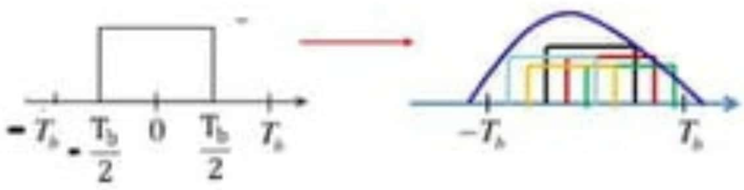
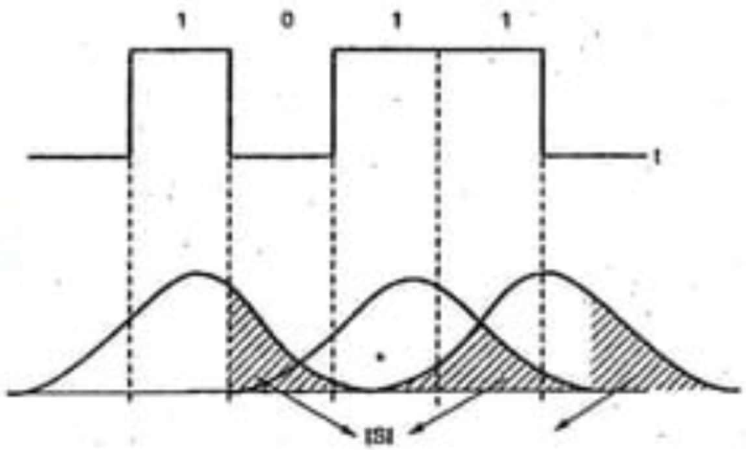
Inter Symbol Interference (ISI)

- The spreading of the pulse beyond its allotted time interval T_b causes it to interfere with neighboring pulses is known as ISI.

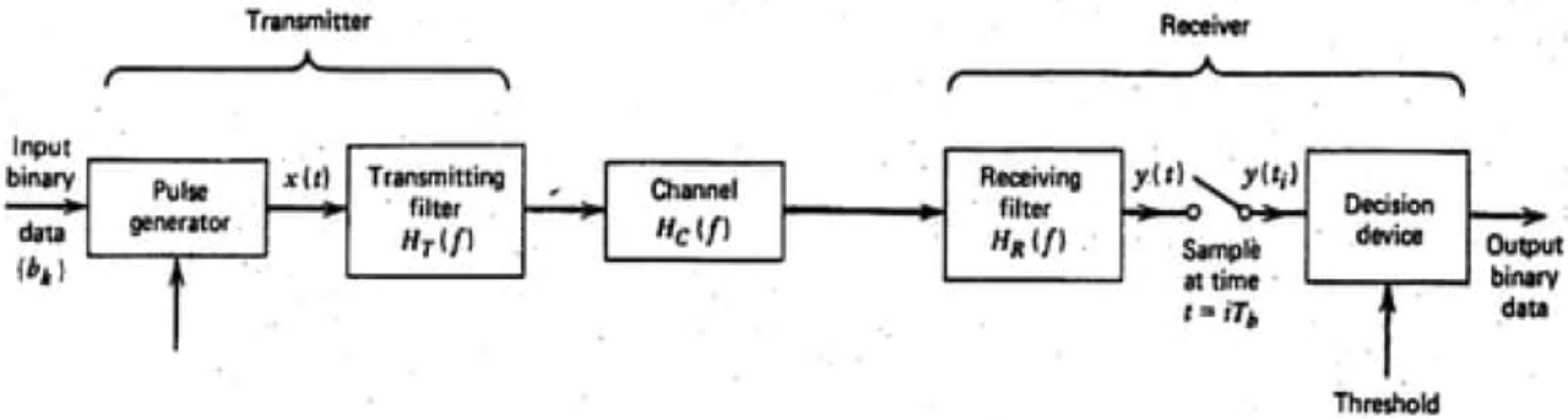




- Signal overlapping may give errors at the receiver. This phenomena of pulse overlapping resulted difficulty of discriminating symbols at the receiver is termed as ISI.

Baseband Transmission of Binary Data

- The basic elements of a baseband binary PAM system is shown in figure below.



Baseband binary data transmission system.

Cont...

- The input binary data sequence $\{b_k\}$ with a bit duration of T_b seconds. This sequence is applied to a pulse generator. Producing the discrete PAM signal.

$$x(t) = \sum_{k=-\infty}^{\infty} a_k v(t - kT)$$

- Where, *v*(t) denotes the basic pulse and normalized such that v(0)=1 &

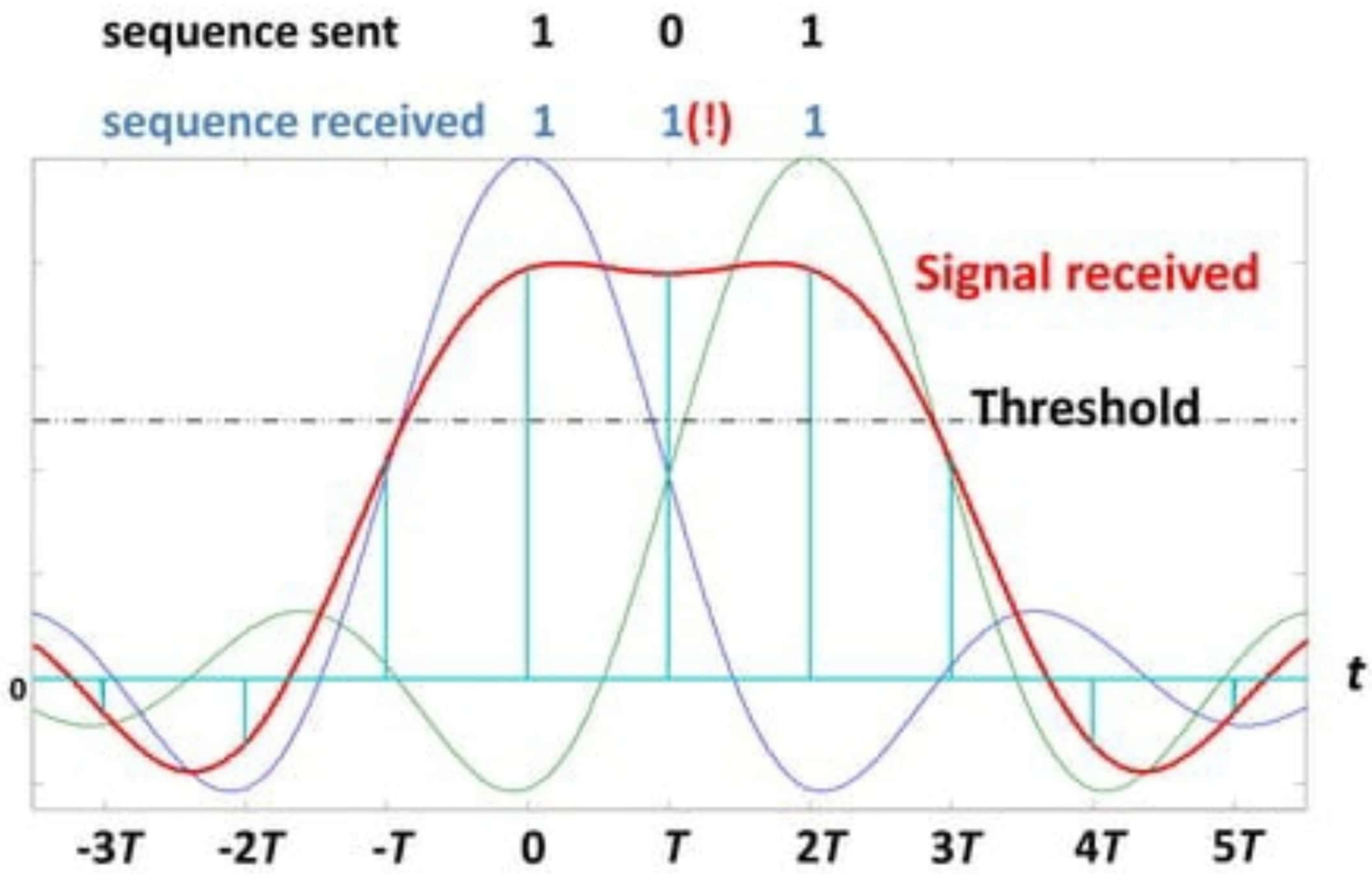
$$a_k = \begin{cases} a & , for\ symbol\ 1 \\ -a & , for\ symbol\ 0 \end{cases}$$

Cont...

- The output of the pulse generator x(t) is passed through a transmitting filter having the frequency response **H_T(f)** & after that it passed through a channel of transfer function **H_c(f)**.
- The channel may be co-axial cable or fibre optic cable. The major source of system degradation is dispersion in channel.
- The signal at the receiver is passed through a receiving filter of transfer function **H_R(f)**.
- The receiving filter output is sampled synchronously with the transmitter at $t = iT_b$ i.e. $y(iT_b)$ and then applied to the decision device.
- The decision device takes the decision based on the magnitude of $y(iT_b)$ as shown below:

If $y(iT_b)$ > Threshold , selects Symbol '1'
If $y(iT_b)$ < Threshold , selects Symbol '0'

Cont...



Sequence of three pulses (1, 0, 1)
sent at a rate $1/T$

Cont...

- Assuming that the channel is noise free. Mathematically, the output of the pulse generator can be described as:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k v(t - kT_b) \quad \text{----------- (1)}$$

- Taking Fourier Transform on both the sides of equation (1), we get

$$X(f) = \sum_{k=-\infty}^{\infty} a_k V(f) e^{-j2\pi k f T_b} \quad \text{----------- (2)}$$

Cont...

- Let the output of the receiving filter be defined by :

$$y(t) = \mu \sum_{k=-\infty}^{\infty} a_k p(t - kT_b) \qquad \text{----------- (3)}$$

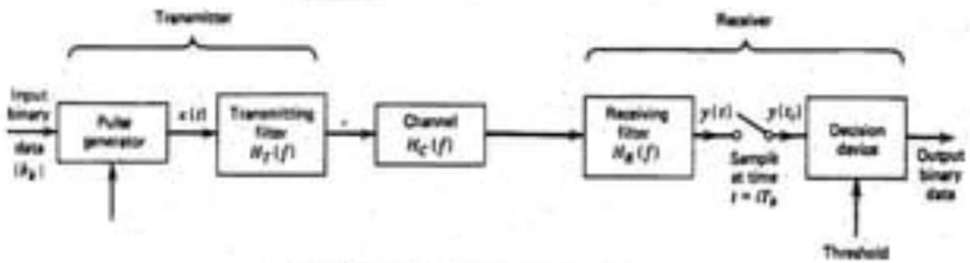
- Where , μ is a scaling factor.

  & p(t) is a pulse shaping function of y(t),

  and normalized such that p(0)=1

- Taking Fourier Transform on both the sides of equation (3), we get

$$Y(f) = \mu \sum^{\infty} a_k P(f) e^{-j2\pi k f T_b} \qquad \text{----------- (4)}$$



Baseband binary data transmission system.

Cont...

- From the block diagram of baseband binary data transmission PAM system, the output of the receiving filter in frequency domain can be written as:

$$Y(f) = X(f).H_T(f).H_c(f).H_R(f) \qquad \text{----------- (5)}$$

- Substituting equation (2) & (4) in (5), we get

$$\mu \sum_{k=-\infty}^{\infty} a_k P(f) e^{-j2\pi k f T_b}$$

$$= \sum_{k=-\infty}^{\infty} a_k V(f) e^{-j2\pi k f T_b}.H_T(f).H_c(f).H_R(f) \qquad \text{----------- (6)}$$

$$\mu P(f) = V(f).H_T(f).H_c(f).H_R(f) \qquad \text{----------- (7)}$$

$$P(f) = \frac{1}{\mu} V(f).H_T(f).H_c(f).H_R(f) \qquad \text{----------- (8)}$$

Taking Inverse Fourier Transform of equation (8) , we get p(t).

Cont...

- y(t) is sampled at time $t=iT_b$. From equation (3), we get

$$y(iT_b) = \mu \sum_{k=-\infty}^{\infty} a_k p(iT_b - kT_b)$$

$$y(iT_b) = \mu \sum_{k=-\infty}^{\infty} a_k p(i-k)\,T_b$$

$$y(iT_b) = \mu a_{k=i} p(0) + \mu \sum_{\substack{k=-\infty \\ k\neq i}}^{\infty} a_k p(i-k)\,T_b$$

$$y(iT_b) = \mu a_i p(0) + \mu \sum_{\substack{k=-\infty \\ k\neq i}}^{\infty} a_k p(i-k)\,T_b$$

$$y(iT_b) = \mu a_i + \underbrace{\mu \sum_{\substack{k=-\infty \\ k\neq i}}^{\infty} a_k p(i-k)\,T_b}_{\text{ISI}} \qquad p(0)=1$$

Cont...

$$y(iT_b) = \mu a_i + \mu \underbrace{\sum_{\substack{k=-\infty \\ k\neq i}}^{\infty} a_k p(i-k) T_b}_{\text{ISI}}$$

- First Term: It is generated by the i^{th} transmitted bit
- Second Term: The residual effect of all other transmitted bits on i^{th} bit. This residual effect is known as ISI.
- ISI occurs because of imperfections in the overall frequency response of the system. The frequency components constituting the input pulses are differently attenuated and differently delayed by the system. Consequently pulse is dispersed at the output.

Cont…

- In the absence of ISI: $y(iT_b) = \mu a_i$
- In the design of the transmitting and receiving filters, the objective is to minimize the effect of ISI and thereby deliver the digital data to its destination with the smallest error rate possible.

Cont...

- The transfer function of the channel and the transmitted pulse shape are specified.
- And Problem is:
  - To determine the transfer functions of the transmitting and receiving filters such that the transmitted data sequence $\{b_k\}$ can be reconstructed.

Nyquist's Criterion for Distortionless Baseband Binary Transmission

- The receiver reconstructs data sequence $\{b_k\}$ by extracting and decoding the corresponding sequence of weights $\{a_k\}$ from the output y(t).
- The extraction involves sampling the output y(t) at some time $t=iT_b$.
- The decoding requires that the weighted pulse contribution as shown in the equation be free from ISI due to overlapping trails of other weighted contributions by $k \neq i$.

$$a_k p(iT_b - kT_b)$$

Nyquist's Criterion for Distortionless Baseband Binary Transmission

- This requires that we control the received pulse p(t).

$$p(iT_b - kT_b) = \begin{cases} 1 & for\ i = k \\ 0 & for\ i \neq k \end{cases} \quad \ldots\ldots\ldots\ldots.(I)$$

Time domain condition forzero ISI

- Therefore,

$$y(iT_b) = \mu a_i + \sum_{\substack{k=-\infty \\ k\neq i}}^{\infty} a_k p(i-k)T_b = \mu a_i$$

- This implies zero ISI condition of equation (I) by assuring perfect reception with absence of noise.

Nyquist's Criterion for Distortionless Baseband Binary Transmission

Frequency domain condition for the perfect reception

- Let $p(nT_b)$ represents the impulses at which $p(t)$ is sampled for decision at a rate of T_b.
- The Fourier Transform of these impulses is given by:

$$P_\delta(f) = \frac{1}{T_b}\sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_b}\right) \dots\dots\dots..(II)$$

$$p_\delta(t) = \sum_{n=-\infty}^{\infty} p(nT_b)\delta(t - nT_b)$$

$$P_\delta(f) = F[p_\delta(t)]$$

$$= \int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty} p(nT_b)\delta(t - nT_b)\, e^{-j2\pi f t}\, dt$$

Nyquist's Criterion for Distortionless Baseband Binary Transmission

- By putting n=i-k

$$P_\delta(f) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} p([i-k]T_b)\delta(t-[i-k]T_b)\, e^{-j2\pi ft} dt$$

- By substituting $p(iT_b - kT_b) = \begin{cases} 1 & for\ i = k \\ 0 & for\ i \neq k \end{cases}$ ……………. (I)
- We get,

$$P_\delta(f) = \begin{cases} \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} 1.\delta(t)\, e^{-j2\pi ft} dt \quad for\ i = k \\ \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} 0.\delta(t)\, e^{-j2\pi ft} dt \quad for\ i \neq k \end{cases}$$

- Therefore, $P_\delta(f) = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi ft} dt = \delta(0) = 1$
- Using Equation (II) , $\frac{1}{T_b} \sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_b}\right) = 1$

Nyquist's Criterion for Distortionless Baseband Binary Transmission

$$\sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_b}\right) = T_b = \frac{1}{R_b}$$

- This is frequency domain condition for zero ISI in the absence of noise (perfect reception) .

Eye Pattern or Eye Diagram

- In a bandlimited channel, the pulse spreading at the output of the system will dispersed over an interval which is longer than that of the transmitted pulse.
- Eye diagram is a very effective tool for digital signal analysis during real time experiments.
- An eye pattern is a practical technique for determining the severity of the degradations introduced by ISI and channel noise into the line coded digital pulses in baseband transmission.

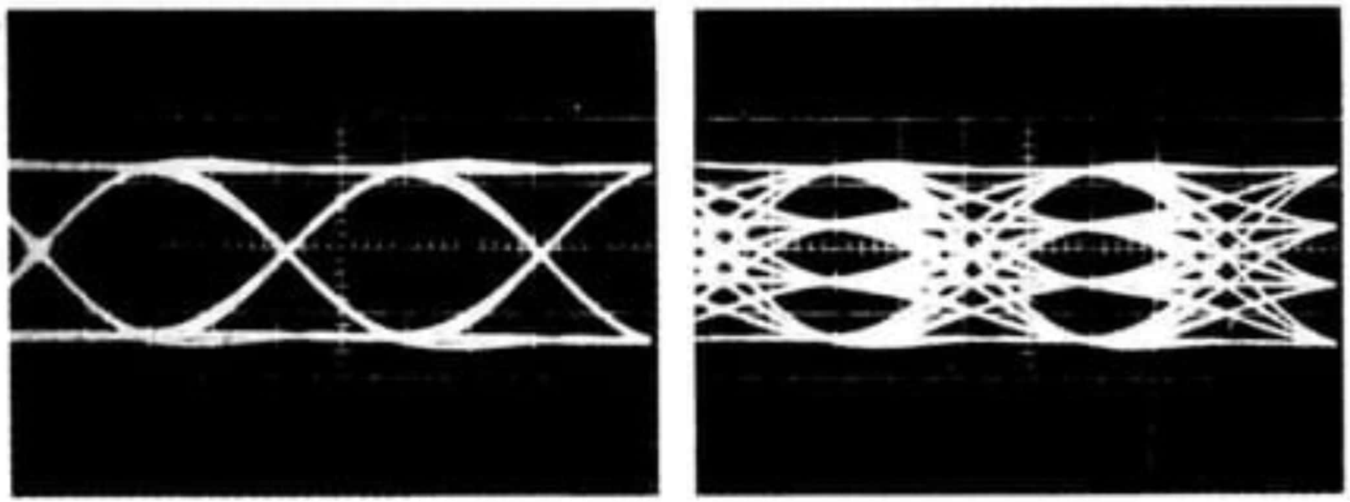
Eye Pattern or Eye Diagram

- Eye diagram is a **simple and convenient engineering tool** applied on received signals for studying the effects of
  - ❑ ISI
  - ❑ Accuracy of timing extraction
  - ❑ Noise Immunity
  - ❑ Determining the bit error rate (BER)
- Eye diagram provides information about the state of the channel and quality of the received pulse.
- This information is useful for faithful detection of received signal and determination of overall performance of digital communication system.

Eye Diagram

- The eye diagram is created by taking the time domain signal and overlapping the traces for a certain number of symbols.
- The **open part** of the signal represents **the time** that we can **safely sample the signal with fidelity**

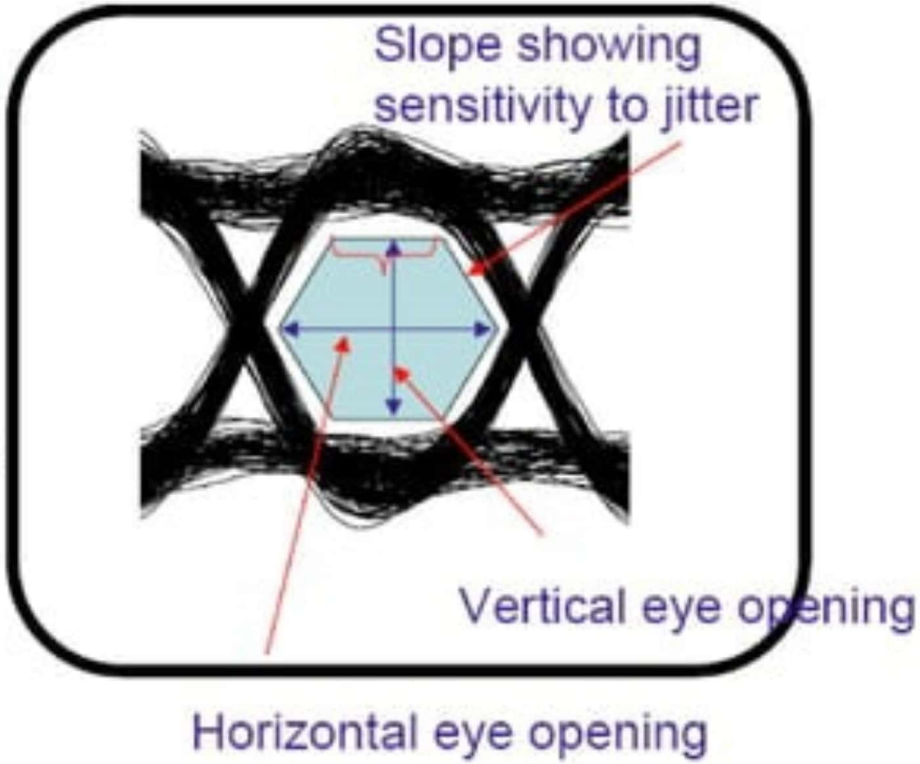
Examples of eye patterns for binary and quaternary amplitude shift keying (or PAM).



BINARY QUATERNARY

Vertical and Horizontal Eye Openings

- **The vertical eye opening or noise margin** is related to the **SNR, and thus the BER**
  - **A large eye opening corresponds to a low BER**
- **The horizontal eye opening** relates **the jitter and the sensitivity of the sampling instant** to jitter
  - The red brace indicates the range of sample instants with good eye opening
  - At other sample instants, the eye opening is greatly reduced, as governed by the indicated slope



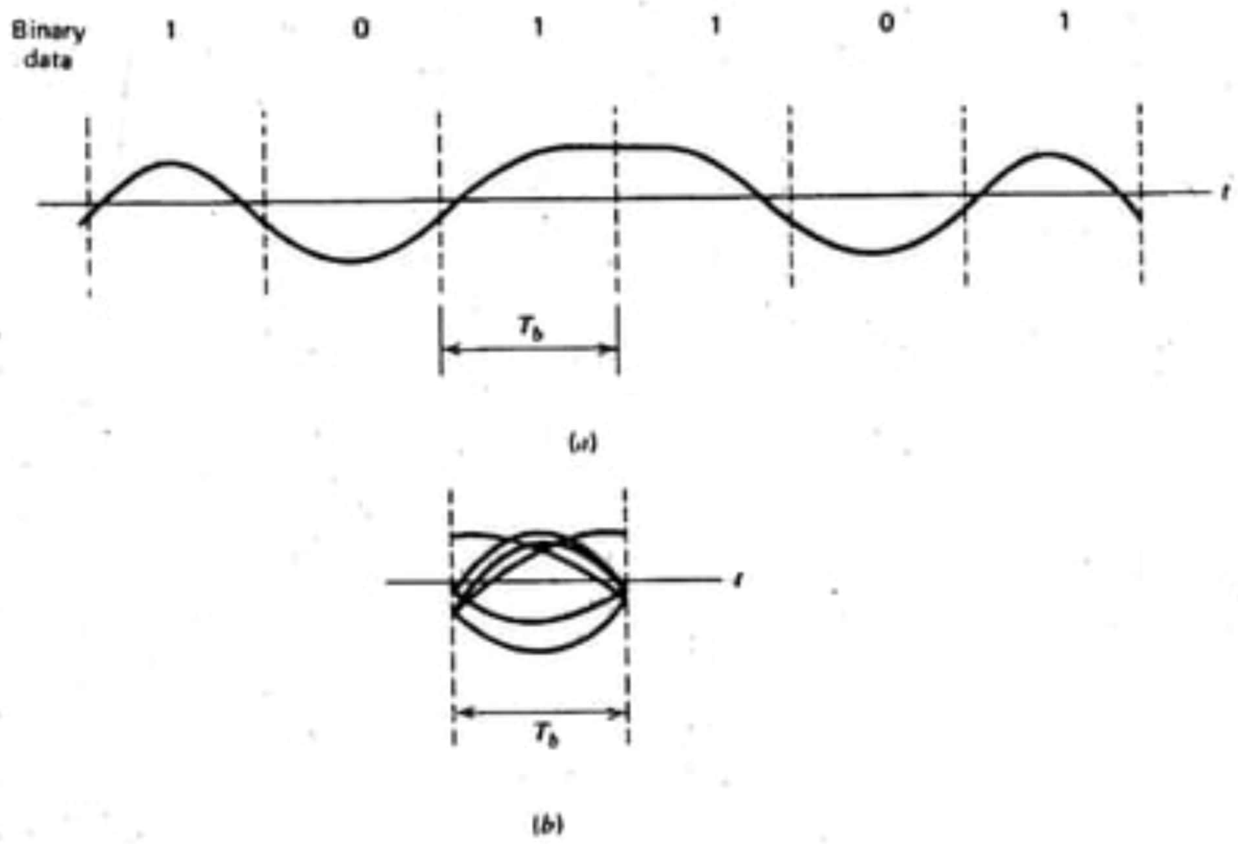
Eye Diagram

- Eye diagram is a means of evaluating the quality of a received "digital waveform"
  - By quality is meant the ability to correctly recover symbols and timing
  - The received signal could be examined at the input to a digital receiver or at some stage within the receiver before the decision stage
- Eye diagrams reveal the impact of ISI and noise.
- Two major issues are

  1) sample value variation, and

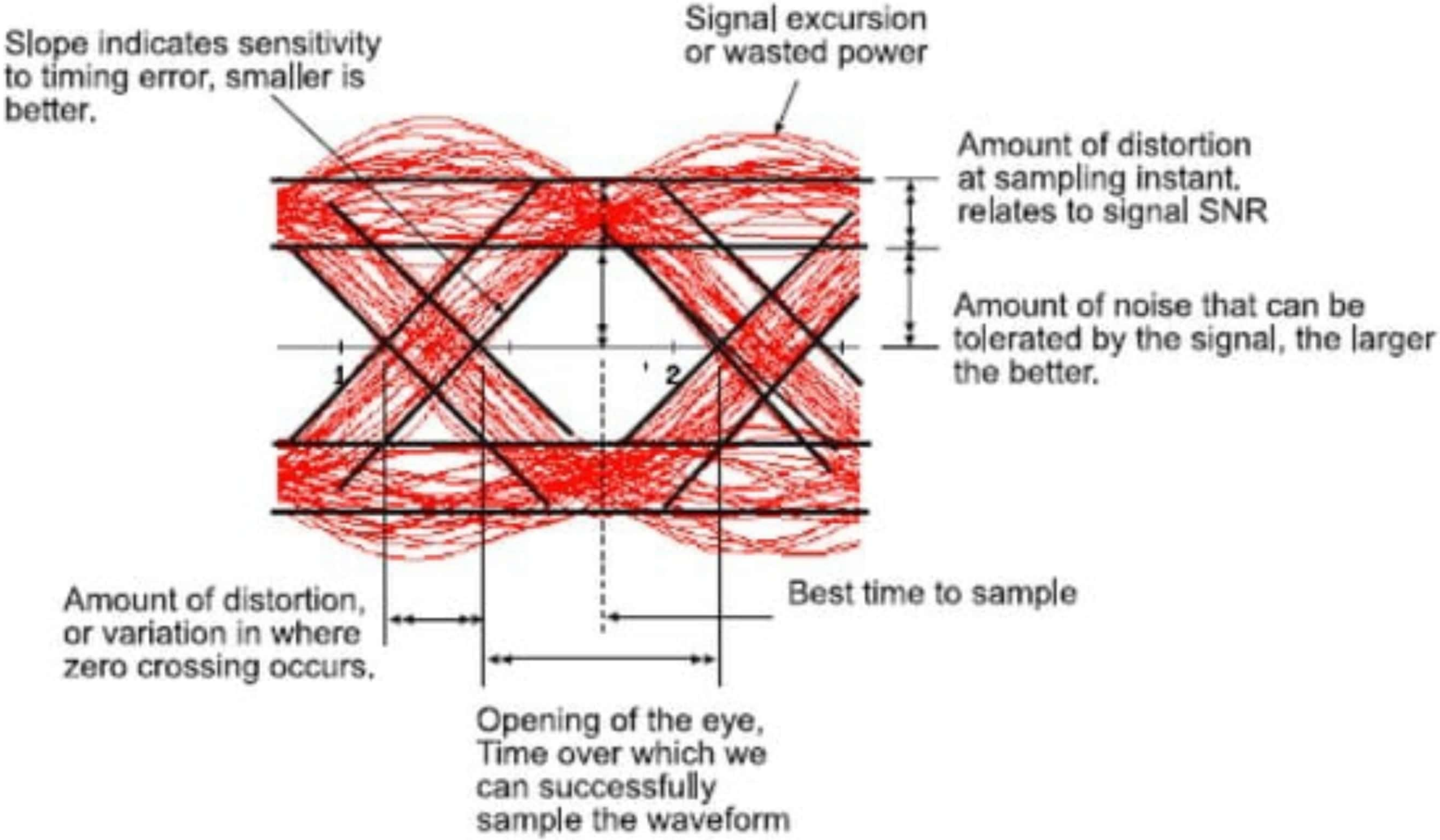
  2) jitter and sensitivity of sampling instant
- Eye diagram reveals issues of both.
- Eye diagram can also give an estimate of achievable BER

Eye Diagram

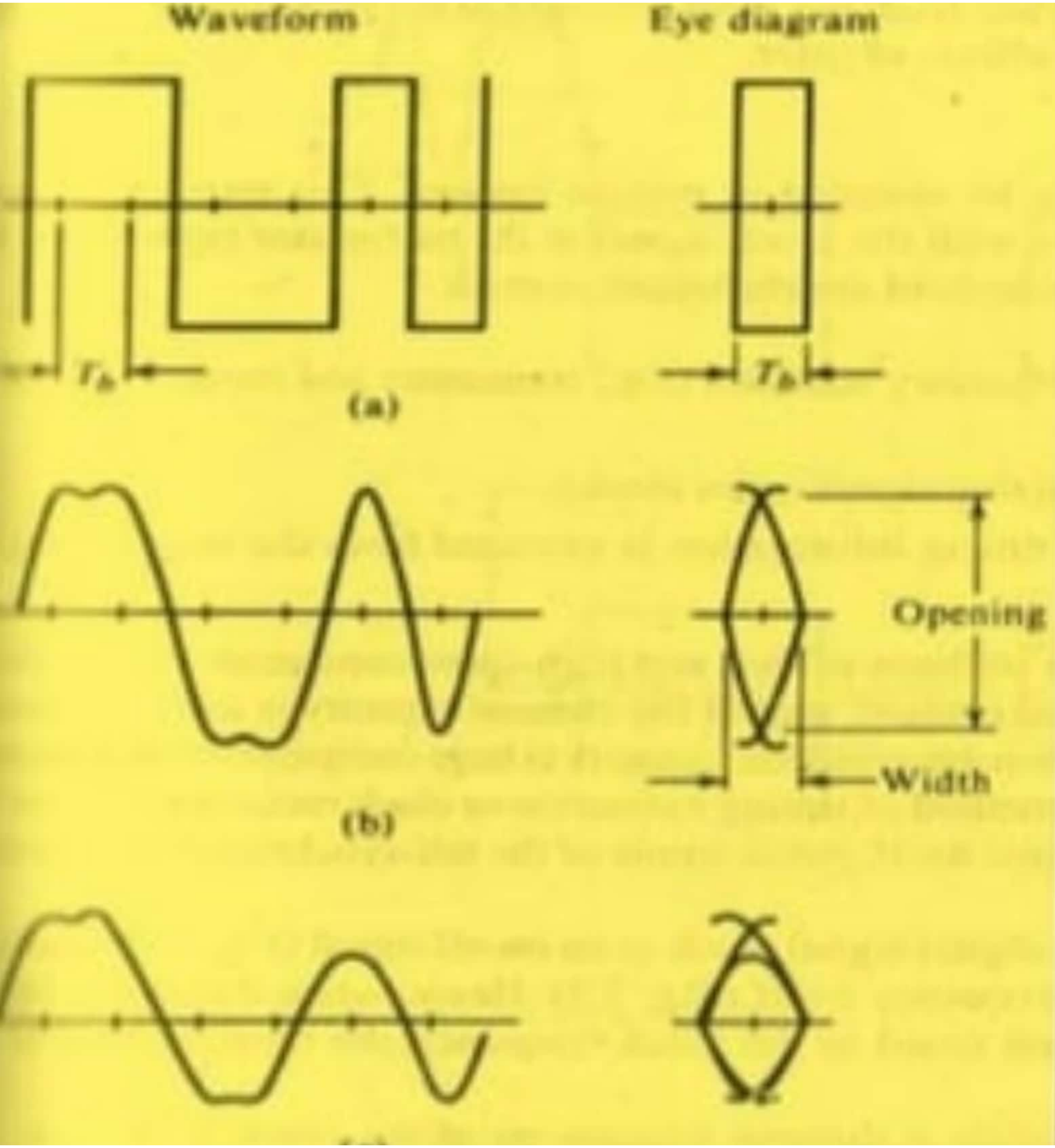


(*a*) Distorted binary wave. (*b*) Eye pattern.

Interpretation of Eye Diagram



- The width of the eye opening defines the time interval over which the received wave can be sampled without error from ISI.
- It is apparent that preferred time for sampling is the instant of time at which the eye is open widest.
- The sensitivity of the system to timing error is determined by the rate of closure of the eye as the sampling time is varied .
- The height of the eye opening , at a specified sampling time, defines the margin over noise.



a. **Ideal channel**, infinite BW, pulse received without distortion.

b. **Distortion channel,** finite BW, received signal will rounded and spread out.
- full opening at mid-pt

c. **Noise channel,** ISI is not zero, the eye close partially at the mid-pt