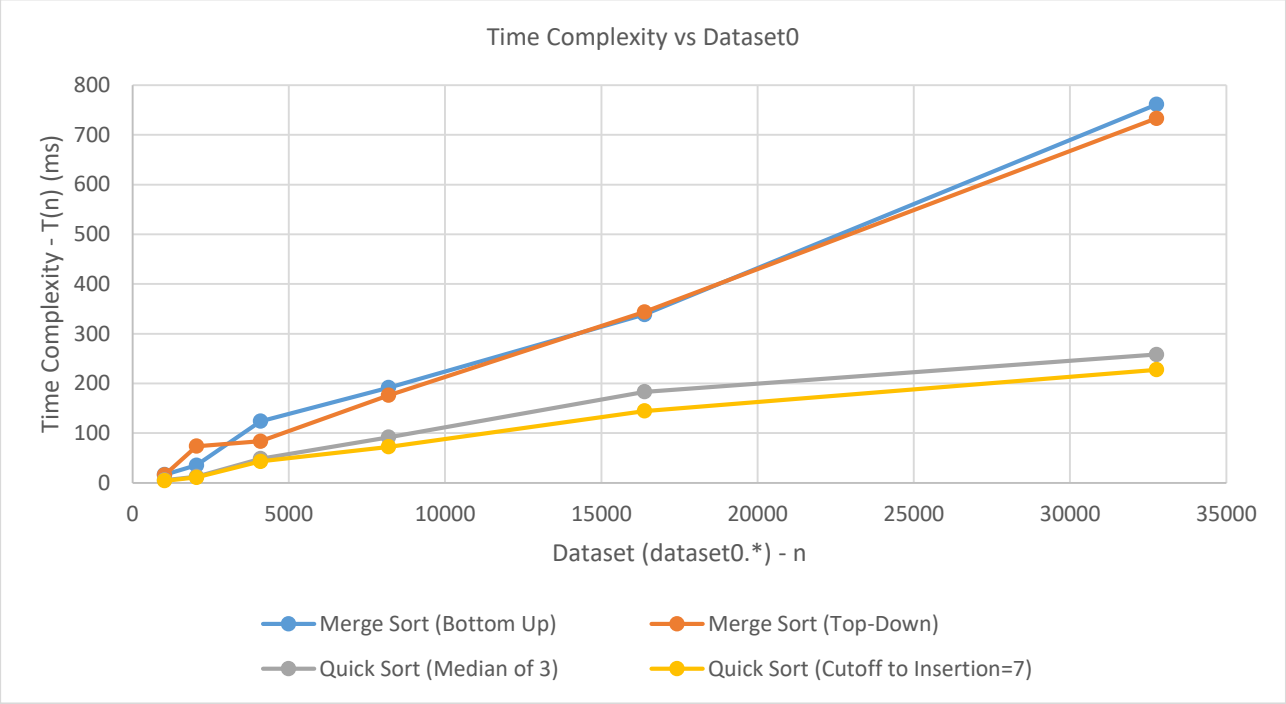
1. Complexity Count v/s Dataset0 for Merge Sort(both types), Quick Sort(Median of 3) and Quick Sort(Cutoff to Insertion Sort with N=7)



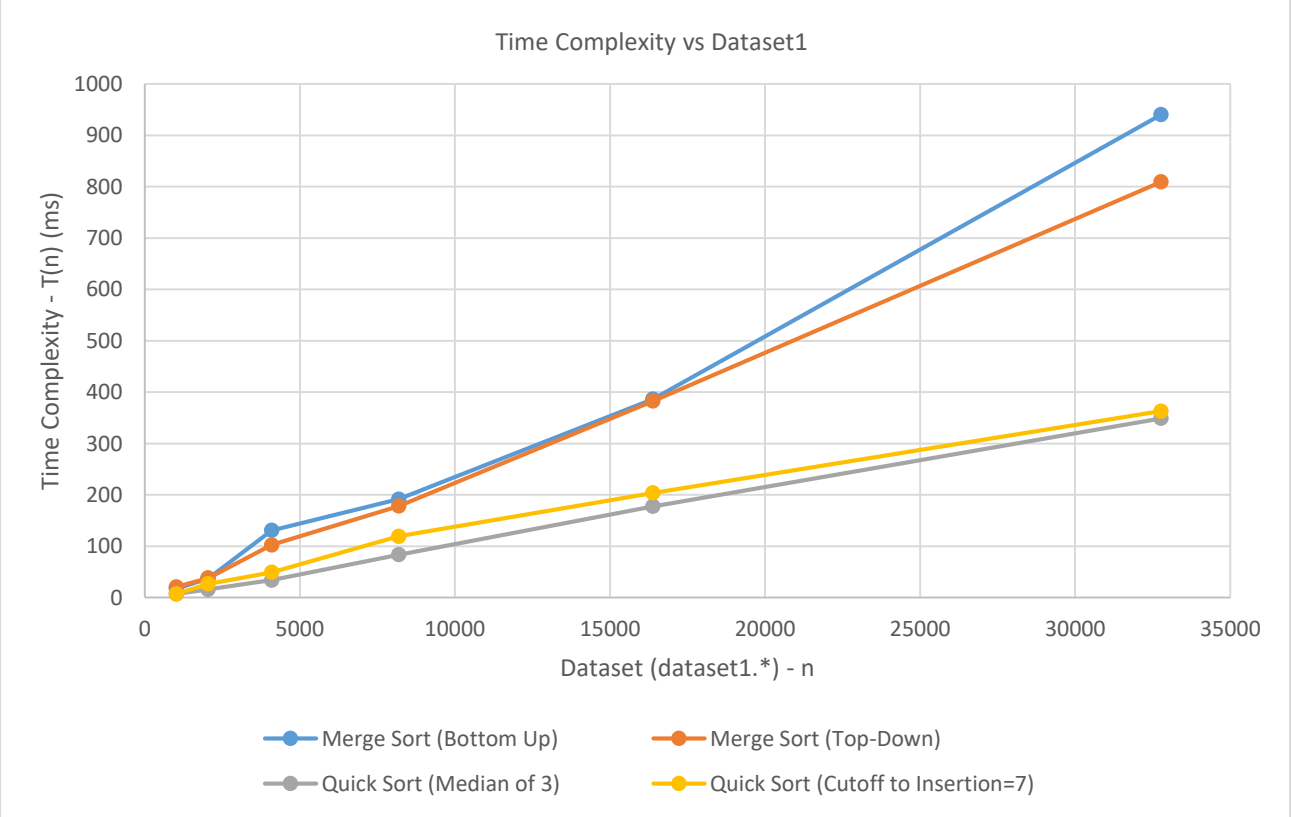
2. Complexity Count v/s Dataset1 for Merge Sort(both types), Quick Sort(Median of 3) and Quick Sort(Cutoff to Insertion Sort with N=7)



3. Time Complexity v/s Dataset0 for Merge Sort (both types), Quick Sort(Median of 3) and Quick Sort(Cutoff to Insertion Sort with N=7)



4. Time Complexity v/s Dataset1 for Merge Sort(both types), Quick Sort(Median of 3) and Quick Sort(Cutoff to Insertion Sort with N=7)



Explanation:
From the results and graphs of Time Complexity (T (n)) or Complexity Count (O (n)), below are the inferences clearly drawn:

- In **dataset0.*** since all the elements are already sorted, the time complexity or complexity count for merge sort (both types) are nearly similar as described in Solution3 too. While, quick sort with median of 3 method, performs better than merge sort. Moreover, quick sort with median of 3 along with Cutoff to Insertion Sort (N=7) performs little better than quick sort.
- The main reason why quick sort with cutoff to insertion sort behaves better than quick sort is because the elements are already sorted and insertion sort on sorted array takes linear time.
- In **dataset1.*** since the elements are unordered/not sorted the performances vary for all the sorting algorithms. Merge Sort (both types) perform nearly same but quick sort with median of 3 performs better than merge sort. Also, quick sort with cutoff to insertion sort (N=7) behaves better than merge sort but a bit low in performance compared to quick sort with median of 3.
- The reason why quick sort with cutoff to insertion sort behaves bad in dataset1.* i.e. unsorted array compared to quick sort with median of 3 is because insertion sort in unsorted array can take quadratic time in worst case. Thus, quick sort with median of 3 performs better and in each recursive call the median of 3 helps in placing the median in almost correct position and reducing overhead while partitioning and sorting.

Part 2: Performance Comparisons between Quick Sort (Median of 3) and Quick Sort (Cutoff to Insertion Sort for different values of N)
Results:

1. Cutoff Analysis for Different Values of Cutoff i.e. N for Dataset0.32768

dataset0.32768		
Time (ms)	Complexity Count	Quick Sort Category
258.18	409618	Quick Sort (Median of 3)
319.14	417809	Quick Sort (Cutoff to Insertion = 5)
292.97	**417809**	**Quick Sort (Cutoff to Insertion = 6)**
227.61	405524	Quick Sort (Cutoff to Insertion = 7)
299.56	405520	Quick Sort (Cutoff to Insertion = 8)
335.06	405520	Quick Sort (Cutoff to Insertion = 9)
472.37	405520	Quick Sort (Cutoff to Insertion = 10)

2. Cutoff Analysis for Different Values of Cutoff i.e. N for Dataset1.32768

dataset1.32768		
Time (ms)	Complexity Count	Quick Sort Category
348.84	286896	Quick Sort (Median of 3)
430.13	286896	Quick Sort (Cutoff to Insertion = 1)
416.94	**291522**	**Quick Sort (Cutoff to Insertion = 2)**
362.08	306126	Quick Sort (Cutoff to Insertion = 4)
487.59	315736	Quick Sort (Cutoff to Insertion = 6)
362.87	320151	Quick Sort (Cutoff to Insertion = 7)
713.04	324436	Quick Sort (Cutoff to Insertion = 8)

Explanation:
For analyzing the performance in quick sort with different values of cutoff, I have considered one dataset each for sorted and unsorted dataset i.e. dataset0.32768 and dataset1.32768. Also, I have not considered time complexity i.e. physical clock time into consideration because the time may change each time the program runs.

- For dataset0.32768 since the data is already sorted, the higher the cutoff value better would be the performance. The cutoff values N = 7,8,9 and so on perform better compared to the Quick Sort Median of 3 method. But for **N = 6** cutoff value, the performance reverts/degrades compared to Quick Sort median of 3 method as seen in table 1 above.
- For dataset1.32768 since the data is not sorted, the lower the cutoff value better would be the performance. Only cutoff value N=1 performs same as that of quick sort median of 3. While, the performance reverts/degrades compared to Quick Sort median of 3 when cutoff value is **N=2** and higher as seen in table 2 above.

Solution 6:
Explanation:
Column 1: Original Array

Column 2: This is the intermediate step of **Merge Sort (Bottom Up method)** as we can see that every 4 elements are sorted which is the result after sorting with size 2.

Column 3: This is the intermediate step of **Quicksort (standard, no shuffle)** as we can see that the first element “navy” is placed correctly at the position same as final position and elements to the left/above are lower than right/below of “navy”. Moreover, to differentiate between standard and 3-way quick sort, we can see that the next higher element to “navy” i.e. “plum” in original array is replaced with the smallest element from the right/end i.e. “mist” after the first iteration and so on.

Column 4: This is the intermediate step of **Knuth Shuffle** algorithm as we can see that all the elements before “silk” are shuffled while elements after it are still in place.

Column 5: This is the intermediate step of **Merge Sort (Top Down method)** as can see that the first 12 elements are sorted i.e. up to “teal” while the for the next 12 elements half of them are sorted i.e. up to “wine” and the next half is also sorted starting from “café”. Thus, next step would be merging and sorting the second half of the column i.e. merging two subarrays of 6 elements (sorted) each.

Column 6: This is the intermediate step of the **Insertion Sort** algorithm as we can see that up to “teal” the data is sorted and this is what Insertion sort does. In each iteration it sorts the earlier part of array i.e. the left sub array.

Column 7: This is the intermediate step of **Heap Sort** algorithm in max-heap form where the largest element “wine” is placed on top of tree/array as root and then followed by two children – “teal” and “silk” less than root and then further these children having their children – “plum”, “sage”, “pink” and “rose” less than them and so on.

Column 8: This is the intermediate step of **Selection Sort** as we can see that the top most element is the smallest element and this is what Selection sort does. In each iteration it finds the smallest array and sorts the array accordingly as visible until “mint”.

Column 9: This is intermediate step of **Quick Sort (3-way, no shuffle)** as we can see that “navy” selected as pivot is placed at correct position with respect to final array and also we can see that during partitioning while it traverses from top/left to bottom/right, it places the elements greater than pivot to the end of array and move the end pointer above 1. Similarly, it swaps pivot with the smaller elements to the top/left of it. This is clearly visible that the elements greater than “navy” – “plum”, ”pink”, ”rose” are moved to the bottom of array in the order of its encounter while moving from left/top to right/bottom of array. Also, “mist”, ”coal”, ”jade”, etc. are swapped with navy while traversing.

Column 10: Sorted Array