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CS 402 - ADVANCED STUDIES - II
HOMEWORK 2

~~1.5~~

1.5 [4] <§1.6> Consider 3 different processors P1, P2 & P3 executing the same instruction set. P1 has a 3GHz clock rate & a CPI of 1.5. P2 has a 2.5 GHz clock rate & a CPI of 1.0. P3 has a 4.0 GHz clock rate & has a CPI of 2.2

(a) Which processor has the highest performance expressed in instructions per second?

(b) If the processors each execute a program in 10 sec, find the no. of cycles & the no. of instructions.

(c) We are trying to reduce the execution time by 30% but this leads to an increase of 20% in the CPI. What clock rate should we have to get this time reduction?

⇒ (a)

$$\text{CPU time}(t) = (\text{Instruction count}) * (\text{clock cycle time}) * (\text{CPI})$$

$$= \frac{\text{Instruction count} * \text{CPI}}{\text{clock rate}}$$

$$t_1 = \frac{1 \times 1.5}{3 \times 10^9} = 5 \times 10^{-10}$$

$$t_2 = \frac{1 \times 1}{2.5 \times 10^9} = 4 \times 10^{-10}$$

$$t_3 = \frac{1 \times 2.2}{4.0 \times 10^9} = 5.5 \times 10^{-10}$$

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For performance - take inverse of the CPU time.

$$P_1 = \frac{1}{t_1} = \frac{1}{5 \times 10^{-10}} = 2 \times 10^9 \text{ instructions/sec.}$$

$$P_2 = \frac{1}{t_2} = \frac{1}{4 \times 10^{-10}} = 2.5 \times 10^9 \text{ instructions/sec}$$

$$P_3 = \frac{1}{t_3} = \frac{1}{5.5 \times 10^{-10}} \approx 1.82 \times 10^9 \text{ instructions/sec}$$

$P_2 > P_1 > P_3$

$\therefore$ P_2 has the highest performance.

(b) $P_{cycles} = \text{Clock Rate} \times t$

$P_{ins} = \dfrac{P_{cycles}}{CPI}$

$P_{1\,cycles} = 3 \times 10^9 \times 10 = 3 \times 10^{10}$

$P_{2\,cycles} = 2.5 \times 10^9 \times 10 = 2.5 \times 10^{10}$

$P_{3\,cycles} = 4.0 \times 10^9 \times 10 = 4.0 \times 10^{10}$

$$P_{1\,ins} = \frac{3 \times 10^{10}}{1.5} = 2 \times 10^{10}$$

$$P_{2\,ins} = \frac{2.5 \times 10^{10}}{1} = 2.5 \times 10^{10}$$

$$P_{3\,ins} = \frac{4.0 \times 10^{10}}{2.2} \approx 1.82 \times 10^{10}$$

(c) $$P_{CR} = \frac{P_{ins} \times 1.2 \times CPI}{0.7 \times t}$$

$$P_{1CR} = \frac{2 \times 10^{10} \times 1.2 \times 1.5}{0.7 \times 10} \approx \cancel{5 \times 14 \times 10^9}$$

$$\approx 5.14 \times 10^9$$

$$P_{2CR} = \frac{2.5 \times 10^{10} \times 1.2 \times 1}{0.7 \times 10}$$

$$\approx 4.29 \times 10^9$$

$$P_{3CR} = \frac{1.82 \times 10^{10} \times 1.2 \times 2.2}{0.7 \times 10}$$

$$\approx 6.86 \times 10^9$$

1.6 Consider two different implementations of the same instruction set architecture. The instructions can be divided into four classes according to their CPI (class A, B, C & D). P1 with a clock rate of 2.5 GHz & CPIs of 1, 2, 3 & 3 & P2 with a clock rate of 3 GHz & CPIs of 2, 2, 2 & 2.

Given a program with a dynamic instruction count of 1.0E6 instructions divided into classes as follows: 10% class A, 20% class B, 50% class C & 20% class D, which implement is faster?

(a) What is the global CPI for each implementatio

(b) Find the clock cycles required in both cases.

⇒ Proportioning the instructions with their respective percentages

$I_A = (0.1) \times (1 \times 10^6) = 1 \times 10^5$

$I_B = (0.2) \times (1 \times 10^6) = 2 \times 10^5$

$I_C = (0.5) \times (1 \times 10^6) = 5 \times 10^5$

$I_D = (0.2) \times (1 \times 10^6) = 2 \times 10^5$

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(a) To find global CPI for each processor, first we will have to calculate its time

$$t_N = \frac{I_N \times CPI_N}{CR_n}$$

$N \rightarrow$ various class

$n \rightarrow$ processor number

For 1st processor :

$$t_A = \frac{1 \times 10^5 \times 1}{2.5 \times 10^9} = 4 \times 10^{-5} \text{ sec}$$

$$t_B = \frac{2.5 \times 10^5 \times 2}{2.5 \times 10^9} = 1.6 \times 10^{-4} \text{ sec}$$

$$t_C = \frac{5 \times 10^5 \times 3}{2.5 \times 10^9} = 6 \times 10^{-4} \text{ sec}$$

$$t_D = \frac{2 \times 10^5 \times 3}{2.5 \times 10^9} = 2.4 \times 10^{-4} \text{ sec}$$

$$CPI_{global} = \frac{CR_n \cdot \sum_{i=0}^{N} t_i}{I_{total}}$$

$$CPI_{1global} = \frac{(0.4 + 1.6 + 6 + 2.4) \times 10^{-4} \times 2.5 \times 10^9}{1 \times 10^6}$$

$$CPI_{1global} = 2.6$$

For 2nd processor :

$$t_A = \frac{1 \times 10^5 \times 2}{3 \times 10^9} = \frac{2}{3} \times 10^{-4} \text{ sec}$$

$$t_B = \frac{2 \times 10^5 \times 2}{3 \times 10^9} = \frac{4}{3} \times 10^{-4} \text{ sec}$$

$$t_C = \frac{5 \times 10^5 \times 2}{3 \times 10^9} = \frac{10}{3} \times 10^{-4} \text{ sec}$$

$$t_D = \frac{2 \times 10^5 \times 2}{3 \times 10^9} = \frac{4}{3} \times 10^{-4} \text{ sec}$$

$$CPI_{2\,global} = \frac{(2/3 + 4/3 + 10/3 + 4/3) \times 10^{-4} \times 3 \times 10^9}{1 \times 10^6}$$

$$CPI_{2\,global} = 2$$

(b) $P_{clock\ cycles} = \sum_{i=0}^{N} I_N \cdot CPI_N$

$$P_1 \text{ clock cycles} = (1 \times 10^5)(1) + (2 \times 10^5)(2) + (5 \times 10^5)(3) + (2 \times 10^5)(3)$$

$$P_1 \text{ clock cycles} = 2.6 \times 10^6$$

$$P_2 \text{ clock cycles} = (1 \times 10^5)(2) + (2 \times 10^5)(2) + (5 \times 10^5)(2) + (2 \times 10^5)(2)$$

$$P_2 \text{ clock cycles} = 2 \times 10^6$$

1.7 Compilers can have a profound impact on the performance of an application. Assume that for a program, compiler A results in a dynamic instruction count of 1.0E9 & has an execution time of 1.1 sec, while compiler B results in a dynamic instructn count of 1.2 E9 & an execution time of 1.5 sec.

(a) Find the avg. CPI for each program given that the processor has a clock cycle time of 1ns.

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(b) Assume the compiled programs run on two different processors. If the execution time on the 2 processors are the same, how much faster is the clock of the processor running compiler A's code versus the clock of the processor running compiler B's code?

(c) A new compiler is developed that uses only 6.0E8 instructions & has an avg. CPI of 1.1. What is the speed up of using this new compiler versus using compiler A or B on the original processor?

⇒ (a)

$$CPI = t_{exec} \cdot \frac{1}{\text{Clock cycles} \times \text{Instruction count}}$$

$$CPI_A = 1.1 \times \frac{1}{10^{-9} \times 10^{9}} = 1.1$$

$$CPI_B = 1.5 \times \frac{1}{1.2 \times 10^{-9} \times 10^{9}} = 1.25$$

(b) To compare two compilers

$$\frac{1.2 \times 10^{9} \times 1.25}{10^{9} \times 1.1} \approx 1.37$$

Compiler B is approximately 1.37 times faster than compiler A.

6 (c) To compare with a new compiler

$$\frac{T_A}{T_{new}} = \frac{1.1 \times 1 \times 10^9}{1.1 \times 6 \times 10^8} \approx 1.67$$

$$\frac{T_B}{T_{new}} = \frac{1.25 \times 1.2 \times 10^9}{1.1 \times 6 \times 10^8} \approx 2.27$$

The speed up with regards to compiler A is approximately 1.67 & for compiler B is approximately 2.27.

1.9

1.9.1

$$t_N = \frac{I_N \cdot CPI_N}{CR_n}$$

$$t_1 = \frac{\left(\frac{2.56 \times 10^9}{0.7} \times 1\right) + \left(\frac{1.28 \times 10^9}{0.7} \times 12\right) + \left(2.56 \times 10^8 \times 5\right)}{2 \times 10^9}$$

$t_1 = 13.44$ sec

$$t_2 = \frac{\left(\frac{2.56 \times 10^9}{0.7 \times 2} \times 1\right) + \left(\frac{1.28 \times 10^9}{0.7 \times 2} \times 12\right) + \left(2.56 \times 10^8 \times 5\right)}{2 \times 10^9}$$

$t_2 = 7.04$ sec

$$t_4 = \frac{\left(\frac{2.56 \times 10^9}{0.7 \times 4} \times 1\right) + \left(\frac{1.28 \times 10^9}{0.7 \times 4} \times 12\right) + \left(2.56 \times 10^8 \times 5\right)}{2 \times 10^9}$$

$t_4 = 3.84$ sec

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$$t_8 = \frac{\left(\frac{2.56 \times 10^9}{0.7 \times 8} \times 1\right) + \left(\frac{1.28 \times 10^9}{0.7 \times 8} \times 12\right) + \left(2.56 \times 10^8 \times 5\right)}{2 \times 10^9}$$

$t_8 = 2.24$ sec

$S_1 = 1$
$S_2 = 1.91$
$S_4 = 3.5$
$S_8 = 6$

1.9.2 Doubling the CPI for the arithmetic value,
$t_1 = 15.27$ sec
$t_2 = 7.95$ sec
$t_4 = 4.30$ sec
$t_8 = 2.47$ sec

1.9.3 It should be reduced to 3.

1.11
1.11.1 $\text{Clock Rate} = \frac{1}{\text{Cycle Time}} = \frac{1}{0.333 \times 10^{-9}}$
$\approx 3 \times 10^9$

$$\text{CPI (biz 2)} = 3 \times 10^9 \times \frac{750}{2389 \times 10^9}$$
≈ 0.94

$\therefore$ Required CPI is approx. 0.94

1.11.2 $\text{SPECratio} = \frac{tr}{IN} = \frac{9650}{750} \approx 12.87$

$\therefore$ SPECratio is approx. 12.87

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1.11.3 We know that,

$$\text{CPU time} = \left(\begin{array}{c}\text{Instruction}\\\text{count}\end{array}\right) \times (\text{CPI}) \times \left(\begin{array}{c}\text{Clock cycle}\\\text{time}\end{array}\right)$$

$$= \frac{\text{Instruction count} \times \text{CPI}}{\text{Clock Rate}}$$

If we assume CPI & clock rate to be constant, there should be a 10% increase in CPU time.

1.11.4
$$\frac{\text{CPU time (after)}}{\text{CPU time (before)}} = \left(\frac{1.1 \times \text{Instruct}^{n}\text{ count} \times 1.05 \times \text{CPI}}{\text{Clock rate}}\right) \times \left(\frac{\text{Clock Rate}}{\text{Instruction count} \times \text{CPI}}\right)$$

$$= 1.1 \times 1.05$$
$$= 1.155$$

$\therefore$ The CPU time increases by 15.5%

1.11.5
$$\frac{\text{SPEC ratio (after)}}{\text{SPEC ratio (before)}} = \left(\frac{t_r}{\text{CPU time (after)}}\right) \times \left(\frac{\text{CPU time (before)}}{t_r}\right)$$

$$= \frac{\text{CPU time (after)}}{\text{CPU time (before)}}$$

$$= \frac{1}{1.155}$$

$$\approx 0.87$$

$\therefore$ The SPECratio decreased by 13%

1.11.6 $CPI = \frac{CPU\ time \times Clock\ rate}{Instruction\ count}$

$= \frac{700 \times 4 \times 10^9}{0.85 \times 2389 \times 10^9} \approx 1.38$

$\therefore$ CPI is approx. 1.38

1.11.7 $\frac{4}{3} \approx 1.33$

$\frac{1.38}{0.94} \approx 1.47$

These 2 values are different due to the fact that although the number of instructions were reduced by 15%, the CPU time was reduced by a lower percentage.

1.11.8

$\frac{700}{750} \approx 0.933$

CPU time reduced by 6.7%

1.11.9

$Instruction\ count = \frac{CPU\ time \times Clock\ Rate}{CPI}$

$= \frac{960 \times 0.9 \times 4 \times 10^9}{1.61}$

$\approx 2.147 \times 10^{12}$

$\therefore$ Instruction count is approx. 2.147×10^{12}

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1.11.10 Clock Rate~~(new)~~ = $\frac{\text{Instruction count} \times \text{CPI}}{\text{CPU time}}$

Rearranging,

$$\text{Clock Rate(new)} = \frac{\text{Clock Rate(old)}}{0.9}$$

$$\approx 3.33 \times 10^9$$

$\therefore$ New Clock Rate is 3.33 GHz

1.11.11 $$\text{Clock Rate(new)} = \frac{0.85}{0.80} \times \text{Clock Rate(old)}$$

$$\approx 3.19 \times 10^9$$

$\therefore$ New Clock Rate is approx. 3.19 GHz

~~1.11.12~~

1.12

1.12.1 Clock Rate = $\frac{\text{Instruction count} \times \text{CPI}}{\text{CPU time}}$

$$t_{P_1} = \frac{5 \times 10^9 \times 0.9}{4 \times 10^9} = 1.125 \text{ sec}$$

$$t_{P_2} = \frac{1 \times 10^9 \times 0.75}{3 \times 10^9} = 0.25 \text{ sec}$$

Clock rate for processor 1 is larger than processor 2 but processor 2 has better performance than processor 1.

$\therefore$ Statement isn't true

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1.12.2 $tp_1 = tp_2$

$$tp_1 = \frac{\text{No. of Instructions} \times CPI}{\text{Clock rate}}$$

~~$tp_1 = 2.53 \times 3$~~

$$tp_1 = \frac{1 \times 10^9 \times 0.9}{4 \times 10^9} = 0.225$$

Number of Instructions executed in this time frame from processor 2.

$$N_2 = \frac{0.225 \times 3 \times 10^9}{0.75} = 9 \times 10^8$$

$\therefore \quad N_2 = 9 \times 10^8$

1.12.3 $$MIPS = \frac{\text{Clock rate}}{CPI} \times 10^{-6}$$

$$MIPS_1 = \frac{4 \times 10^9 \times 10^{-6}}{0.9} \approx 4.44 \times 10^3$$

$$MIPS_2 = \frac{3 \times 10^9 \times 10^{-6}}{0.75} = 4 \times 10^3$$

$\therefore$ The MIPS value is greater in processor 1, yet we still have the performance to be better for processor 2.

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1.12.4 $$\text{MFLOPS} = \frac{\text{No. of FP operations}}{t_{exec} \times 1 \times 10^6}$$

$$\text{MFLOPS}_1 = \frac{0.4 \times 5 \times 10^9}{1.125 \times 10^6} \approx 1.78 \times 10^3$$

$$\text{MFLOPS}_2 = \frac{0.4 \times 1 \times 10^9}{0.25 \times 10^6} = 1.6 \times 10^3$$

∴ MFLOPS_1 is greater than MFLOPS_2 yet we still have the performance to be better for processor 2.

1.13

1.13.1 Reducing FP operations by 20%, we get

$$t_{FP} = 70 \times 0.8 = 56 \text{ sec}$$

$$t_{total} = 56 + 85 + 40 + 55 = 236 \text{ sec}$$

∴ We get a 5.6 % reduction from reducing the FP operations by 20 %

1.13.2 Taking total time & reducing it by 20%

$$t_{new} = 250 \times 0.8 = 200 \text{ sec}$$

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$t_{new} - (t_{FP} + t_{L/S} + t_{branch}) = 200 - 165$
$= 35$ sec

∴ We get a 58.8% reduction for the INT operations.

1.13.3 $t_{new} = 200$ sec
$t_{branch} = 0$

$t_{FP} + t_{L/S} + t_{INT} = 210 \text{ sec} > 200$ sec
Since this is greater than the threshold this is not possible.

1.14

1.14.1 Clock cycles $= (CPI_{FP} \times N_{FP}) + (CPI_{INT} \times N_{INT}) + (CPI_{L/S} \times N_{L/S}) + (CPI_{branch} \times N_{branch})$

$$t_{cpu} = \frac{512 \times 10^6}{2 \times 10^9} = 0.256 \text{ sec}$$

So we want to run the program twice as fast with only changing the CPI of the FP instructions.

$$CPI_{new\,FP} = \frac{256 - 462}{50} = -4.12$$

∴ Since the number would be negative, this is impossible.

1.14.2 $CPI_{new\ L/S} = \frac{256 - 198}{80}$

$= 0.725$

To run the program twice as fast, we would need L/S CPI to be 0.725

1.14.3 $CPI_{FP} = CPI_{INT} = 0.6 \times 1 = 0.6$

$CPI_{L/S} = 0.7 \times 4 = 2.8$

$CPI_{branch} = 0.7 \times 2 = 1.4$

$t_{cpu} = 0.171$ sec

∴ we get 1.5 times speed up from the CPI improvements

1.15 Data ~~sh~~ should be below, where primes are the ideal number

$t_1 = 100$ sec | $t'_1 = 100$ sec
$t_2 = 54$ sec | $t'_2 = 50$ sec
$t_4 = 29$ sec | $t'_4 = 25$ sec
$t_8 = 16.5$ sec | $t'_8 = 12.5$ sec
$t_{16} = 10.25$ sec | $t'_{16} = 6.25$ sec
$t_{32} = 7.125$ sec | $t'_{32} = 3.125$ sec
$t_{64} = 5.5625$ sec | $t'_{64} = 1.5625$ sec
$t_{128} = 4.78125$ sec | $t'_{128} = 0.78125$ sec

Speed ups for these values would be

$S_1 = \frac{100}{100} = 1$ $\qquad$ $S_1' = \frac{100}{100} = 1$

$S_2 = \frac{100}{54} = 1.85$ $\qquad$ $S_2' = \frac{100}{50} = \cancel{0.93}\ 2$

$S_4 = \frac{100}{29} = 3.44$ $\qquad$ $S_4' = \frac{100}{25} = 4$

$S_8 = \frac{100}{16.5} = 6.06$ $\qquad$ $S_8' = \frac{100}{12.5} = 8$

$S_{16} = \frac{100}{10.25} = 9.76$ $\qquad$ $S_{16}' = \frac{100}{6.25} = 16$

$S_{32} = \frac{100}{7.125} = 14.03$ $\qquad$ $S_{32}' = \frac{100}{3.125} = 31.007 \approx 32$

$S_{64} = \frac{100}{5.56225} = 17.98$ $\qquad$ $S_{64}' = \frac{100}{1.5625} = 64$

$S_{128} = \frac{100}{4.78125} = 20.91$ $\qquad$ $S_{128}' = \frac{100}{0.78125} = 128$

∴ Required Ratios would be

$\frac{S_1}{S_1'} = 1$ $\qquad$ $\frac{S_{16}}{S_{16}'} = 0.61$

$\frac{S_2}{S_2'} = 0.93$ $\qquad$ $\frac{S_{32}}{S_{32}'} = 0.44$

$\frac{S_4}{S_4'} = 0.86$ $\qquad$ $\frac{S_{64}}{S_{64}'} = 0.28$

$\frac{S_8}{S_8'} = 0.75$ $\qquad$ $\frac{S_{128}}{S_{128}'} = 0.16$