ST. XAVIER'S COLLEGE (AUTONOMOUS) , KOLKATA

DEPARTMENT OF STATISTICS

TITLE : ASYMTOTIC BEHAVIOUR OF SOME STATISTICS FOR DIFFERENT PARENT DISTRIBUTIONS

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DECLARATION :

I affirm that I have identified all my sources and that no part of my dissertation paper uses unacknowledged materials.

Partha Pratim Das.

Signature of the student

CONTENTS-

INTRODUCTION:

In statistics ;asymptotic theory or large sample theory is a framework for assessing properties of estimators and statistical tests. Within this framework, it is often assumed that the sample size n may grow indefinitely; the properties of estimators and tests are then evaluated under the limit of n-->infinity. In practice , a limit evaluation is considered to be approximately valid for large finite sample sizes too.
The asymptotic theory proceeds by assuming that it is possible (in principal) to keep collecting additional data, thus that the sample size grows infinitely, i.e. n-->infinity. Under the assumption many results can be obtained that are unavailable for samples of finite size. An example is the WLLN. The law states that for a sequence of independent and identically distributed (IID) random variables X1,X2,...,if one value is drawn from each random variable and the average of the first n values is computed as Xnbar , then Xnbar converges in probability to the population mean E(Xi) as n-->infinity. A primary goal of asymptotic analysis is to obtain a deeper qualitative understanding of quantitative tools. The conclusions of an asymptotic analysis often supplement the conclusions which can be obtained by numerical methods.

The pivotal concept:
The behaviour of a phenomenon on a large scale is generally stable.(The concept of large sample comes from here).

Statistical Regularity:

A notation in statistics and probability theory that random events exhibit regularity when repeated enough times . Under more or less identical situation. A repeated performence of any study is preferable before drawing any conclusion.
Some concepts of limit theorems: Consider a sequence {Xn} ; as n-->infinity, will be studied in terms of distribution ; probability; moments etc. Hence the different modes of convergence of {Xn} are
i) Convergence in Distribution.
ii) Convergence in Probability.
iii) Convergence in rth mean/rth order raw moments.

Convergence in Distn (Law):

Consider a sequence {Xn} of R.V.s with the corresponding sequence of D.F's {Fn(x)}; actually {Fn(x)} is a sequence of function and
limn-->infinity (Fn(x)=F(x)) is called the limit function. Here the limit function F(x) may not be a D.F. If F(x) is a D.F, we associate a R.V. X. Then we say that "{Xn} converges in distribution to X" & we write
Xn-L-->X and αFn(x)--w-->F(x).

Defn:A sequence {Xn} of R.V.s with the corresponding sequence of {Fn(x)} is said to converge in distn to a R.V. X with D.F. F(x) ;
if limn-->infinityFn(x)=F(x); at every continuity point of F(x).

ILLUSTRATION THROUGH AN EXAMPLE:

Q) Let X1,X2,......Xn,..... be a seqn of i.i.d R.V.s each having R(0,α) distn. a) Find the limiting distn of X(n) .b) Find the limiting distn of Y(n)=n(α-X(n))

Soln:i)

0; x<=0

D.F. of X(n): Gn(x)=P[X(n)<=x]={Fx1(x)}n= (x/α); 0<x<α

1; x=>α

now, limn-->infinityGn(x)= 0; x<α

1;x=>α

=G(x) (say) --------Degenerated D.F.

hence X(n)--L-->X; where X is a R.V. degenerated at x=α

Gn(y)=P[Yn <=y]=P[n(α-X(n)<=y]=P[X(n)=>α-y/n]=

1-FX(n)(α-y/n)=1-0 ; α-y/n<=0

1-(1-y/nα)n ; 0<α-y/n<α

1 ; y=>nα

limn-->infinityGn(y)= 0; y<=0

1-e^-(y/α); 0<y<infinity

=G(y)

hence Yn--L-->Y ; Y is a R.V. having EXP(mean=α) distn.

Remark:** **here X(n) = is the MLE of . We can see that if X1 ,X2 ….,Xn follows iid U(0 , α) then n(α- X(n)). EXP(mean= α) as n◊infinity . Hence MLE does not have an assymptotic normal distn.

Querry: So Does here property of of MLE seems to be false?

***Ans:** No. property of MLE is not false neither X(n) disobeys the property that MLE follows asymptotic normal distn. ,as to follow the property MLE must obey certain regularity conditions among them one is that the range of the distribution must be independent of the population parameter .But here the range is dependent on the parameter.*

***ii)Convergence in Probability:** Let {Xn} be a sequence of R.V.s defined on the same probability space (W , A , P). Then we say that {Xn} converges in probability to a R.V. X defined on (W ,A, P); if P(|Xn-X|< eps)→as n→infinity.*

An example :

Q) Let P[Xn=0]=1-1/n, P[Xn=1]=1/n , n=1,2,3… i.e., {Xn} be a sequence of Bernoulli R.V.s .Does {Xn} converge in probability to some distn?

Ans:

Intuitively , it is very much clear that as n→infinity P[|Xn=1]→0 and P[Xn=0]→1 so we can say that Xn converges to X, where X is a R.V. degenerated at x=0.

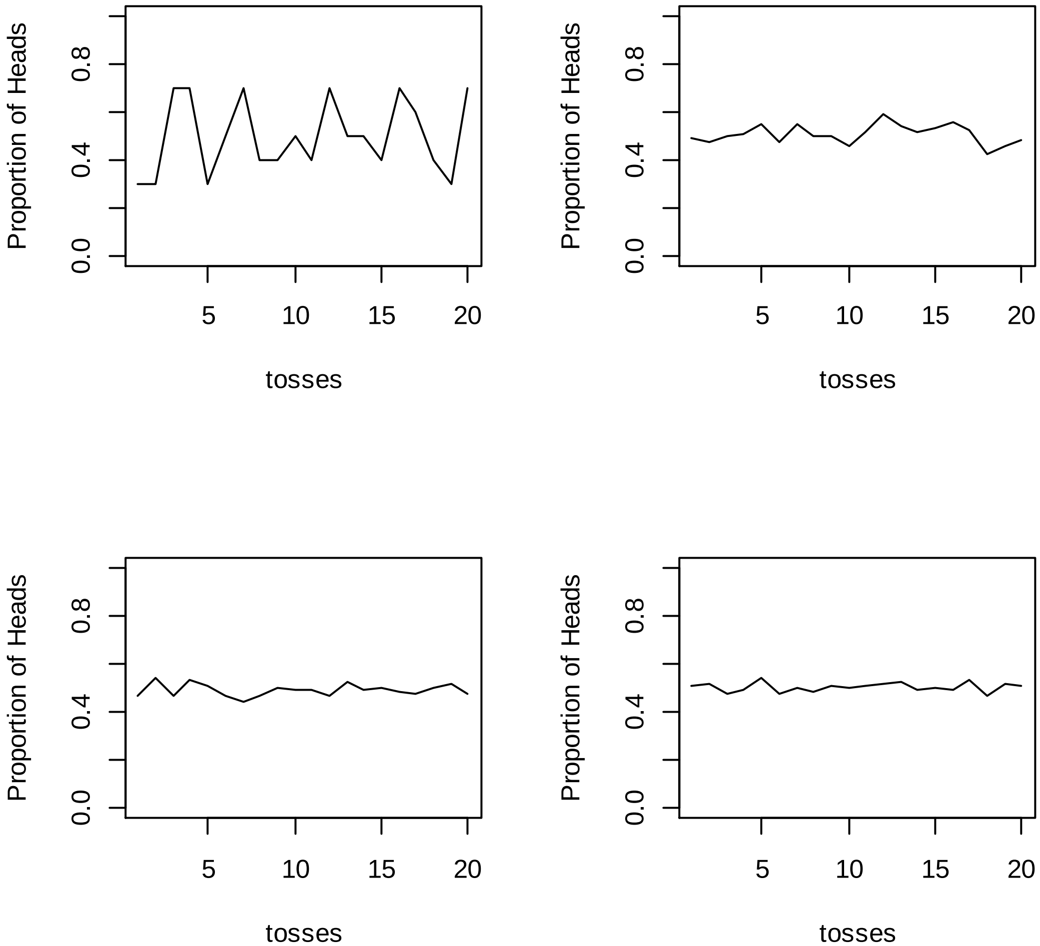
So, P[|Xn-0|<eps]=P[Xn<eps]=P [Xn=0] ; 0< eps <1
P [Xn=0]+P[Xn=1]; eps =>1
=1-1/n ; 0< eps <1
1 ;
P[|Xn-0|<eps]→1as, n→infinity for every eps >0.
So , by defn, Xn converges in probability to X,where X is a R.v. Degenerated at x=0.

An Real life example:

Q) I)Suppose I toss a coin 10 times and observe the no. of heads and repeat the experiment for 20 times , again I toss the coin 200 times and repeat this experiment for 20 times , next I toss it for 500 (repeat the experiment 20 times)and 1000(repeat for 20 times) times.
II) Now fix the number of toss to 20 times and increase the repeatition of experiment from 10 to 200, 200 to 500,500 to 1000.
1)Find the frequency distn and A.M. , S.D. in both of (i), (ii).
2) Give your interpretation ,What if you fixed the number of toss to 200 times in part (ii)

Soln-The table , graph etc needed are given in the next pages .

```
rm(list=ls())
y=c(10,200,500,1000)
par(mfrow=c(2,2))
z=seq(1:20)
z
for(j in y)
{
 x=rbinom(20,j,0.5)
print(table(x))
print(mean(x))
print(sqrt(19/20)*sd(x))
plot(z,(x/j),type="l",ylab="Proportion of Heads",xlab="tosses",ylim=c(0,1))
```



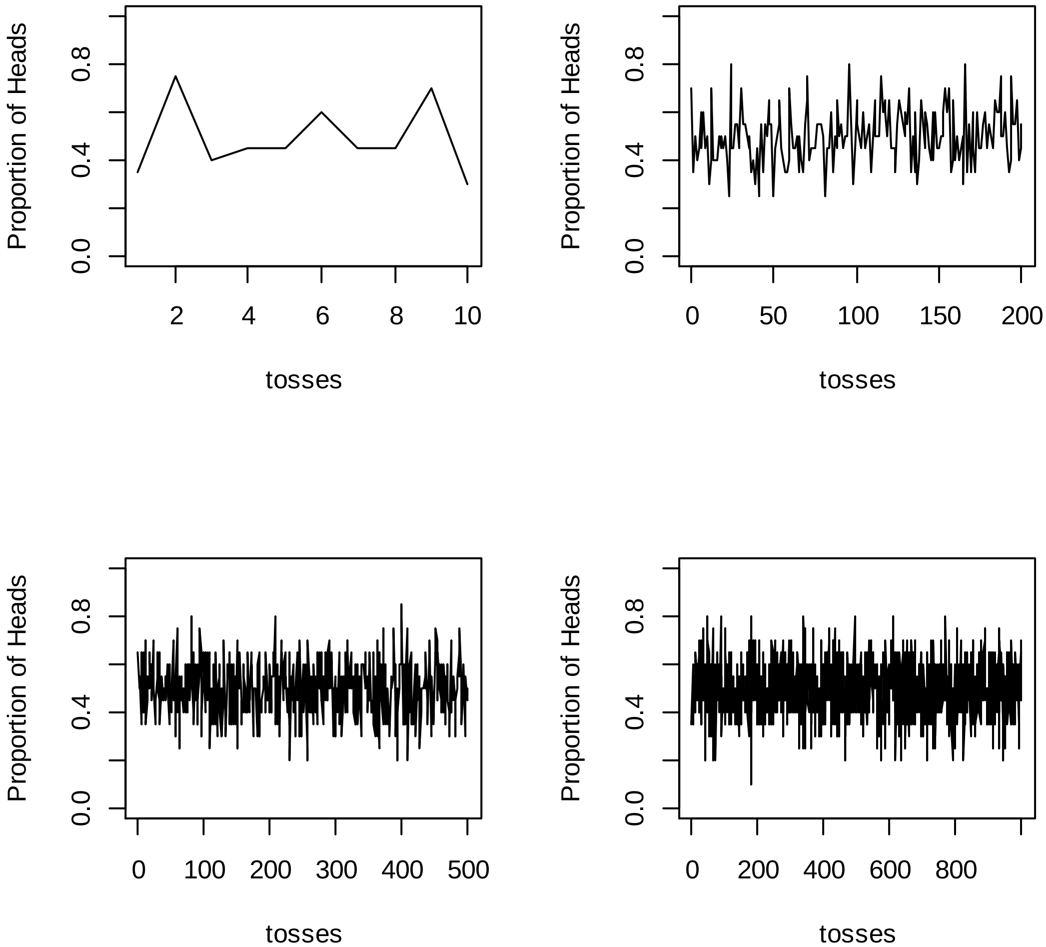
Interpretation: ***Clearly, as for the first case as the no. of tosses is very low i.e. 10, so the proportion of heads is fluctuating around 0.5 , as we increase the number of tosses , to 200,500, 1000, the proportion of heads seems to stabilize around 0.5 . Hence we can conclude that as the no. of tosses increases the proportion of heads seems to stabilize around 0.5,i.e. sample proportion converges in probability to 0.5.***

x
6 7 8 9 12 14 15
1 1 1 4 1 1 1
[1] 9.8
[1] 2.862012
x
5 6 7 8 9 10 11 12 13 14 15 16
4 5 18 19 42 34 31 20 13 7 4 3
[1] 10.01
[1] 2.193619

```
x
 4  5  6  7  8  9 10 11 12 13 14 15 16 17
 4  6 27 44 60 74 84 70 59 47 14  7  3  1
[1] 9.914
[1] 2.274471
x
  2   4   5   6   7   8   9  10  11  12  13  14  15  16          [1] 9.994
 1  11  15  28  76 132 145 180 156 121  71  46  11   7    [1]2.2129863

rm(list=ls())
y=c(10,200,500,1000)
par(mfrow=c(2,2))

for(j in y)
{z=seq(1:j)
 x=rbinom(j,20,0.5)
print(table(x))
print(mean(x))
print(sqrt(19/20)*sd(x))
plot(z,(x/20),type="l",ylab="Proportion of Heads",xlab="tosses",ylim=c(0,1))
}
```



<u>Interpretation:</u> As we are fixing the number of tosses to 20, which is a low value ,so the value seems to fluctuate around 0.5 .

METHODOLOGY

In statistics asymptotic theory or large sample theory , is a frame work for assessing properties of estimators or test statistics. With in this frame work it is often assumed that the sample size n may grow indefinitely; the properties of estimators and tests are then evaluated under the limit n tends to infinity. Although in regular practice we deal with finite data set there is a lot of application of "asymptotic nature of statistics" in real life data set. The objective of the project is mainly shedding light on different asymptotic behaviour of some basic statistics and making a well comparison between their asymptotic behaviour. There are mainly two kinds of asymptotic behaviour of a statistics1) Consistency(related to convergence in probability) 2) Convergence in distribution or law. In this project The asymptotic convergence in probability and distribution of some statistics from parent populations (like Normal, Exponential, Cauchy with certain parameters)like sample mean, sample median, sample midrange, sample minimum, sample maximum are established through simulation and with the help of R(the well known statistical software) For each statistics a guess limit is found at first and then convergence in probability and convergence is tested. i)Firstly, For a statistic keeping the replication number fixed, the sample size are increased(like n=100, 500,1000 for fixed R=1000). ii)Then keeping the sample size fixed , the replication number is increased (like R=100,500,1000 for fixed n=1000) iii)Next, For a fixed sample size (say n=1000) and a fixed replication number(R=1000) the parameters of the parent population are changed . Thus the asymptotic behaviour of the statistics are observed in all the above mentioned cases and corresponding observation and findings are noted.

RESULTS & DISCUSSIONS-

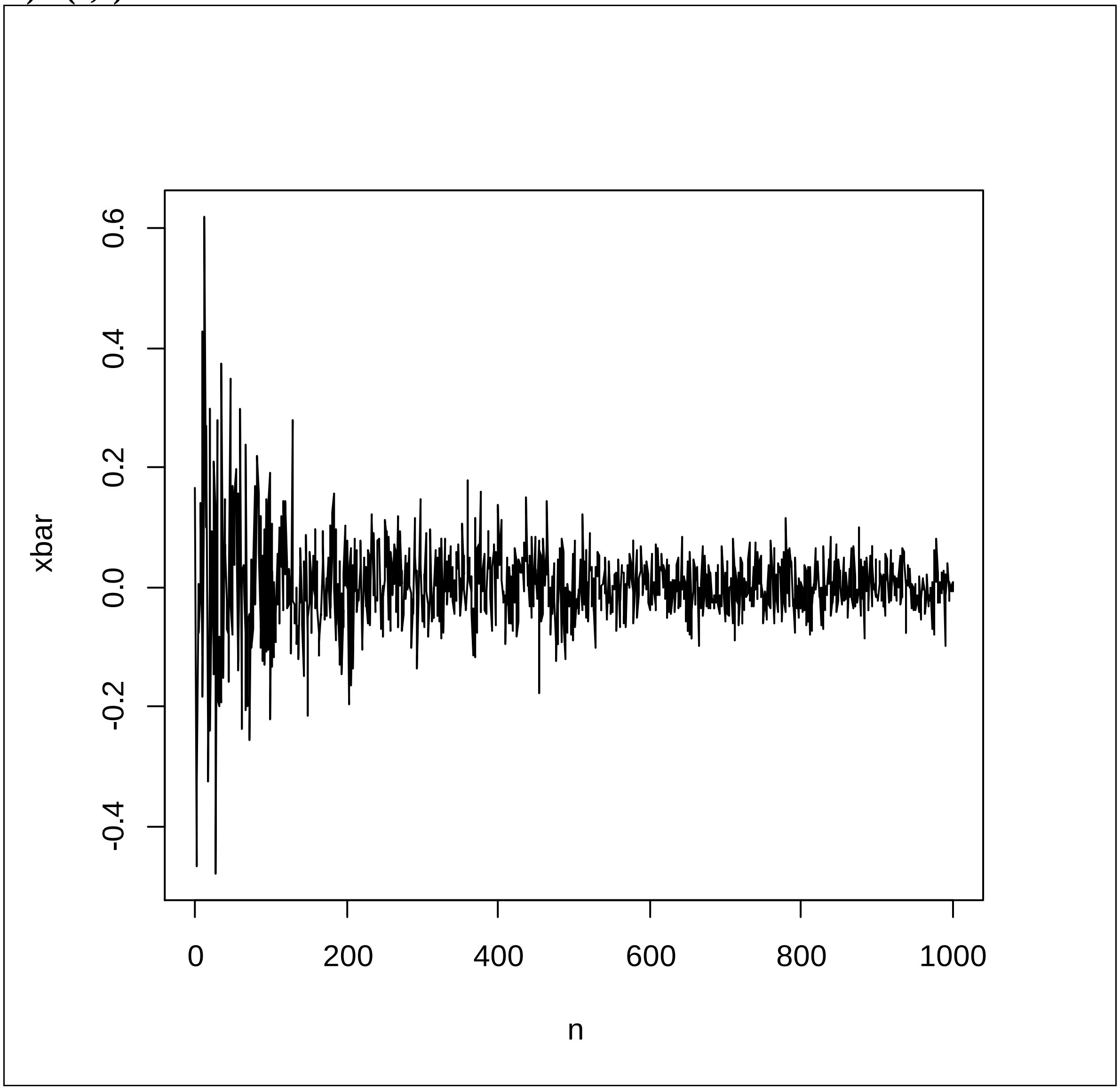
NORMAL DISTRIBUTION-

Here we will find guess limits-

The term "guess limit" is used to represent that particular value to which the statistics tends to converge as the sample size n tends to infinity. Here we are trying to find guess limits for sample mean , median , X(n) , X(1) from the population N(0,1) , N(5,1) , N(5,2) to make a well defined comparison between the statistical behaviour of the statistics.

A)MEAN-

i) N(0,1)-

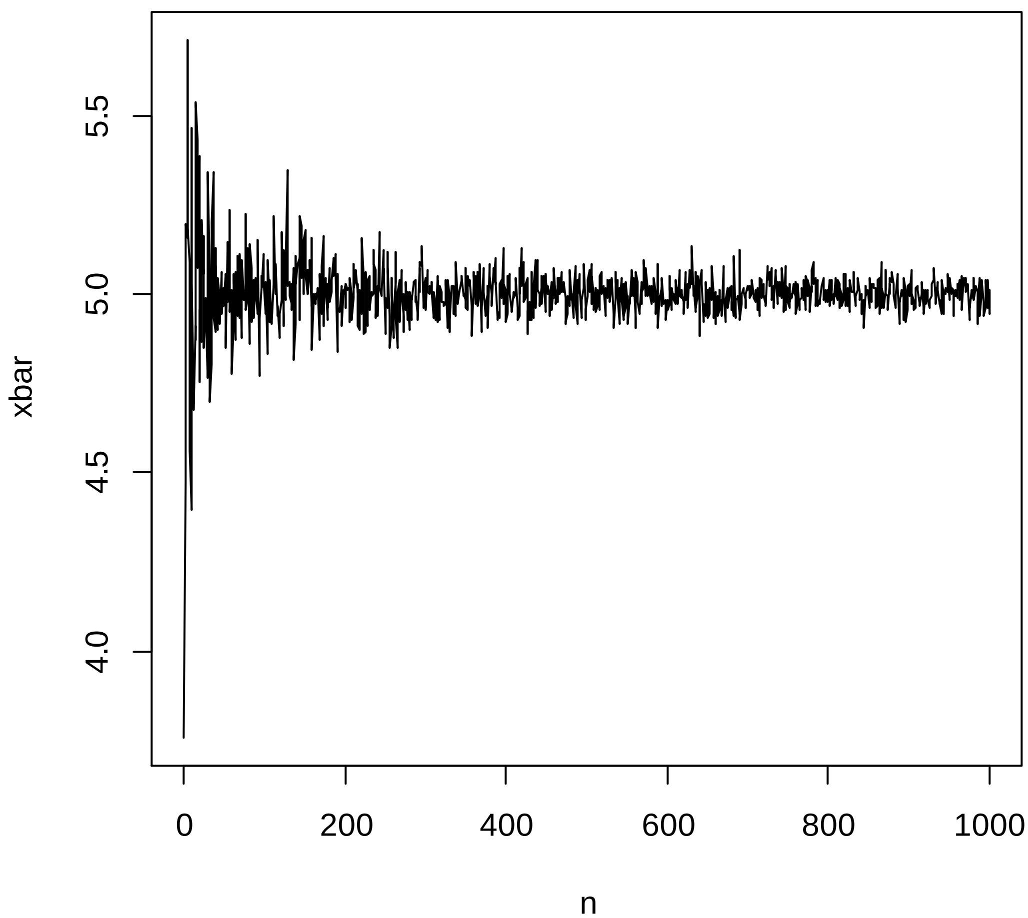


Findings-

1) For N(0,1) , the sample mean Xbar tends to converge to its location parameter

2)As the sample size n increases the convergence becomes more clear and rapid.

ii)N(5,1)-

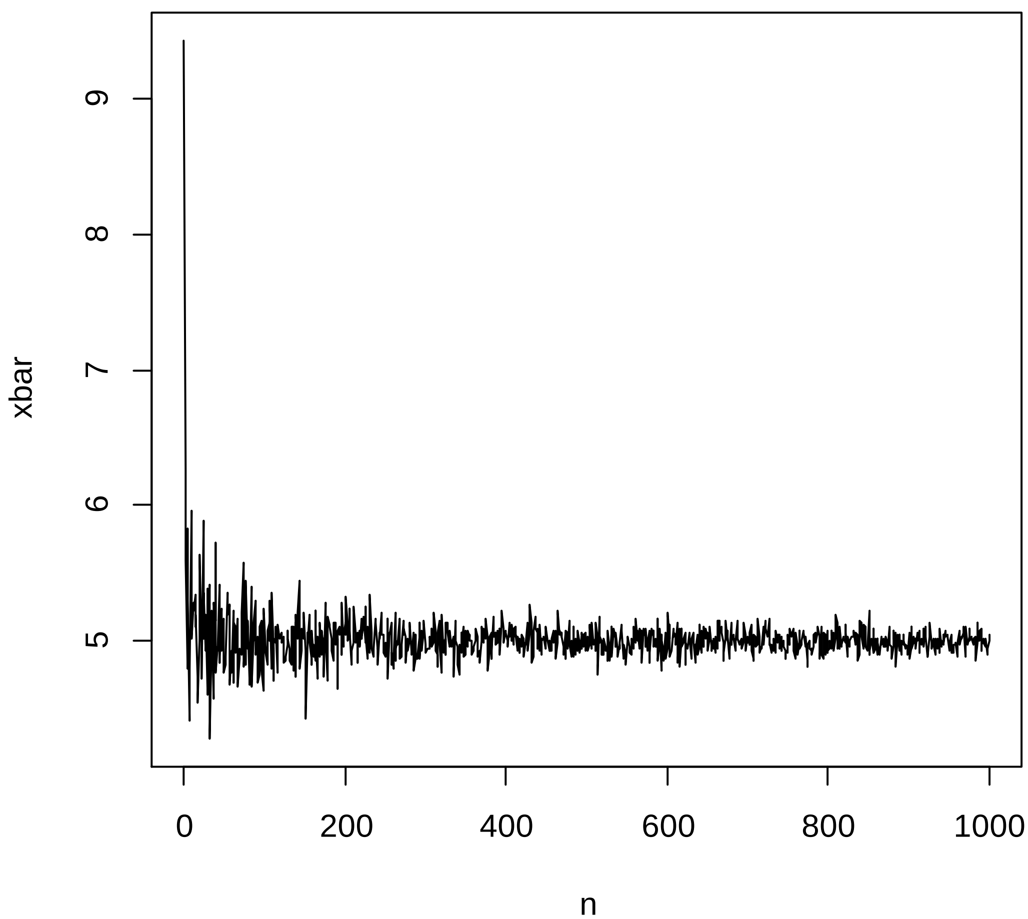


Findings-

1) ***From the diagram it is clear that for N(5,1) , the sample mean Xbar tends to its location parameter 5 as n becomes large enough.***

2) *The convergence becomes more clear and fast as the sample size goes on increasing.*
3) *The convergence of Xbar is clearer in N(5,1) than the convergence of Xbar in N(0,1) for large sample size (say, n= 600 to 1000).*

iii)N(5,2)-



Findings-

1) *The sample mean Xbar converges to the location parameter 5 although the scale parameter is now changed to '2' from '1' as n tends to infinity.*
2) *As the sample size increases, the convergence of xbar becomes more rapid.*

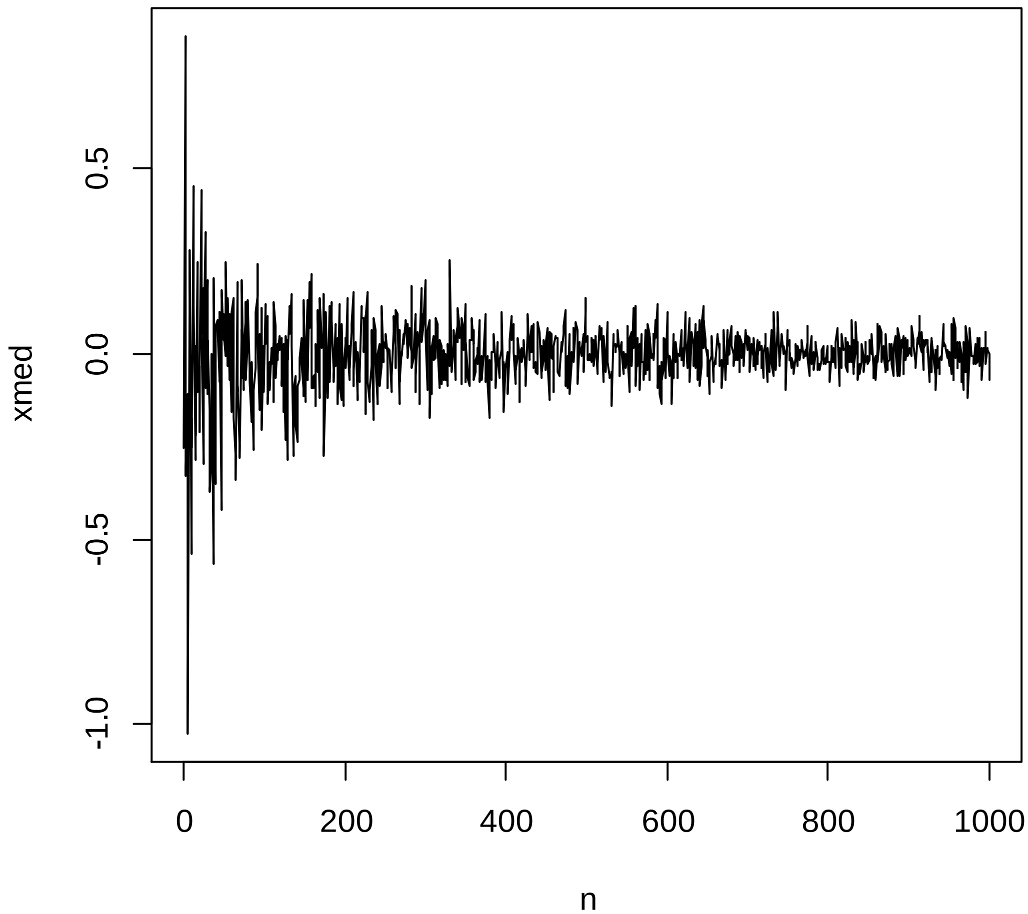
3) ***For N(5,2) the convergence of xbar occurs before than for the cases N(0,1) or N(5,1). That is for comparatively lesser sample size (say n=400 to 600).***
4) ***As scale parameter sigma is increased in N(5,2) case fro N(5,1) case , the fluctuation of Xbar around location parameter , mew=5 , is decreased for comparatively lesser sample size say n=200 – 300***.

Conclusion-

- ***Hence in general we can conclude that for any N(u,A^2) population, as n tends to infinity, the sample mean converges to population mean u,(which is the location parameter), irrespective of the population variance A^2, A is scale parameter.***
- ***u belongs to real line , as u is deviated from 0 , the convergence becomes more rapid.***
- ***A>0, as A increases, the fluctuation of Xbar around location parameter decreases.***

B) MEDIAN-

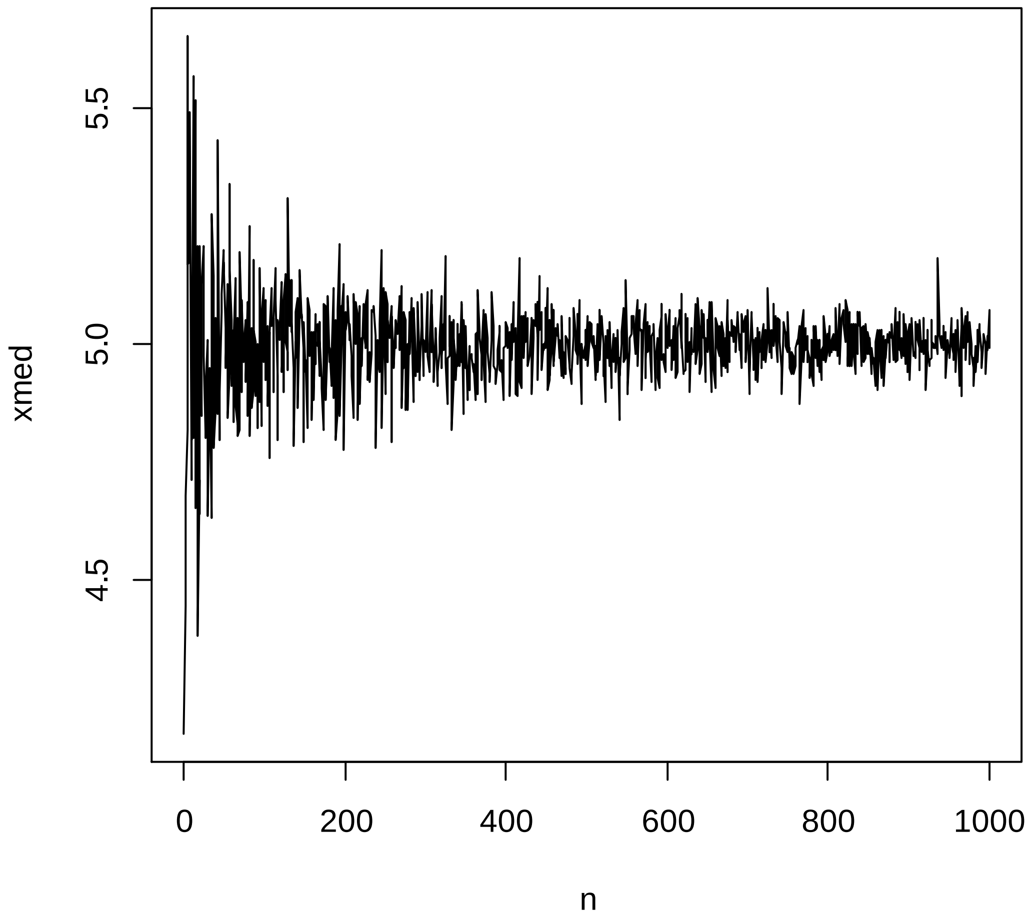
i)N(0,1)-



Findings-

1) ***For N(0,1) as n tends to infinity, the sample median tends to converge to location parameter '0'.***
2) ***As the sample size increase, (say , n=600 to 1000), the convergence becomes more clear.***

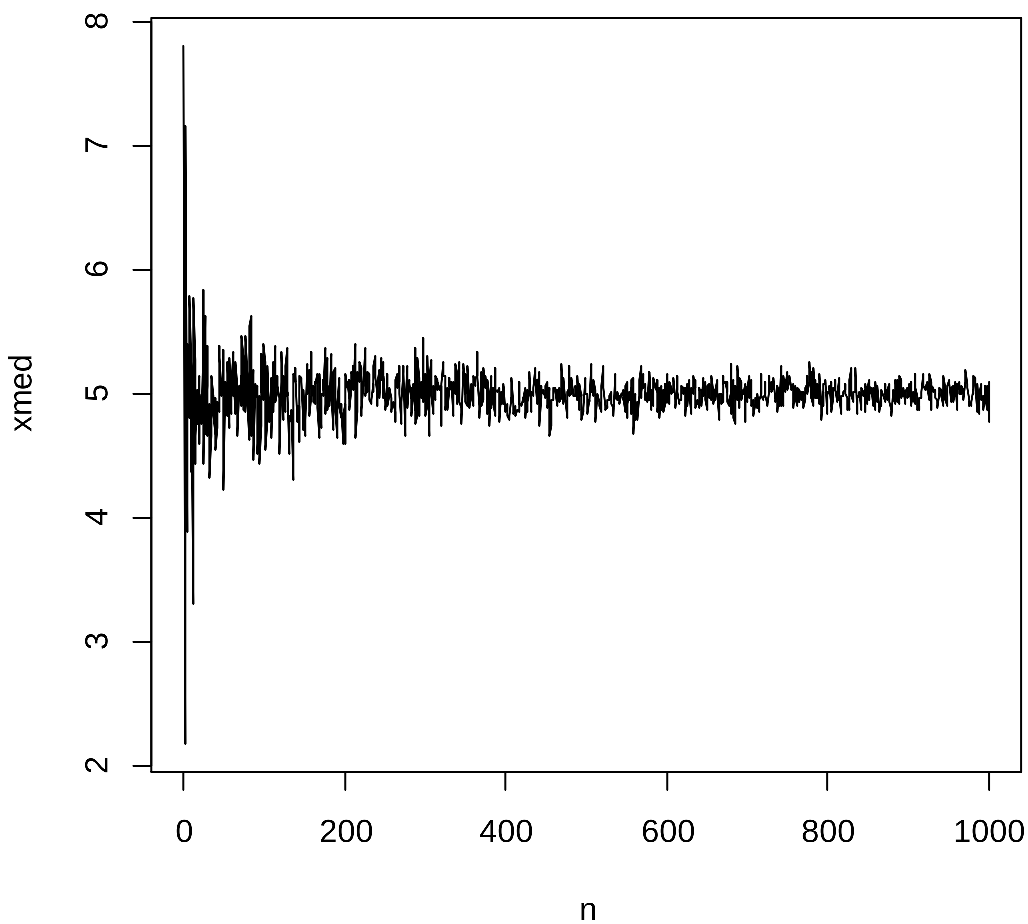
ii)N(5,1)-



Findings-

1) ***As n increases, sample median Xmed tends to the location parameter,u=5.***
2) ***With the increase in sample size, the convergence becomes more clear.***
3) ***There is no significant difference in fluctuation of Xmed around location parameter in the two cases N(0,1) and N(5,1).***

iii)**N(5,2)-**



Findings-

1)***As n tends to infinity, for N(5,2) the sample median Xmed tends to the location parameter u=5.***   ***2) Larger the sample size, clearer is the convergence.***   ***3) The sample median tends to location parameter more rapidly when the population is N(5,2) rather than when the population is N(0,1) or N(5,1).***   ***4) Clearly the fluctuation of Xmed around location parameter is lesser in N(5,2) rather when the population is N(5,1) or N(0,1).***

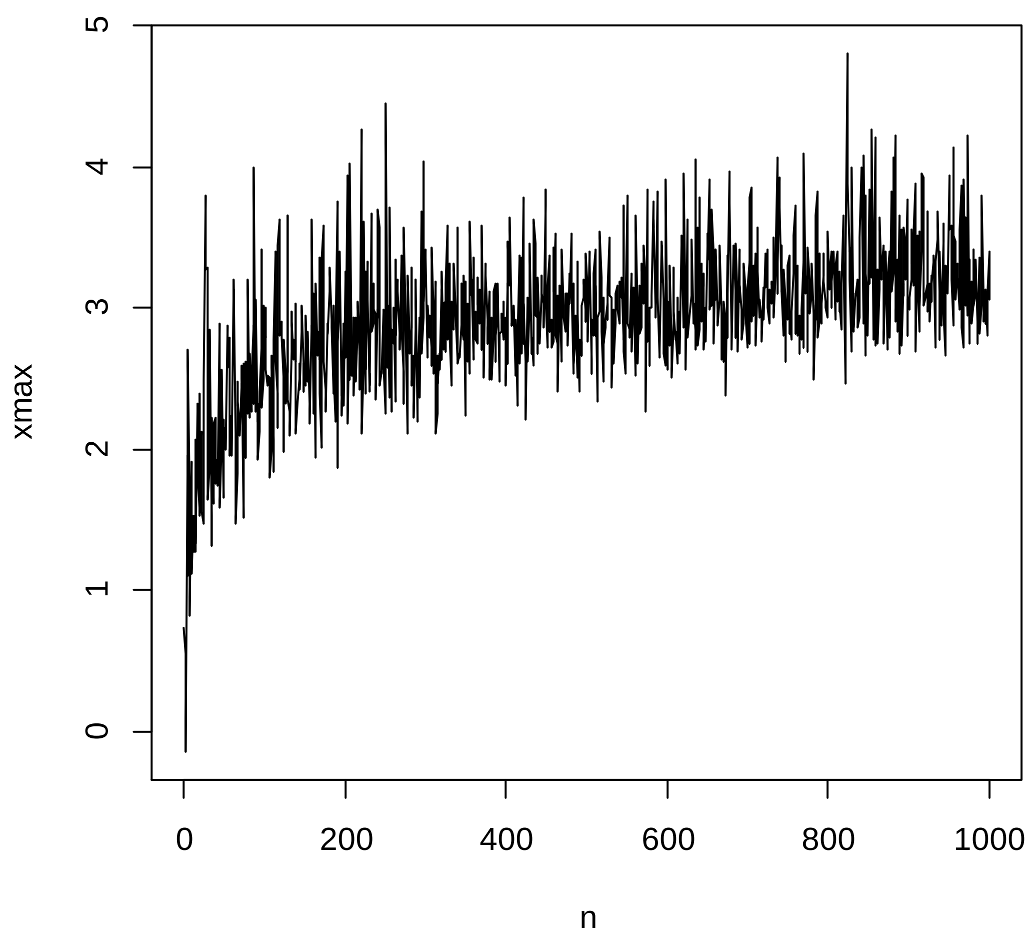
Conclusion-

- ***Hence in general we can conclude that for any N(u,A^2) population, as n tends to infinity, the sample median converges to the population median u(which is***

- *the location parameter), irrespective of the population variance A^2 , A is thw scale parameter.*
- ***Keeping the scale parameter constant if we change the location parameter, there is no significant change in convergence of Xmed.***
- ***Keeping the location parameter constant, if we increase scale parameter (A>0), the fluctuation of Xmed around scale parameter decreases.***

C) X(n)-

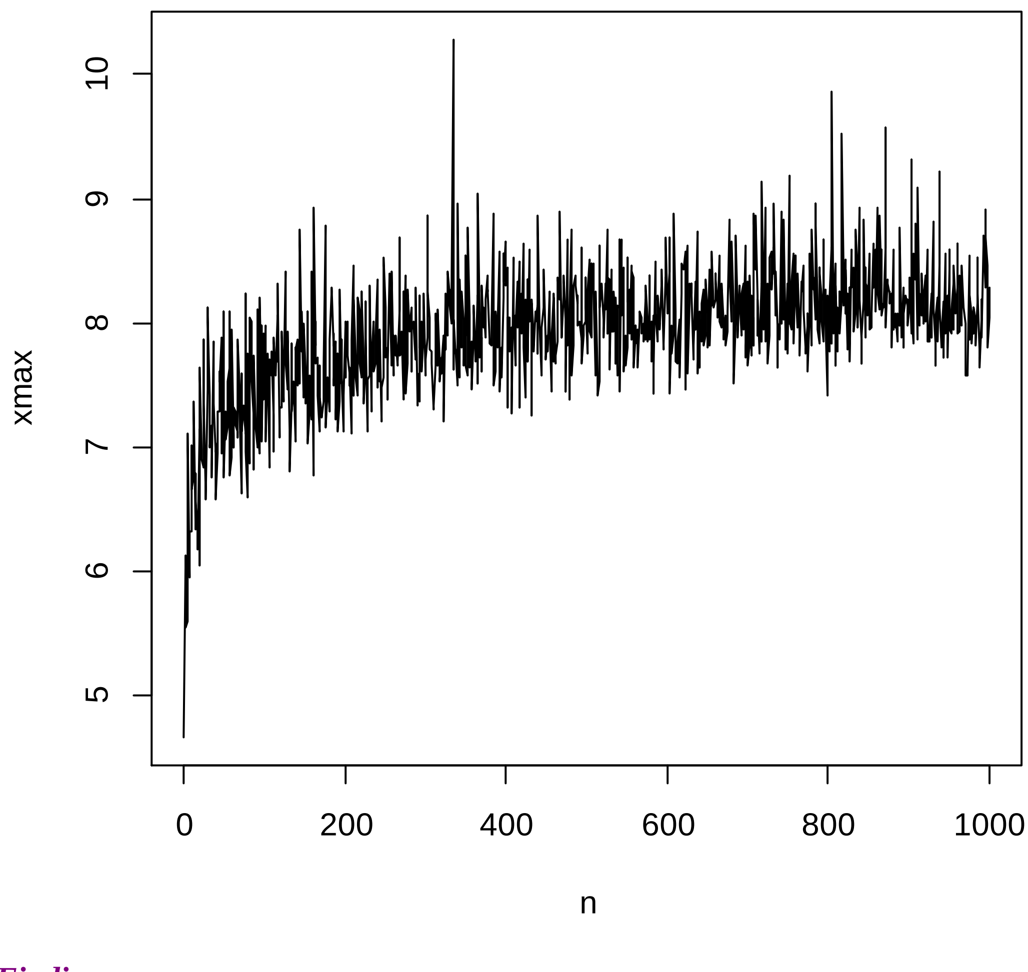
i)N(0,1)-



Findings-

1) ***Clearly as n tends to infinity , the sample maximum, Xmax does not seems to tend to any particular limit.***
2) ***Rather as sample size increases, the fluctuation increases proportionally.***
3) ***Although average Xmax values seems to be close to 3 but, it may differ from sample to sample , hence it is not a proper guess limit.***

ii) N(5,1)-

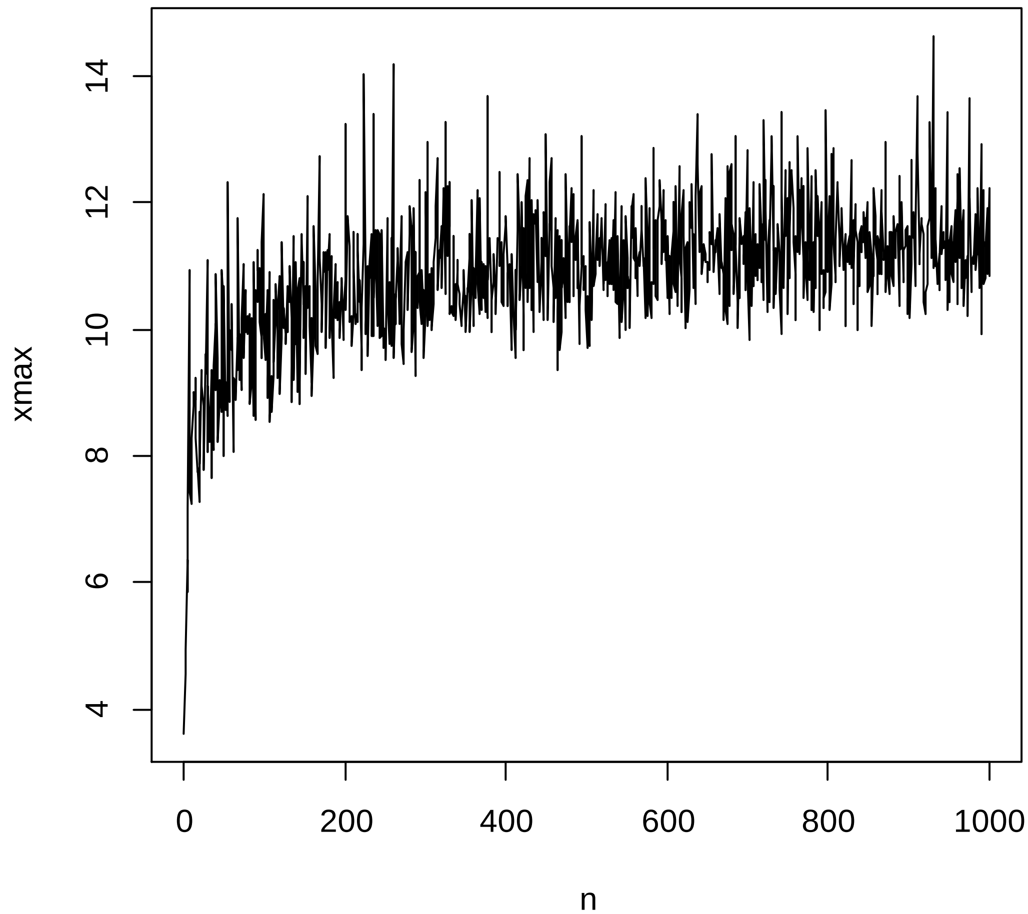


Findings-

1)***With the increase in sample size, Xmax does not seem to converge any particular limit.***
2)The oscillation increases rapidly as the sample size increase .

3)Apparently it may be seemed that Xmax values are clustering around 8 but for different sample sizes, it may differ from sample to sample, also, the fluctuation around 8 does not seem to reduce as n tends to infinity.

iii) N(5,2)-



Findings-

1) ***As the sample size goes on increasing, X(n) or Xmax does not seem to converge to any particular value.***
2) ***Rather as sample size increases the fluctuation increases rapidly.***
3) ***Apparently it may be seemed that the Xmax values are tending to 11, but it may differ from sample to sample and fluctuation of Xmax around 11 increases proportionately.***

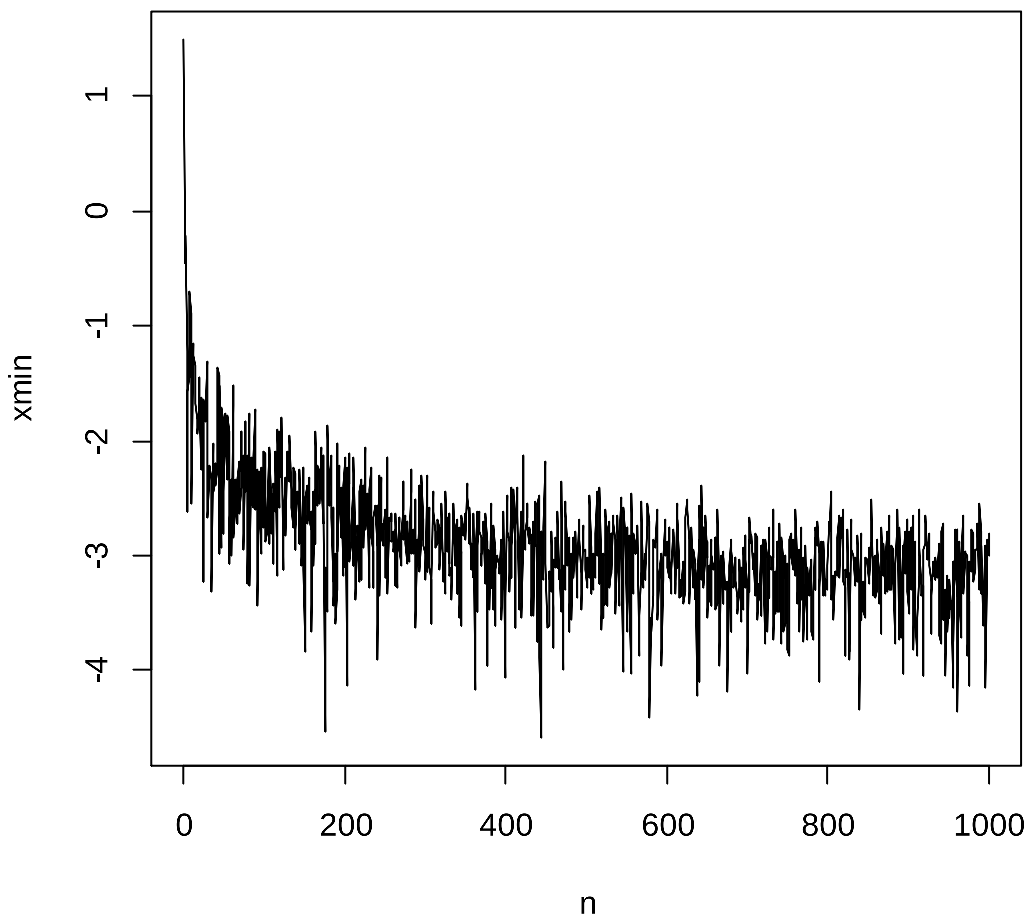
Conclusions-

- ***Clearly Whatever the normal population be, whatever may be its location or scale parameter, sample maximum or X(n) does not seem to tend to any particular value.***
- ***As we increase sample size , the randomness in sample X(n) values increases.***

- ***Although the sample X(n) values seem to cluster to any value apparently, the value changes from sample to sample, hence in general we can not find any guess limit for sample X(n).***

D) X(1)-

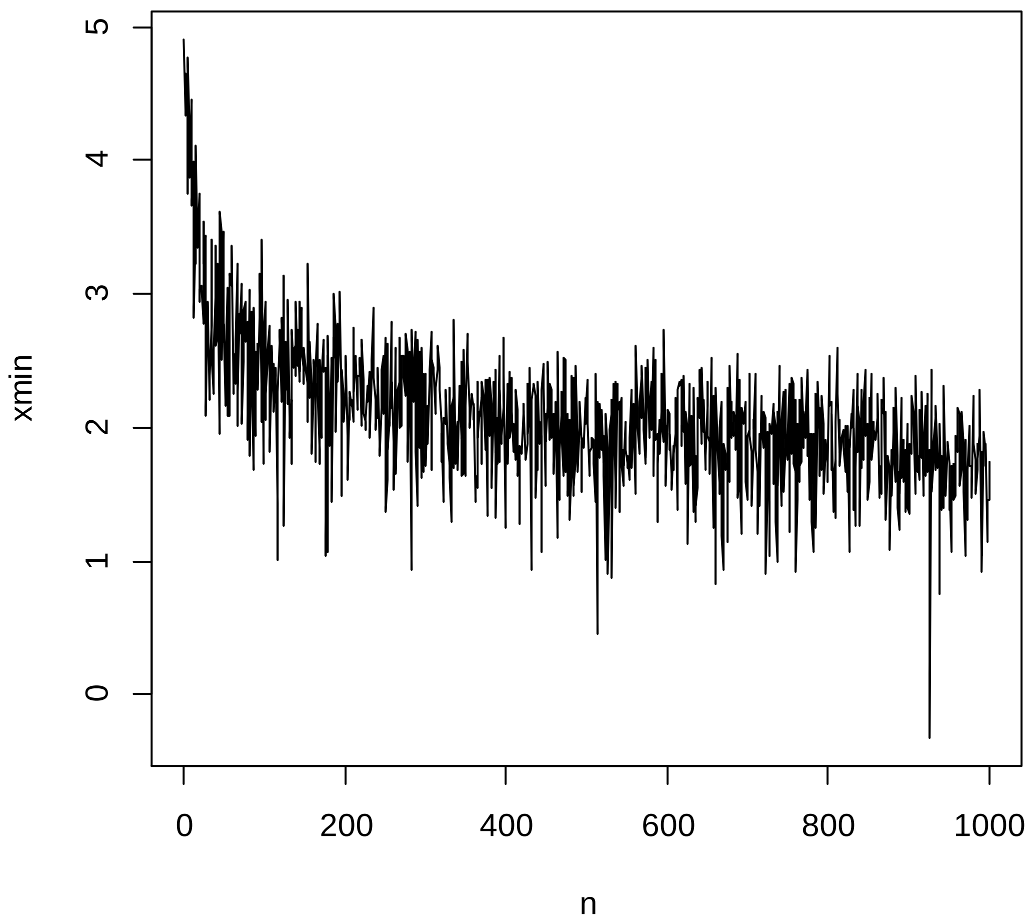
i) N(0,1)-



Findings-

1) ***As n tends to infinity, the sample minimum X(1) or Xmin does not seem to tend to any particular value.***
2) ***As sample size increases, the fluctuation also increases.***
3) ***Apparently it might seem to us that the values of X(1) are clustering around -3, but the value changes from sample to sample.***

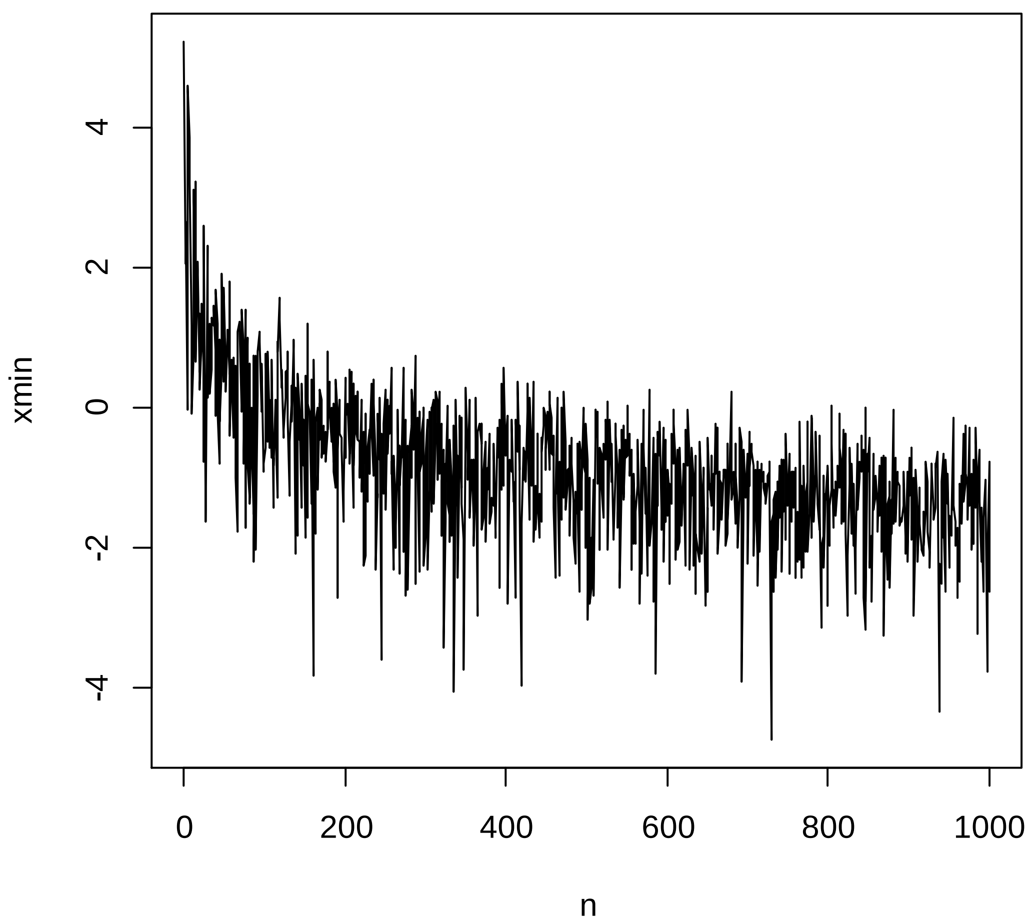
ii) N(5,1)-



Findings-

1) ***As n is becomes large, the sample X(1) does not seem to converge to any particular limit.***
2) ***As the sample size increases , the oscillation of sample X(1) increases.***
3) ***Also, it might appear to the viewer that X(1) is clustering around 2 but the value changes from sample to sample.***

iii) N(5,2)-



Findings-

1) As n tends to infinity, sample X(1) does not seem to converge to any particular limit.
2) The fluctuation increases as n goes on increasing.
3) Apparently it might be seen that sample X(1) values are clustering around -1, but the value may differ from sample to sample.

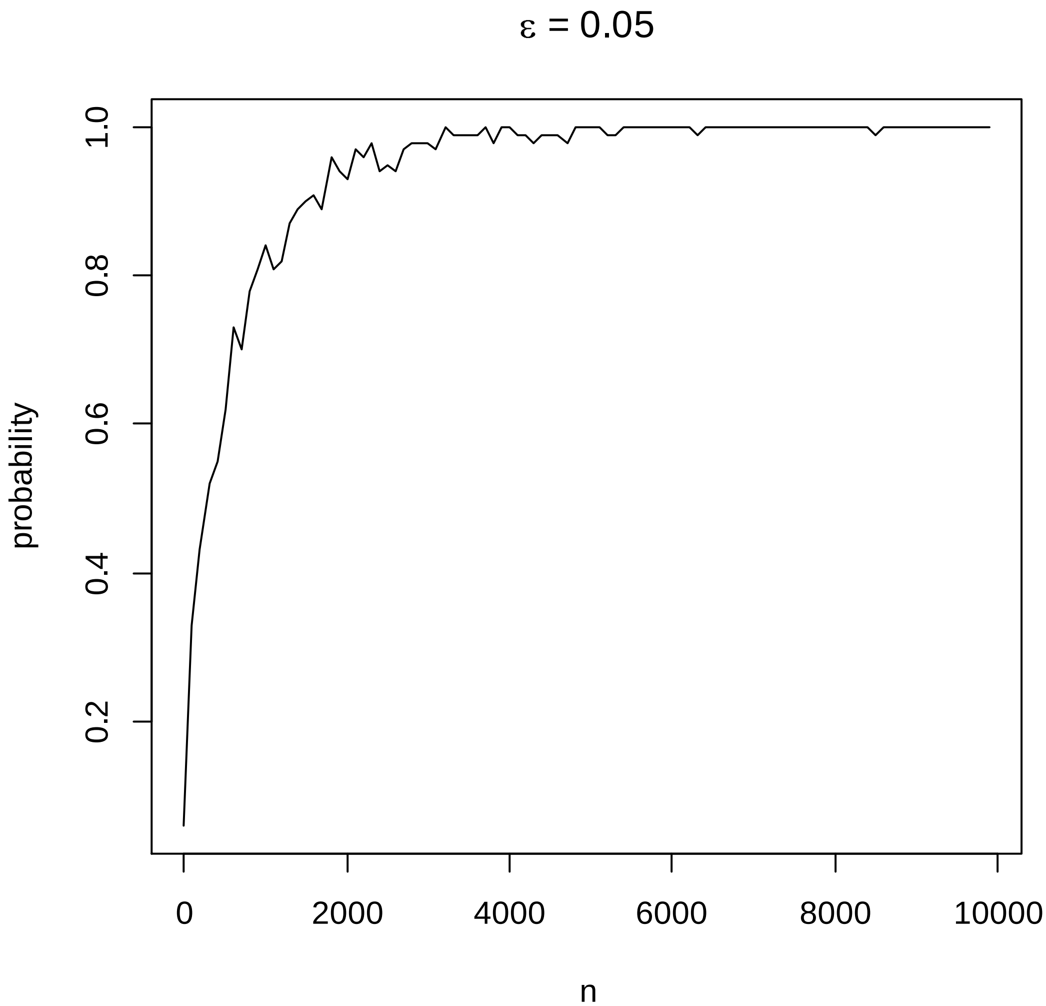
Conclusion-

- ***Clearly Whatever the normal population be, whatever may be its location or scale parameter, sample minimum or X(1) does not seem to tend to any particular value.***
- ***As we increase sample size , the randomness in sample X(1) values increases.***
- ***Although the sample X(1) values seem to cluster to any value apparently, the value changes from sample to sample, hence in general we can not find any guess limit for sample minimum.***

Now we shall discuss convergence in probability for n=100 , 200, 300,……..,1000 and R=100

A. MEDIAN-

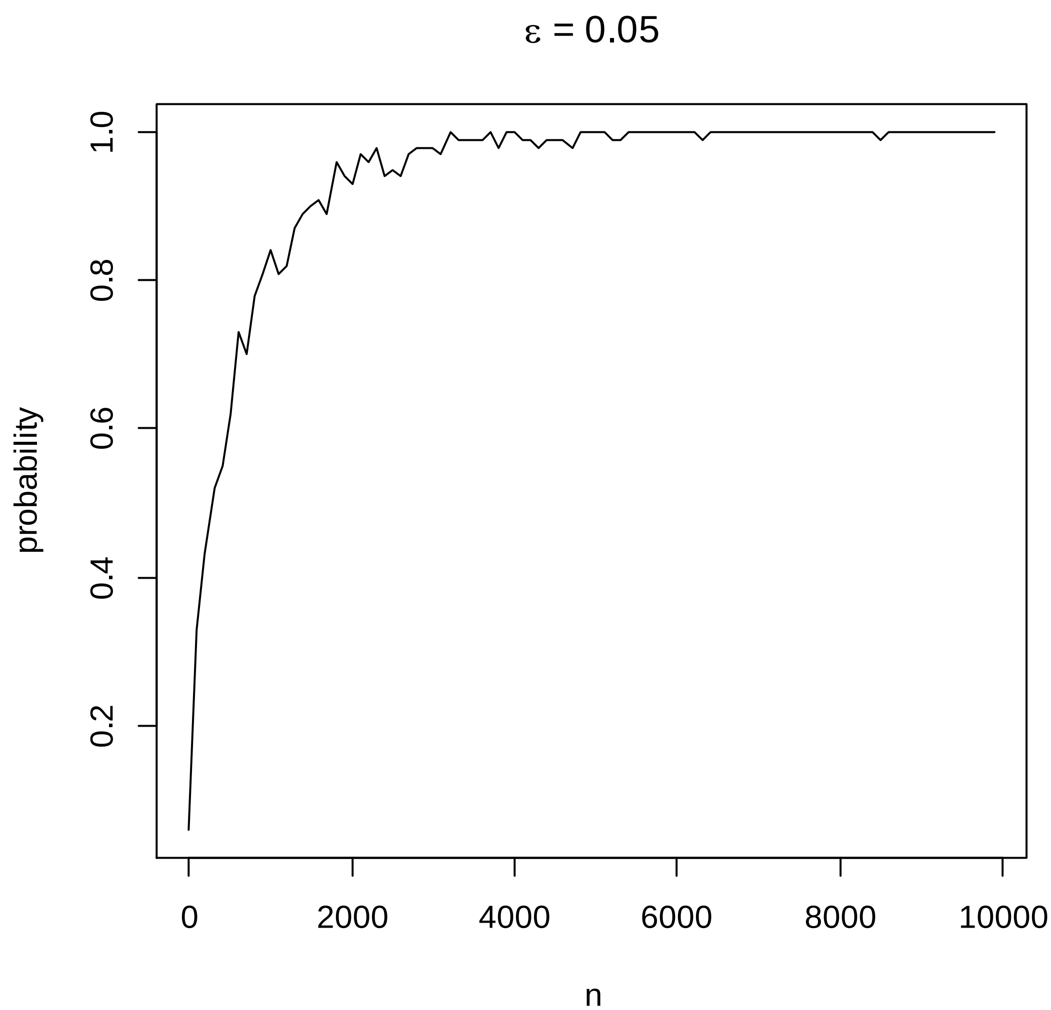
i) N(0,1)-



Findings-

1) ***We have got the guess limit as '0' for N(0,1) population.***
2) ***Clearly as n is increasing , P[|Xmed-0|<eps=0.05]→1, hence Xmed converges to '0' in probability as n tends to infinity.***
3) ***In other words for N(0,1) distribution, sample Xmed is consistent for population median '0'.***

ii) N(5,1)-



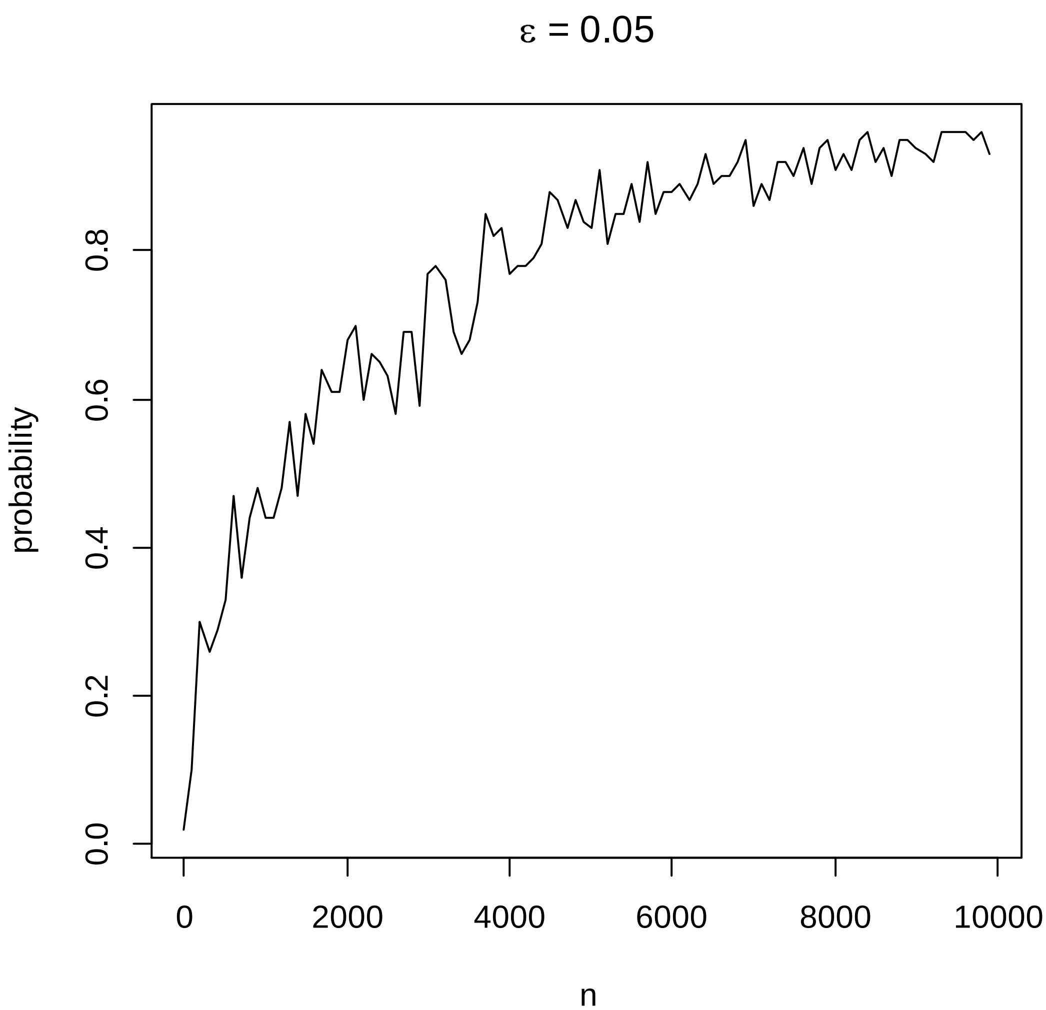
Findings-

1) ***We have got the guess limit as '5' for N(5,1) population.***
2) ***It is clear from the above graph that as n tends to infinity,***

P[|Xmed – 5|<eps=0.05]→1, indicating that sample Xmed converges in probability to population median '5'.

3) ***In other words sample Xmed is consistent for population median '5'.***
4) ***The convergence rate for Xmed to corresponding population median in both the cases N(0,1) and N(5,1) are more or less equal as for both the cases, Xmed converges in probability to population median between the sample size n=2000 to 4000.***

iii)N(5,2)



Findings-

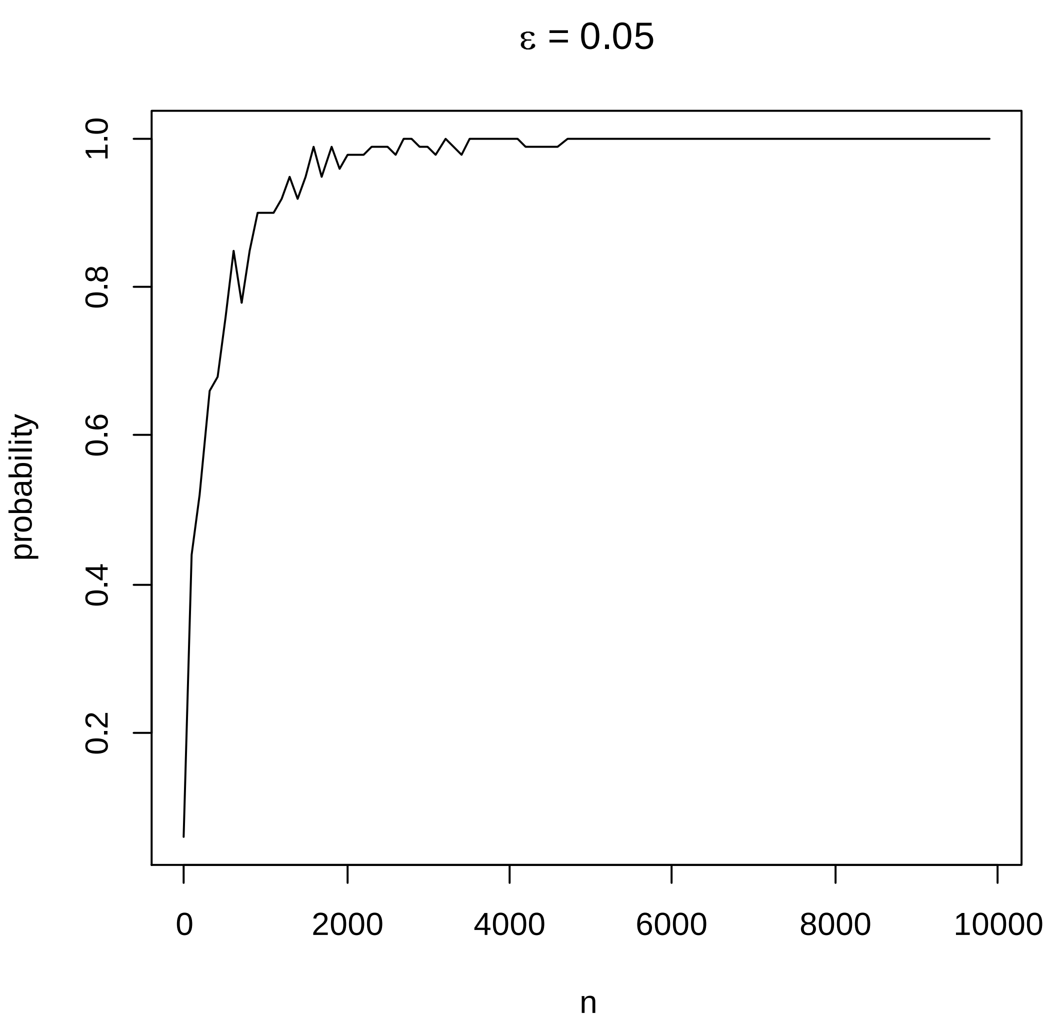
1) ***We have got the guess limit as '5' for N(5,2) population.***
2) ***It is clear from the above graph that as n tends to infinity, P[|Xmed – 5|<eps=0.05]→1, indicating that sample Xmed converges in probability to population median '5'.***
3) ***In other words sample Xmed is consistent for population median '5'.***
4) ***For N(5,2) case, the convergence rate of Xmed to population median is lesser than that of the cases N(0,1) and N(5,1) as here the probability convergence of Xmed occurs between the sample size n=8000 to 1000, unlike in N(0,1) and N(5,1) cases(where it happened for n=2000 to 4000).***

Conclusions-

- ***We have the guess limit as 'u', the population median.***
- ***It is clear that as n tends to infinity, P[|Xmed-u|<eps=0.05]→1, indicating that sample median Xmed converges in probability to population median 'u'.***
- ***That is Xmed is consistent for population median 'u'.***
- ***As we change the location parameter of a Normal parent population, keeping the scale parameter fixed, there is no significant change in the rate of convergence, but as we change the scale parameter keeping the location parameter fixed, the convergence rate decreases.***

B.MEAN-

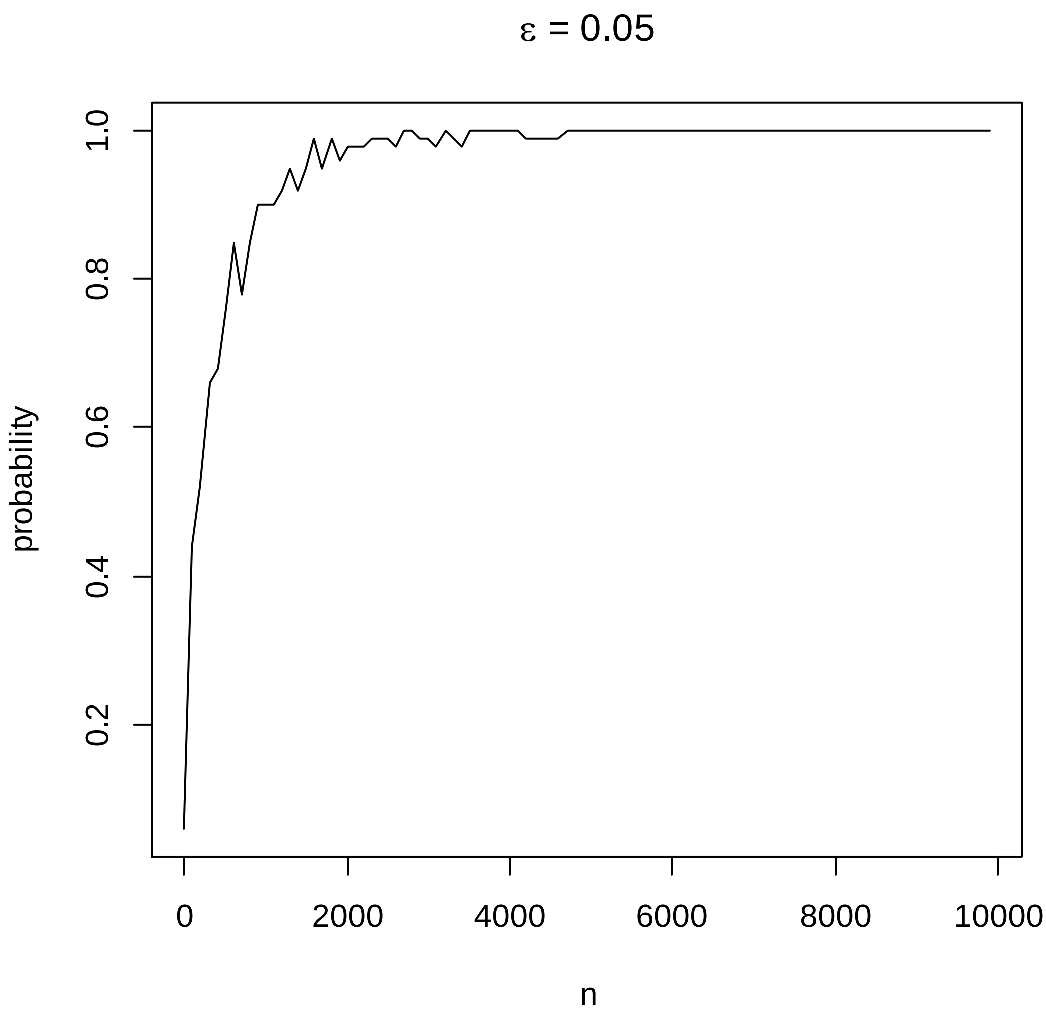
i) N(0,1)-



Findings-

1) ***We have got the guess limit as '0' for N(0,1) population.***
2) ***Clearly as n is increasing , P[|Xbar-0|<eps=0.05]→ 1, hence Xbar converges to '0' in probability as n tends to infinity.***
3) ***In other words for N(0,1) distribution, sample Xbar is consistent for population mean '0'.***

ii) N(5,1)-



Findings-

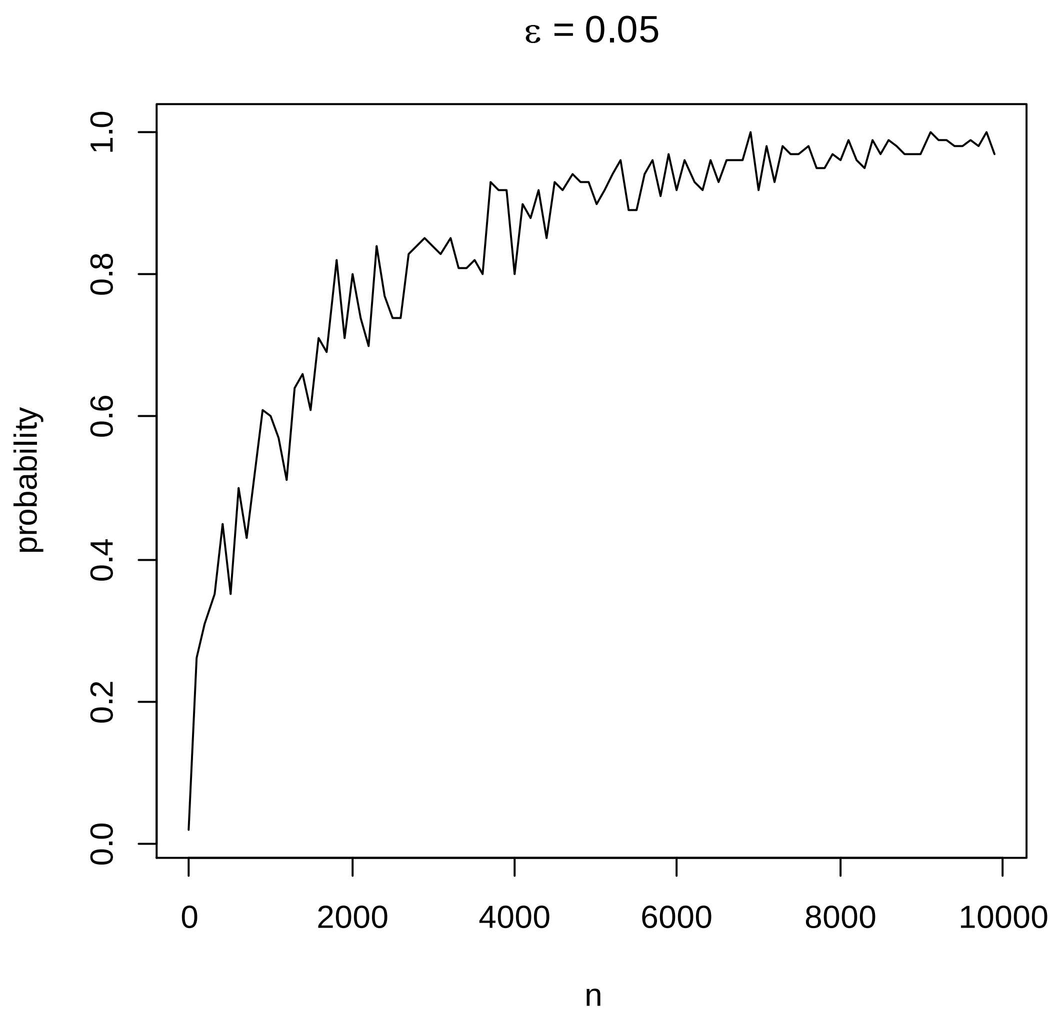
1) ***We have got the guess limit as '5' for N(5,1) population.***
2) ***It is clear from the above graph that as n tends to infinity,***

P[|Xbar– 5|<eps=0.05]→1, indicating that sample Xbar converges in probability to population mean '5'.

3)In other words sample Xbar is consistent for population mean '5'.

4)The convergence rate for Xbar to corresponding population mean in both the cases N(0,1) and N(5,1) are more or less equal as for both the cases, Xbar converges in probability to population median between the sample size n=2000 to 4000.

iii) N(5,2)-



Findings-

1) We have got the guess limit as '5' for N(5,2) population.
2) It is clear from the above graph that as n tends to infinity, P[|Xbar – 5|<eps=0.05]→1, indicating that sample Xbar converges in probability to population mean '5'.
3)In other words sample Xbar is consistent for population mean'5'.
4)For N(5,2) case, the convergence rate of Xbar to population mean is lesser than that of the cases N(0,1) and N(5,1) as here the probability convergence

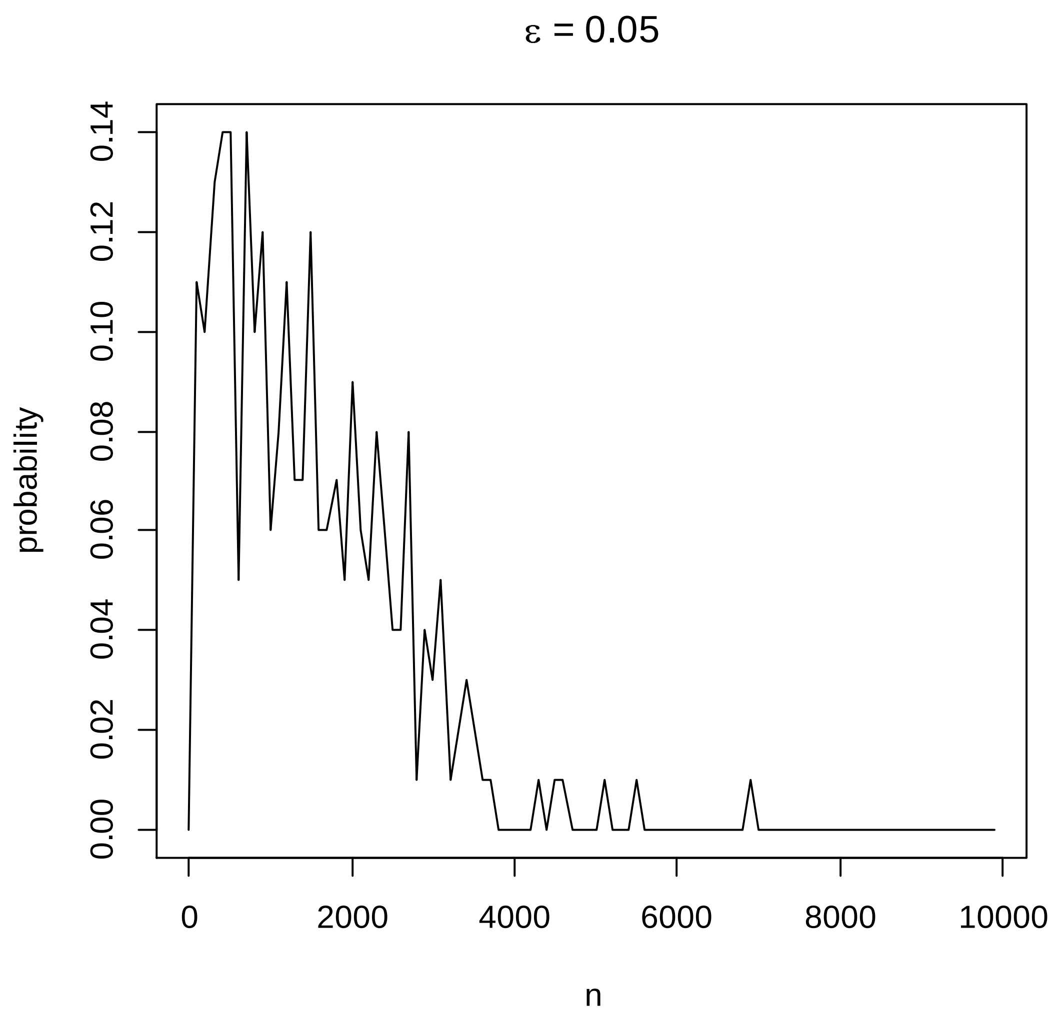
of Xbar occurs between the sample size n=8000 to 1000, unlike in N(0,1) and N(5,1) cases(where it happened for n=2000 to 4000).

Conclusions-

- ***We have the guess limit as 'u', the population mean.***
- ***It is clear that as n tends to infinity, P[|Xbar -u|<eps=0.05]→1, indicating that sample mean Xbar converges in probability to population mean 'u'.***
- ***That is Xbar is consistent for population mean 'u'.***
- ***As we change the location parameter of a Normal parent population, keeping the scale parameter fixed, there is no significant change in the rate of convergence, but as we change the scale parameter keeping the location parameter fixed, the convergence rate decreases.***

C. X(1)-

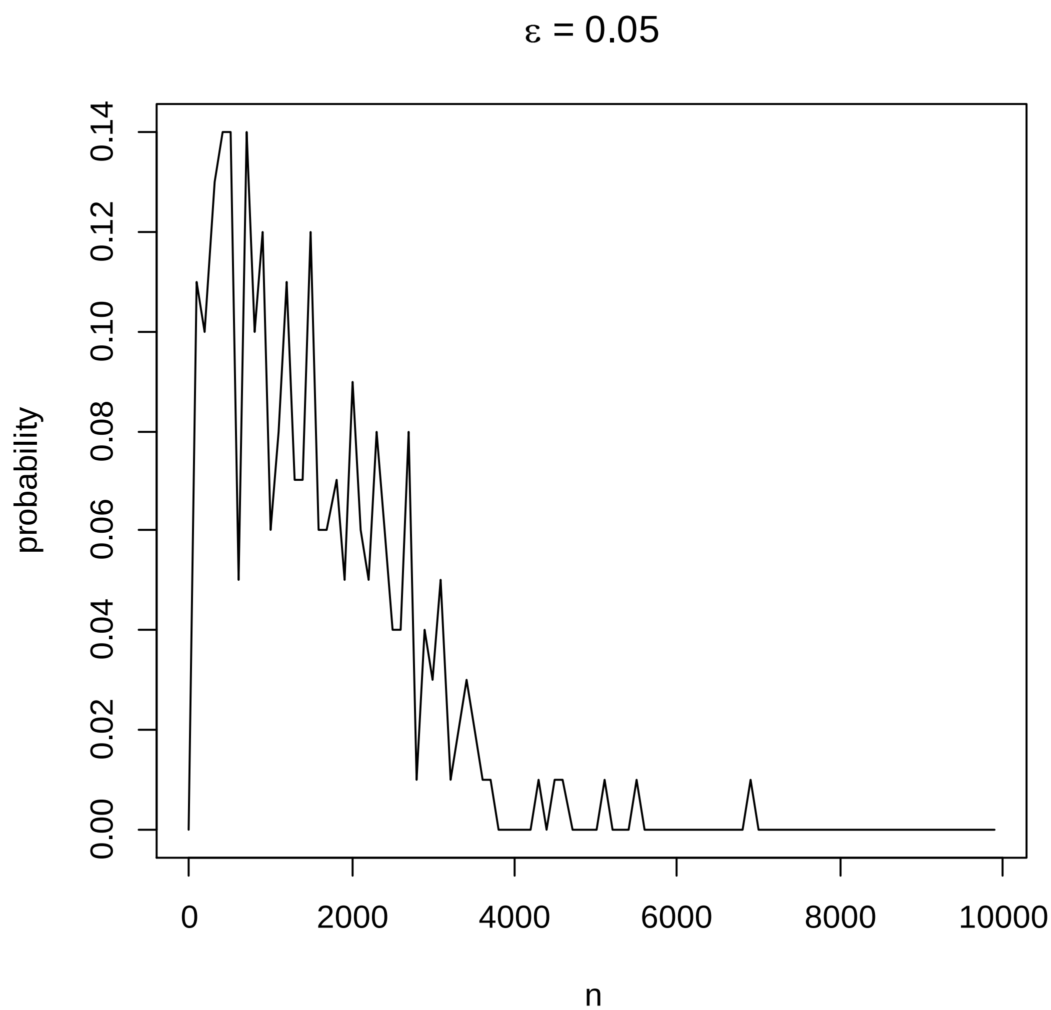
i) N(0,1)-



Findings-

1) ***We have not found any proper guess limit for X(1) for N(0,1) still as based on the sample we see most of the values are clustering around -3, we take it as a guess limit and try to check the convergence in probability of X(1).***
2) ***It is very much clear from the above graph that as n tends to infinity, P[|X(1)+3|<eps=0.05] does not tend to 1 rather between n = 6000 to 10000 it tends to 0.***
3) ***Hence X(1) does not converge in probability to '-3'.We can also check by taking different guess limits but will find that the probability is not going to 1 as n tends to infinity.***

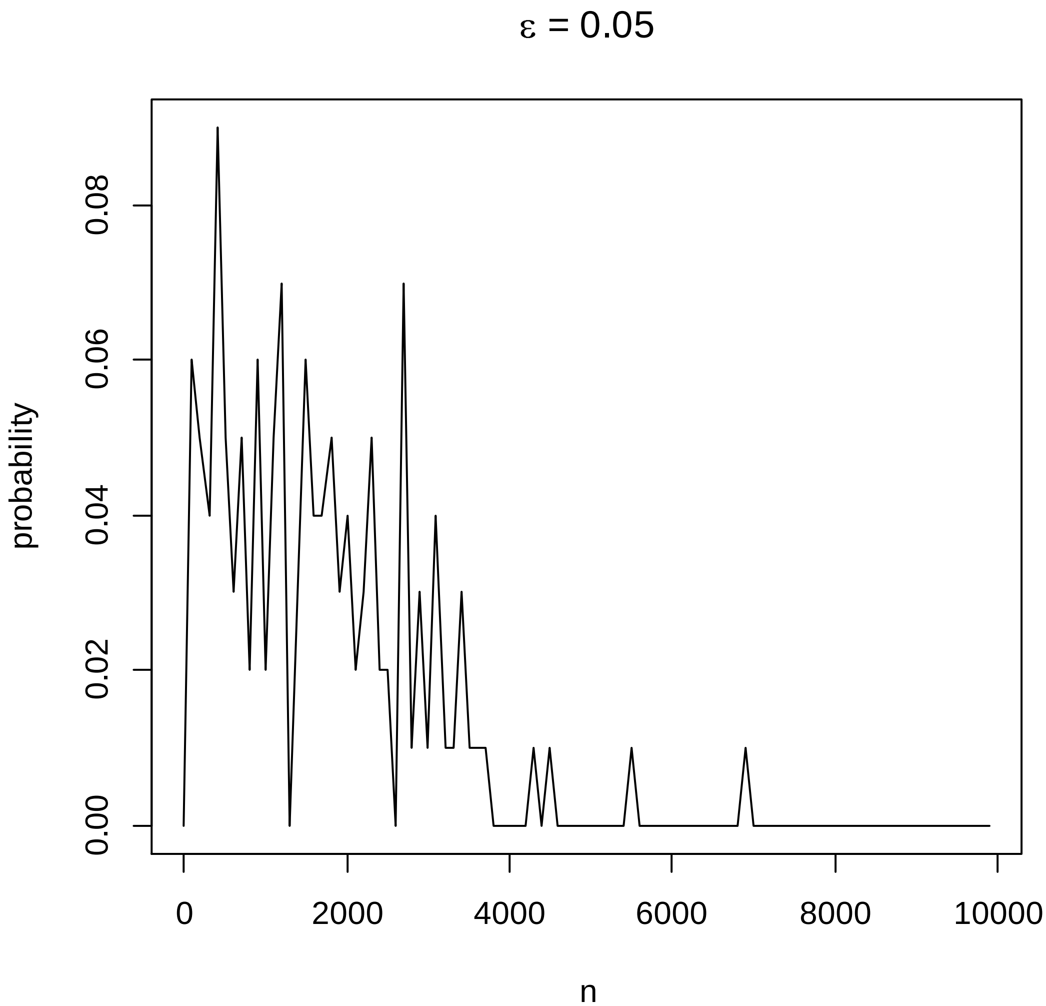
ii) N(5,1)-



Findings-

1. ***We have not found any proper guess limit for X(1) for N(5,1) still as based on the sample we see most of the values are clustering around 2, we take it as a guess limit and try to check the convergence in probability of X(1).***
2. ***It is very much clear from the above graph that as n tends to infinity, P[|X(1)-2|<eps=0.05] does not tend to 1 rather between n= 6000 to 10000 it tends to 0.***
3. ***Hence X(1) does not converge in probability to '2'. We can also check by taking different guess limits but will find that the probability is not going to 1 as n tends to infinity.***

iii) **N(5,2)-**



Findings-

1. ***We have not found any proper guess limit for X(1) for N(5,2) still as based on the sample we see most of the values are clustering around '-1' we take it as a guess limit and try to check the convergence in probability of X(1).***

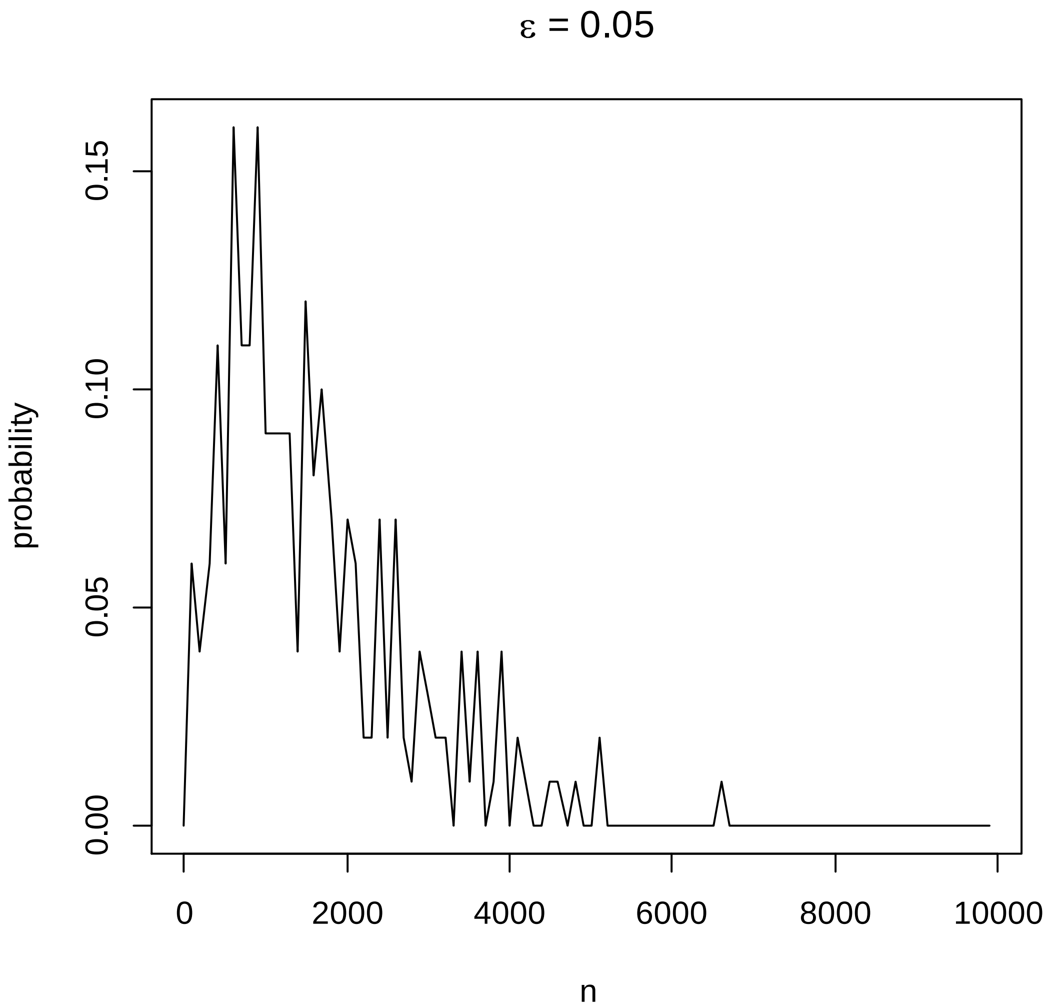
2. ***It is very much clear from the above graph that as n tends to infinity, P[|X(1)+1|<eps=0.05] does not tend to 1 rather between n=6000 to 10000 it tends to 0.***
3. ***Hence X(1) does not converge in probability to '-1'. We can also check by taking different guess limits but will find that the probability is not going to 1 as n tends to infinity..***
4. ***As keeping the location parameter fixed variance is increased, the probability value in y axis has also decreased ,i.e., becomes closer to '0' .***

Conclusions-

1) ***We have not found any guess limit for X(1) for whatever the location and scale parameter of the population be, so, by an eye estimate, we fix a number and check convergence in probability to that number for X(1).***
2) ***But we can see P[|X(1)-u|<eps=0.05] is not tending towards 1 as n tends to infinity, rather irrespective of location or scale parameter for large n, (n=6000 to 10000), it tends to '0' so we can say X(1) does not converge in probability to 'u',u belongs to real line.***
3) ***That is , in other words X(1) is not consistent estimator of 'u'.***

D. X(n)-

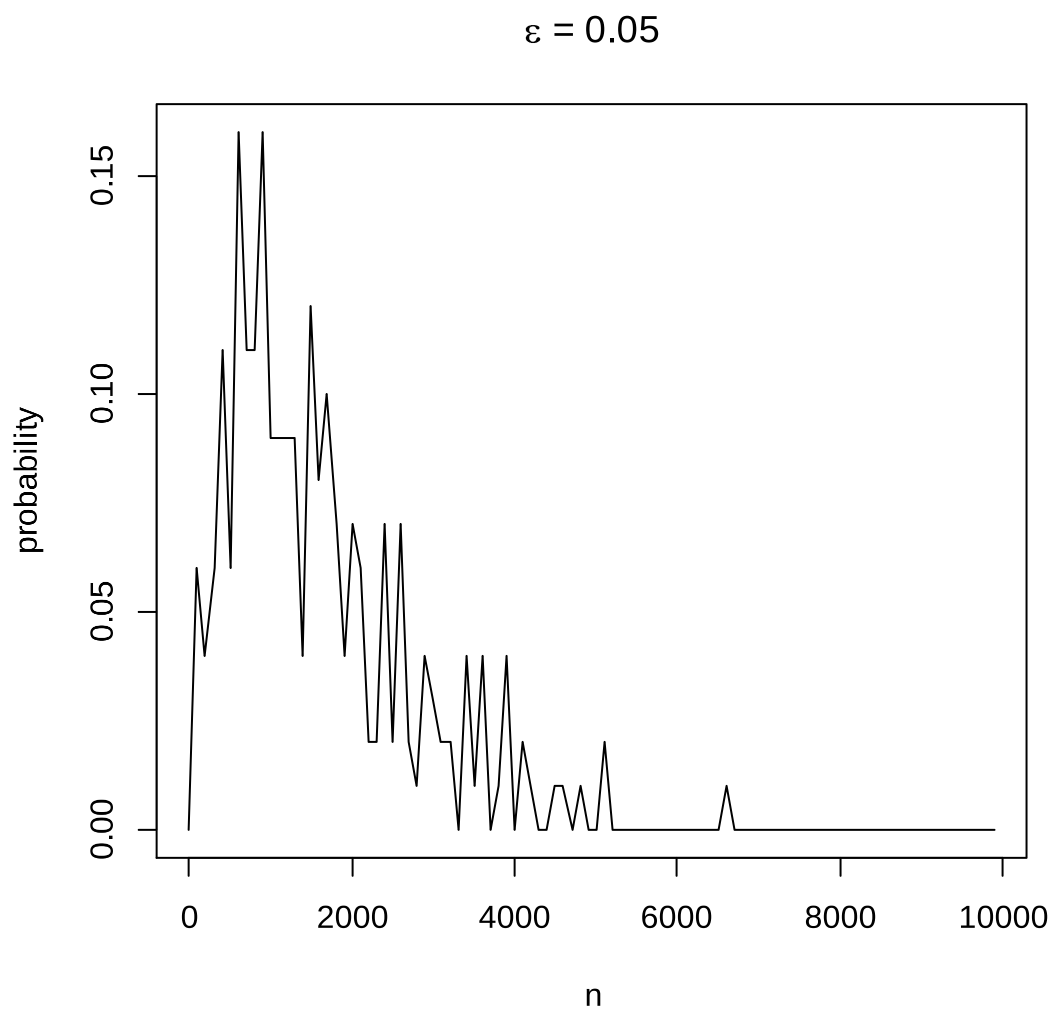
i) N(0,1)-



Findings-

1) ***We have not found any proper guess limit for X(n) for N(0,1) still as based on the sample we see most of the values are clustering around 3, we take it as a guess limit and try to check the convergence in probability of X(n).***
2) ***It is very much clear from the above graph that as n tends to infinity, P[|X(1)-3|<eps=0.05] does not tend to 1, rather between n=6000 to 10000, it tends to 0.***
3) ***Hence X(n) does not converge in probability to '3' We can also check by taking different guess limits but will find that the probability is not going to 1 as n tends to infinity.***

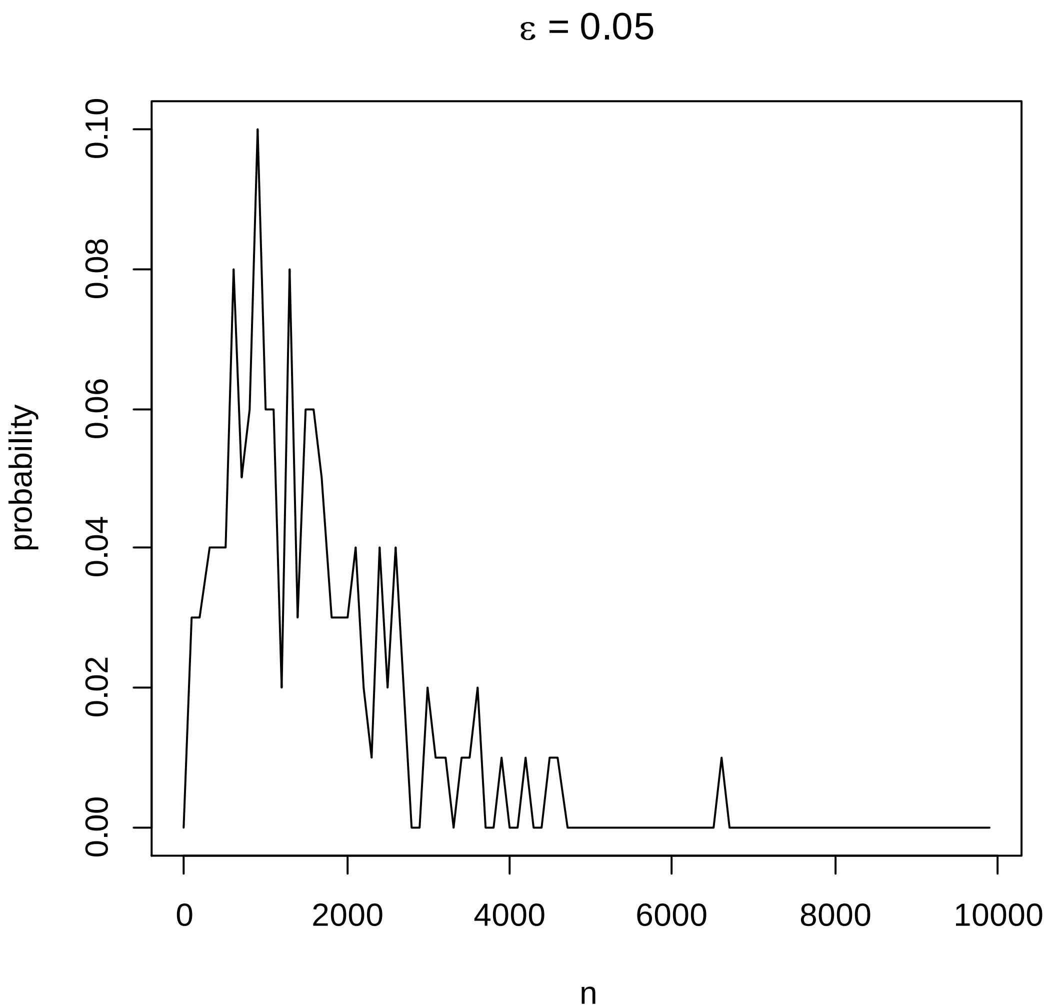
ii) N(5,1)-



Findings-

1) ***We have not found any proper guess limit for X(n) for N(5,1) still as based on the sample we see most of the values are clustering around 8, we take it as a guess limit and try to check the convergence in probability of X(n).***
2) ***It is very much clear from the above graph that as n tends to infinity, P[|X(n)-8|<eps=0.05] does not tend to 1, rather between n=6000 to 10000 , it seems to tend to '0'.***
3) ***Hence X(n) does not converge in probability to '8'. We can also check by taking different guess limits but will find that the probability is not going to 1 as n tends to infinity.***

iii) N(5,2)-



Findings-

1) ***We have not found any proper guess limit for X(n) for N(5,2) still as based on the sample we see most of the values are clustering around '11' we take it as a guess limit and try to check the convergence in probability of X(n).***
2) ***It is very much clear from the above graph that as n tends to infinity, P[|X(n)-11|<eps=0.05] does not tend to 1, rather as n is between 6000 to 10000, it becomes small.***

3) ***Hence X(n) does not converge in probability to '11'. We can also check by taking different guess limits but will find that the probability is not going to 1 as n tends to infinity.***
4) ***As keeping the location parameter fixed variance is increased, the probability value in y axis has also decreased ,i.e., becomes closer to '0' .***

Conclusions-

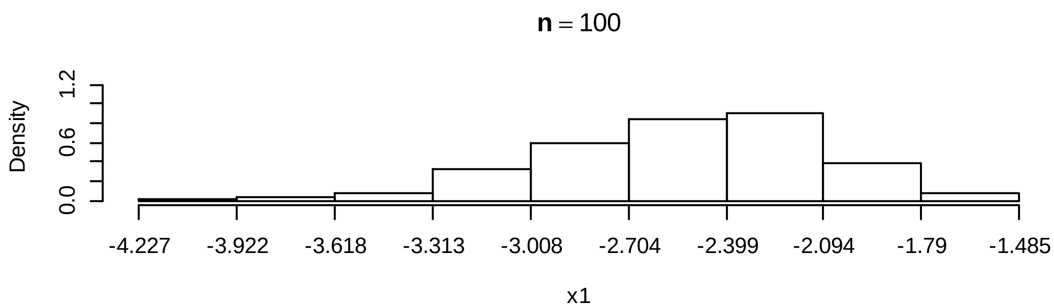
1) ***We have not found any guess limit for X(n) for whatever the location and scale parameter of the population be, so, by an eye estimate, we fix a number and check convergence in probability to that number for X(n).***
2) ***But we can see P[|X(n)-u|<eps=0.05] is not tending towards 1 as n tends to infinity, rather irrespective of location or scale parameter, for large n (say n= 6000 to 10000), it tends to '0', so we can say X(n) does not converge in probability to 'u',u belongs to real line.***
3) ***In other words we can say X(n) is not a consistent estimator of 'u'.***

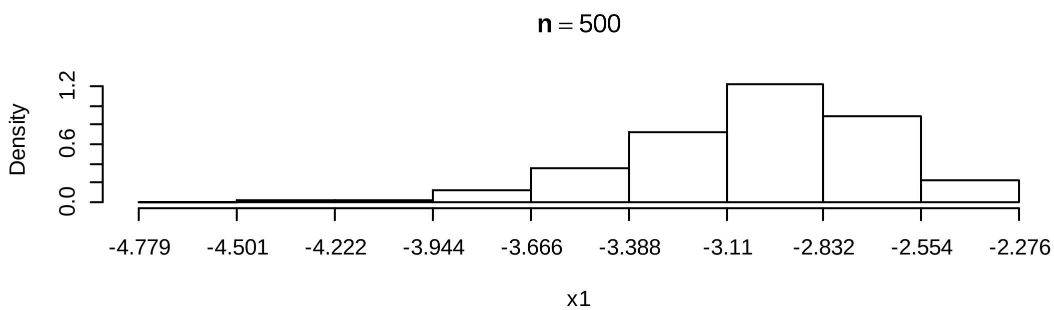
Here we will check convergence in Distribution for R=1000, n=100,500,1000 (R=no. of repetition, n=sample size)

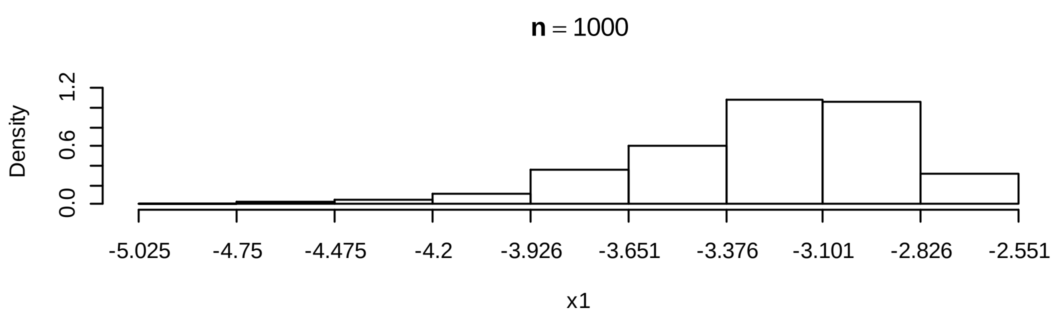
Here we will check the convergence in distribution for some well known statistics like sample mean , median , minimum , maximum , midrange etc for normal population with certain mean and variance and make a significant comparison between their large sample behaviour and their dependency on parameters for fixed repetition number R=1000 and n=100,500,1000 .

A. X(1)-

i) For N(0,1)



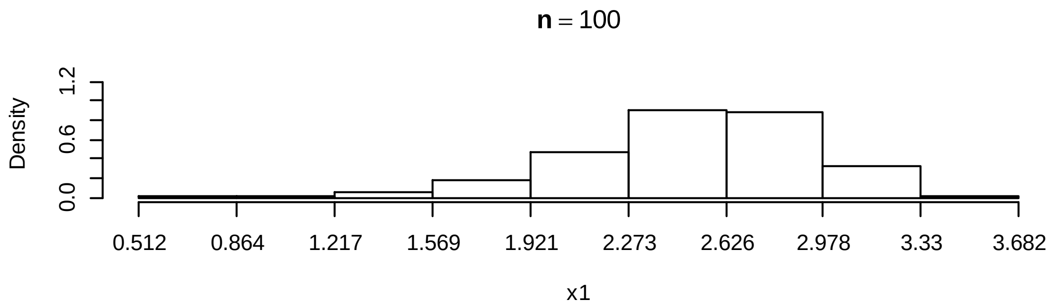


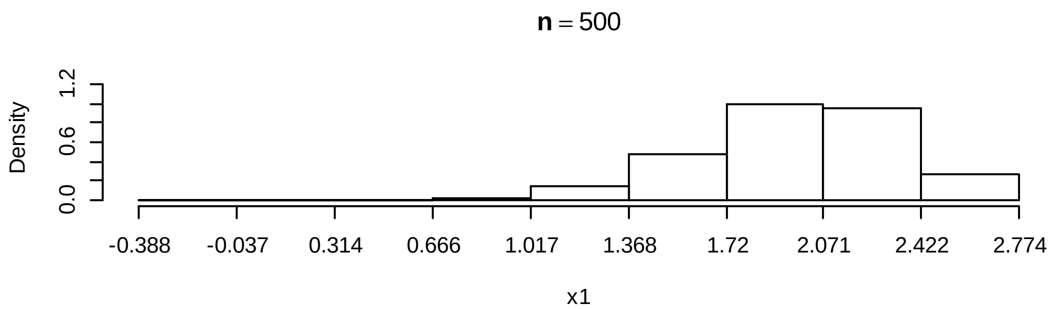


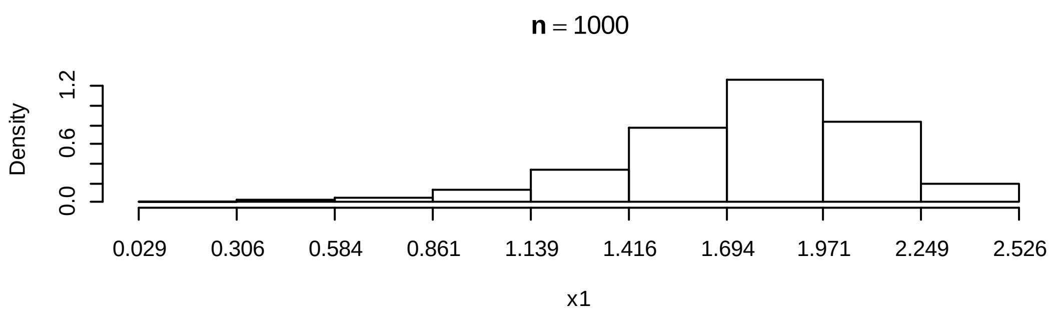
Observations-

1)For comparatively small sample size say n=100 the frequency density histogram does not seem to seem to be symmetric rather it is becoming negatively skewed.
2) As we increase the sample size to n=500 or 1000, then also the histogram is not becoming symmetric.
3) Failure to attain symmetric frequency density histogram indicates a deviation from Normal distribution for sample X(1) for large sample size.

ii) For N(5,1)







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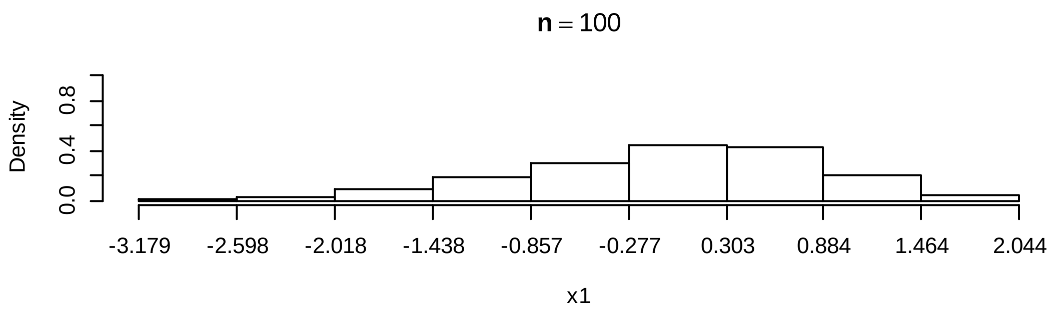
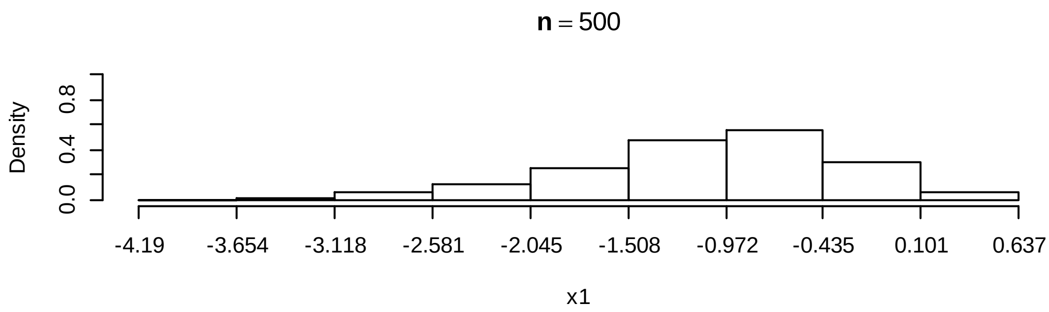
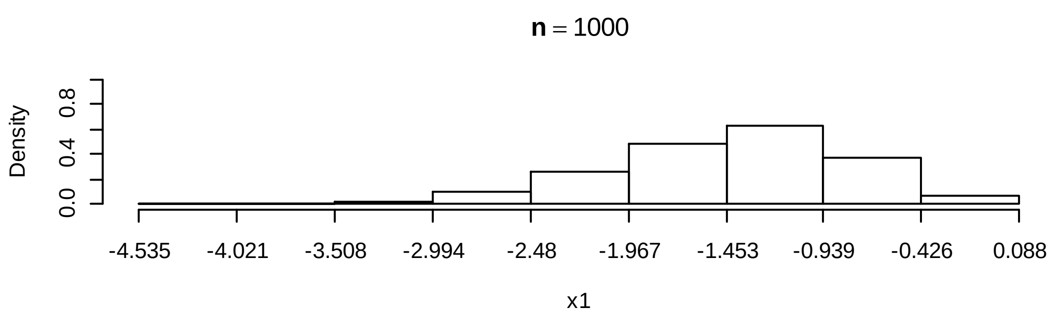
iii) N(5,2)-







Observations-

1) For comparatively small sample size say n=100 the frequency density histogram does not seem to seem to be symmetric rather it is becoming negatively skewed.
2) As we increase the sample size to n=500 or 1000, then also the histogram is not becoming symmetric. As we increase the sample size , the skewness is becoming more clear unlike the case of N(0,1) or N(5,1). As variance is increasing keeping location parameter fixed, the height of the histograms are decreasing.
3) Failure to attain symmetric frequency density histogram indicates a deviation from Normal distribution for sample X(1) for large sample size.

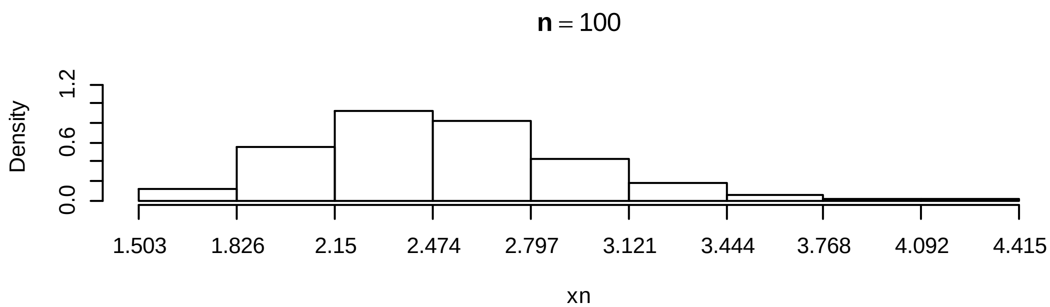
Conclusions-

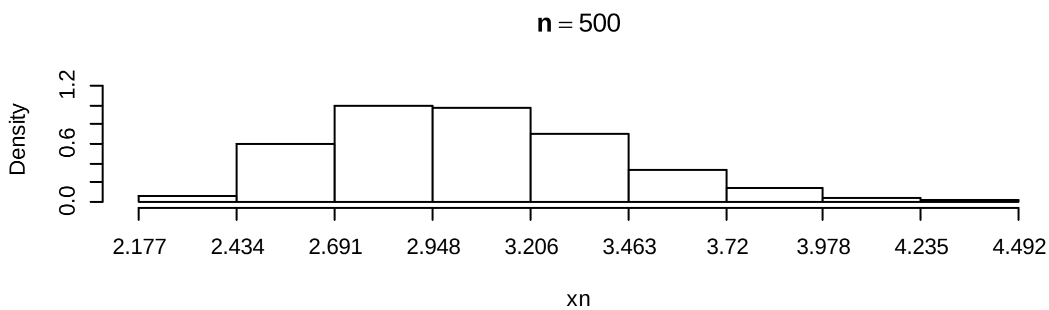
- ***For comparatively small sample size say n=100 the frequency density histogram does not seem to seem to be symmetric rather it is becoming negatively skewed.***
- ***As we increase the sample size to n=500 or 1000, then also the histogram is not becoming symmetric***

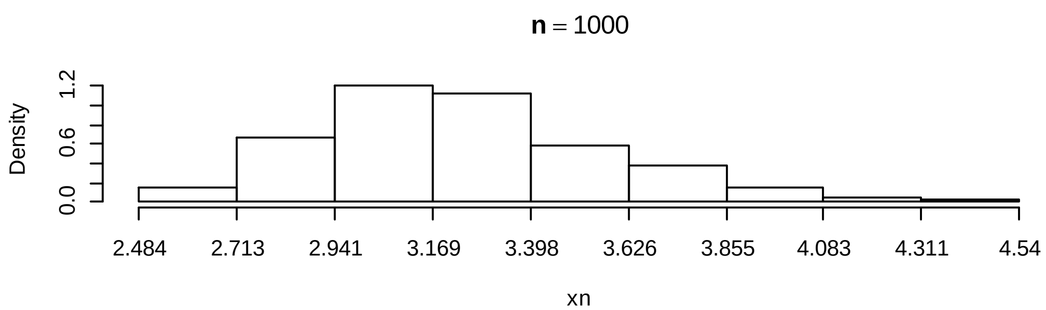
- ***Failure to attain symmetric frequency density histogram indicates a deviation from Normal distribution for sample X(1) for large sample size irrespective of the location or scale parameter of the Normal population.***
- ***Although location parameter has no such meaningful effect on asymptotic behaviour of X(1), scale parameter has slightly better effect on it for fixed repetition number R=1000.***

B) X(n)-

i) For N(0,1)





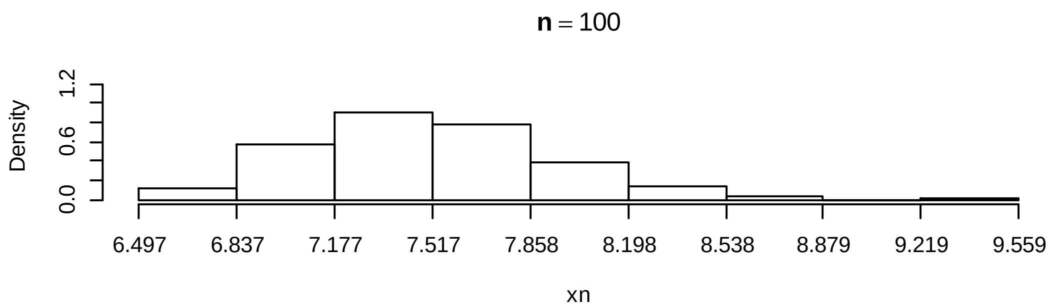


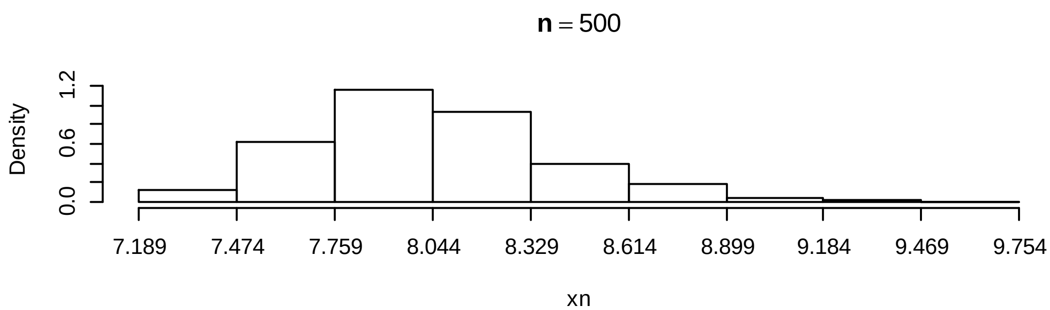
Observations-

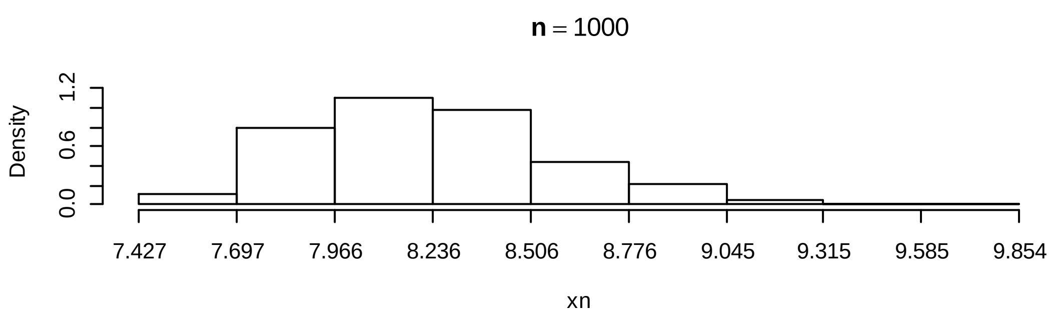
1) For comparatively small sample size say n=100 the frequency density histogram does not seem to seem to be symmetric rather it is becoming positively skewed.

2) *As we increase the sample size to n=500 or 1000, then also the histogram is not becoming symmetric.*
3) *Failure to attain symmetric frequency density histogram indicates a deviation from Normal distribution for sample X(n) for large sample size.*

ii) For N(5,1)-







Ob

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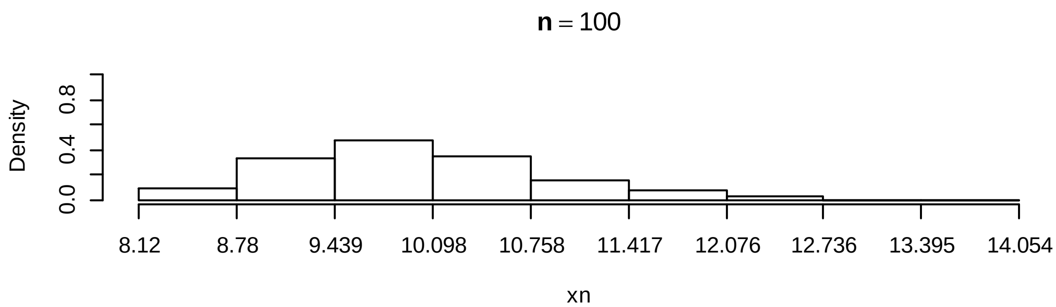
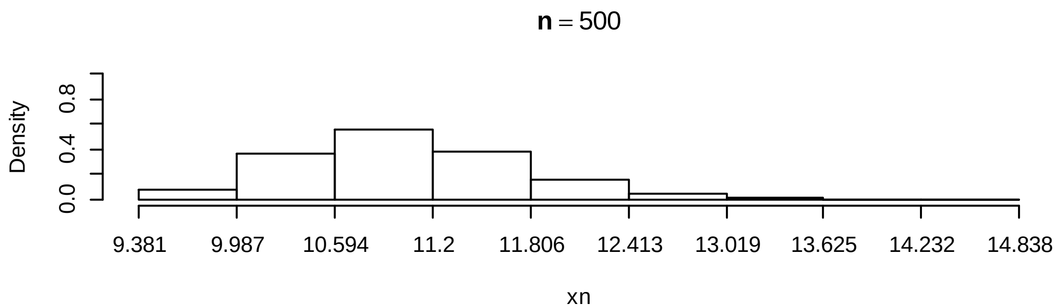
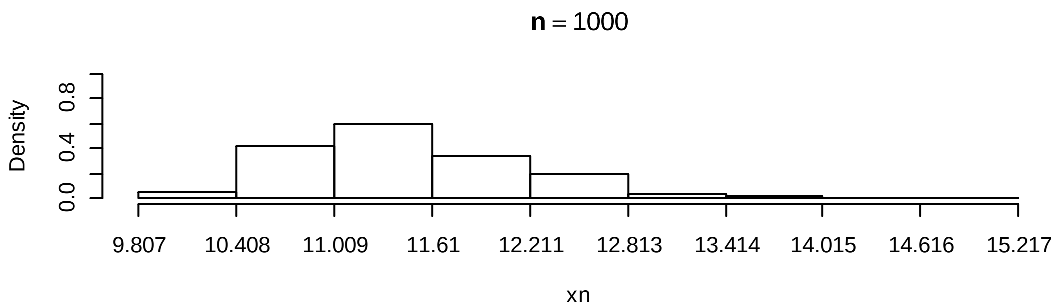
1) ***For comparatively small sample size say n=100 the frequency density histogram does not seem to seem to be symmetric rather it is becoming positively skewed.***

2) *As we increase the sample size to n=500 or 1000, then also the histogram is not becoming symmetric.*
3) *Failure to attain symmetric frequency density histogram indicates a deviation from Normal distribution for sample X(n) for large sample.*

iii) N(5,2)-







Observations-

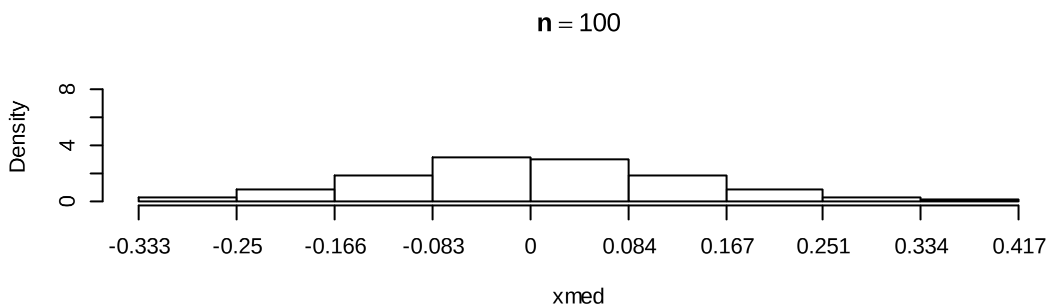
1) For comparatively small sample size say n=100 the frequency density histogram does not seem to seem to be symmetric rather it is becoming positively skewed.
2) As we increase the sample size to n=500 or 1000, then also the histogram is not becoming symmetric. As we increase the sample size , the skewness is becoming more clear unlike the case of N(0,1) or N(5,1). As variance is increasing , keeping location parameter fixed, the height of the histograms are decreasing.
3) Failure to attain symmetric frequency density histogram indicates a deviation from Normal distribution for sample X(n) for large sample size.

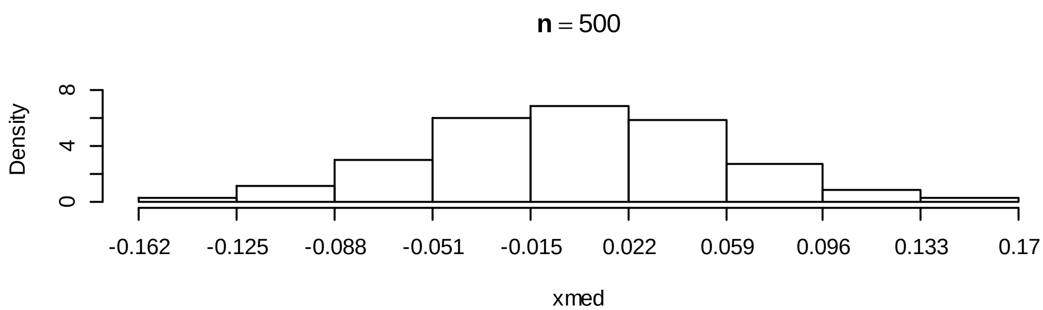
Conclusions-

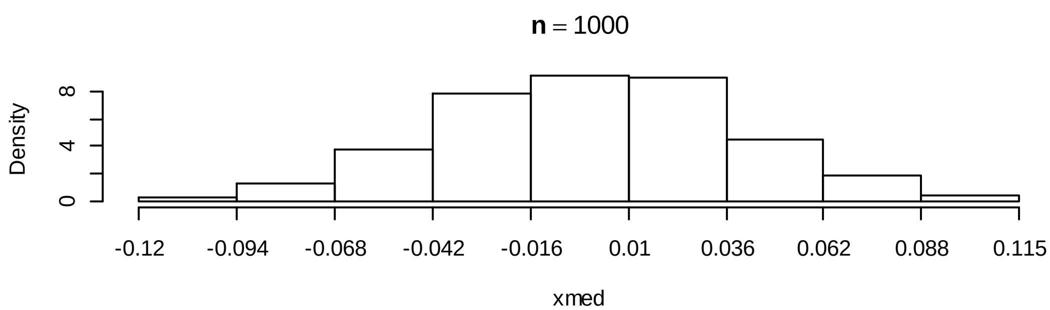
- ***For comparatively small sample size say n=100 the frequency density histogram does not seem to seem to be symmetric rather it is becoming positively skewed.***
- ***As we increase the sample size to n=500 or 1000, then also the histogram is not becoming symmetric***
- ***Failure to attain symmetric frequency density histogram indicates a deviation from Normal distribution for sample X(n) for large sample size irrespective of the location or scale parameter of the Normal population.***
- ***Although location parameter has no such meaningful effect on asymptotic behaviour of X(n), scale parameter has slightly better effect on it for fixed repetition number R=1000.***

C. MEDIAN-

i) For N(0,1)



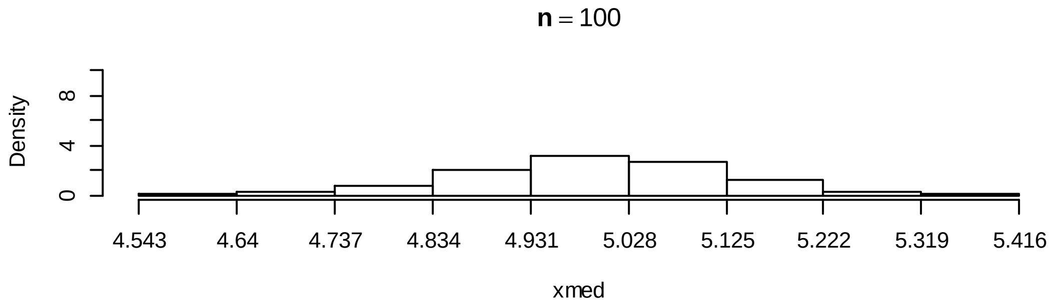


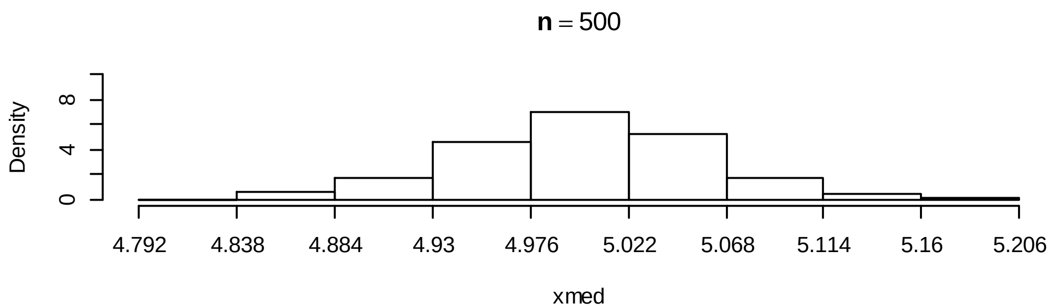


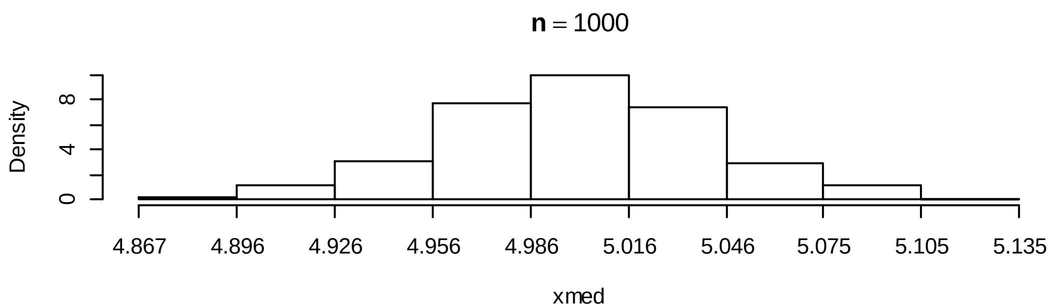
Observations-

1) ***For comparatively small sample size, n=100 , the frequency density histogram seems to be symmetric .***
2) ***As we increase the sample size(say n= 500 or 1000), the asymptotic symmetric nature of Xmed is becoming more obvious.***
3) ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xmed might have an asymptotic normal distribution.***
4) ***For fixed location and scale parameters of the normal population, the height of the histograms are increasing as we are increasing the sample size, indicating the asymptotic variance of Xmed is decreasing, i.e., it is inversely proportionate to sample size.***

ii) N(5,1)



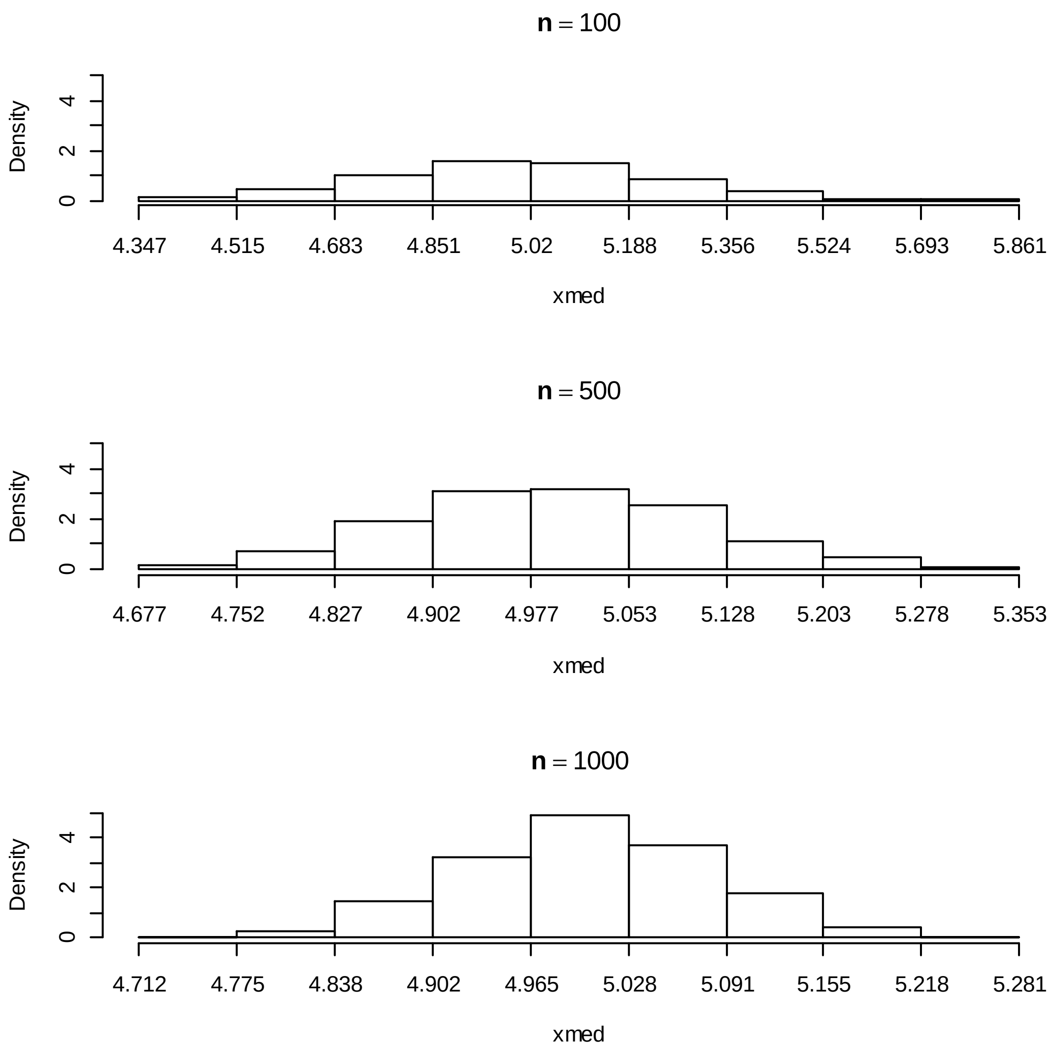




Observations-

1) ***For comparatively small n=100,the frequency density histogram becomes symmetric.***
2) ***As we increase the sample size(say n= 500 or 1000), the asymptotic symmetric nature of Xmed is becoming more obvious.***
3) ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xmed might have an asymptotic normal distribution.***
4) ***For fixed location and scale parameters of the normal population, the height of the histograms are increasing as we are increasing the sample size, indicating the asymptotic variance of Xmed is decreasing, i.e., it is inversely proportionate to sample size.***

iii) N(5,2)-



Observations-

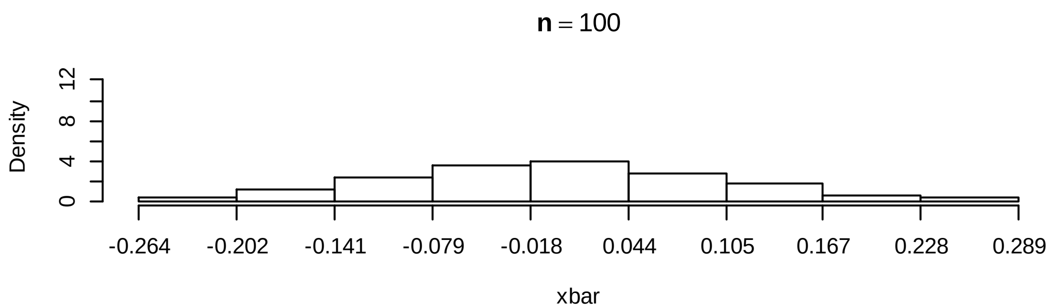
1) ***For comparatively small n=100,the frequency density histogram becomes symmetric***
2) ***As we increase the sample size(say n= 500 or 1000), the asymptotic symmetric nature of Xmed is becoming more obvious.***
3) ***As variance is increased, keeping the location fixed, height of the histogram is decreasing proportionately.***
4) ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xmed might have an asymptotic normal distribution.***
5) ***For fixed location and scale parameters of the normal population, the height of the histograms are increasing as we are increasing the sample size, indicating the asymptotic variance of Xmed is decreasing, i.e., it is inversely proportionate to sample size.***

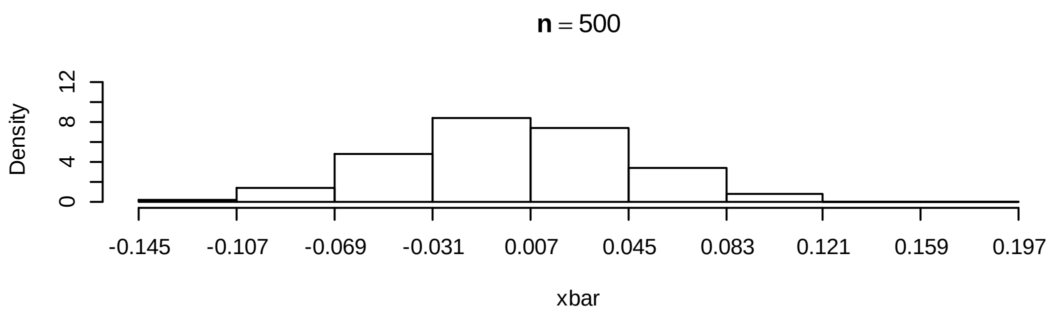
Conclusions-

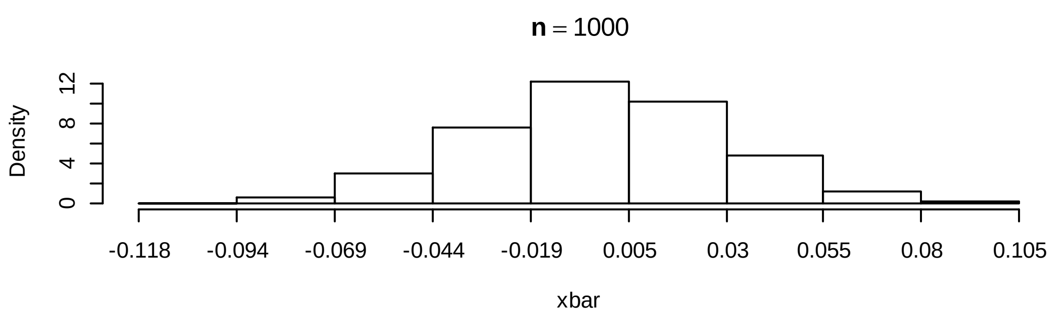
- ***For comparatively small n(say, n=100), the frequency densiy histogram becomes symmetric, as we increase the sample size, keeping the repetition constant, the symmetric behaviour of Xmed becomes more clear.***
- ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xmed might have an asymptotic normal distribution.***
- ***The scale parameter has more effect than location parameter on the height of the histograms.***
- ***For fixed location and scale parameters of the normal population, the height of the histograms are increasing as we are increasing the sample size, indicating the asymptotic variance of Xmed is decreasing, i.e., it is inversely proportionate to sample size.***

D. MEAN-

i) N(0,1)



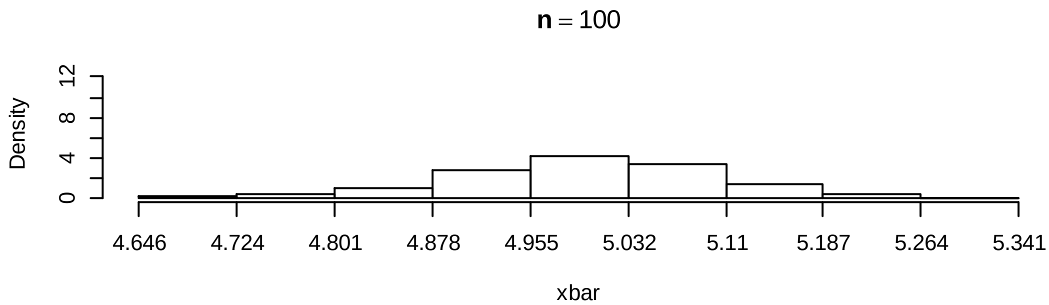


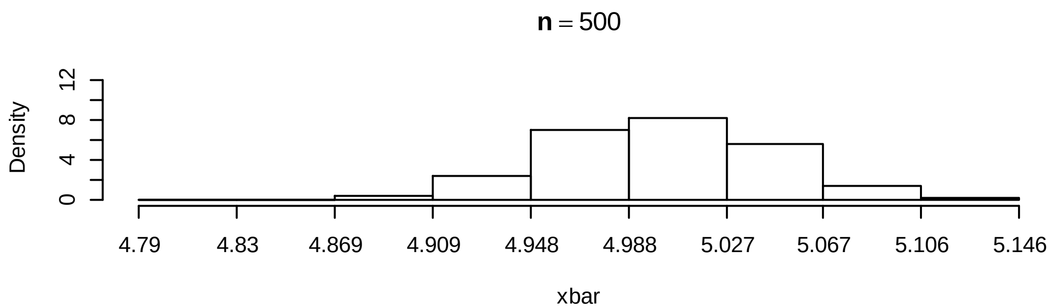


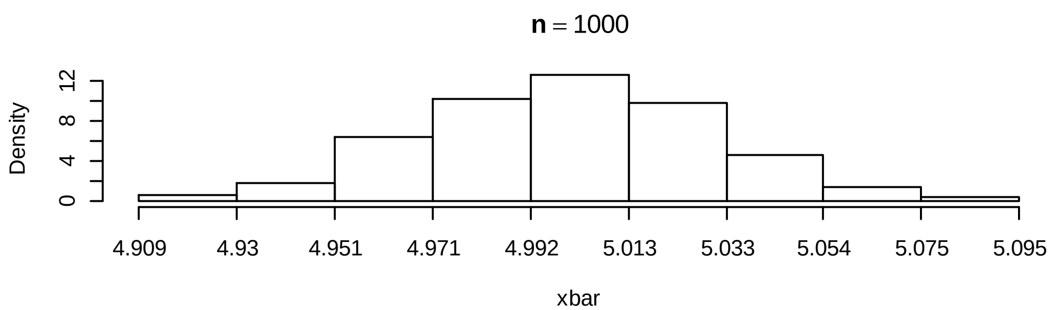
Observations-

- ***For comparatively small sample size, n=100 , the frequency density histogram seems to be symmetric .***
- ***As we increase the sample size(say n= 500 or 1000), the asymptotic symmetric nature of Xbar is becoming more obvious.***
- ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xbar have an asymptotic normal distribution.***
- ***For fixed location and scale parameter, as we increase the sample size, the height of the histograms are increasing, indicating the asymptotic variance of Xbar is decreasing, i.e, asymptotic variance of Xbar might be inversely proportionate to sample size n.***

ii) N(5,1)



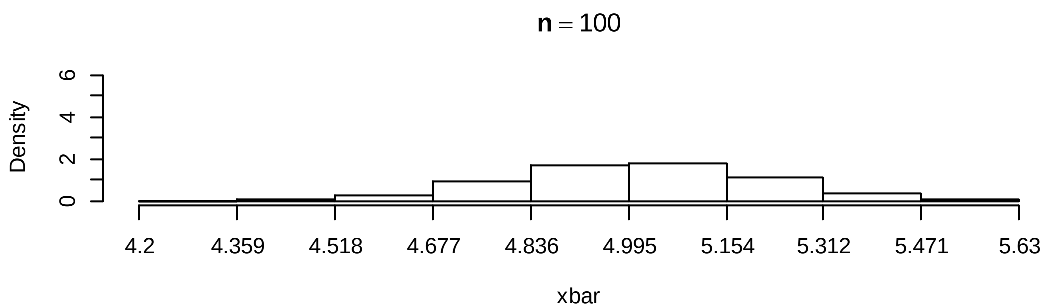


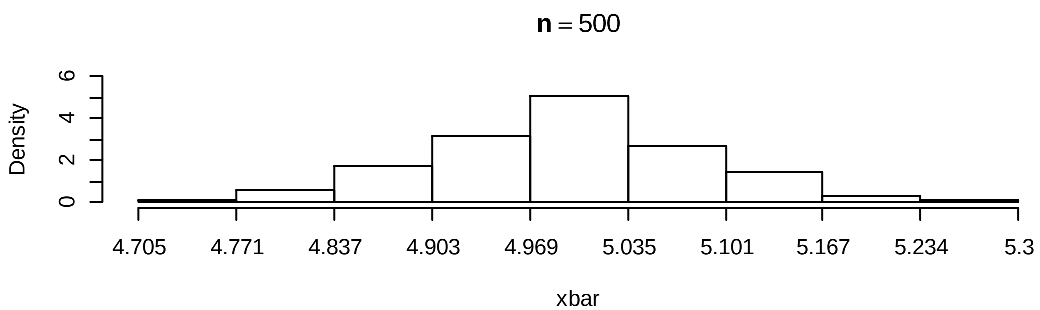


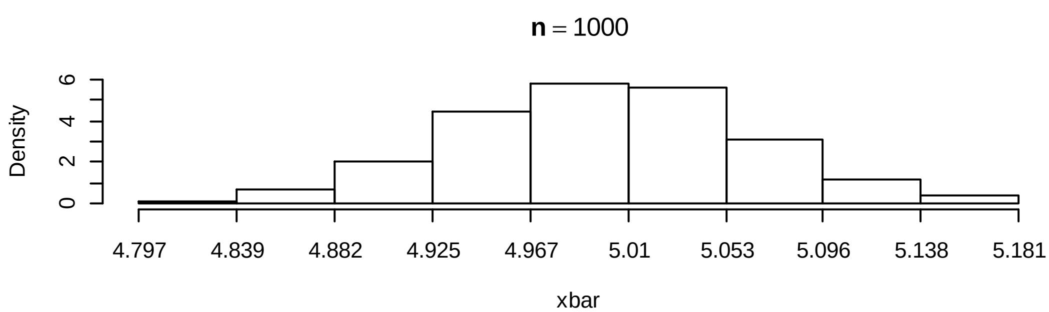
Observations-

- ***For comparatively small sample size, n=100 , the frequency density histogram seems to be symmetric .***
- ***As we increase the sample size(say n= 500 or 1000), the asymptotic symmetric nature of Xbar is becoming more obvious.***
- ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xbar have an asymptotic normal distribution irrespective of the parameter of parent population.***
- ***For fixed location and scale parameter, as we increase the sample size, the height of the histograms are increasing , indicating the asymptotic variance of Xbar is decreasing, i.e, asymptotic variance of Xbar might be inversely proportionate to sample size n.***

iii) N(5,2)-







Observations-

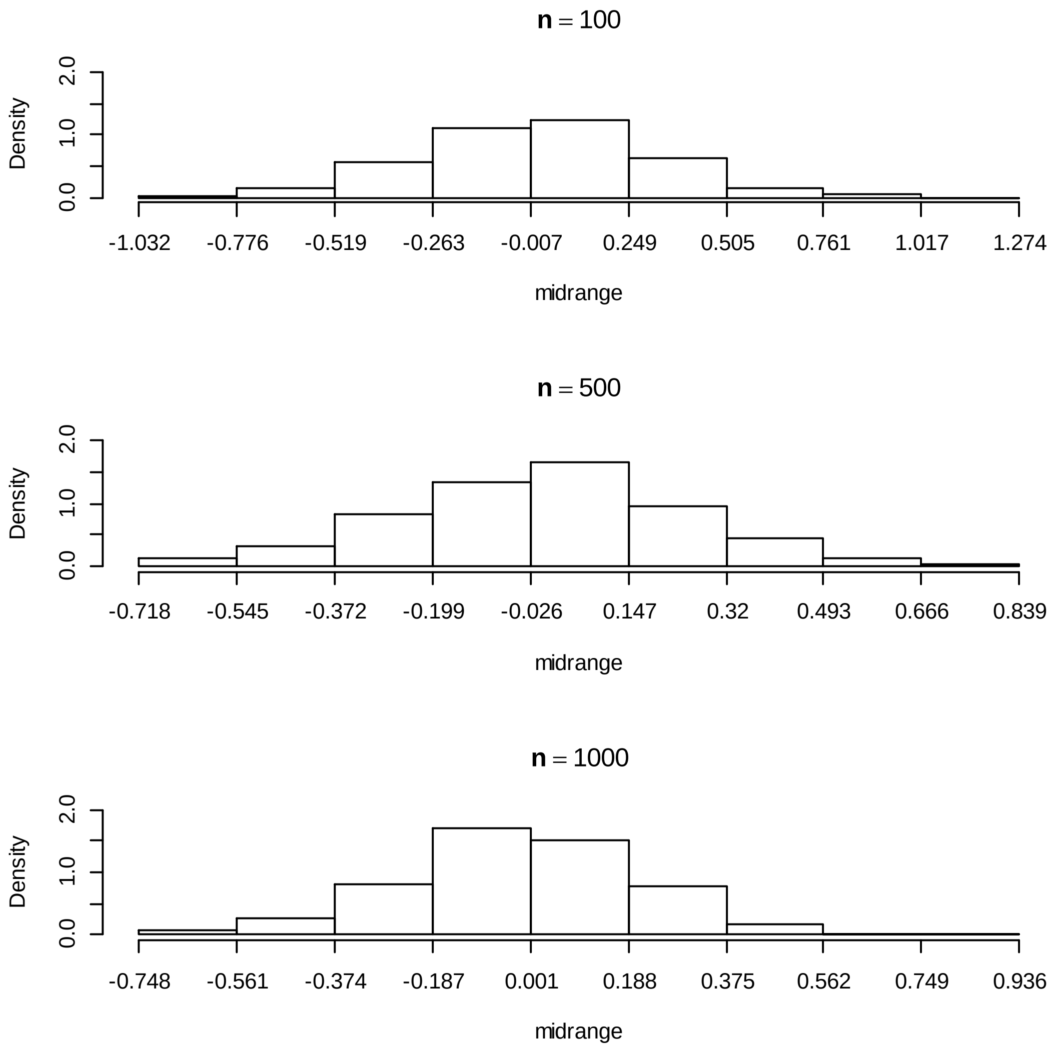
- ***For comparatively small n=100,the frequency density histogram becomes symmetric***
- ***As we increase the sample size(say n= 500 or 1000), the asymptotic symmetric nature of Xbar is becoming more obvious.***
- ***As variance is increased, keeping the location fixed, height of the histogram is decreasing proportionately.***
- ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xbar might have an asymptotic normal distribution.***
- ***For fixed location and scale parameters of the normal population, the height of the histograms are increasing as we are increasing the sample size, indicating the asymptotic variance of Xbar is decreasing ,i.e., asymptotic variance of Xbar is inversely proportionate to sample size n.***

Conclusions-

- ***For comparatively small n(say, n=100), the frequency densiy histogram becomes symmetric, as we increase the sample size, keeping the repetition constant, the symmetric behaviour of Xbar becomes more clear.***
- ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xbar might have an asymptotic normal distribution irrespective of the parameters of the parent population.***
- ***The scale parameter has more effect than location parameter on the height of the histograms.***
- ***For fixed location and scale parameters of the normal population, the height of the histograms are increasing as we are increasing the sample size, indicating the asymptotic variance of Xbar is decreasing, i.e., it is inversely proportionate to sample size n.***
- ***On an average when R is fixed at 1000, for any n, irrespective of location and scale parameter, the height of the density histogram is lesser for Xmed than Xbar indicating V(Xmed)>V(Xbar) as n tends to infinity.***

E. MIDRANGE-

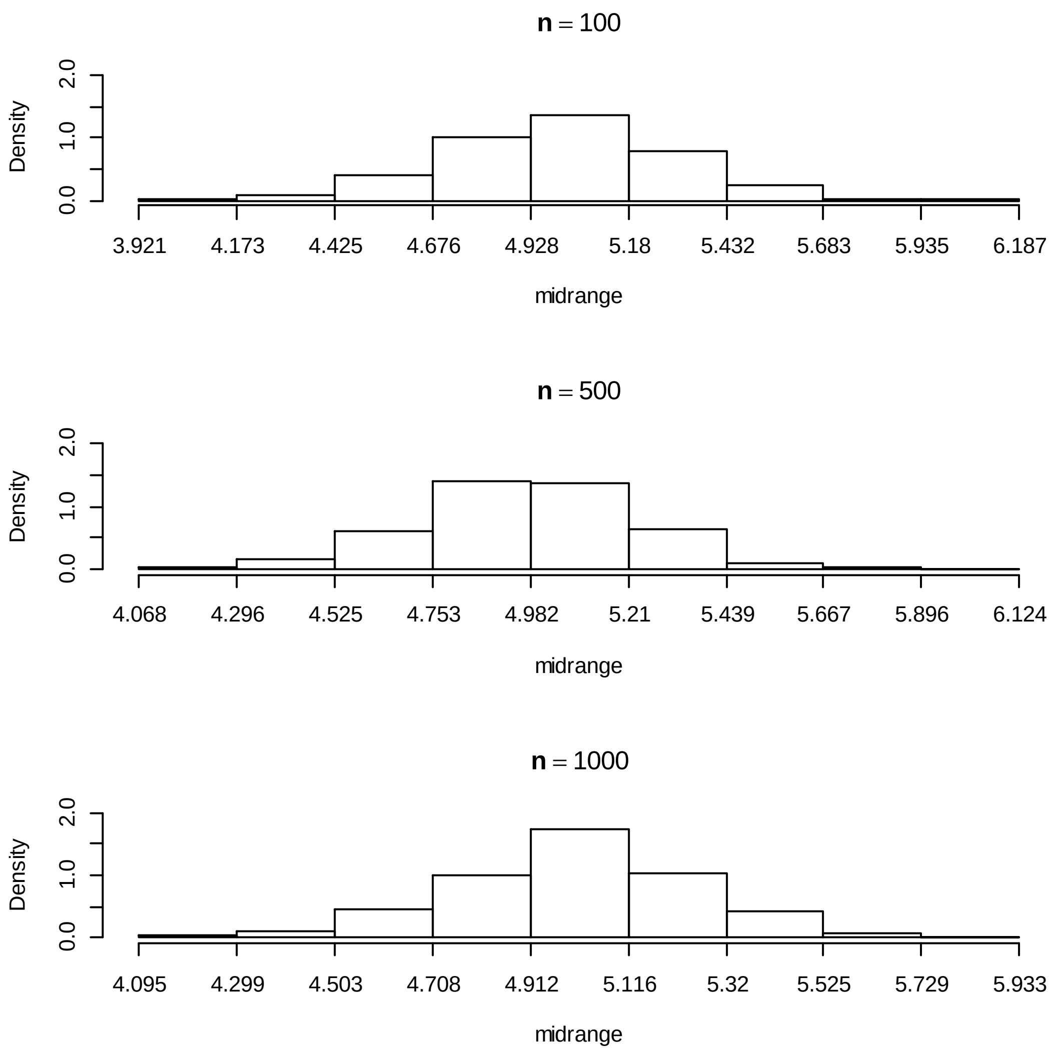
i) N(0,1)-



Observations-

- ***For comparatively small sample size, n=100 , the frequency density histogram seems to be symmetric .***
- ***As we increase the sample size(say n= 500 or 1000), the asymptotic symmetric nature of midrange is becoming more obvious.***
- ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample midrange has an asymptotic normal distribution.***
- ***For fixed location and scale parameter, as we increase the sample size, the height of the histograms are more or less same indicating the asymptotic variance of midrange remains almost constant.***

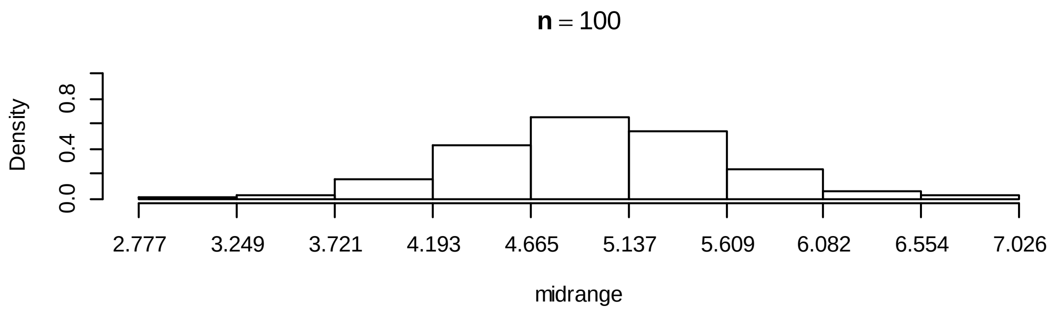
ii) N(5,1)

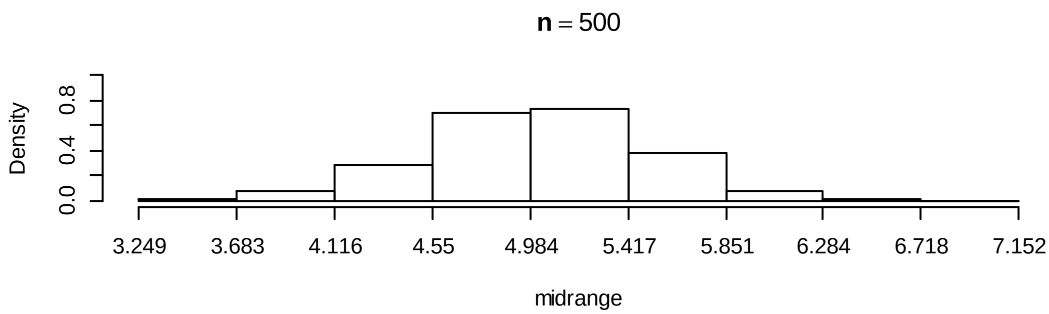


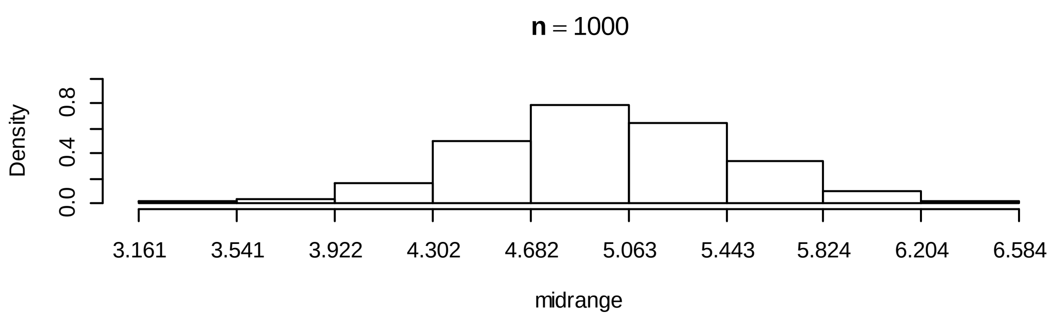
Observations-

- ***For comparatively small sample size, n=100 , the frequency density histogram seems to be symmetric .***
- ***As we increase the sample size(say n= 500 or 1000), the asymptotic symmetric nature of midrange is becoming more obvious.***
- ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample midrange has an asymptotic normal distribution.***
- ***For fixed location and scale parameter, as we increase the sample size, the height of the histograms are slightly increasing or more or less same, indicating the asymptotic variance of midrange is decreasing, i.e. , it might be inversely proportionate to sample size n, but here the decrement asymptotic variance is lesser than Xbar or Xmed as changing height of histograms with increase in n is not so significant.***

iii) N(5,2)-







Observations-

- ***For comparatively small sample size, n=100 , the frequency density histogram seems to be symmetric .***
- ***As we increase the sample size(say n= 500 or 1000), the asymptotic symmetric nature of midrange is becoming more obvious.***
- ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample midrange has an asymptotic normal distribution.***
- ***For fixed location and scale parameter, as we increase the sample size, the height of the histograms are slightly increasing or more or less constant, , indicating the asymptotic variance of midrange is decreasing, i.e. , it might be inversely proportionate to sample size n, but here the decrement***

asymptotic variance is lesser than Xbar or Xmed as changing height of histograms with increase in n is not so significant.

Conclusion-

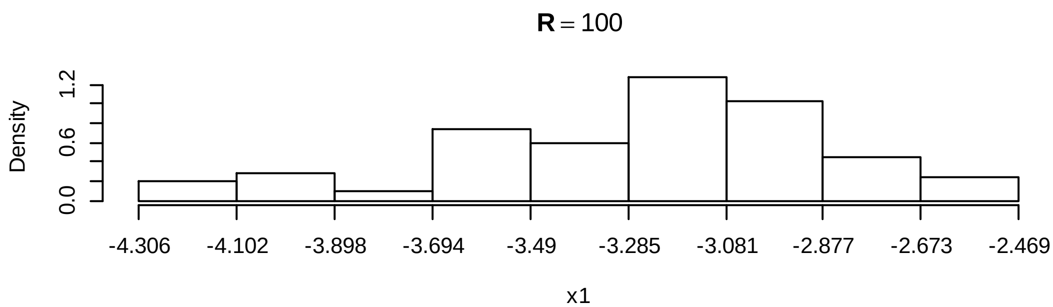
- ***For comparatively small n(say, n=100), the frequency densiy histogram becomes symmetric, as we increase the sample size, keeping the repetition constant, the symmetric behaviour of midrange becomes more clear.***
- ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample midrange might have an asymptotic normal distribution irrespective of the parameters of the parent population***
- ***The scale parameter has more effect than location parameter on the height of the histograms.***
- ***For fixed location and scale parameters of the normal population, the height of the histograms are more or less same, indicating the asymptotic variance of midrange remains almost constant***
- ***When R is fixed at 1000, for any n=100,500 or 1000, irrespective of the location and scale parameter, the height of the density histogram is in the order H(midrange)<H(median)<H(mean),where H(statistic) denotes height of the histogram of the statistic for a particular x value. Hence V(midrange)>V(Xmed)>V(Xbar).***

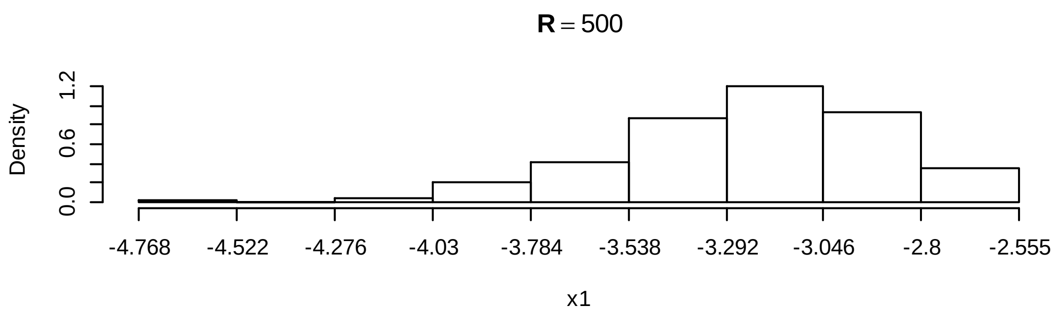
Now we will check convergence in distribution for n=1000,R=100,500,1000.

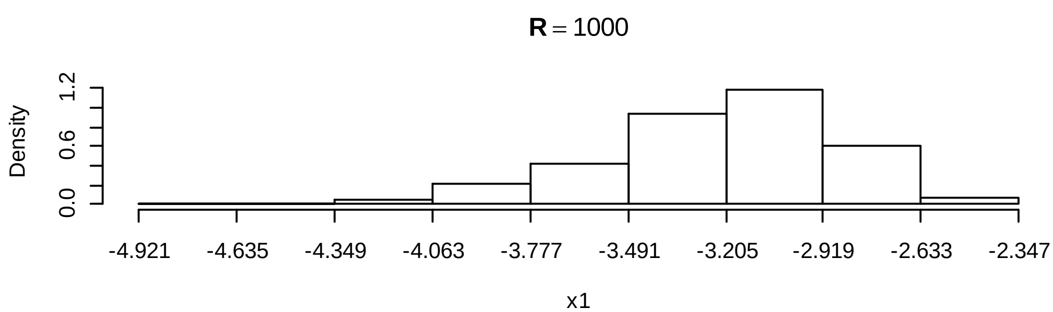
Here we will check the convergence in distribution for some well known statistics like sample mean , median , minimum , maximum , midrange etc for normal population with certain mean and variance and make a significant comparison between their large sample behaviour and their dependency on parameters for fixed sample size n=1000 and repetition numbers R=100,500,1000.

A. X(1)-

i) N(0,1)-



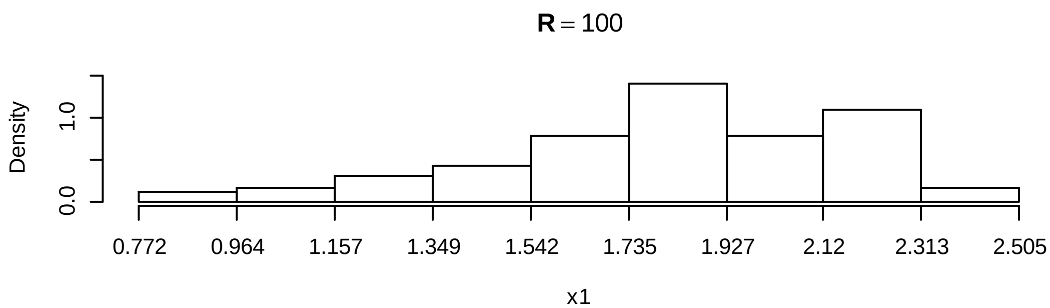


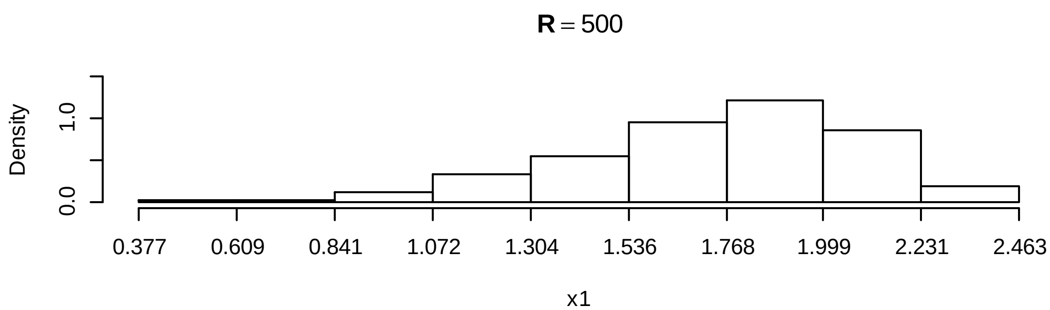


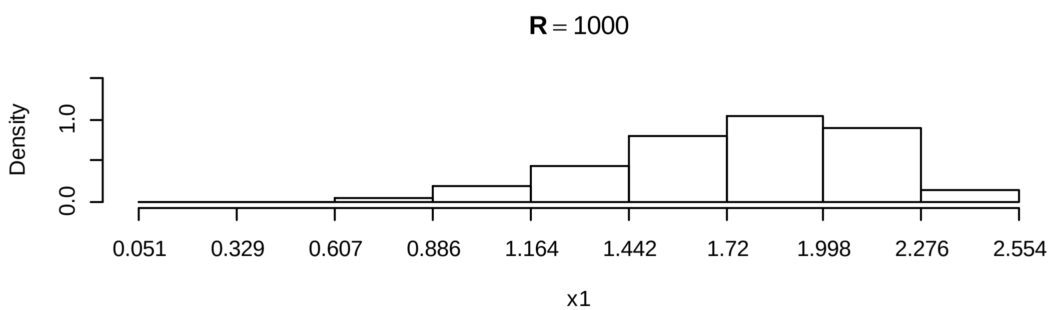
Observations-

1) ***For relatively small repetition number (say, R=100), the frequency density histogram is more or less negatively skewed but the skewness is not as significant as relatively small sample size (say n=100).***
2) ***As the no. of repetition is increased say R=500 or 1000, the skewness becomes clearer***
3) ***Deviation from symmetric nature indicates a clear deviation from asymptotic normal distribution of sample X(1) for fixed large sample size, n=1000.***

ii) N(5,1)



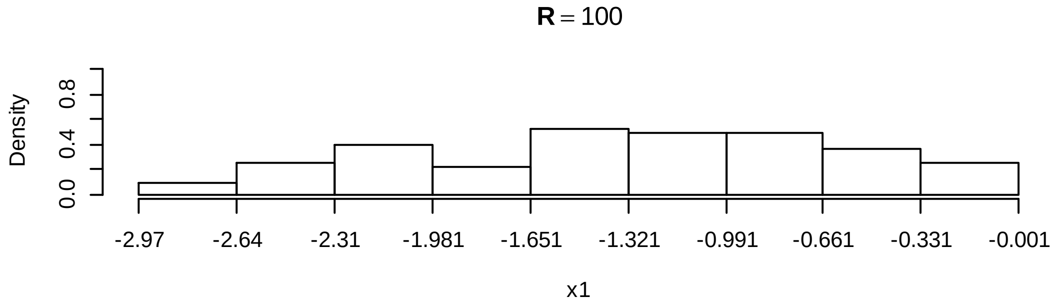


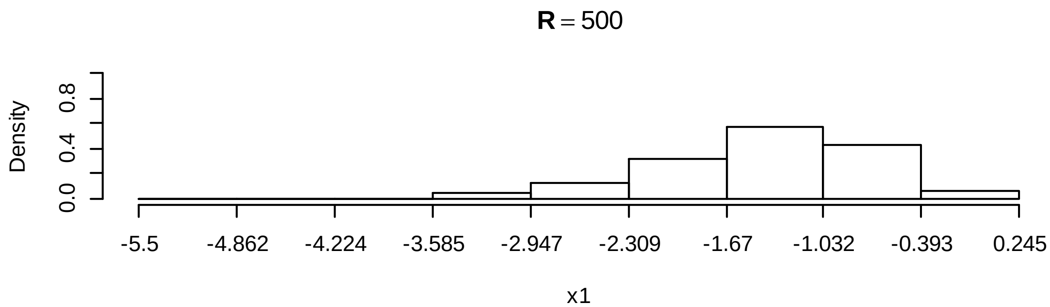


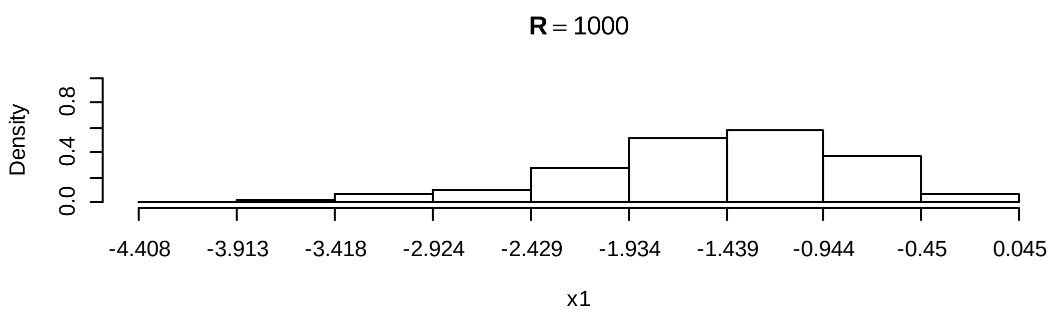
Observations-

1) ***For relatively small repetition number (say, R=100), the frequency density histogram is more or less negatively skewed but the skewness is not as significant as relatively small sample size (say n=100).***
2) ***As the no. of repetition is increased say R=500 or 1000, the skewness becomes clearer.***
3) ***Deviation from symmetric nature indicates a clear deviation from asymptotic normal distribution of sample X(1) for fixed large sample size, n=1000.***

iii) N(5,2)-







Observations-

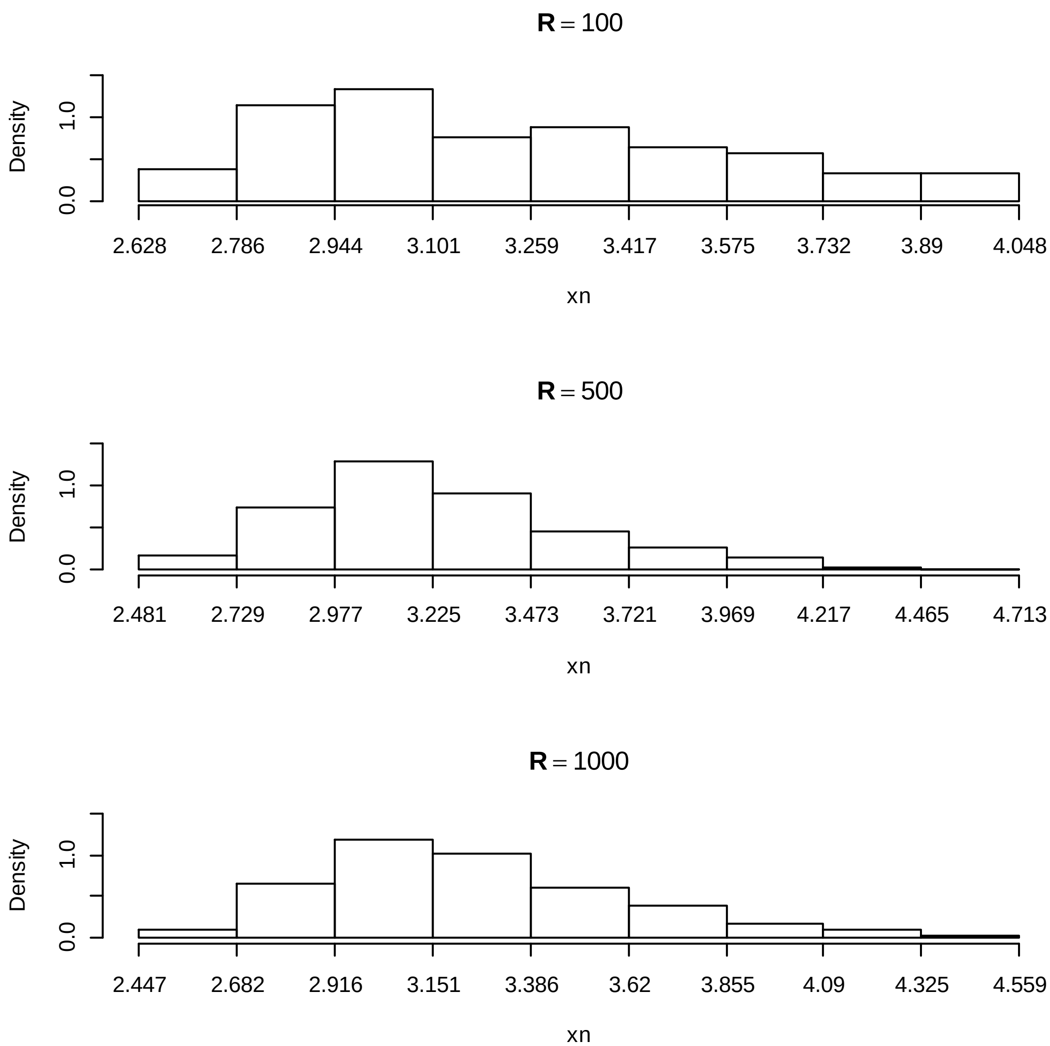
1) ***For relatively small repetition number (say R=100), the frequency density histogram becomes more or less symmetric ,from here one might wrongly interpret about asymptotic normal behaviour of X(1).***
2) ***Keeping the location parameter fixed as we have increased the variance, as a result, the height of the histograms decreases.***
3) ***As we increase the repetition number (say R=500,1000), the frequency density histogram becomes negatively skewed.***
4) ***Deviation from symmetric nature of the histograms is a clear indication of deviation from asymptotic normal distribution of sample X(1) for fixed large n=1000.***

Conclusion-

1) ***For relatively small repetition number (say, R=100), the frequency density histogram is more or less negatively skewed but the skewness is not as significant as relatively small sample size (say n=100) and fixed repetition number (say R=1000 in previous case).***
2) ***As the no. of repetition is increased say R=500 or 1000, the skewness becomes clearer, irrespective of the location and scale parameter of normal population.***
3) ***Deviation from symmetric nature of the histograms is a clear indication of deviation from asymptotic normal distribution of sample X(1) for fixed large n=1000.***
4) ***Although the location parameter has no such significant effect on the asymptotic behaviour of X(1), the scale parameter seems to have remarkable effect on it, as, with the increment of variance, for small repetition number, (say R=100), the histogram becomes more or less symmetric.***

B. X(n)-

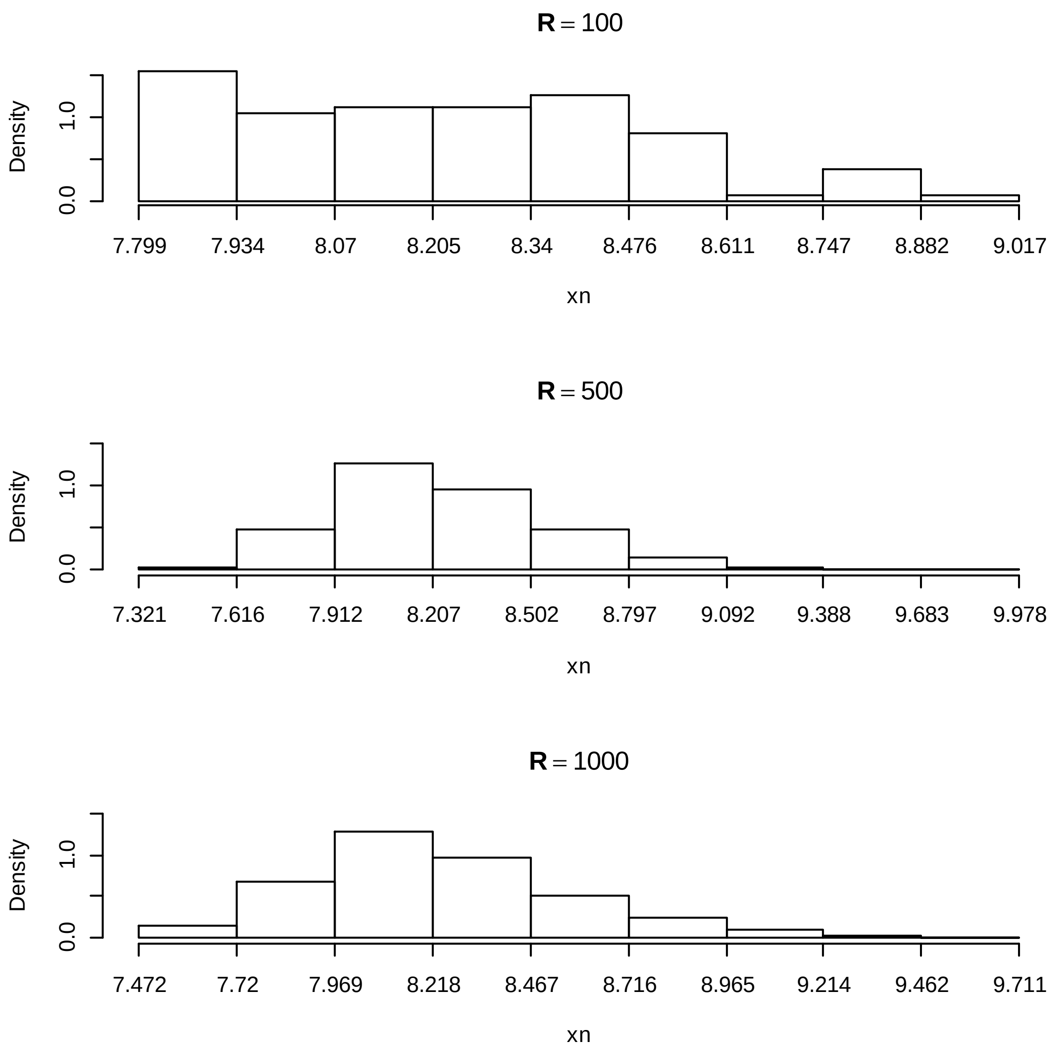
i) N(0,1)-



Observations-

1) ***For relatively small repetition number (say, R=100), the frequency density histogram is more or less positively skewed but the skewness is not as significant as relatively small sample size (say n=100).***
2) ***As the no. of repetition is increased say R=500 or 1000, the skewness becomes clearer.***
3) ***Deviation from symmetric nature indicates a clear deviation from asymptotic normal distribution of sample X(n) for fixed large sample size, n=1000***

ii) N(5,1)-



Observations-

1) ***For relatively small repetition number (say, R=100), the frequency density histogram is more or less positively skewed but the skewness is not as significant as relatively small sample size (say n=100).***
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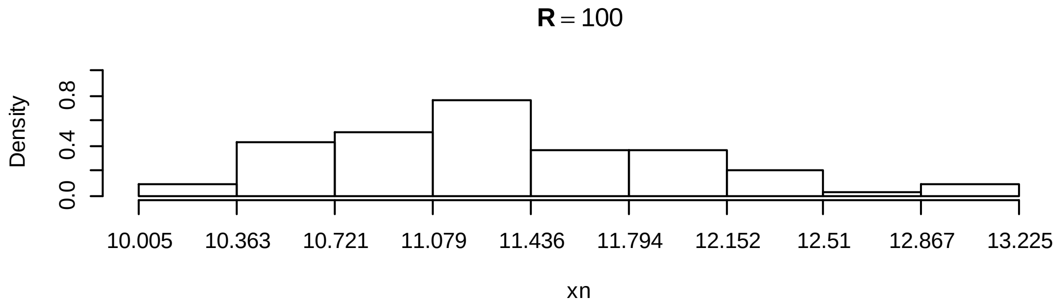
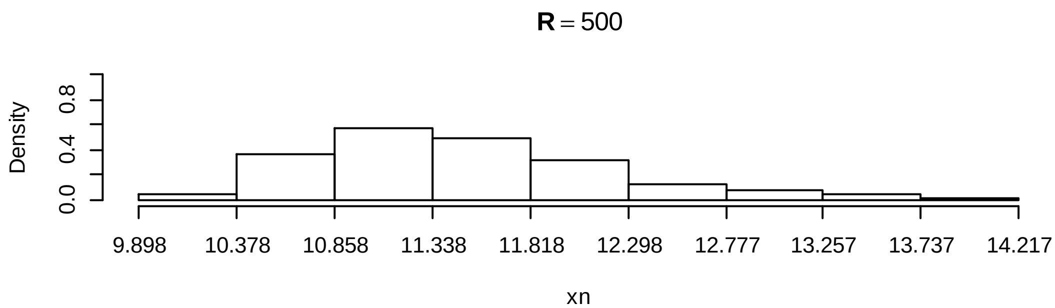
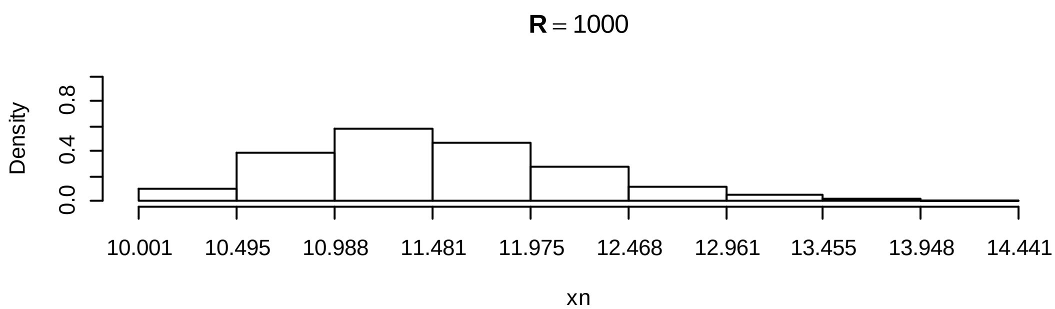
iii) N(5,2)-







Observations-

1) ***For relatively small repetition number (say R=100), the frequency density histogram becomes more or less positively skewed for sample X(n),unlike sample X(1).***
2) ***Keeping the location parameter fixed as we have increased the variance, as a result, the height of the histograms decreases.***
3) ***As we increase the repetition number (say R=500,1000), the frequency density histogram becomes positively skewed.***
4) ***Deviation from symmetric nature of the histograms is a clear indication of deviation from asymptotic normal distribution of sample X(n) for fixed large n=1000.***

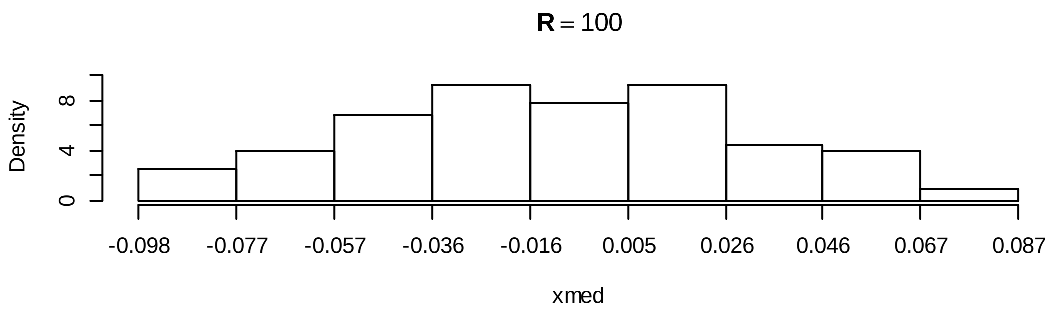
Conclusions-

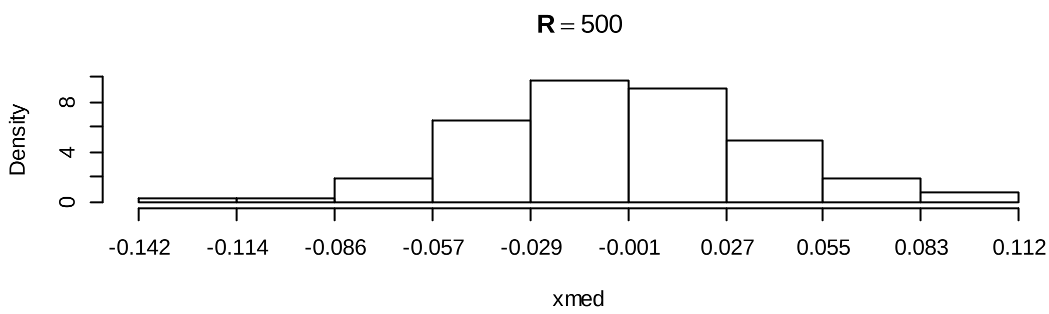
- ***For relatively small repetition number (say, R=100), the frequency density histogram is more or less negatively skewed but the skewness is not as significant as relatively small sample size (say n=100) and fixed repetition number (say R=1000 in previous case).***
- ***As the no. of repetition is increased say R=500 or 1000, the skewness becomes clearer, irrespective of the location and scale parameter of normal population.***
- ***Deviation from symmetric nature of the histograms is a clear indication of deviation from asymptotic normal distribution of sample X(n) for fixed large n=1000.***

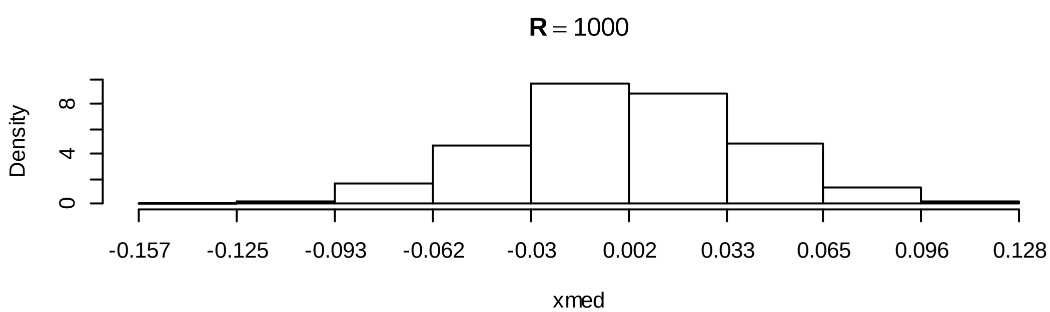
- ***Although the location parameter has no such significant effect on the asymptotic behaviour of X(n), the scale parameter seems to have remarkable effect on it, as, with the increment of variance, results in decrement of height of the histograms.***

C. MEDIAN-

i) N(0,1)-



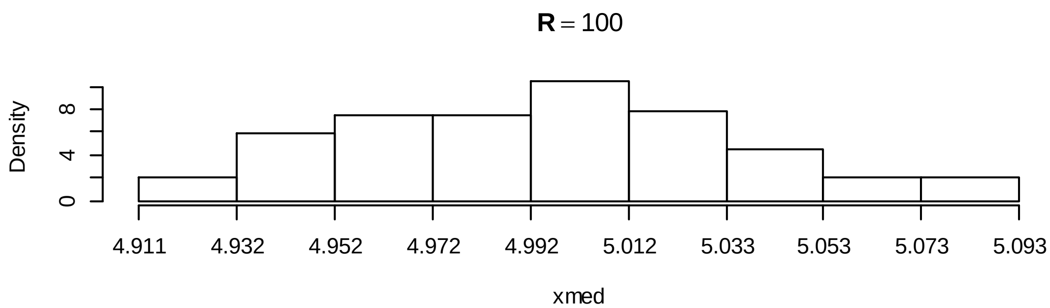


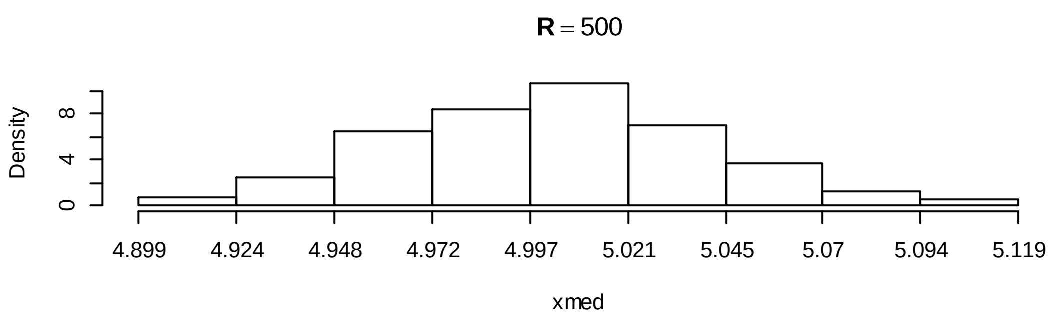


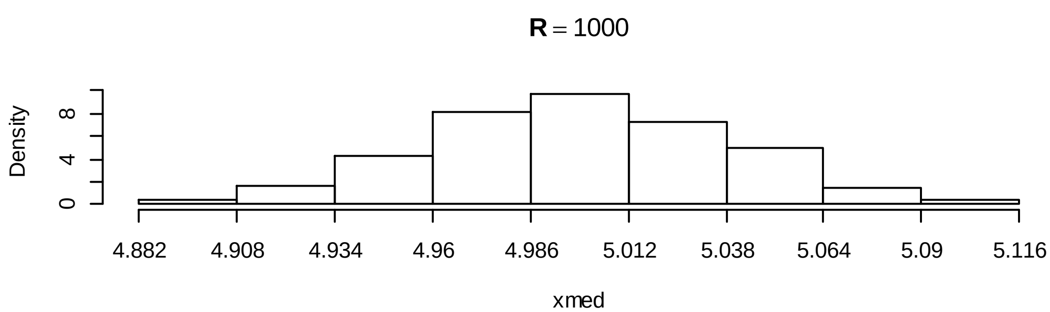
Observations-

1) ***For relatively small repetition number (say R=100), the frequency density histogram is more or less symmetric.***
2) ***As the number of repetition is increased,(R=500 or 1000), keeping the sample size fixed(n=1000), the symmetric nature is becoming more obvious.***
3) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample median Xmed.***
4) ***Clearly the height of the histogram remains almost same for any repetition number (R=100, 500, 1000)and fixed n(say n=1000), for any normal population with fixed location and scale parameter. This indicates that asymptotic variance of Xmed is uncorrelated with repetition R (unlike sample size n).***

ii) N(5,1)-







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1) ***For relatively small repetition number (say R=100), the frequency density histogram is more or less symmetric.***
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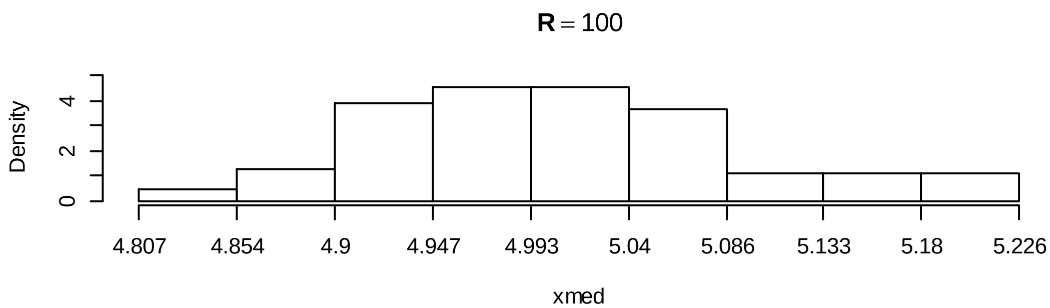
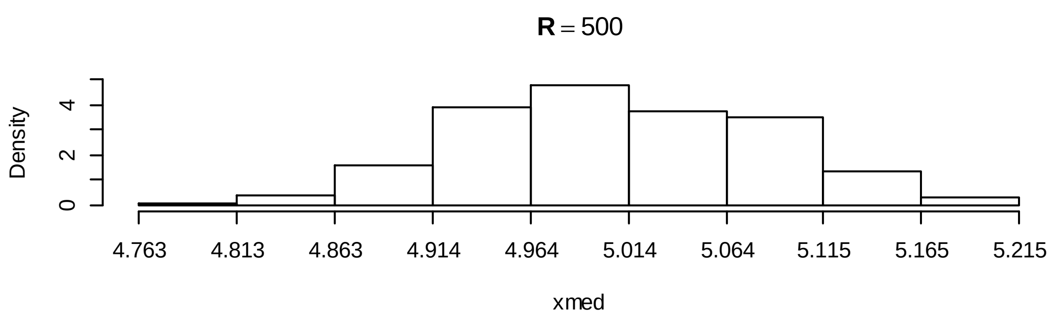
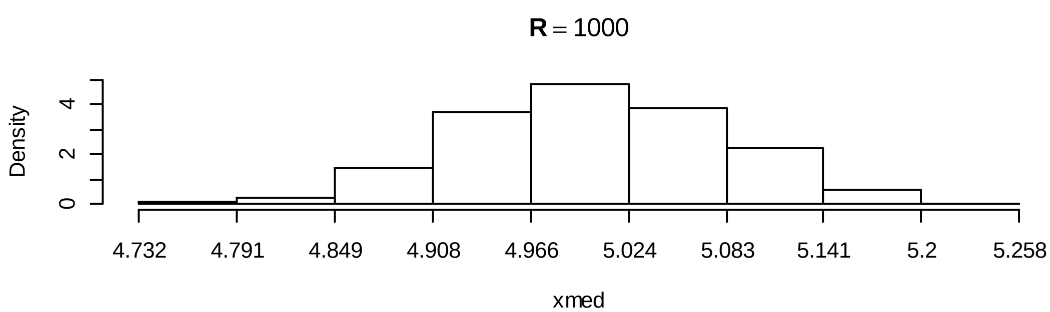
iii) N(5,2)-







Observations-

1) ***For relatively small repetition number (say R=100), the frequency density histogram is more or less symmetric.***
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4) ***Clearly the height of the histogram remains almost same for any repetition number (R=100, 500, 1000)and fixed n(say n=1000), for any normal population with fixed location and scale parameter. This indicates that asymptotic variance of Xmed is uncorrelated with repetition R (unlike sample size n)***

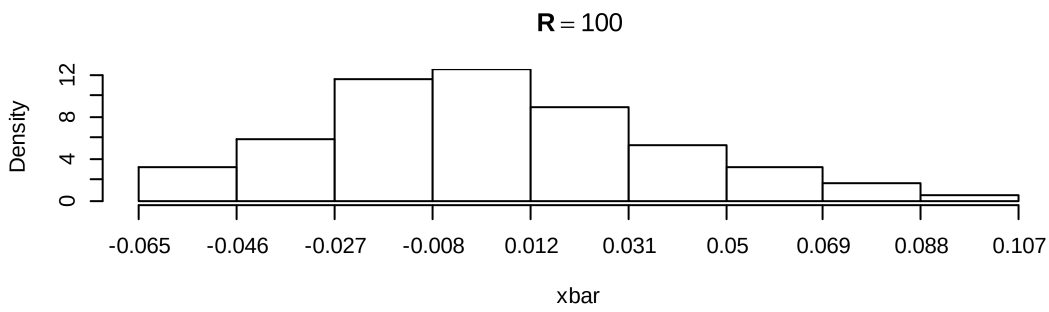
5) ***Keeping the location parameter fixed in '5' as variance is increased by '1' unit, clearly the height of the histograms is decreased.***

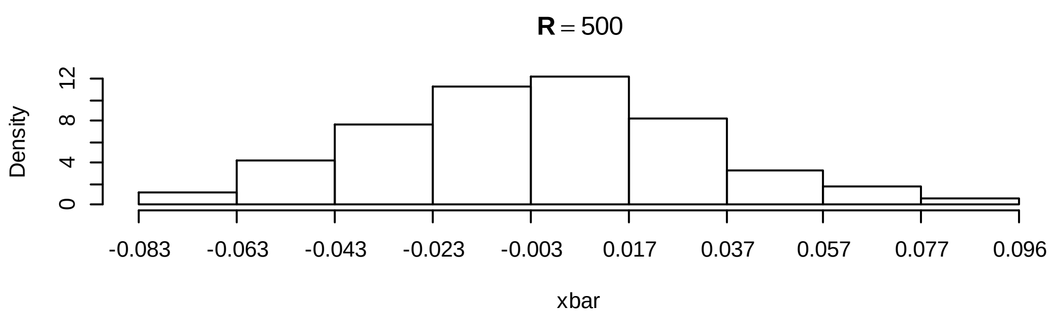
Conclusions-

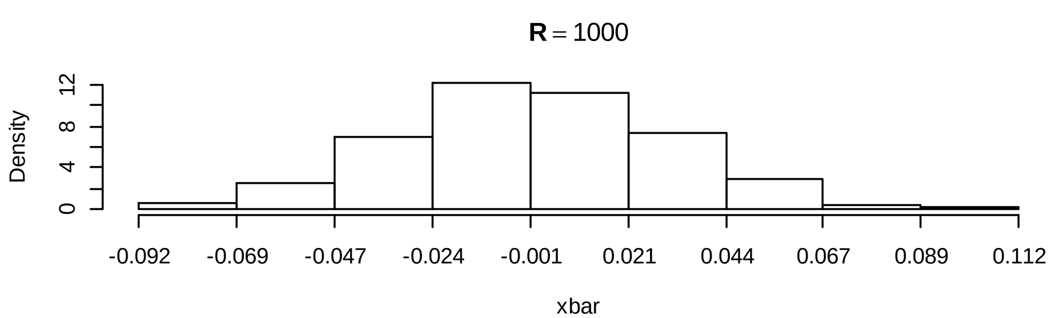
1) ***Irrespective of location and scale parameter of the normal population, For relatively small repetition number (say R=100), the frequency density histogram is more or less symmetric.As the number of repetition is increased,(R=500 or 1000), keeping the sample size fixed(n=1000), the symmetric nature is becoming more obvious.***
2) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample median Xmed.***
3) ***Clearly the height of the histogram remains almost same for any repetition number (R=100, 500, 1000)and fixed n(say n=1000), for any normal population with fixed location and scale parameter. This indicates that asymptotic variance of Xmed is uncorrelated with repetition R (unlike sample size n)***
4) ***Clearly the scale parameter have better significant on height of the histograms than the location parameter.***

D. MEAN-

i) N(0,1)-



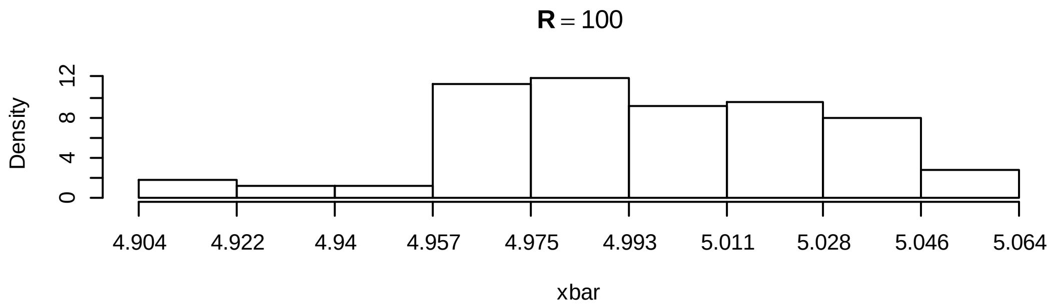


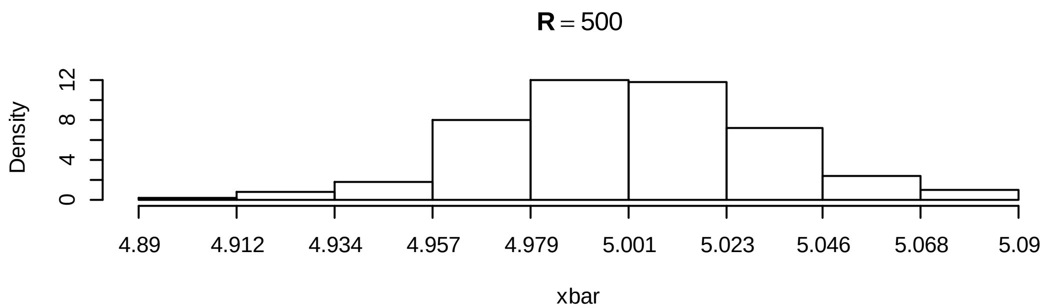


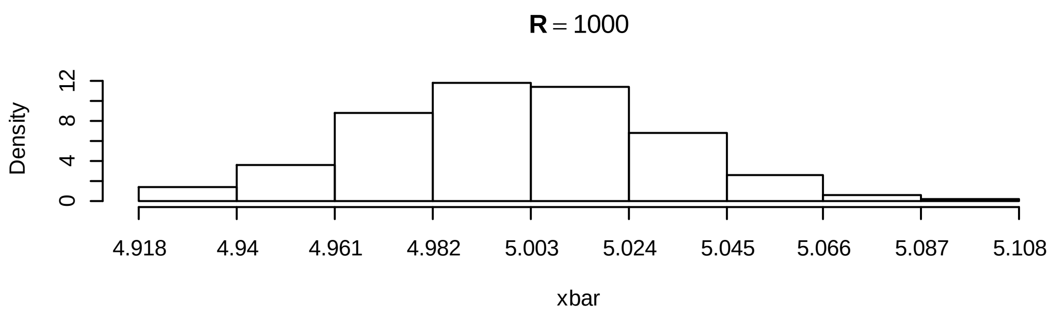
Observations-

1) ***For relatively small repetition number (R=100), the frequency density histogram is slightly positively skewed.***
2) ***As the number of repetition is increased,(R=500 or 1000), keeping the sample size fixed(n=1000), the symmetric nature is becoming more obvious.***
3) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample mean Xbar.***
4) ***Clearly the height of the histogram remains almost same for any repetition number (R=100, 500, 1000)and fixed n(say n=1000), for any normal population with fixed location and scale parameter. This indicates that asymptotic variance of Xbar is uncorrelated with repetition R (unlike sample size n).***

ii) N(5,1)-





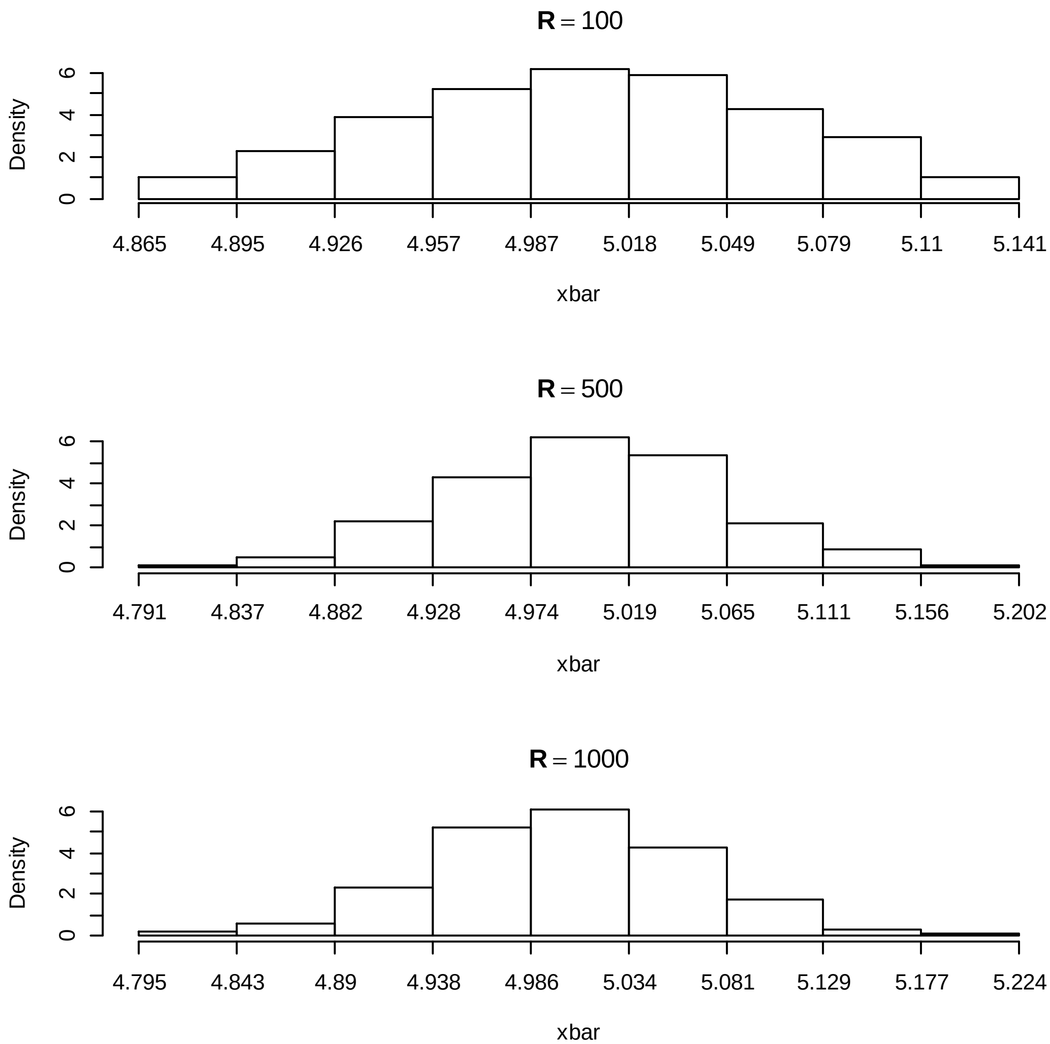


Observations-

1) ***For relatively small R=100, there is no such particular pattern for the histogram.***
2) ***As the number of repetition is increased,(R=500 or 1000), keeping the sample size fixed(n=1000), the symmetric nature is becoming more obvious.***
3) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample mean Xbar.***
4) ***Clearly the height of the histogram remains almost same for any repetition number (R=100, 500, 1000)and fixed n(say n=1000), for any normal population with fixed location and scale parameter. This indicates that***

asymptotic variance of Xbar is uncorrelated with repetition R (unlike sample size n).

iii) N(5,2)-



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3) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample mean Xbar.***
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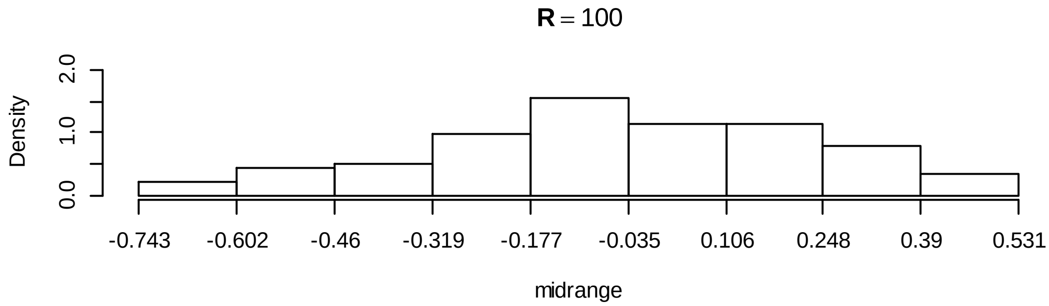
5) ***Keeping the location parameter fixed in '5' as variance is increased by '1' unit, clearly the height of the histograms is decreased.***

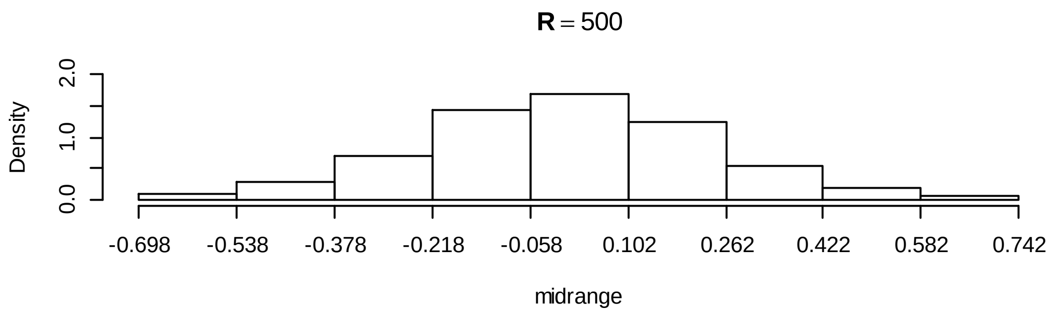
Conclusions-

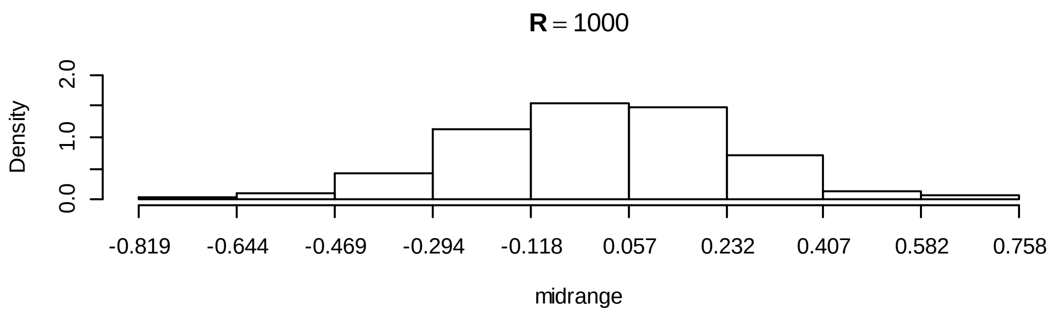
- ***Irrespective of location and scale parameter of the normal population, For relatively small repetition number (say R=100), the frequency density histogram is more or less symmetric. As the number of repetition is increased,(R=500 or 1000), keeping the sample size fixed(n=1000), the symmetric nature is becoming more obvious.***
- ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample mean Xbar.***
- ***Clearly the height of the histogram remains almost same for any repetition number (R=100, 500, 1000)and fixed n(say n=1000), for any normal population with fixed location and scale parameter. This indicates that asymptotic variance of Xbar is independent of with repetition R (unlike sample size n).***
- ***Clearly the scale parameter have better significant on height of the histograms than the location parameter.***
- ***On an average when n is fixed at 1000, for any R,irrespective of location and scale parameter, the height of the density histogram is lesser for Xmed than Xbar indicating V(Xmed)>V(Xbar) as R tends to infinity.***

E. MIDRANGE-

i) N(0,1)-



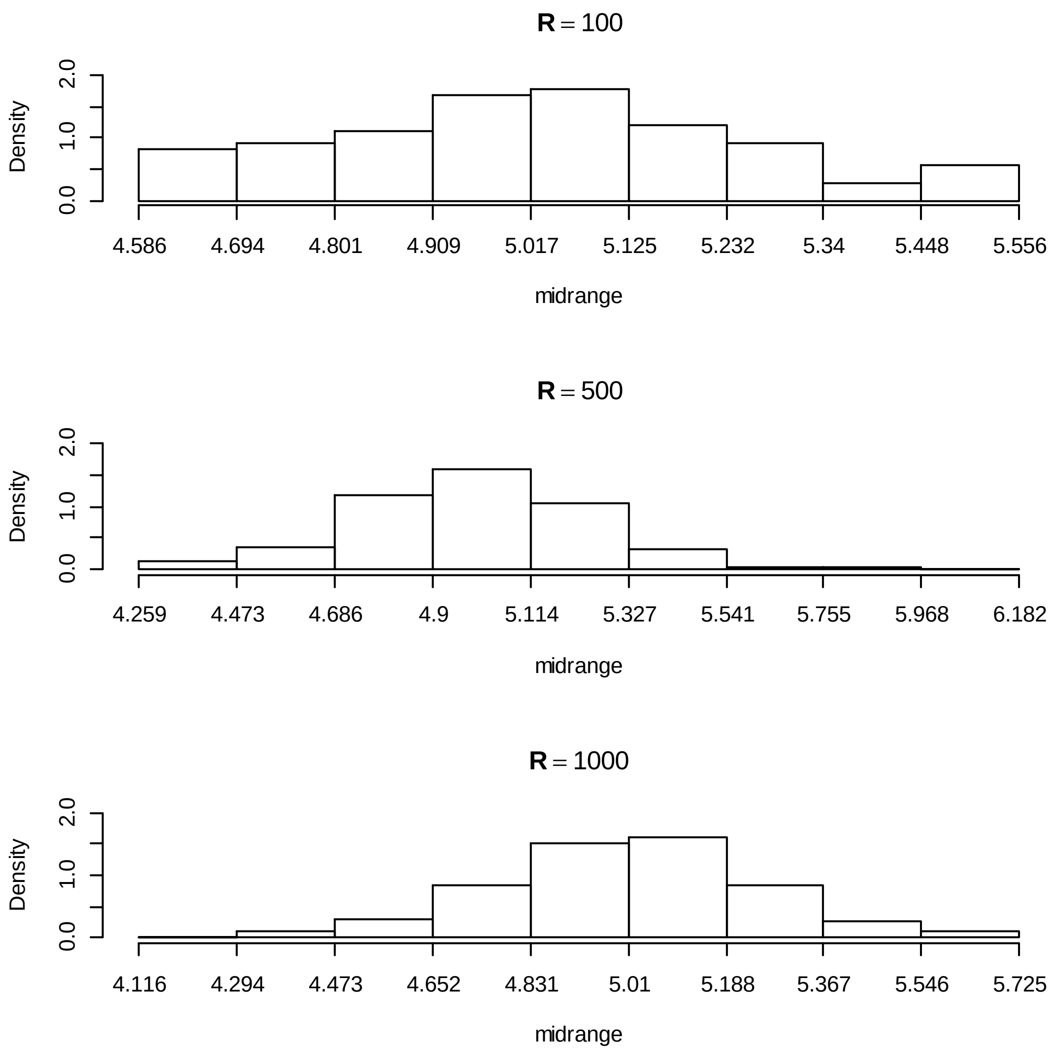




Observations-

1) ***For relatively small repetition number (say R=100), the frequency density histogram is more or less symmetric.***
2) ***As the number of repetition is increased,(R=500 or 1000), keeping the sample size fixed(n=1000), the symmetric nature is becoming more obvious.***
3) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample midrange.***
4) ***Clearly the height of the histogram remains almost same for any repetition number (R=100, 500, 1000)and fixed n(say n=1000), for any normal population with fixed location and scale parameter. This indicates that asymptotic variance of midrange is uncorrelated with repetition R (unlike sample size n).***

iii)N(5,1)



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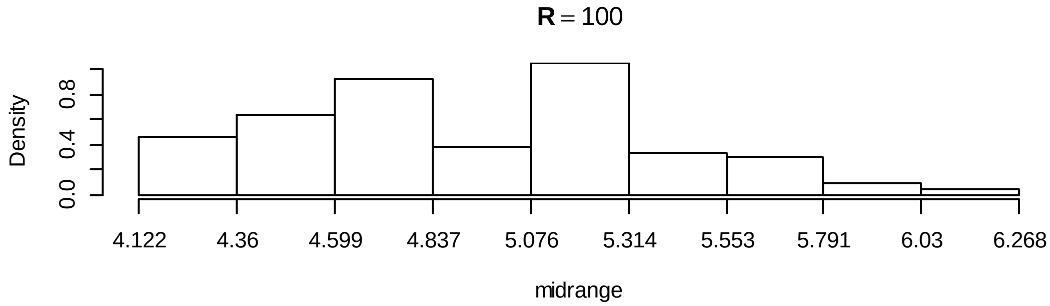
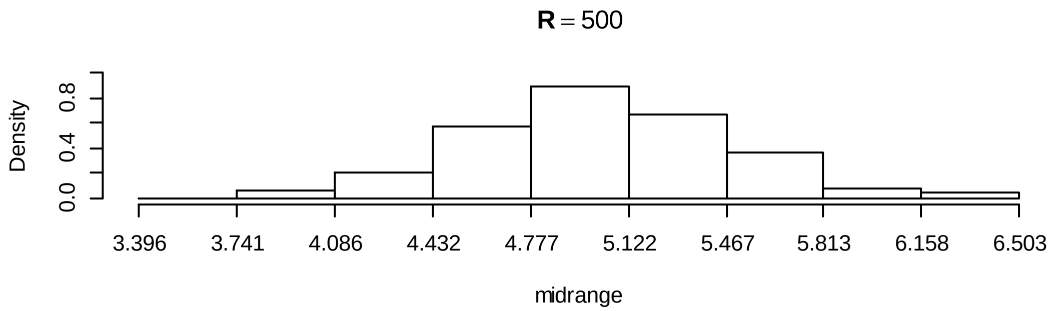
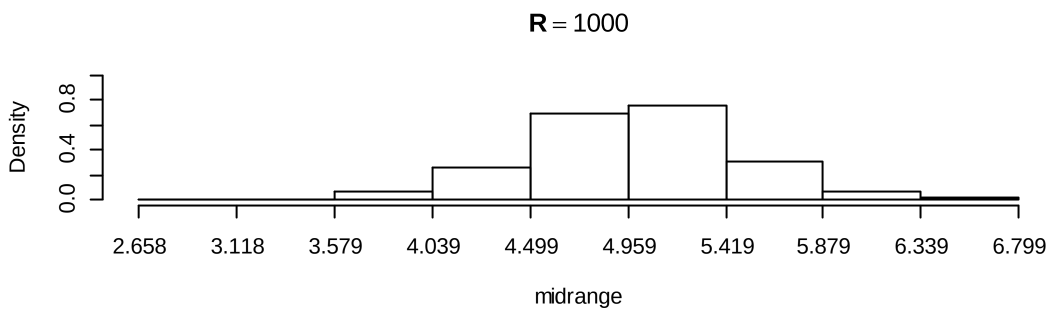
4) ***Clearly the height of the histogram remains almost same for any repetition number (R=100, 500, 1000)and fixed n(say n=1000), for any normal population with fixed location and scale parameter. This indicates that asymptotic variance of midrange is independent of repetition R (unlike sample size n).***

iii) N(5,2)-







Observations-

1) ***There is no such particular pattern in the histogram for relatively small R(say R=100).***
2) ***As the number of repetition is increased,(R=500 or 1000), keeping the sample size fixed(n=1000), the symmetric nature is becoming more obvious.***
3) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample midrange.***
4) ***Clearly the height of the histogram remains almost same for any repetition number (R=100, 500, 1000)and fixed n(say n=1000), for any normal population with fixed location and scale parameter. This indicates that asymptotic variance of midrange is independent of repetition R (unlike sample size n)***

5) ***Keeping the location parameter fixed in '5' as variance is increased by '1' unit, clearly the height of the histograms is decreased.***

Conclusions-

- ***Irrespective of location and scale parameter of the normal population, For relatively small repetition number (say R=100), the frequency density histogram is more or less symmetric. As the number of repetition is increased,(R=500 or 1000), keeping the sample size fixed(n=1000), the symmetric nature is becoming more obvious.***
- ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample midrange.***
- ***Clearly the height of the histogram remains almost same for any repetition number (R=100, 500, 1000)and fixed n(say n=1000), for any normal population with fixed location and scale parameter. This indicates that asymptotic variance of midrange is independent of with repetition R (unlike sample size n).***
- ***Clearly the scale parameter have better significant on height of the histograms than the location parameter.***
- ***When n is fixed at 1000, for any R=100,500 or 1000, irrespective of the location and scale parameter, the height of the density histogram is in the order H(midrange)<H(median)<H(mean),where H(statistic) denotes height of the histogram of the statistic for a particular x value. Hence V(midrange)>V(Xmed)>V(Xbar).***

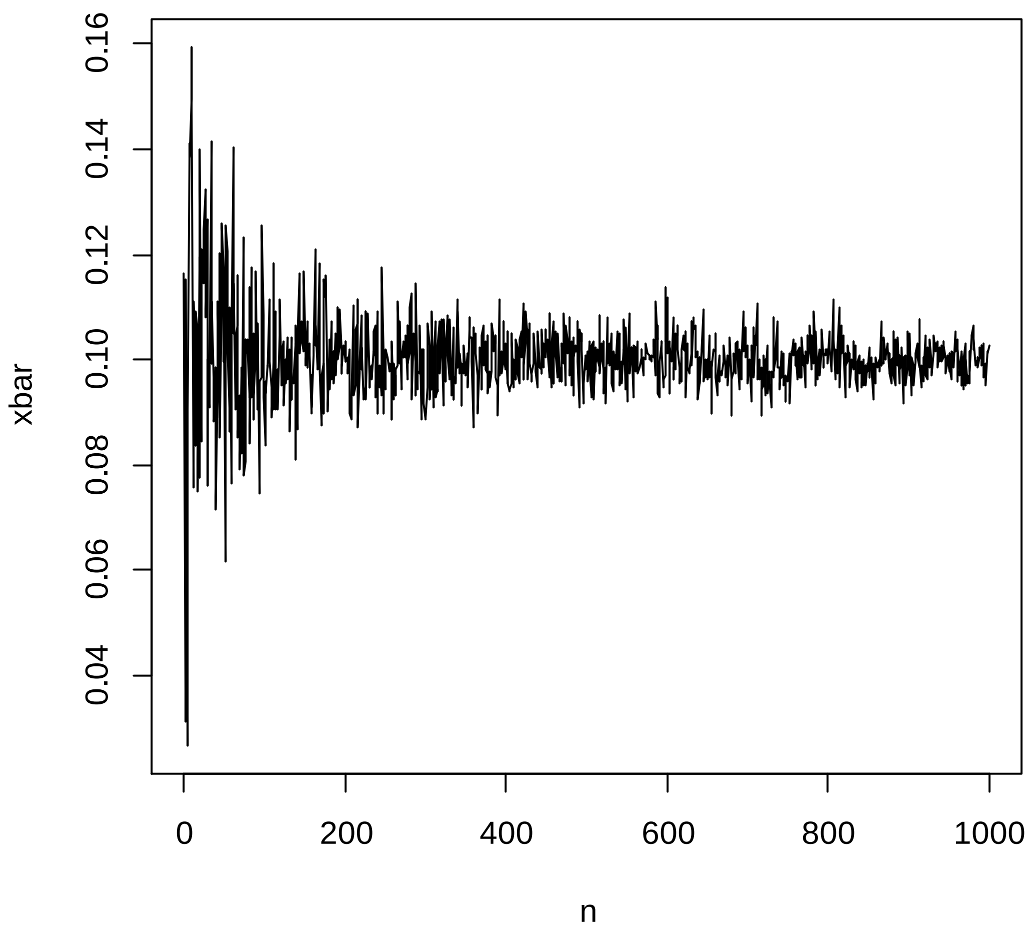
EXPONENTIAL DISTRIBUTION

HERE WE WILL FIND THE GUESS LIMITS-

The term "guess limit" is used to represent that particular value to which the statistics tends to converge as the sample size n tends to infinity. Here we are trying to find guess limits for sample mean , median , X(n) , X(1) from the population Exp(mean=0.1), Exp(mean=1) to make a well defined comparison between the statistical behaviour of the statistics.

A. ***MEAN-***

i) Exp(mean=0.1)

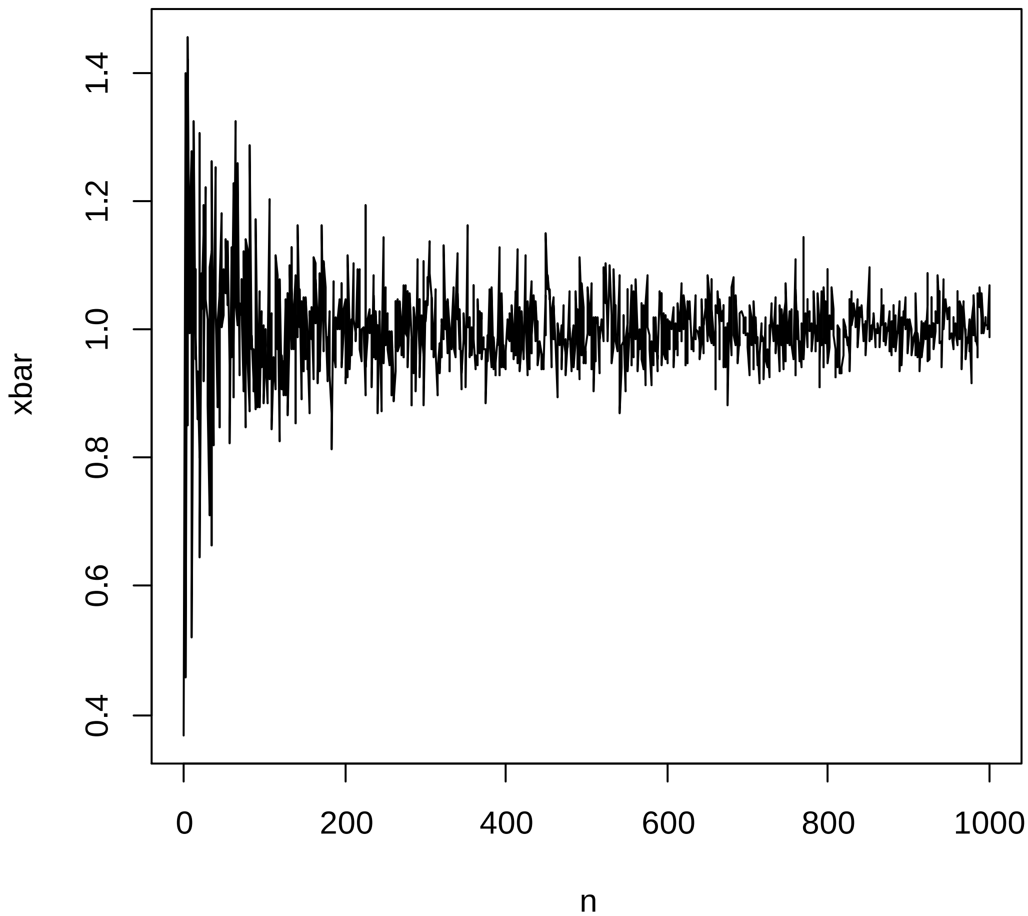


Findings-

1) ***For Exp(mean=0.1), the sample mean Xbar tends to converge to the population mean '0.1'***

2) ***As the sample size increases, the fluctuation of sample Xbar values around population Xbar values reduces , i.e., the convergence becomes clearer and rapid.***

ii) Exp(mean=1)-



Findings-

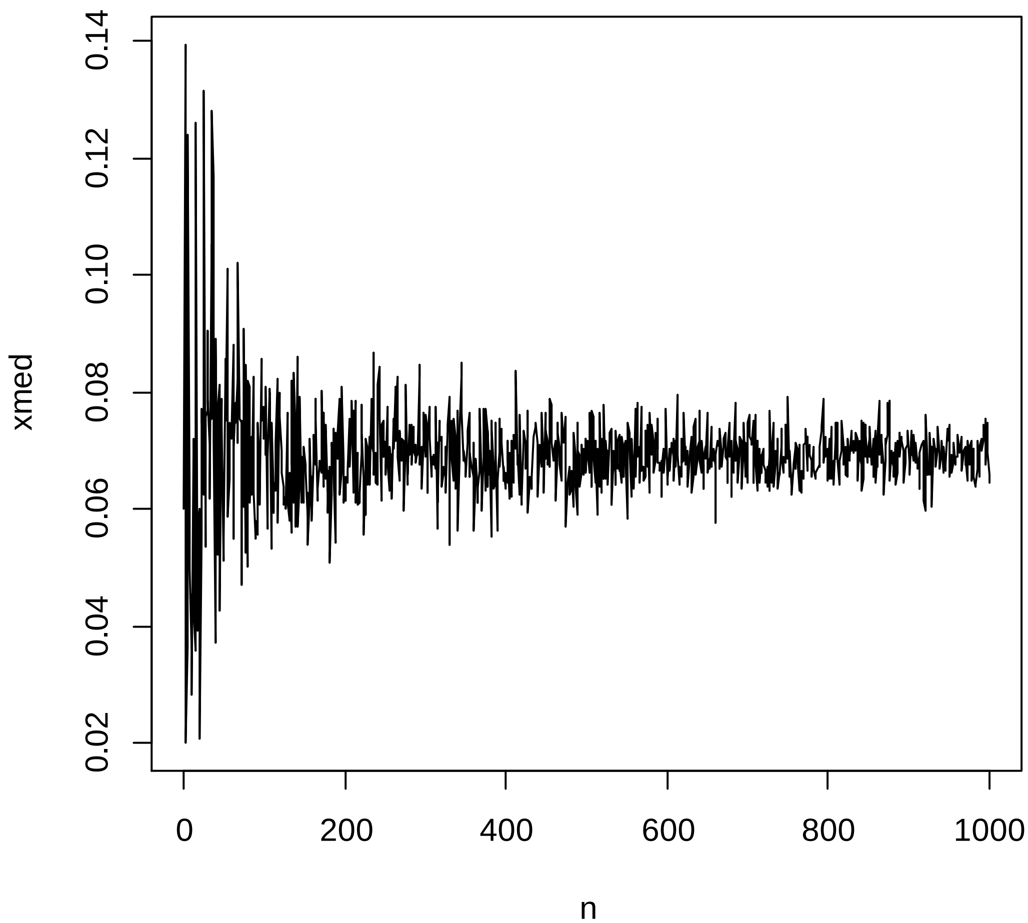
1) ***From the diagram it is clear that for Exp(mean=1),the sample mean Xbar tends to the population mean '1', as n becomes large enough.***
2) ***The convergence becomes more clear and fast, as the sample size goes on increasing.***
3) ***The convergence of Xbar is less clear in Exp(mean=1) distn than that of Exp(mean=0.1) distn, as fluctuation around its population mean is more in the 2nd case than the 1st one(The reason may be smaller variance of Exp(mean=0.1) distn than Exp(mean=1) distn.***

Conclusion-

- ***Hence in general, we can conclude that for any Exp(mean=u) , u>0, population, as n tends to infinity, the sample mean converges to population mean 'u', irrespective of the population parameter '1/u'.***
- ***As u is increasing, the convergence becomes slower because with the increment of u, the fluctuation of Xbar values around 'u' increases.***

B. MEDIAN-

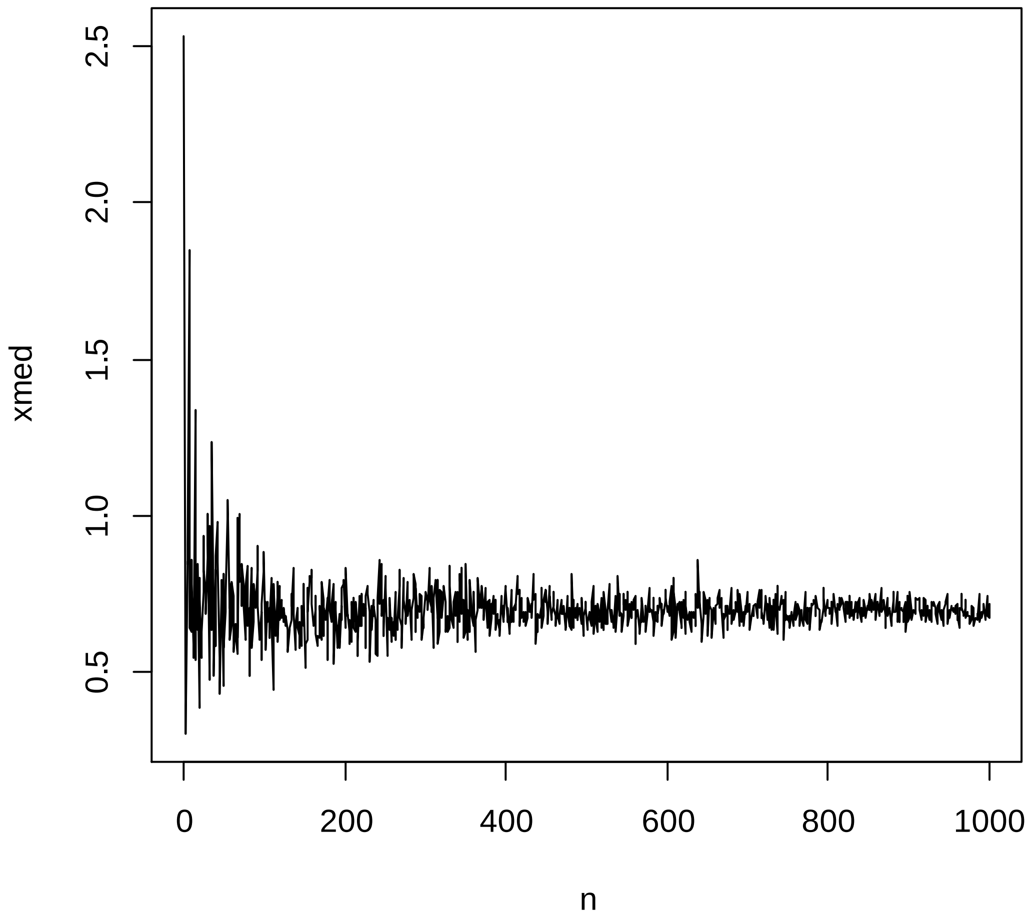
i) Exp(mean=0.1)-



Findings-

1) ***As n tends to infinity, the sample median Xmed tends to converge to ln(2)/10=0.06931.***
2) ***As the sample size is increased, say , n= 600 to 1000, the convergence becomes clearer and faster.***

ii) Exp(mean=1)-



Findings-

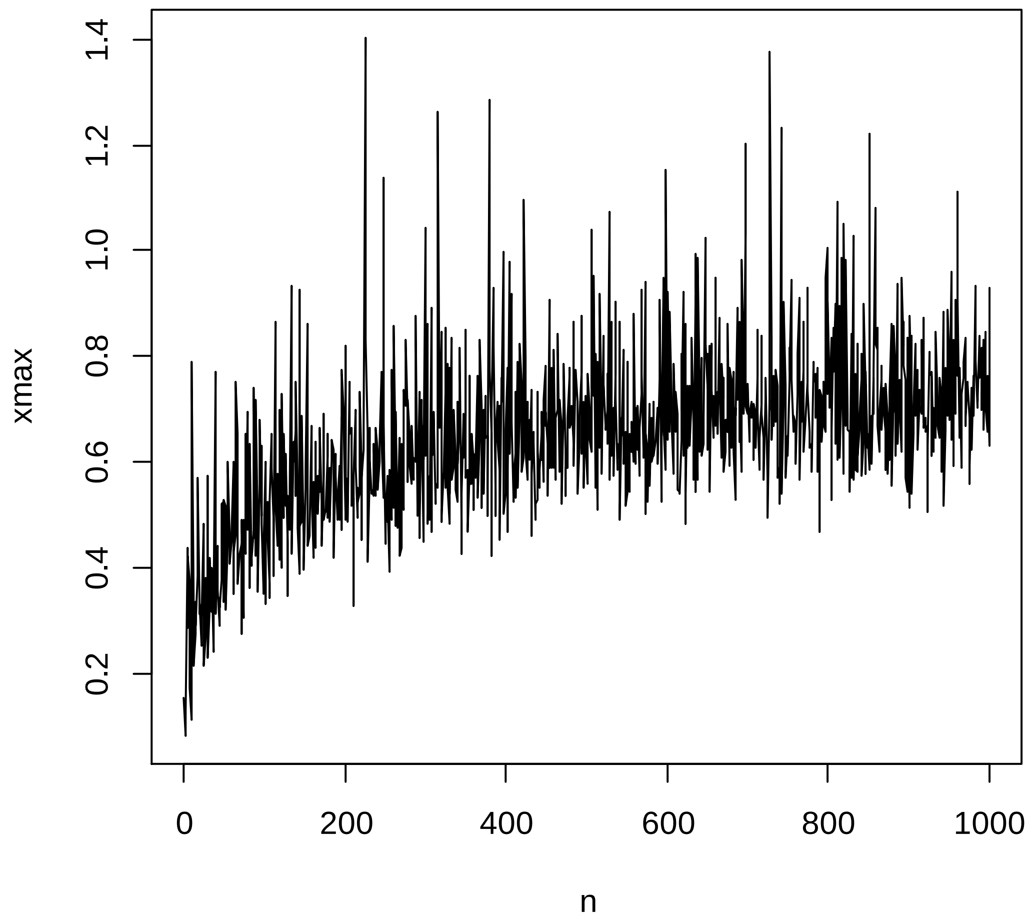
1)As n tends to infinity, the sample median Xmed tends to ln(2)=0.6931.   ***2) With the increment in the sample size, the rate of convergence also increases.***   ***3) The fluctuation reduces, hence convergence becomes fast and clear than Exp(mean=0.1) distn.***

Conclusions-

- ***Hence in general, we can conclude that for any Exp(mean=u) , u>0, population, as n tends to infinity, the sample mean converges to population mean 'u', irrespective of the population parameter '1/u'.***
- ***With the increment in the sample size, the rate of convergence also increases, the fluctuation reduces, hence convergence becomes fast and clear as we increase 'u'.***

C. X(n)-

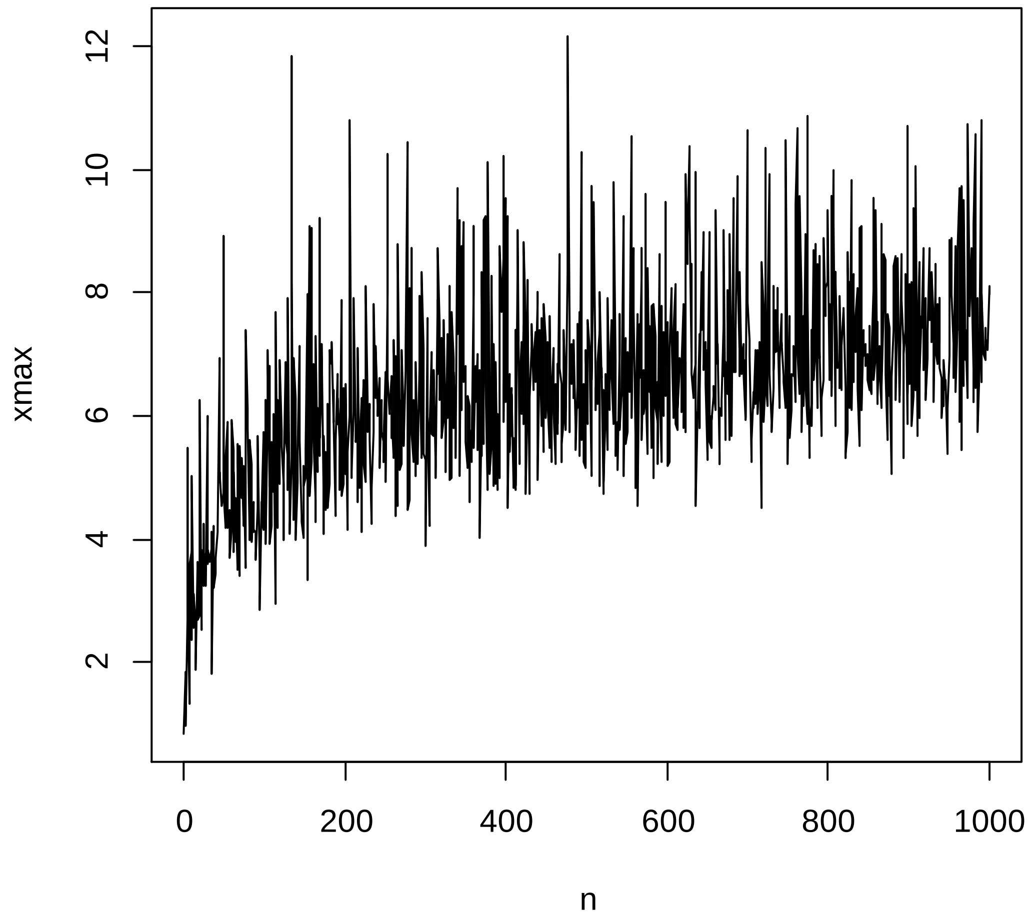
i) Exp(mean=0.1)-



Findings-

1) ***For Exp(mean=0.1), as n tends to infinity, Xmax does not seem to converge to any particular value ,i.e., we fail to find any guess limit here.***
2) ***Though most of the Xmax values are likely to cluster around '0.7', the value will vary from sample to sample, hence, we will check convergence in probability by taking '0.7' as guess limit.***

ii) Exp(mean=1)-



Findings-

1) ***For Exp(mean=1), as n tends to infinity, Xmax does not seem to converge to any particular value ,i.e., we fail to find any guess limit here.***
2) ***Though most of the Xmax values are likely to cluster around '7', the value will vary from sample to sample, hence, we will check convergence in probability by taking '7' as a guess limit.***

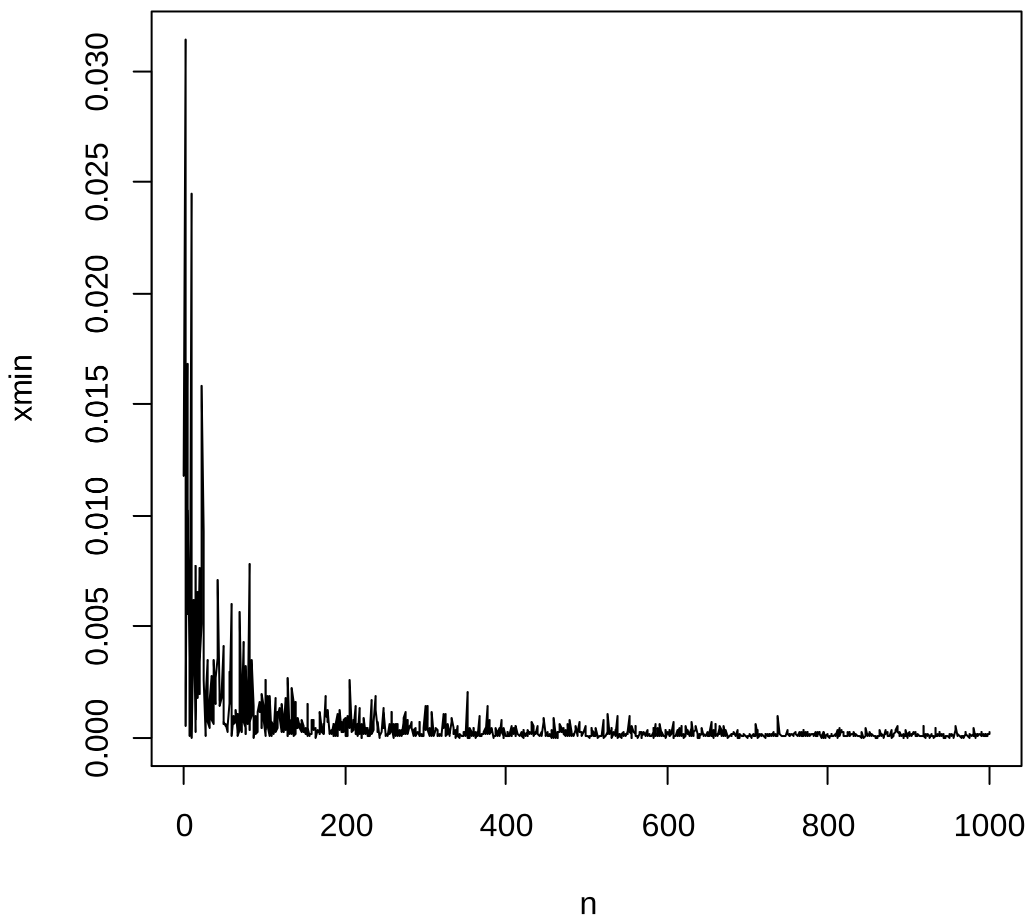
Conclusions-

- ***So whatever the parameter of the population be, sample maximum, or X(n) does not seem to tend to any particular value.***
- ***As we increase the sample size, the randomness in the X(n) values increases.***

- ***Although the sample X(n) values seem to cluster around any value apparently, the value changes from sample to sample , hence in general we can not find any guess limit for sample X(n).***

D. X(1)-

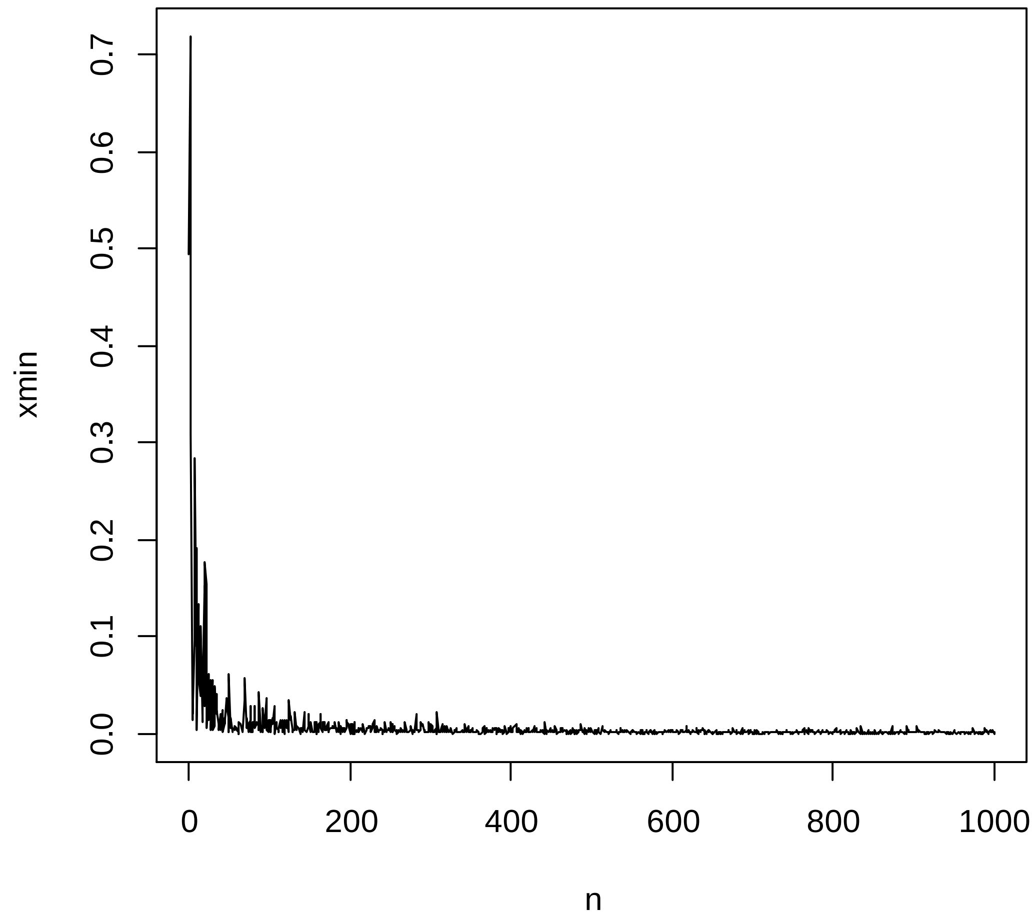
i) Exp(mean=0.1)-



Findings-

1) ***From the above graph , it is clear that X(1) tends to 0 as n goes on increasing.***
2) ***As the sample size increases the rate of convergence also increases.***

ii) Exp(mean=1)-



Findings-

1) ***From the above graph , it is clear that X(1) tends to 0 as n goes on increasing.***
2) ***As the sample size increases the rate of convergence also increases.***
3) ***For relatively small sample size say n=100 or 200, the convergence rate of X(1)is faster for Exp(mean=1) than that of Exp(mean=0.1).***

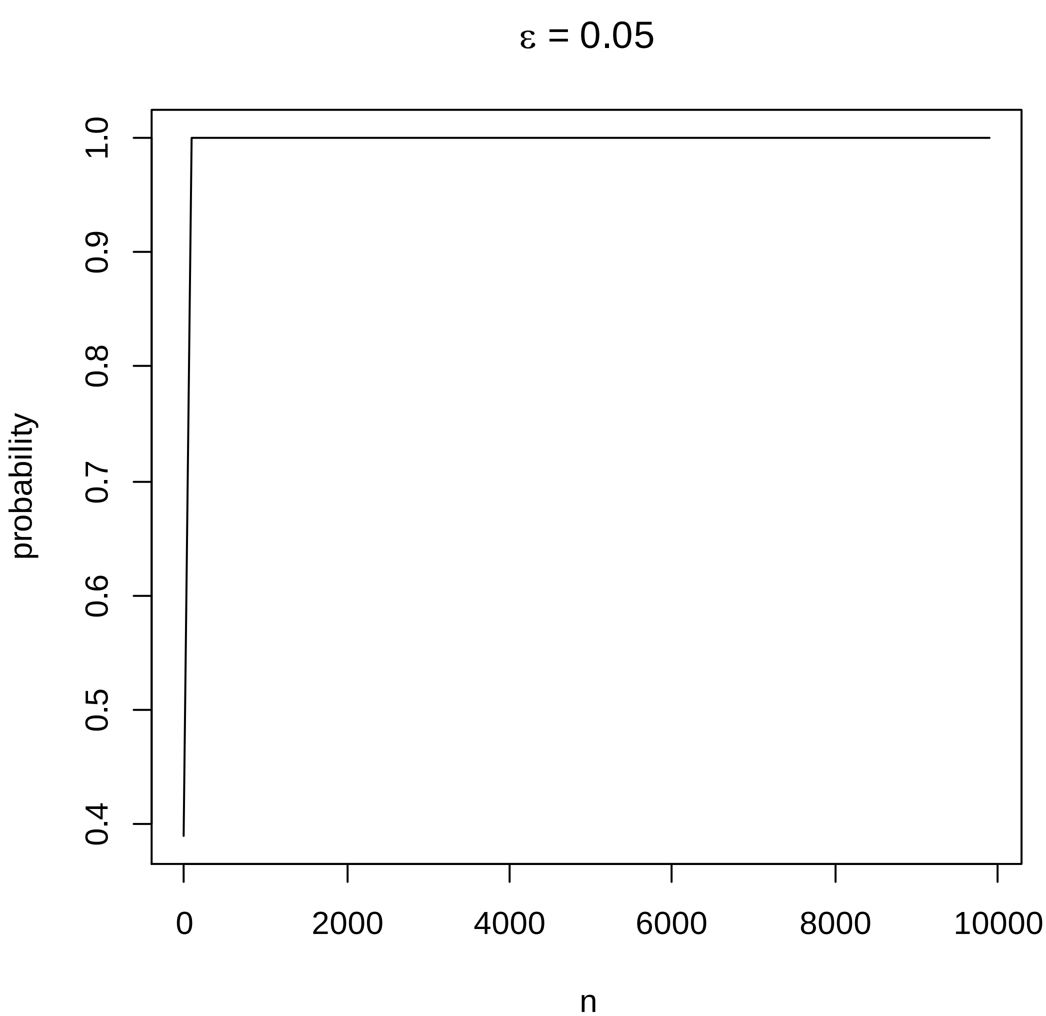
Conclusions-

- ***Irrespective of the population mean(u), X(1) tends to 0 as n tends to infinity.***
- ***As sample size increases, the rate of convergence increases for an Exponential population with mean 'u' , u>0***
- ***For relatively small sample size, the convergence rate of X(1) is faster for an Exponential distribution with larger mean.***

NOW WE WILL CHECK FOR CONVERGENCE IN PROBABILITY FOR R=100, n=100,200,……..,1000.

A.MEAN-

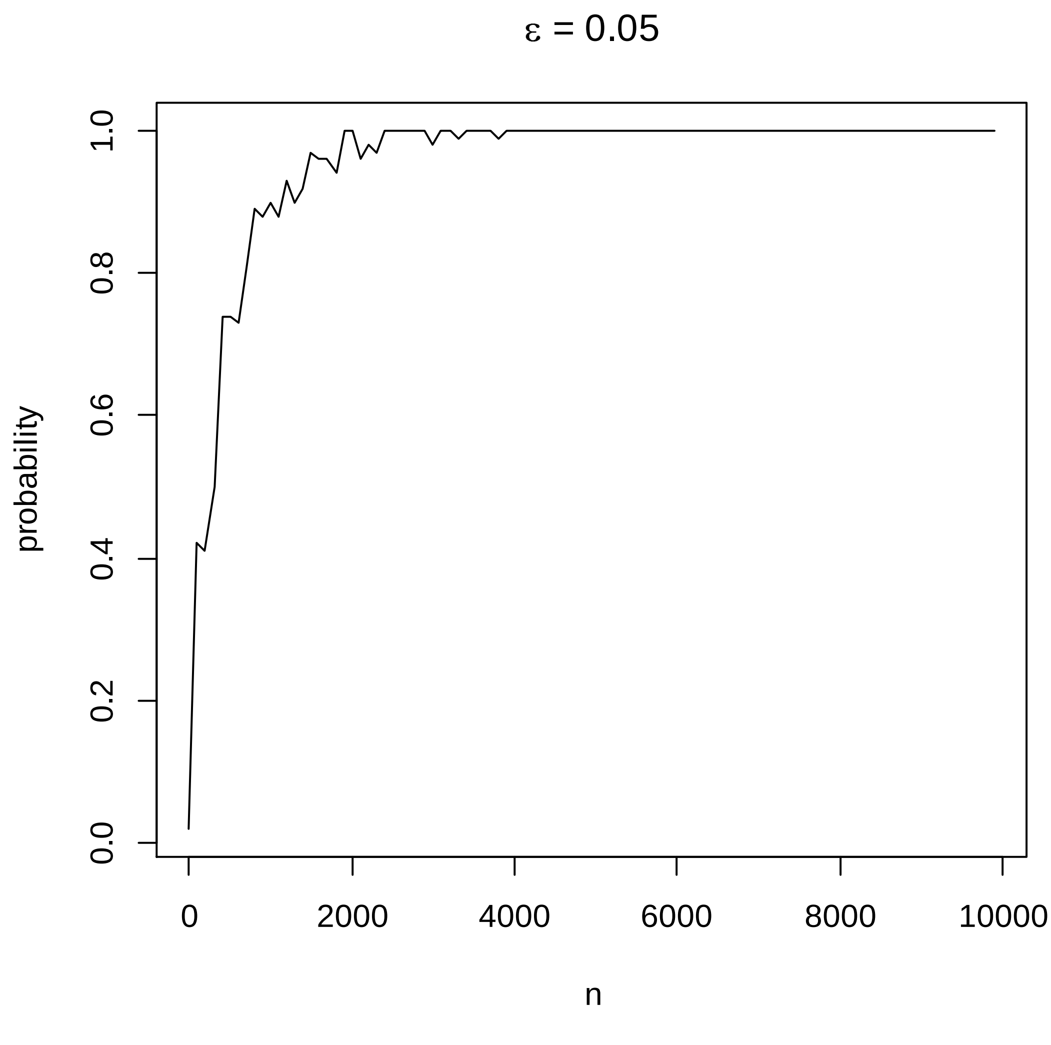
i) Exp(mean=0.1)-



Findings-

1) ***We got the guess limit of Xbar as '0.1' for Exponential distribution with mean=0.1.***
2) ***Clearly as n is increasing, P[|Xbar-0.1|<eps=0.05]→1, hence Xbar converges to '0.1' in probability, as n tends to infinity.***
3) ***In other words for Exp(mean=0.1), sample Xbar is consistent for population mean=0.1.***

ii) Exp(mean=1)-



Findings-

1) ***We got the guess limit of Xbar as '1' for Exponential distribution with mean=1.***

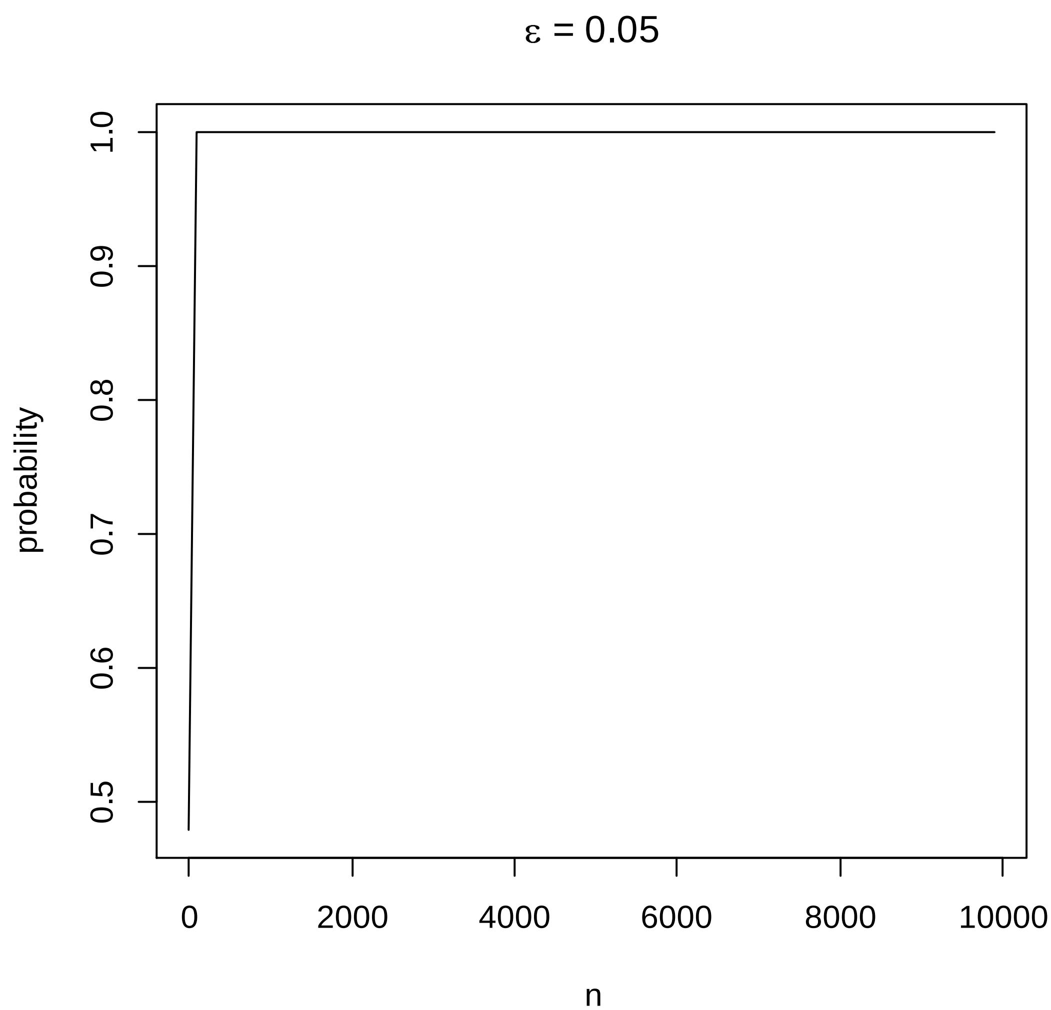
2) ***Clearly as n is increasing, P[|Xbar-1|<eps=0.05]→1, hence Xbar converges to '1' in probability, as n tends to infinity.***
3) ***In other words for Exp(mean=1), sample Xbar is consistent for population mean=1.***
4) ***The convergence rate is more for Xbar in probability in Exp(mean=0.1) distn than that of in Exp(mean=1) distn, as in the previous one the probability reaches 1 (before n=2000), before , that in the later one (here it reaches 1 after n=2000).***

Conclusions-

- ***We will get the guess limit of Xbar as 'u' , u>0, for an Exponential distribution with mean =u.***
- ***Clearly as n is increasing, P[|Xbar-u|<eps=0.05]→1, hence Xbar converges in probability to u as n tends to infinity, in other words Xbar is consistent for 'u'.***
- ***The convergence rate of Xbar in probability to u increases as u decreases.***

B.MEDIAN

i)Exp(mean=0.1)

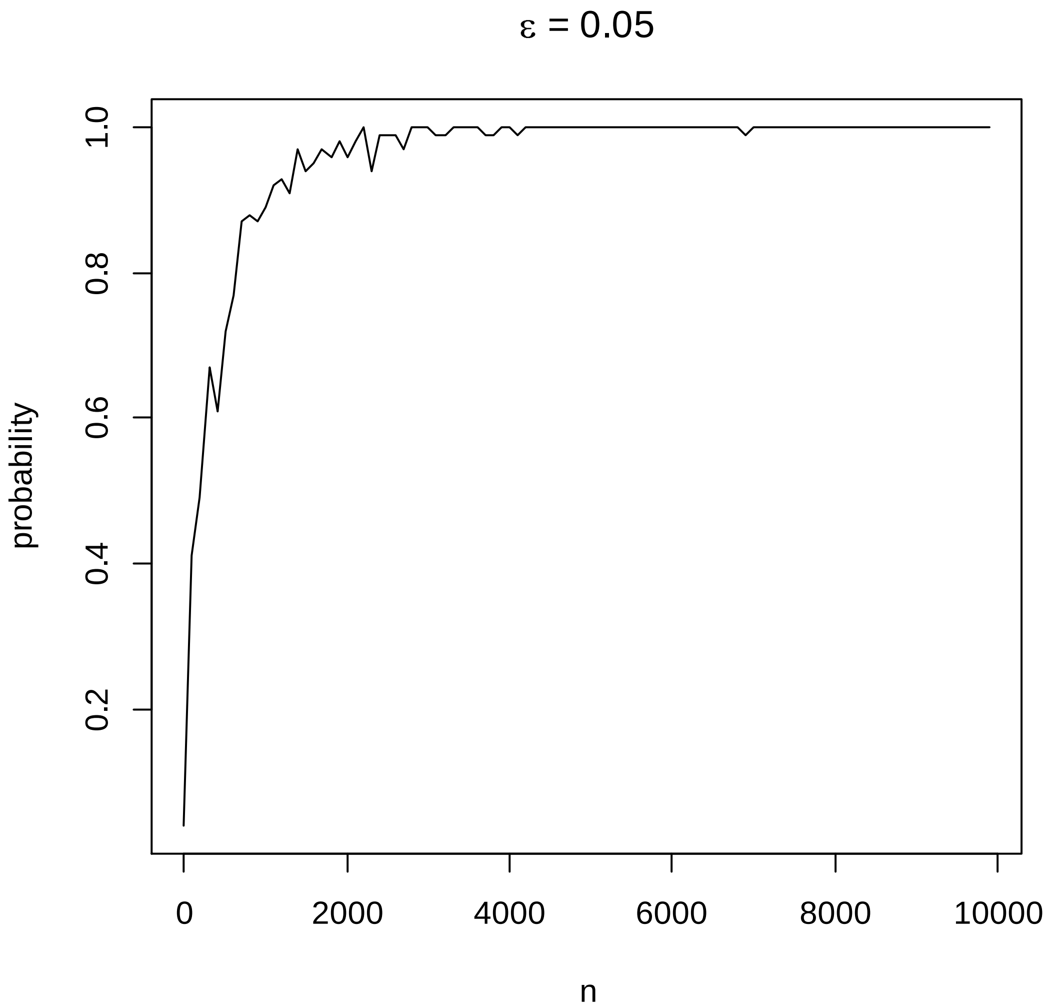


Findings-

1) ***We got the guess limit of Xmed as '0.1' for Exponential distribution with mean=0.1.***

2) ***Clearly as n is increasing, P[|Xmed-0.1|<eps=0.05]→1, hence Xmed converges to '0.1' in probability, as n tends to infinity.***
3) ***In other words for Exp(mean=0.1), sample Xmed is consistent for population mean=0.1.***

ii) Exp(mean=1)



Findings-

1) ***We got the guess limit of Xmed as '1' for Exponential distribution with mean=1.***
2) ***Clearly as n is increasing, P[|Xmed -1|<eps=0.05]→1, hence Xmed converges to '1' in probability, as n tends to infinity.***
3) ***In other words for Exp(mean=1), sample Xmed is consistent for population mean=1.***
4) ***The convergence rate is more for Xmed in probability in Exp(mean=0.1) distn than that of in Exp(mean=1) distn, as in the previous one the probability reaches 1 (before n=2000), before , that in the later one (here it reaches 1 after n=2000).***

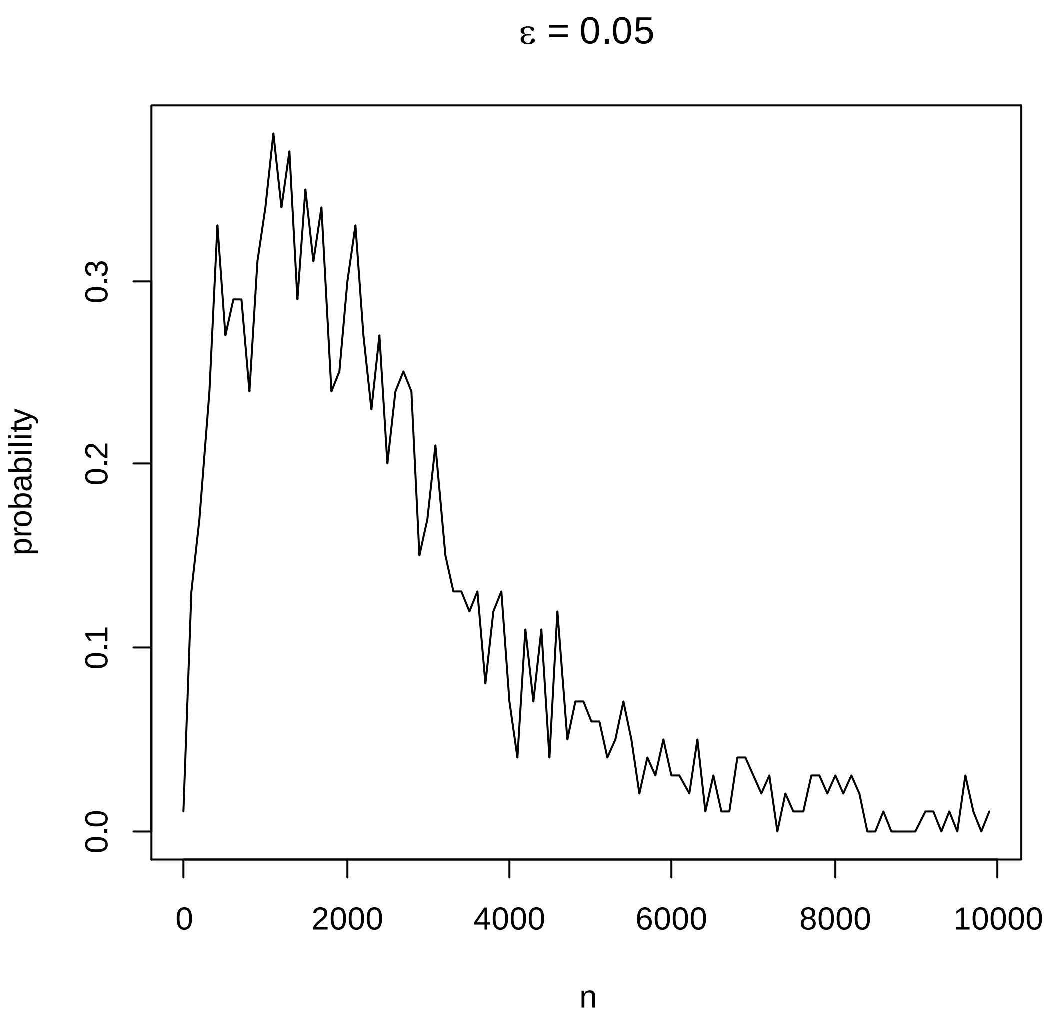
Conclusions-

- ***We will get the guess limit of Xmed as 'u' , u>0, for an Exponential distribution with mean =u.***

- ***Clearly as n is increasing, P[|Xmed-u|<eps=0.05]→1, hence Xmed converges in probability to u as n tends to infinity, in other words Xmed is consistent for 'u'.***
- ***The convergence rate of Xmed in probability to u increases as u decreases.***

C.X(n)-

i) Exp(mean=0.1)-

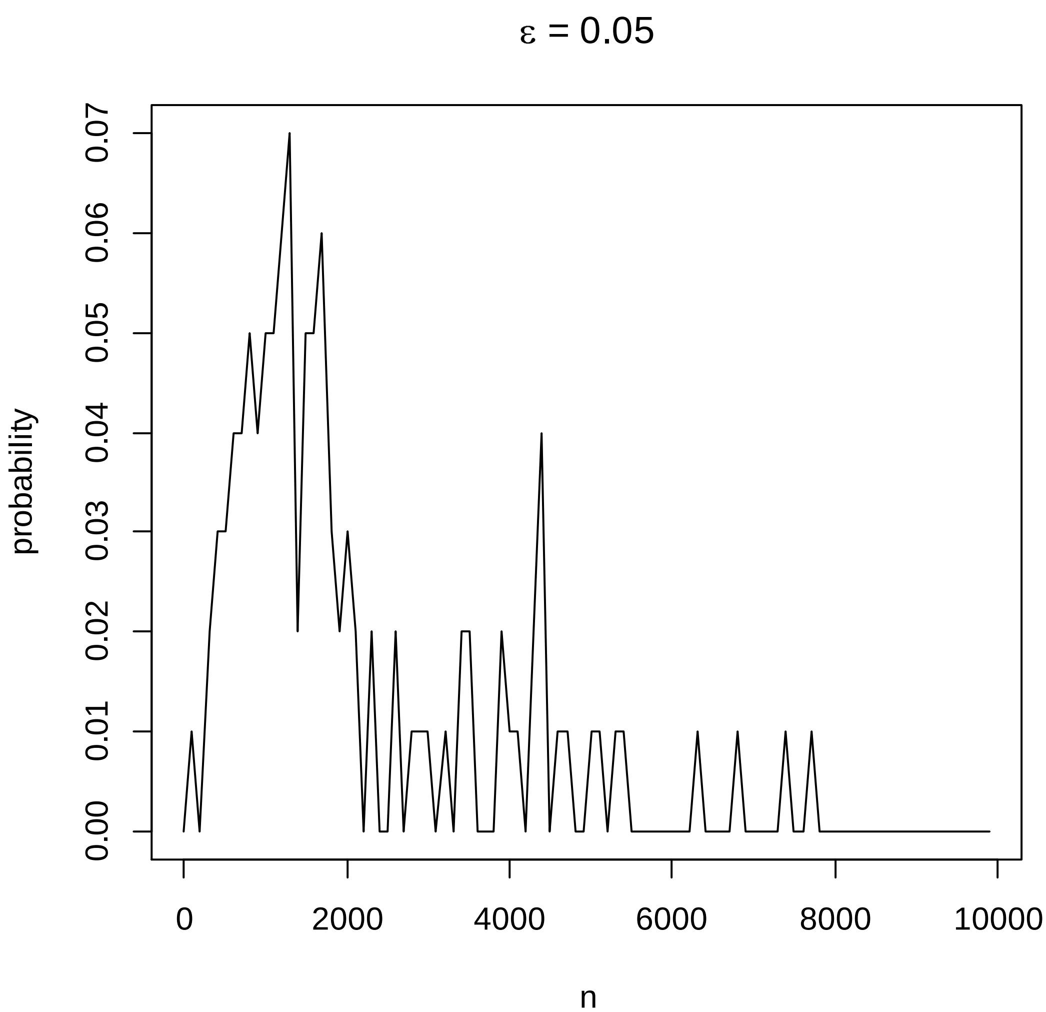


Findings-

1) ***We can not find any guess limit still we try to check convergence in probability by taking the guess limit as 0.7***
2) ***Clearly as n tends to infinity, P[|X(n)-0.7|<eps=0.05] does not tend to '1', Hence X(n) does not converge in probability to 0.7.***

3) ***We can also check by taking different guess limits but will still find that probability is not going to 1 as n tends to infinity.***

ii) Exp(mean=1)-



Findings-

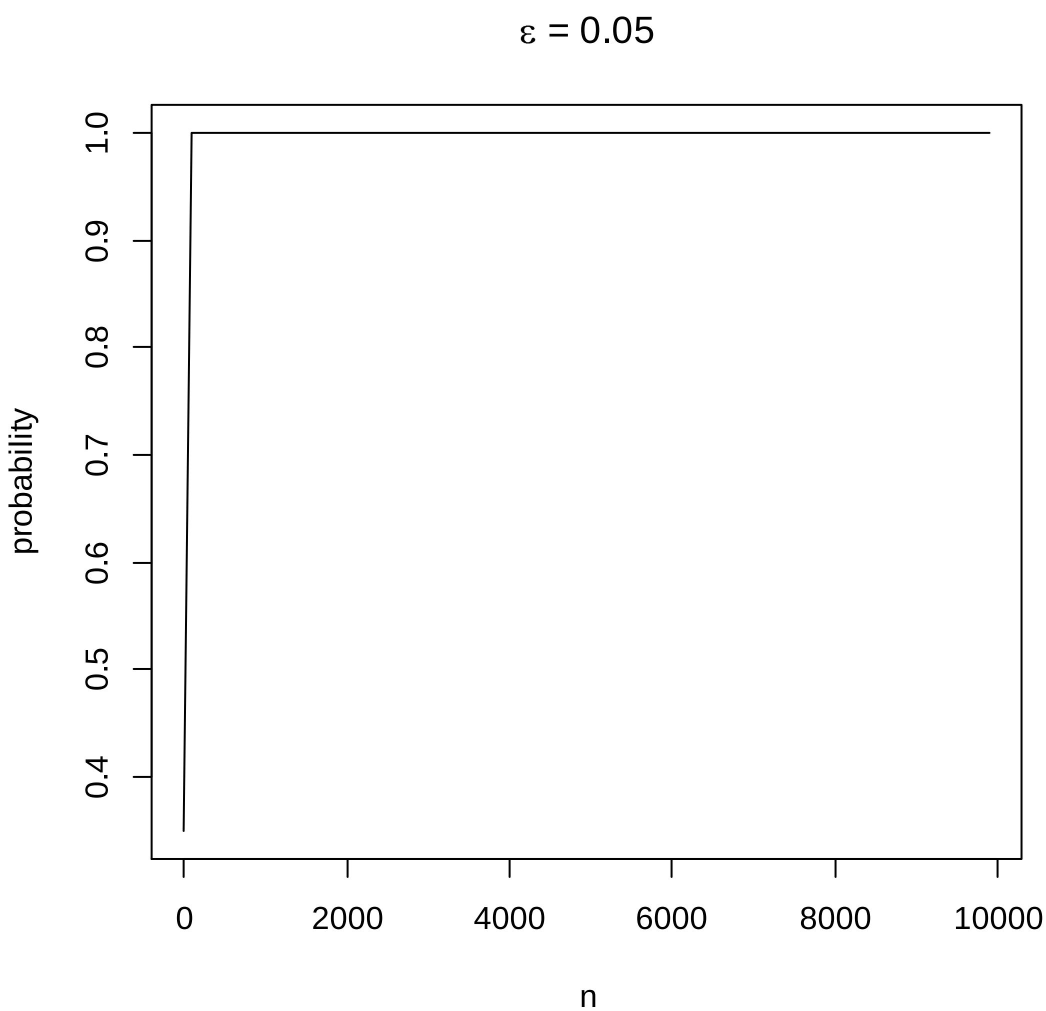
1) ***We can not find any guess limit still we try to check convergence in probability by taking the guess limit as 7.***
2) ***Clearly as n tends to infinity, P[|X(n)-7|<eps=0.05] does not tend to '1', Hence X(n) does not converge in probability to 7.***
3) ***We can also check by taking different guess limits but will still find that probability is not going to 1 as n tends to infinity.***

Conclusions-

1) ***We can not find any guess limit still we try to check convergence in probability by taking the guess limit as a.***
2) ***Clearly as n tends to infinity, P[|X(n)-a|<eps=0.05] does not tend to '1', Hence X(n) does not converge in probability to a.***
3) ***We can also check by taking different guess limits but will still find that probability is not going to 1 as n tends to infinity.***

D.X(1)-

i) Exp(mean=0.1)-

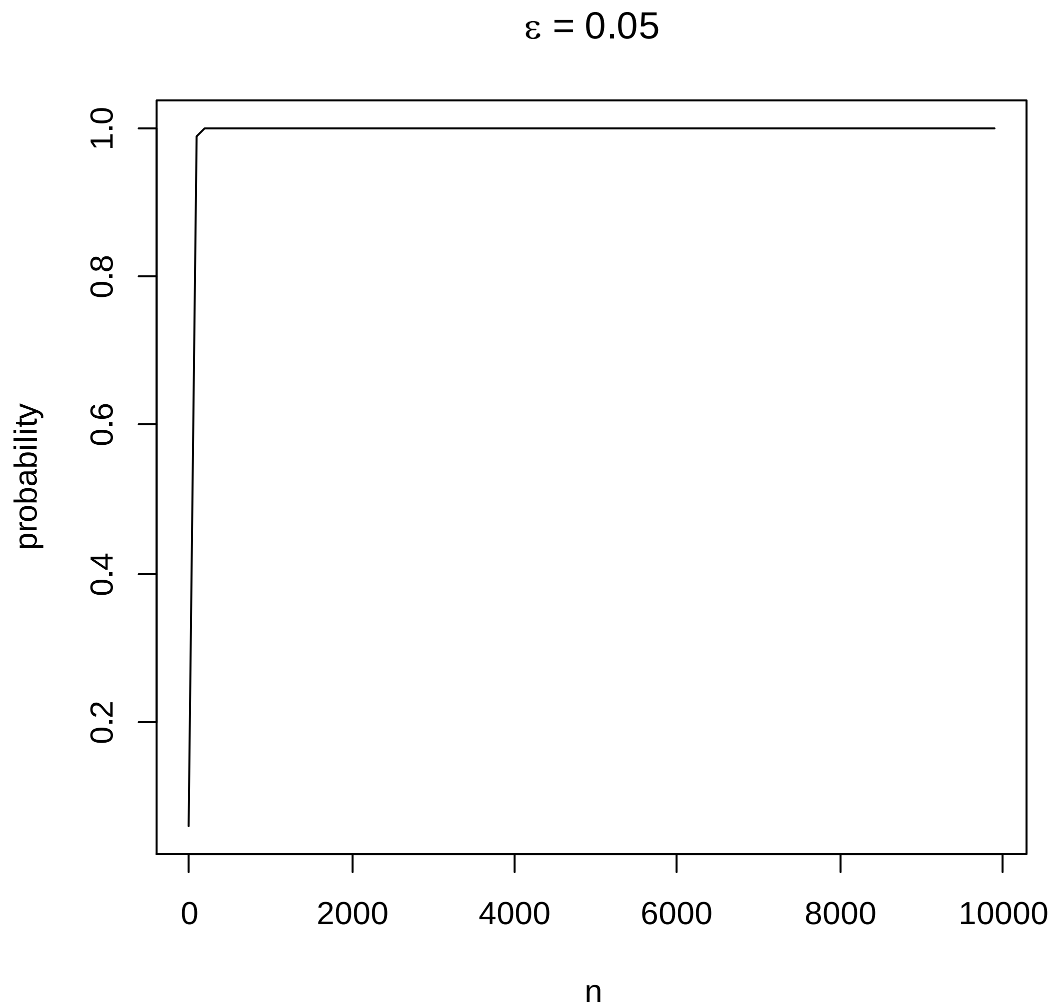


Findings-

1) ***Here we get the guess limit as '0'.***

2) ***Clearly as n tends to infinity P[|X(1)-0|<eps=0.05]→1 , implies that X(1) is convergent in probability to '0'.***
3) ***Hence X(1) is consistent for '0'.***

ii) Exp(mean=1)-



Findings-

1) ***Here we get the guess limit as '0'.***
2) ***Clearly as n tends to infinity P[|X(1)-0|<eps=0.05]→1 , implies that X(1) is convergent in probability to '0'.***
3) ***Hence X(1) is consistent for '0'.***

Conclusions-

1) ***Here the guess limit is '0', whatever the parameter of the population be.***

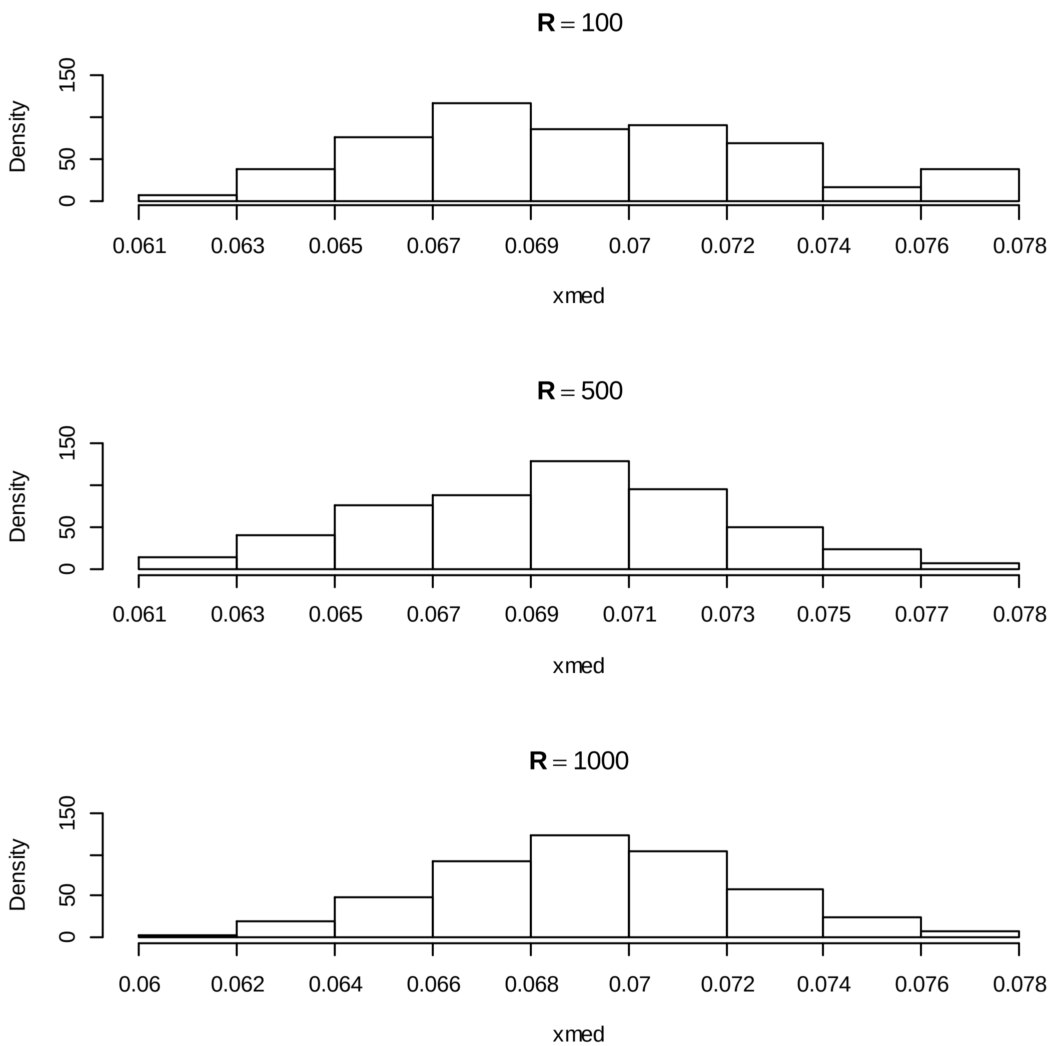
2) ***Clearly as n tends to infinity P[|X(1)-0|<eps=0.05] → 1 , implies that X(1) is convergent in probability to '0'.***
3) ***Hence X(1) is consistent for '0'.***
4) ***The convergence rate is same for X(1) , whatever the mean of the distribution be.***

NOW WE WILL CHECK CONVERGENCE IN DISTRIBUTION FOR n=1000,R=100,500,1000.

Here we will check the convergence in distribution for some well known statistics like sample mean , median , minimum , maximum , midrange etc for Exponential population with certain mean and make a significant comparison between their large sample behaviour and their dependency on parameters for fixed sample size n=1000 and repetition numbers R=100,500,1000.

A.MEDIAN-

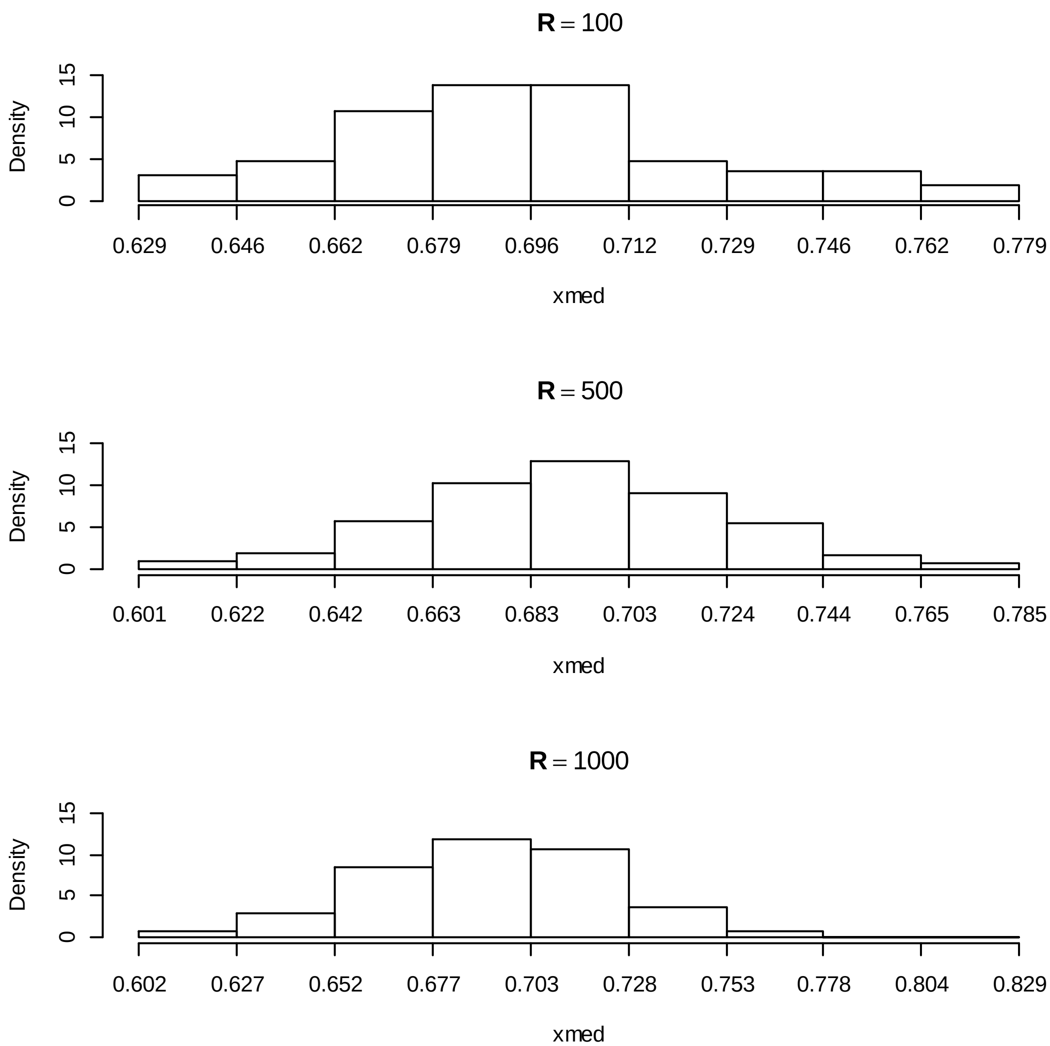
i)Exp(mean=0.1)



Findings-

1) ***For relatively small replication number, (say R=100), the frequency density histogram is more or less symmetric.***
2) ***As the number of repetition is increased, (R=500 or 1000), keeping the sample size fixed(n=1000), the symmetric nature is becoming more obvious.***
3) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of of the sample median Xmed.***
4) ***Clearly the height of of the histogram remains almost same for any repetition number say R=100,500 or 1000 and any fixed n=1000. Thus for any Exponential distn with fixed mean,it is an indication that asymptotic variance of Xmed is independent with repetition number.***

ii) Exp(mean=1) -



Findings-

1) ***For relatively small replication number, (say R=100), the frequency density histogram is more or less symmetric.***
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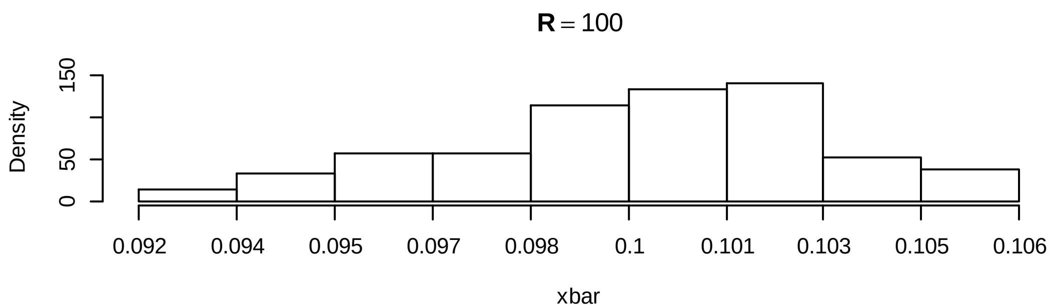
3) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of of the sample median Xmed.***
4) ***Clearly the height of of the histogram remains almost same for any repetition number say R=100,500 or 1000 and any fixed n=1000. Thus for any Exponential distn with fixed mean,it is an indication that asymptotic variance of Xmed is independent with repetition number.***
5) ***As the mean is increased , the height of the histograms are decreasing , it may be an indication that increasing population mean of an Exponential distribution also increases the variance of Xmed.***

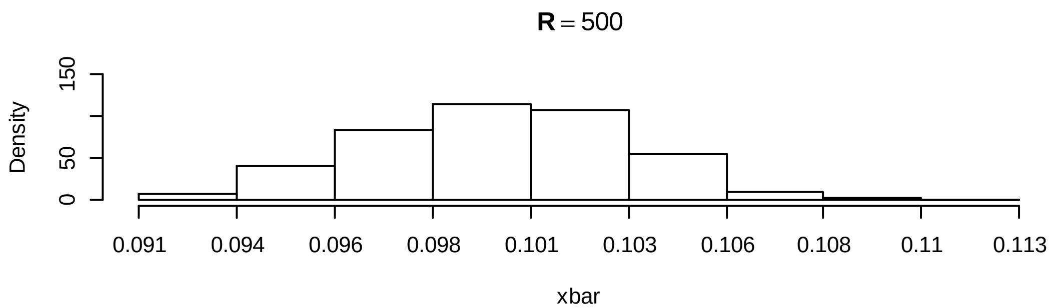
Conclusions-

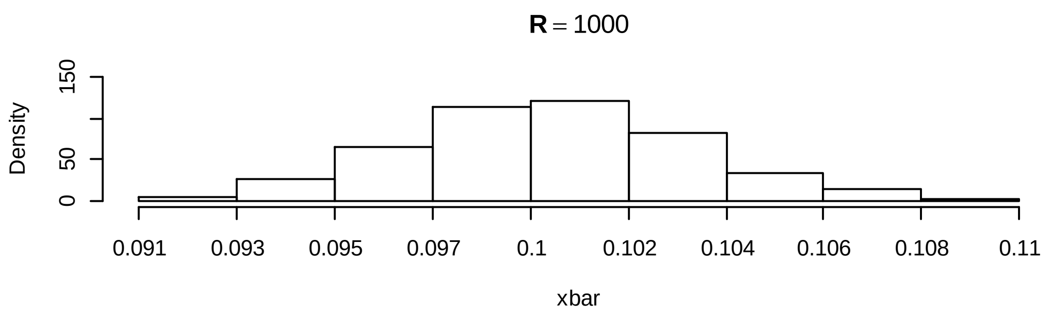
1) ***Irrespective of the population mean of Exponential distn, for relatively small repetition number(say R=100), the frequency density histogram is more or less symmetric, as the number of repetition is increased, (R=500 or 1000), keeping the sample size fixed (n=1000),the symmetric nature is becoming more obvious.***
2) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample Xmed.***
3) ***The height of the histogram remains almost same for any repetition number (R=100,500,1000) and fixed n(say n=1000), for any Exponential population with fixed mean. This indicates that asymptotic variance of Xmed is independent of repetition R.***

B.MEAN-

i) Exp(mean=0.1)



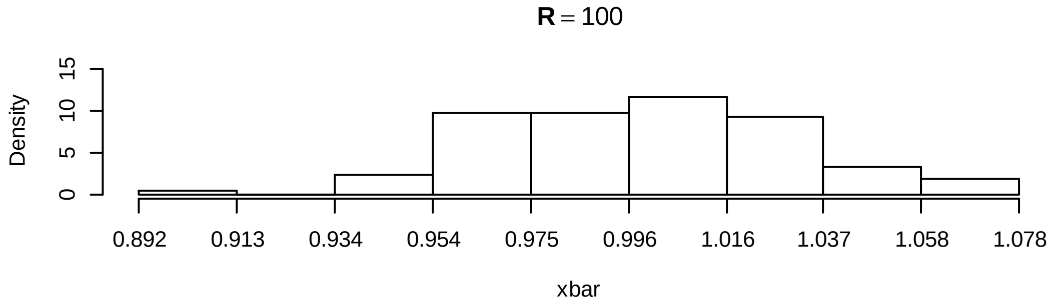


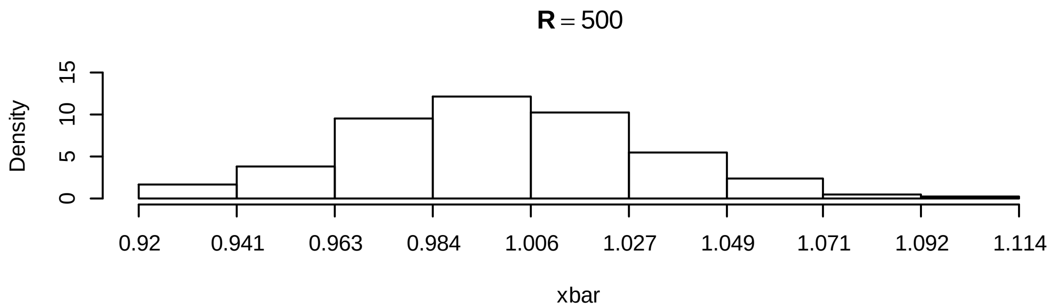


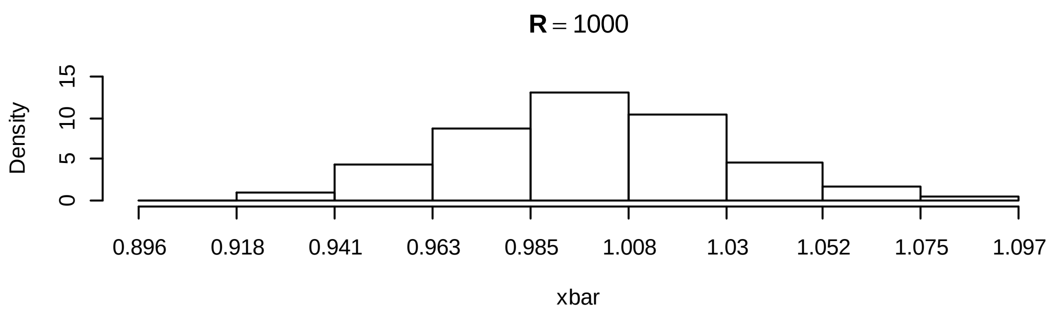
Findings-

1) ***For small replication number, (R=100), the frequency density histogram is slightly negatively skewed.***
2) ***As the repetition number is increased keeping the sample size n fixed, the histogram becomes symmetric.***
3) ***The symmetric nature of the histogram for large R is an indication of normality.***
4) ***The height of the histograms remains same for any R and a fixed n, for any Exponential population with certain mean, this indicates that asymptotic variance of Xbar is independent with R.***

ii) Exp(mean=1) -







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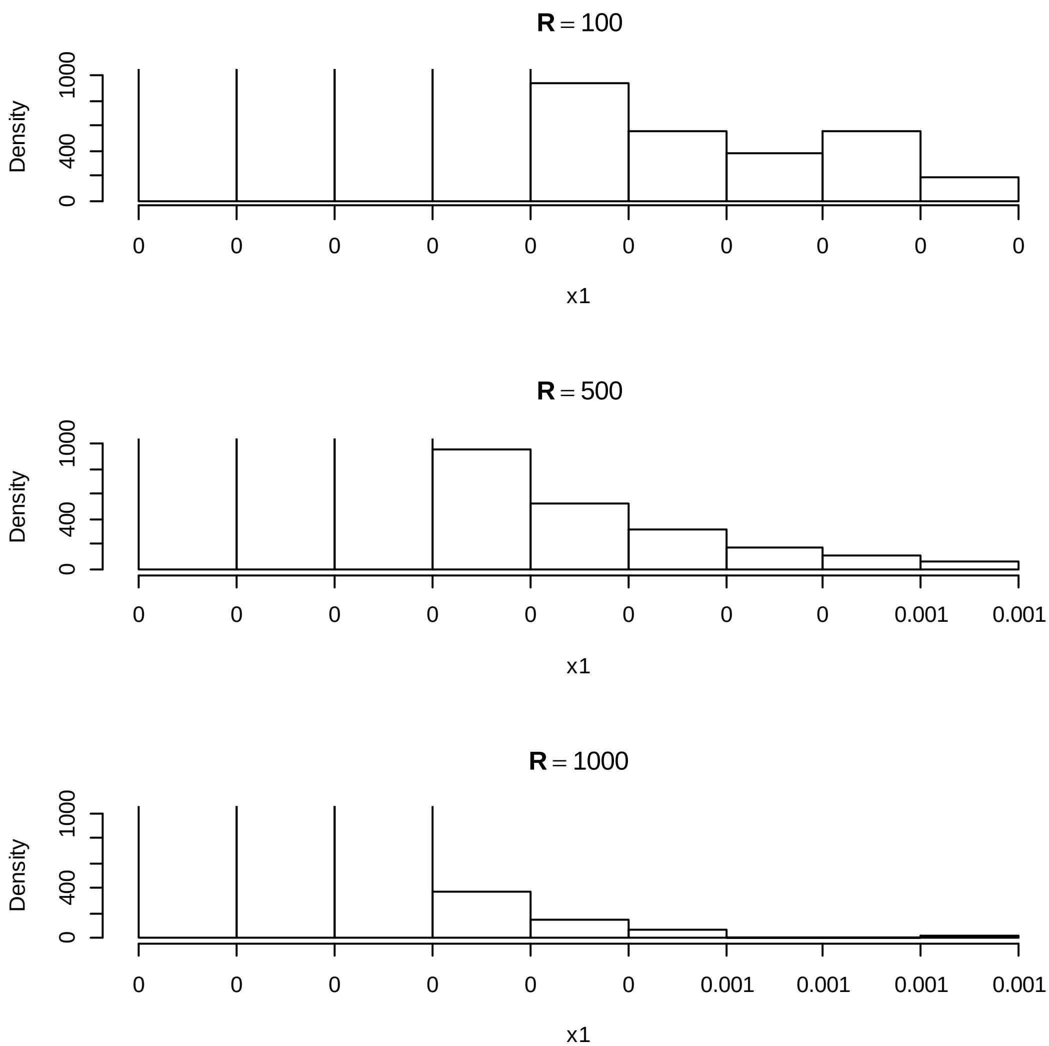
5) ***As the mean is increased , the height of the histograms are decreasing , it may be an indication that increasing population mean of an Exponential distribution also increases the variance of Xbar.***

Conclusions-

1) ***Irrespective of the population mean of Exponential distn, for relatively small repetition number(say R=100), the frequency density histogram is more or less symmetric, as the number of repetition is increased, (R=500 or 1000), keeping the sample size fixed (n=1000),the symmetric nature is becoming more obvious.***
2) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample Xbar..***
3) ***The height of the histogram remains almost same for any repetition number (R=100,500,1000) and fixed n(say n=1000), for any Exponential population with fixed mean. This indicates that asymptotic variance of Xbar is independent of repetition R.***

C.X(1)-

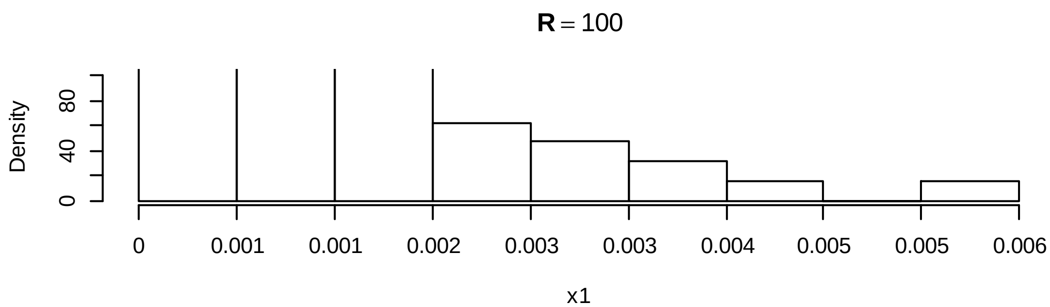
i) Exp(mean=0.1) -

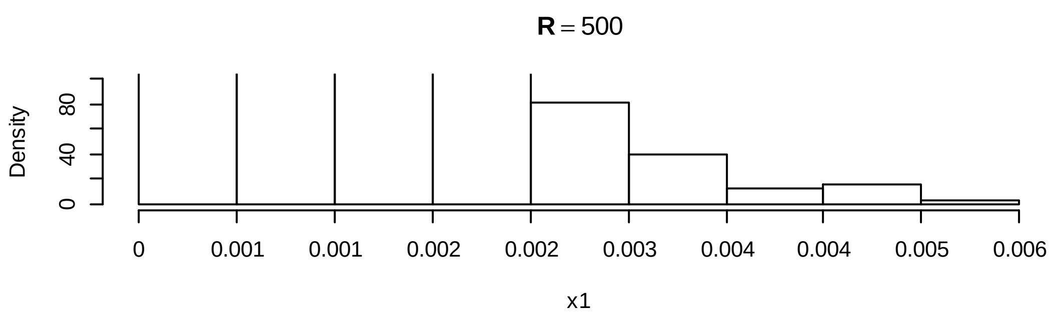


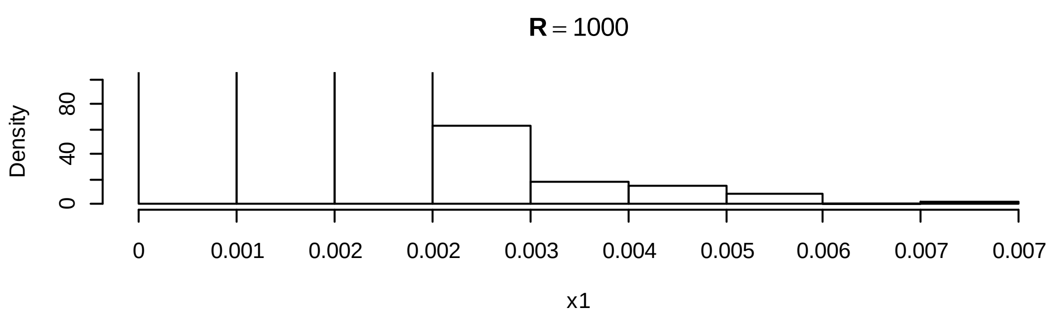
Findings-

1) ***For relatively small replication number (say , R=100), the frequency density histogram is unbounded above, i.e., for some x(1) values density is undefined.***
2) ***As repetition no. is increased there is no such change in the behaviour.***
3) ***Deviation from symmetric nature indicates a clear deviation from asymptotic normal distribution of sample X(1) for fixed large sample size , n=1000.***

ii) Exp(mean=1) -







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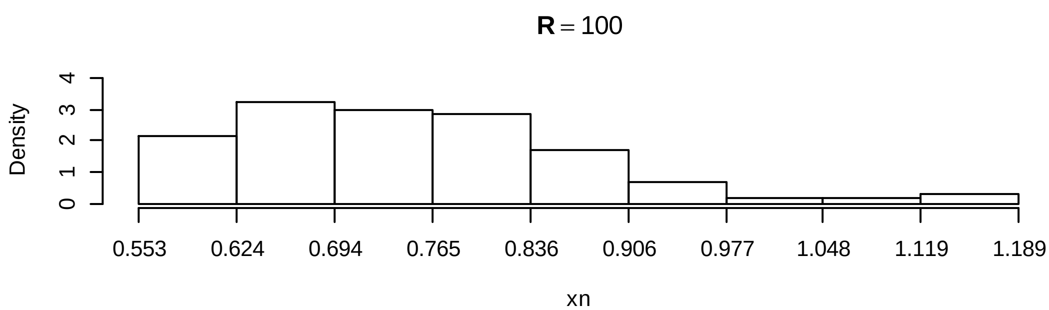
Conclusions-

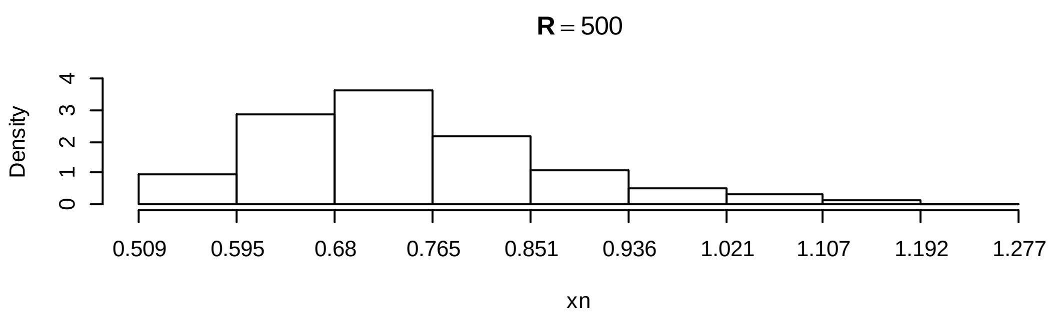
1) ***Irrespective of the population mean For relatively small replication number (say , R=100), the frequency density histogram is unbounded above, i.e., for some x(1) values density is undefined.***
2) ***As repetition no. is increased there is no such change in the behaviour.***

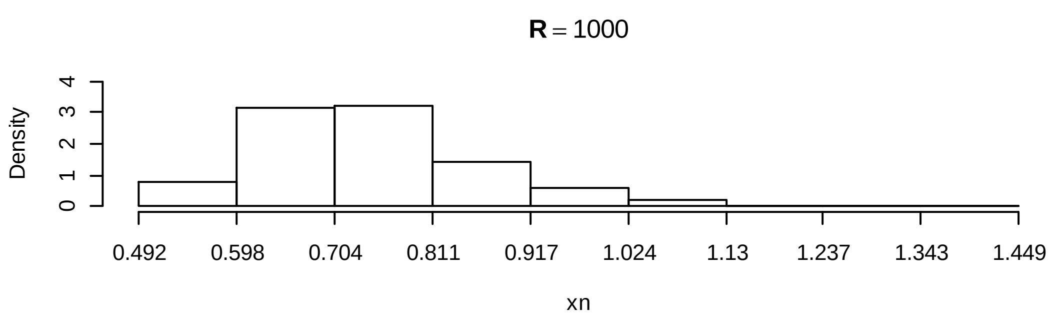
3) Deviation from symmetric nature indicates a clear deviation from asymptotic normal distribution of sample X(1) for fixed large sample size , n=1000.

D.X(n)-

i) Exp(mean=0.1)





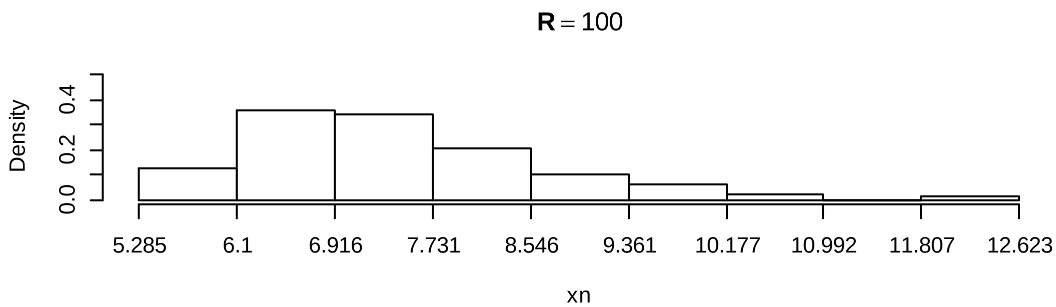


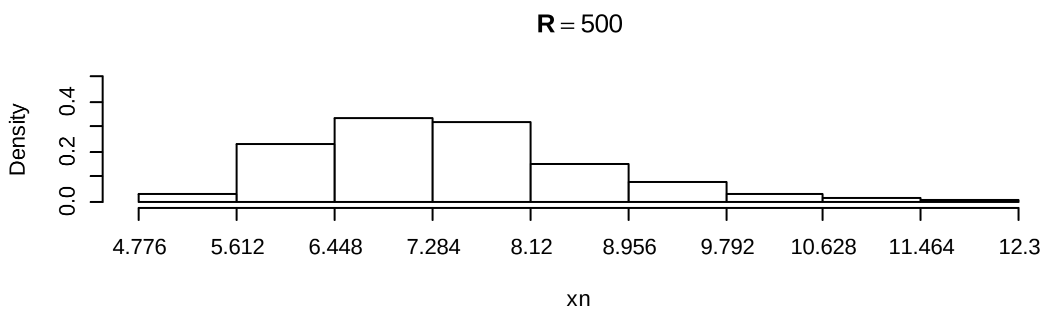
Findings-

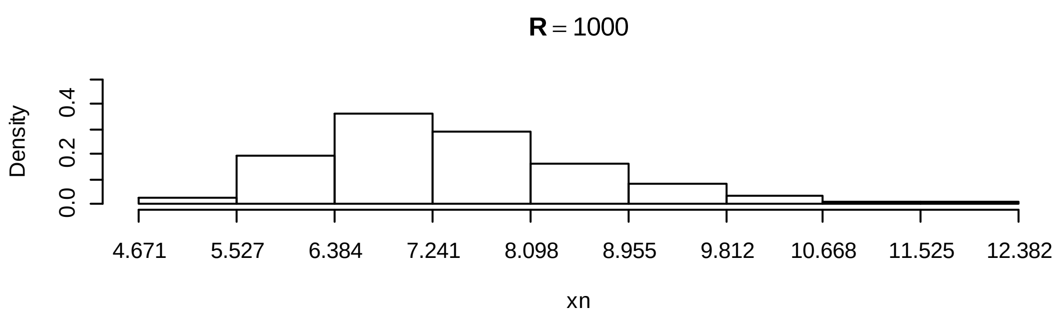
1) For relatively small replication number (say , R=100), the frequency density histogram is more or less positively skewed .

2) *As the no. of repetition is increased say R=500 or 1000, the skewness becomes clearer.*
3) *Deviation from symmetric nature indicates a clear deviation from asymptotic normal distribution of sample X(n) for fixed large sample size , n=1000.*

ii) Exp(mean=1) -







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Conclusions-

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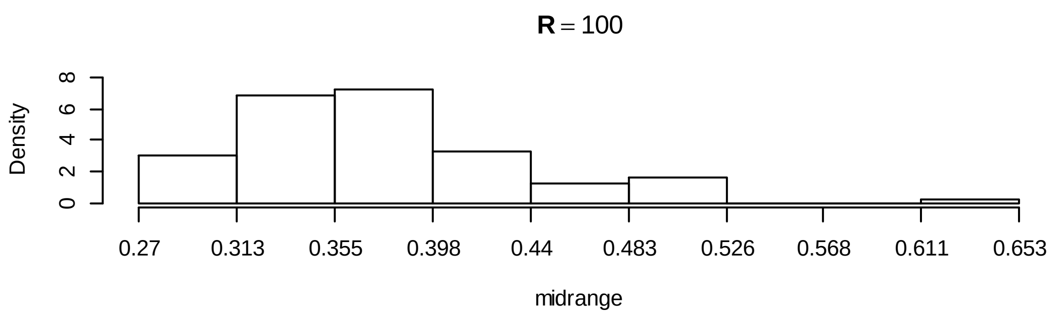
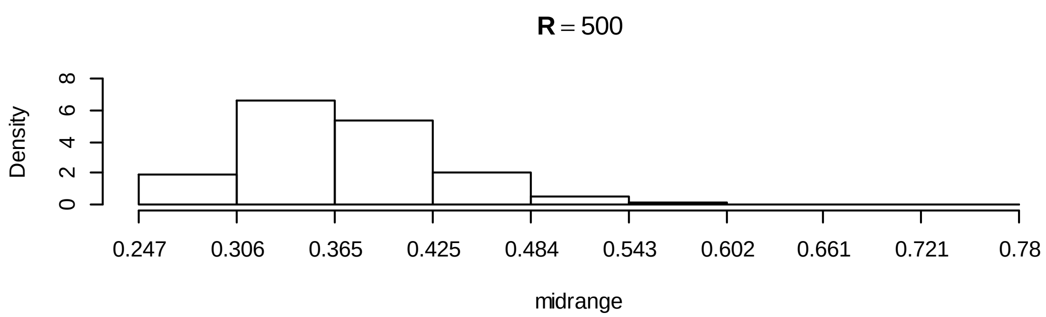
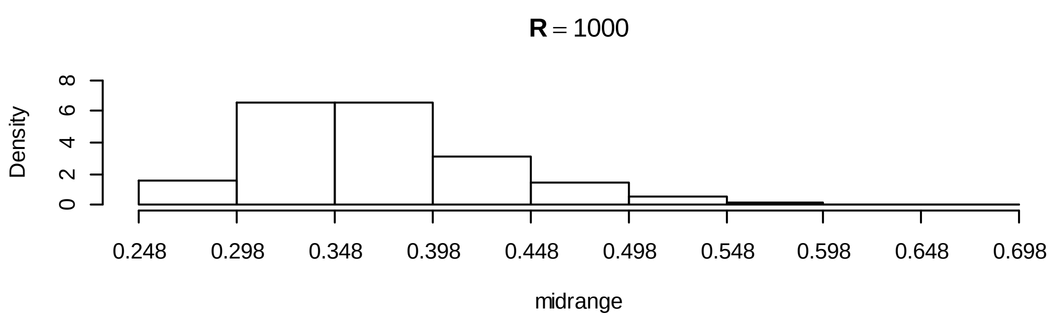
E.MIDRANGE-

i) Exp(mean=0.1) -







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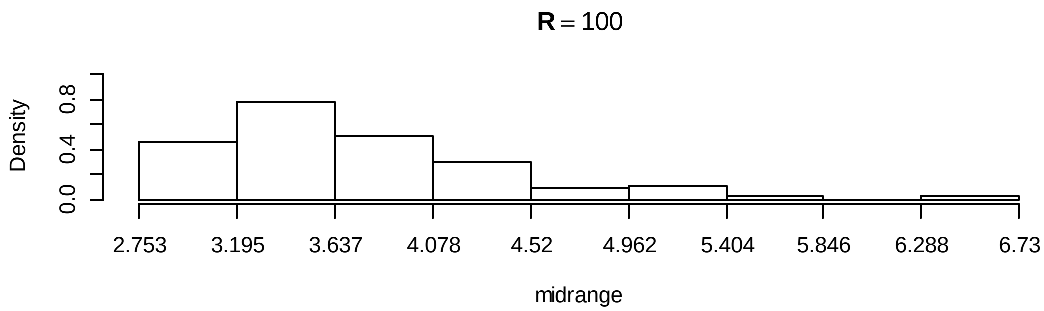
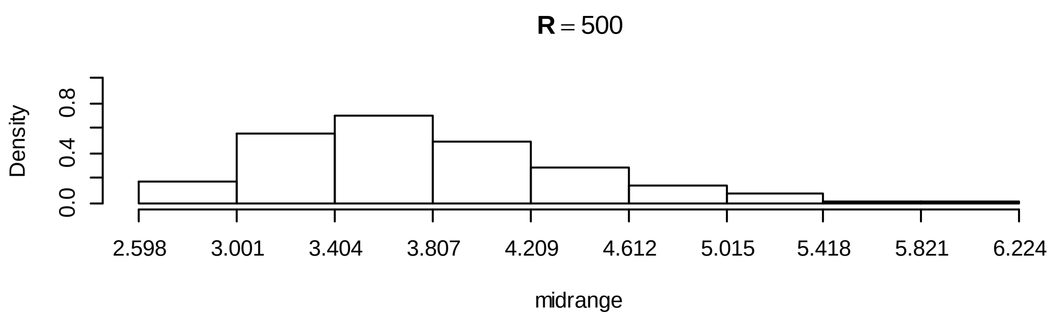
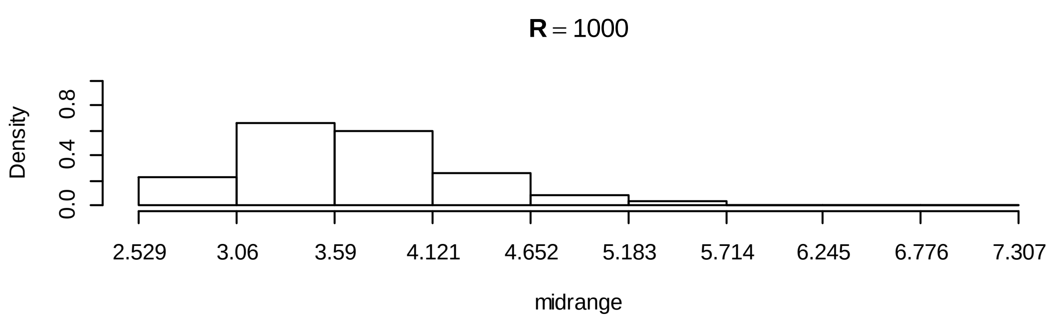
ii) Exp(mean=1) -







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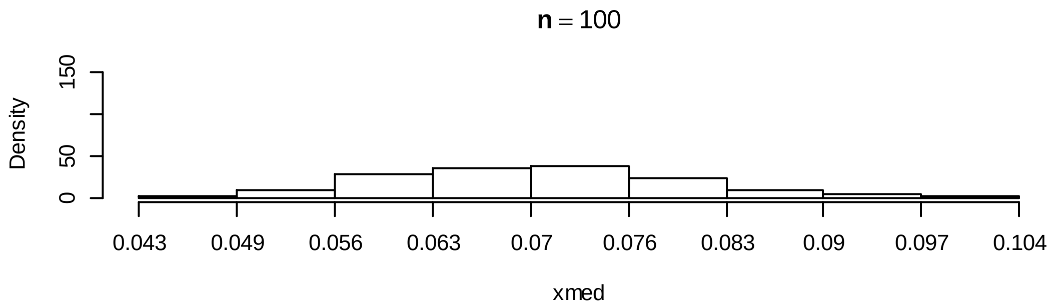
4) *As the mean is increased, the height of the histograms are decreased.*

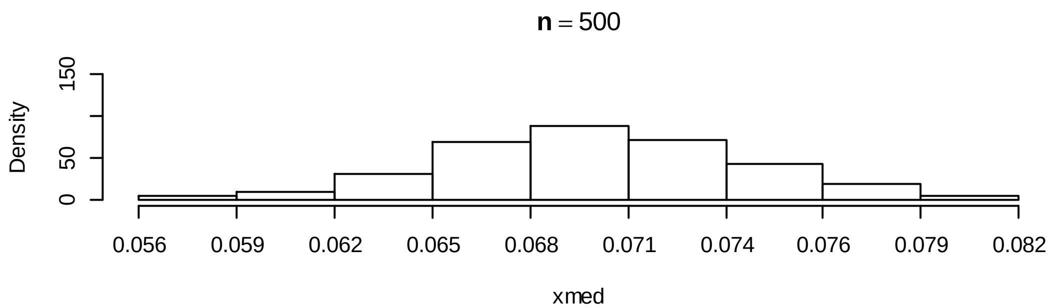
Now we will calculate Convergence in distribution for R=1000, n=100,500,1000

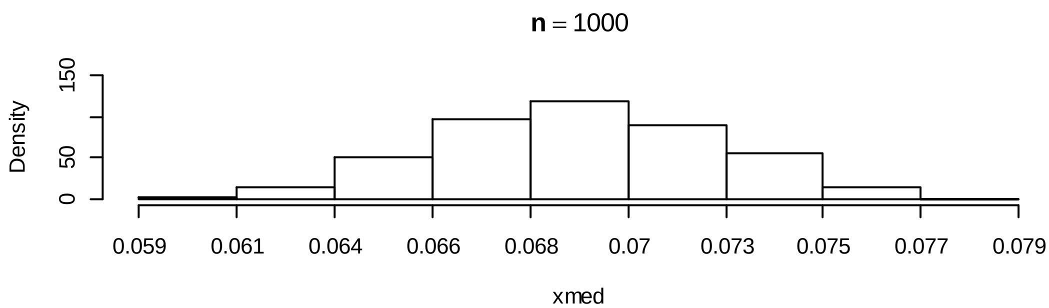
Here we will check the convergence in distribution for some well known statistics like sample mean , median , minimum , maximum , midrange etc for Exponential population with certain mean and make a significant comparison between their large sample behaviour and their dependency on parameters for fixed repetition number R=1000 and n=100,500,1000 .

A.MEDIAN-

i) Exp(mean=0.1) -



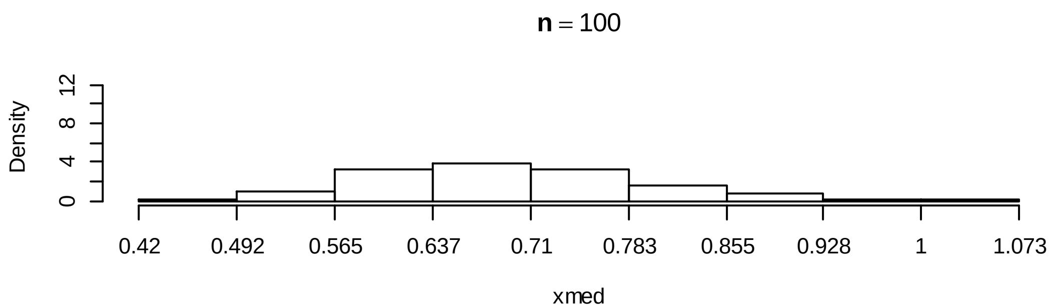


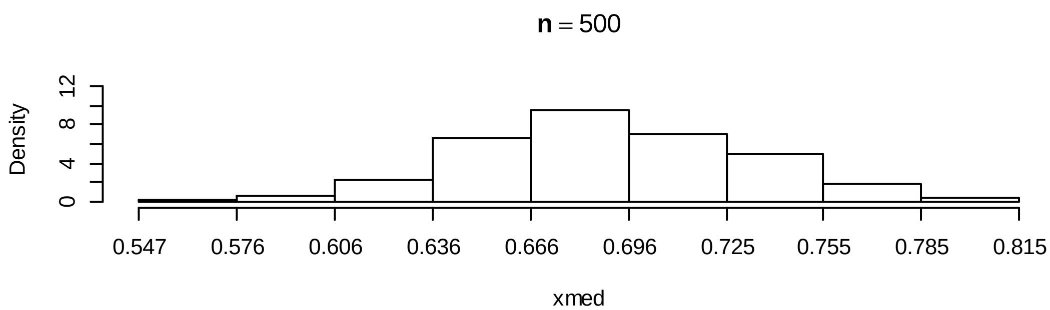


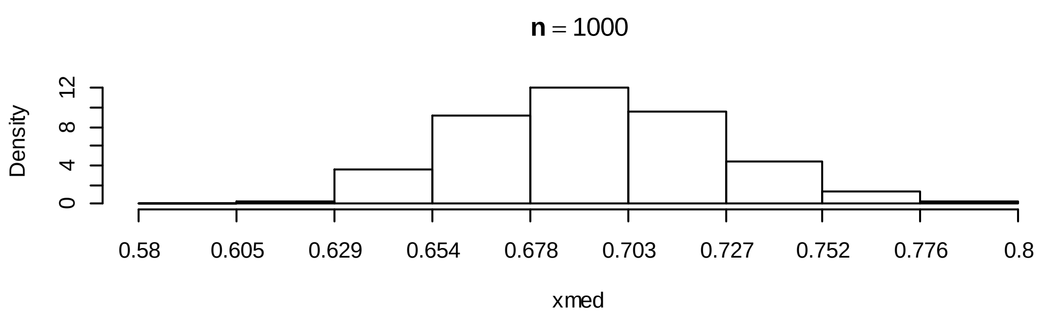
Findings-

1) ***For relatively small sample size , say n=100 , the frequency density histogram is more or less symmetric.***
2) ***As the sample size n is increased, (n=500 or 1000), keeping the repetition number fixed(R=1000), the symmetric nature is becoming more obvious.***
3) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of of the sample median Xmed.***
4) ***For an Exponential distn with fixed mean, as the sample size n is increasing , keeping repetition number R fixed, the height of the histograms is also increasing (unlike in the case of fixed n and varying R).***

ii) Exp(mean=1) -







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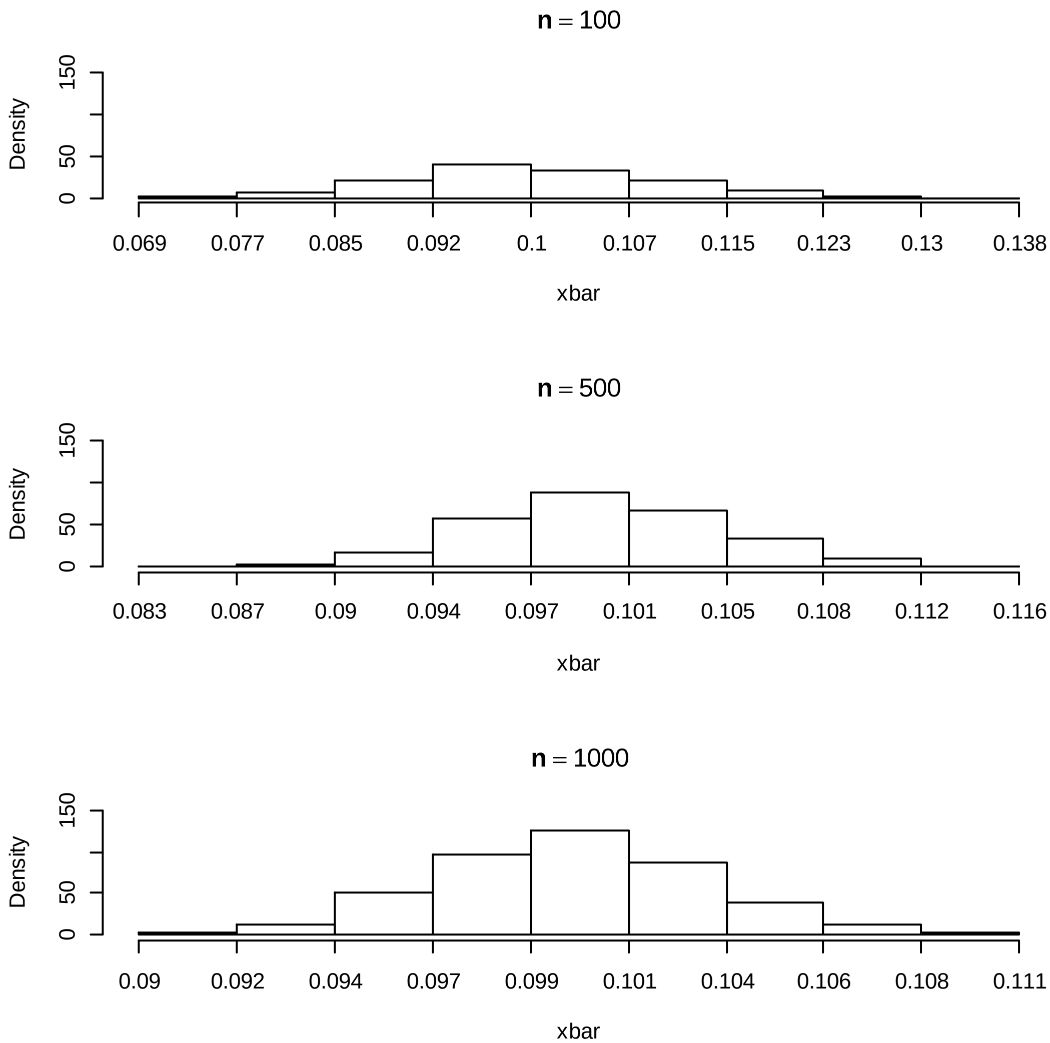
Conclusions-

1) ***Irrespective of the population mean of Exponential distn, for relatively small sample size, say n=100,for fixed R=1000, the frequency density histogram is more or less symmetric, as the sample size n is icreased, keeping the repetition no. R fixed,(R=1000), the symmetric nature is becoming more obvious.***
2) ***The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample Xmed.***
3) ***For any Exponential population as we increase the population mean, the variance of Xmed also increases.***

B.MEAN-

i) Exp(mean=0.1) –


n = 100
Density
0
50
150
0.069
0.077
0.085
0.092
0.1
0.107
0.115
0.123
0.13
0.138
xbar
n = 500
Density
0
50
150
0.083
0.087
0.09
0.094
0.097
0.101
0.105
0.108
0.112
0.116
xbar
n = 1000
Density
0
50
150
0.09
0.092
0.094
0.097
0.099
0.101
0.104
0.106
0.108
0.111
xbar

Findings-

1) ***For relatively small sample size , say n=100 , the frequency density histogram is more or less symmetric.***
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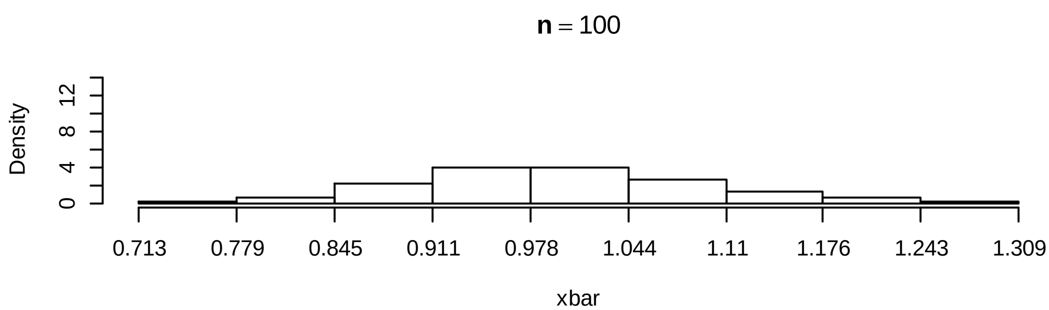
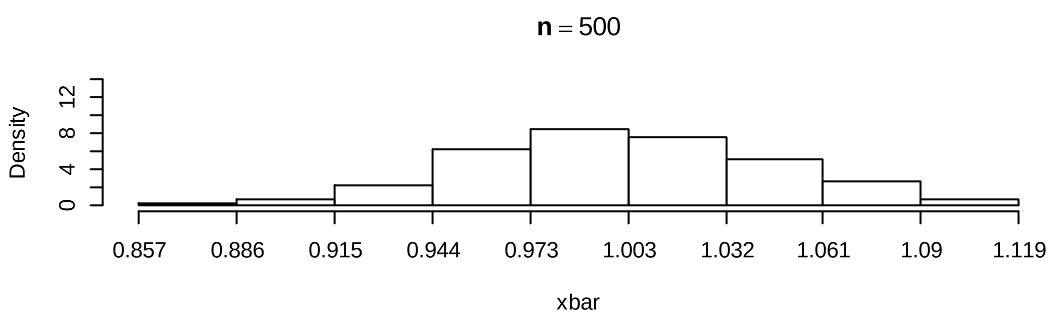
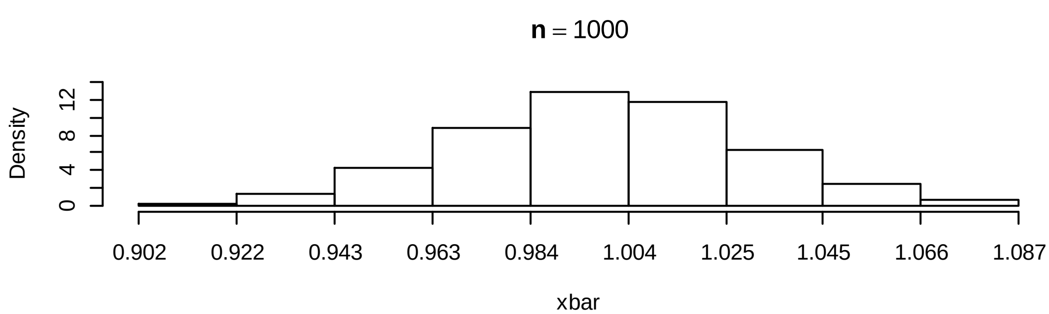
ii) Exp(mean= 1) -







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Conclusions-

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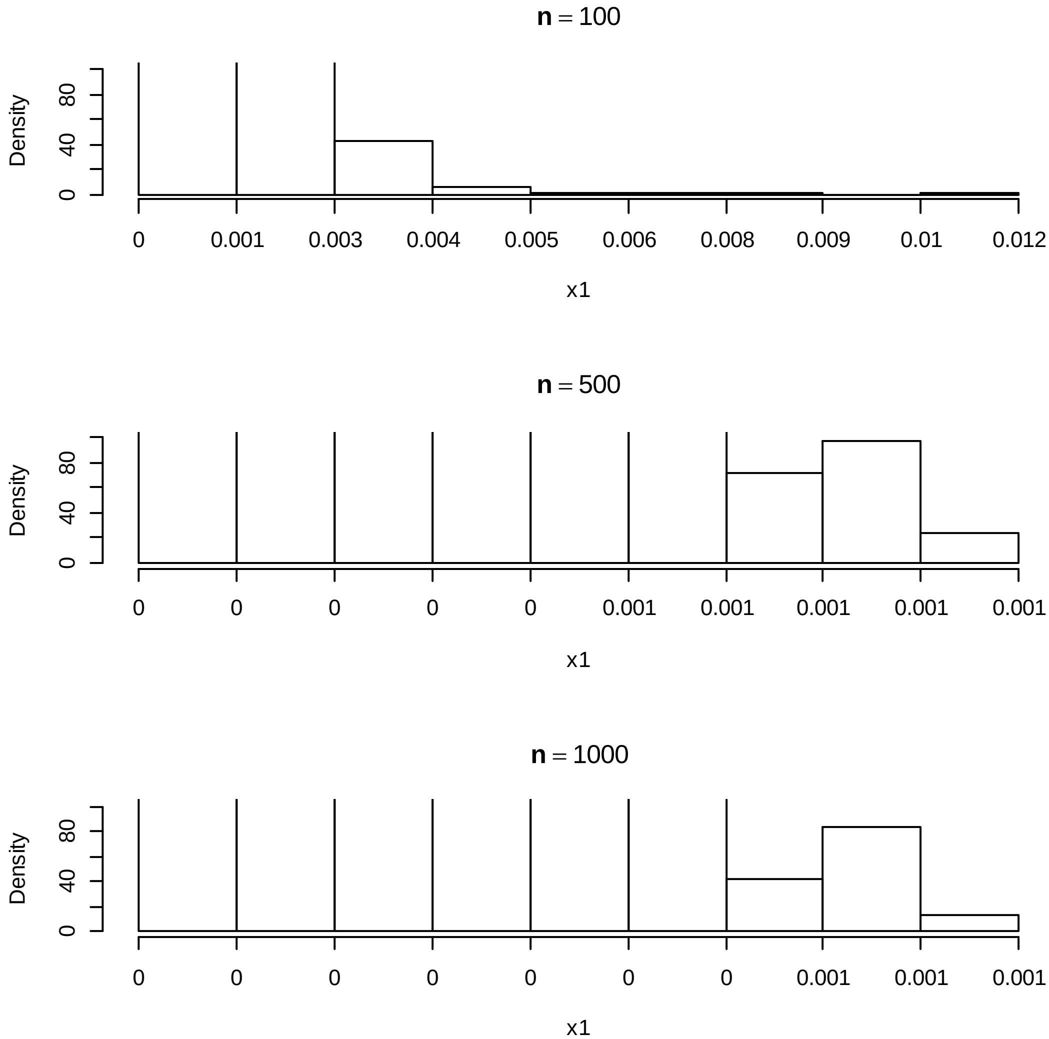
- *The symmetric nature of the histogram indicates asymptotic normal behaviour of the sample Xbar.*
- *For any Exponential population as we increase the population mean, the variance of Xbar also increases.*

C.X(1)-

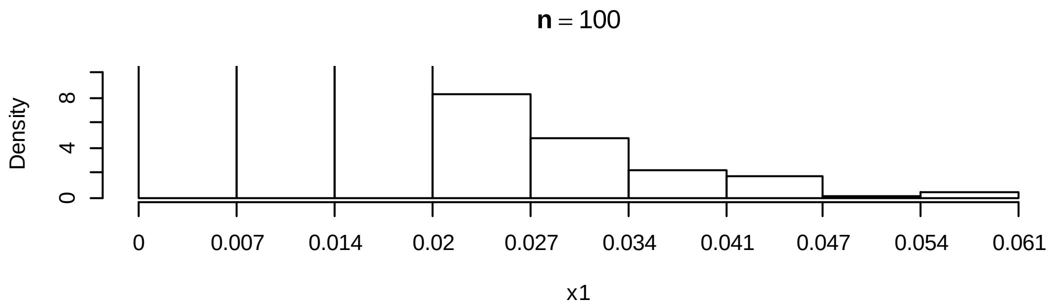
i) Exp(mean=0.1) -

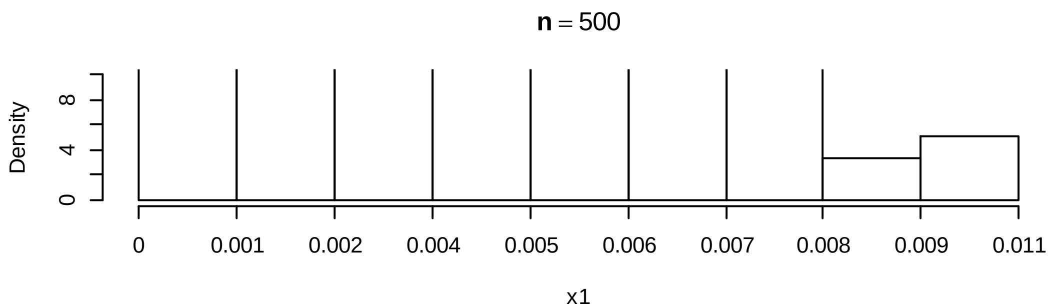


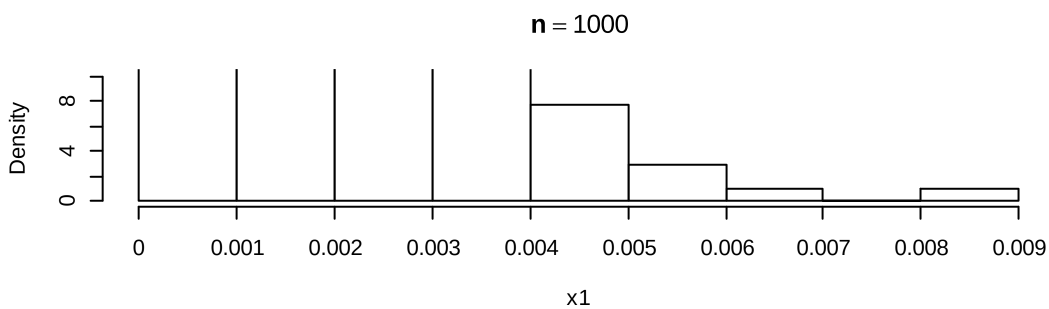
Findings-

1) ***For relatively small sample size n (say , n=100), the frequency density histogram is unbounded above, i.e., for some x(1) values density is undefined.***
2) ***As repetition no. is increased there is no such change in the behaviour.***
3) ***Deviation from symmetric nature indicates a clear deviation from asymptotic normal distribution of sample X(1) for fixed Replication number say , R=1000.***

ii) Exp(mean=1)







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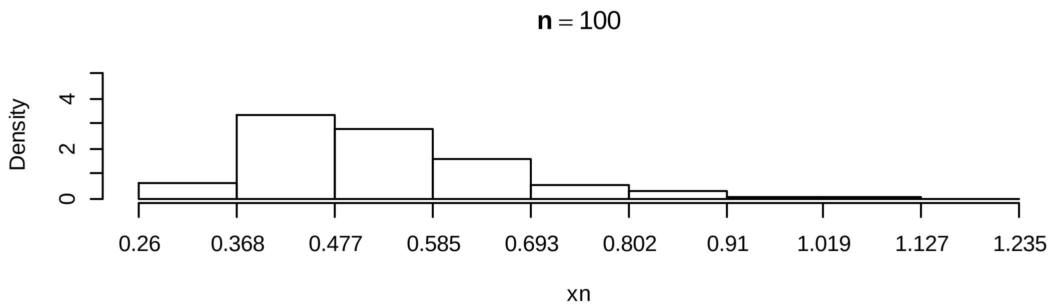
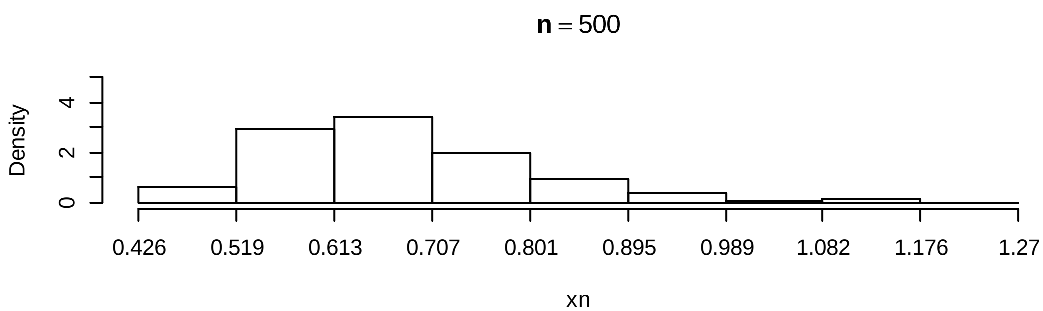
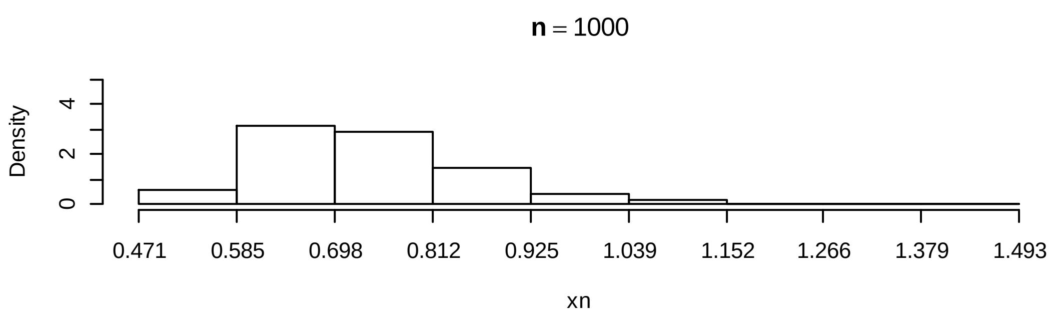
D.X(n)-

i) Exp(mean=0.1) -







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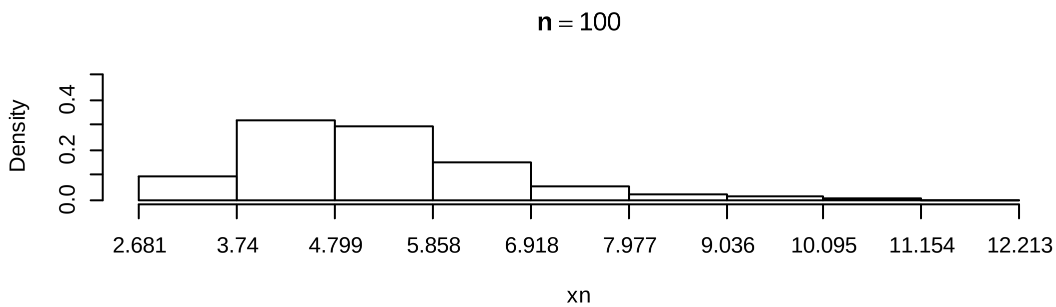

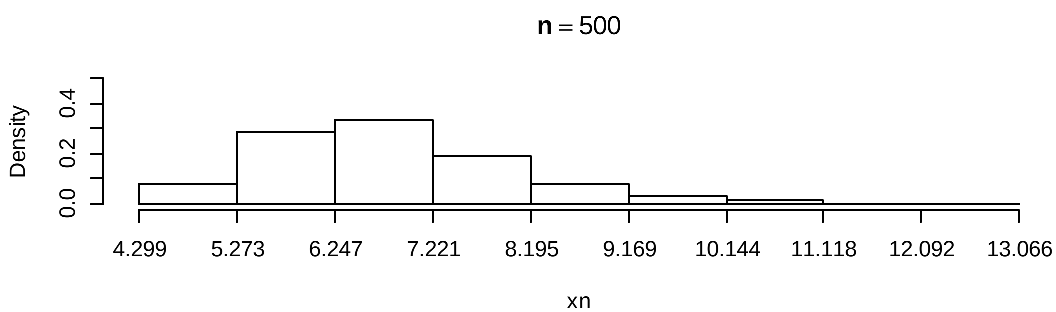

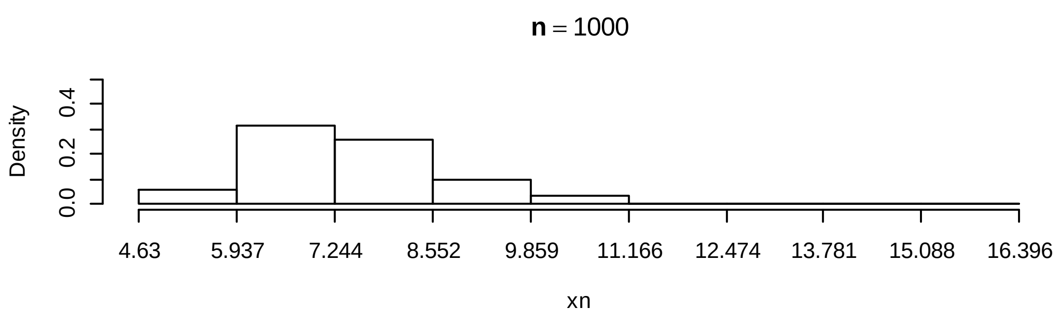

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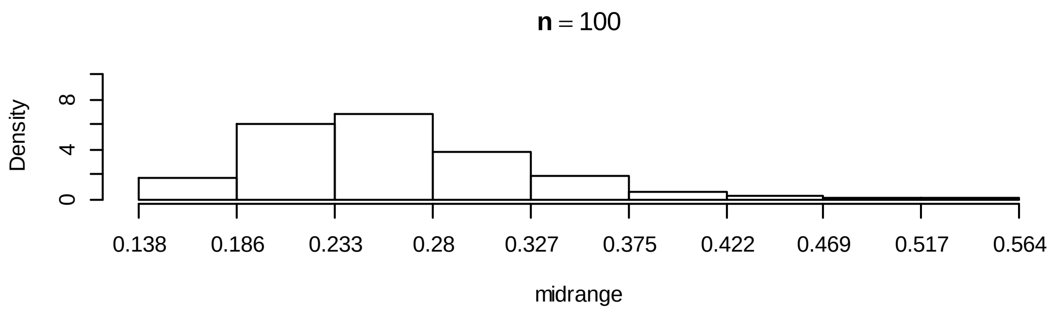
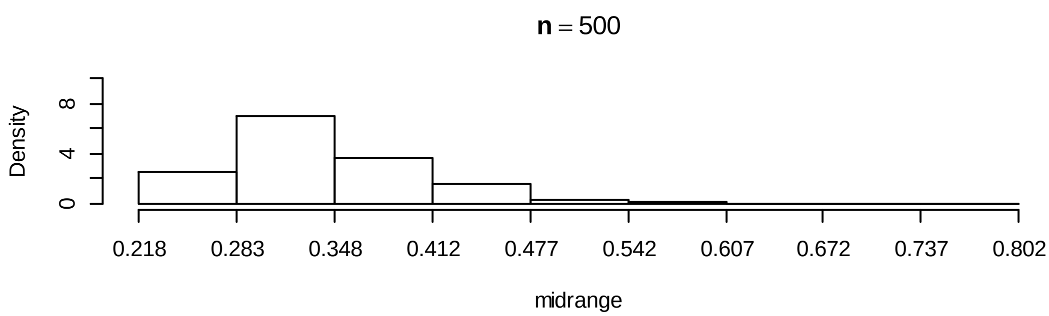
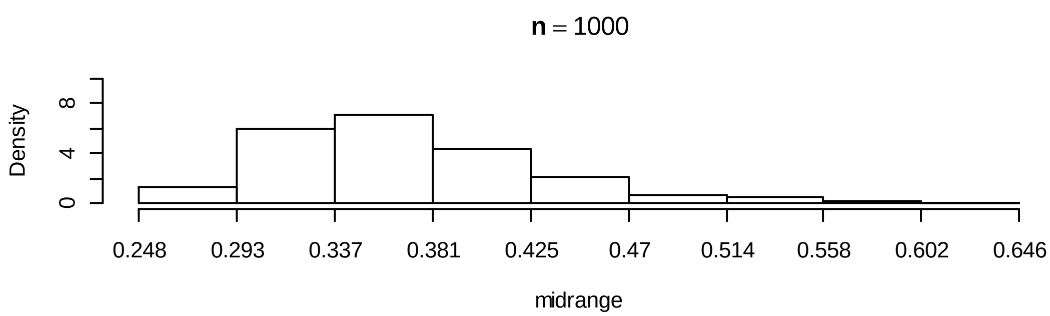
i) MIDRANGE-

i) Exp(mean=0.1) -







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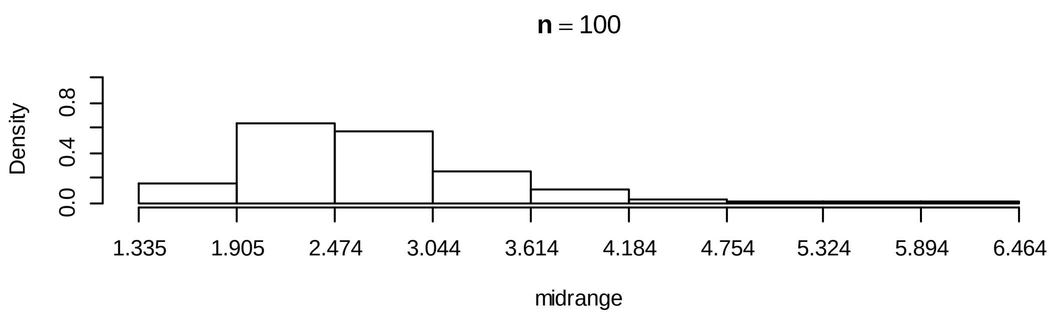
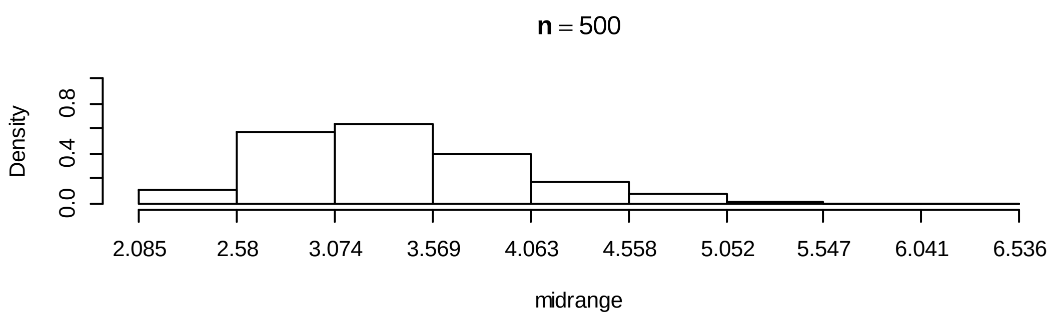
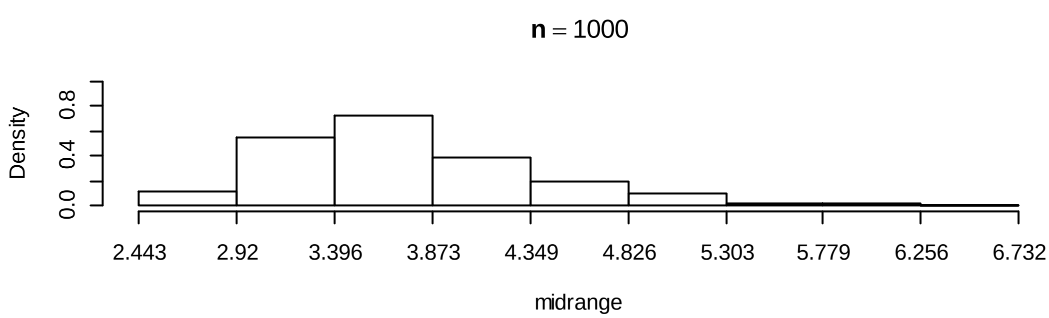
ii) Exp(mean=1) -







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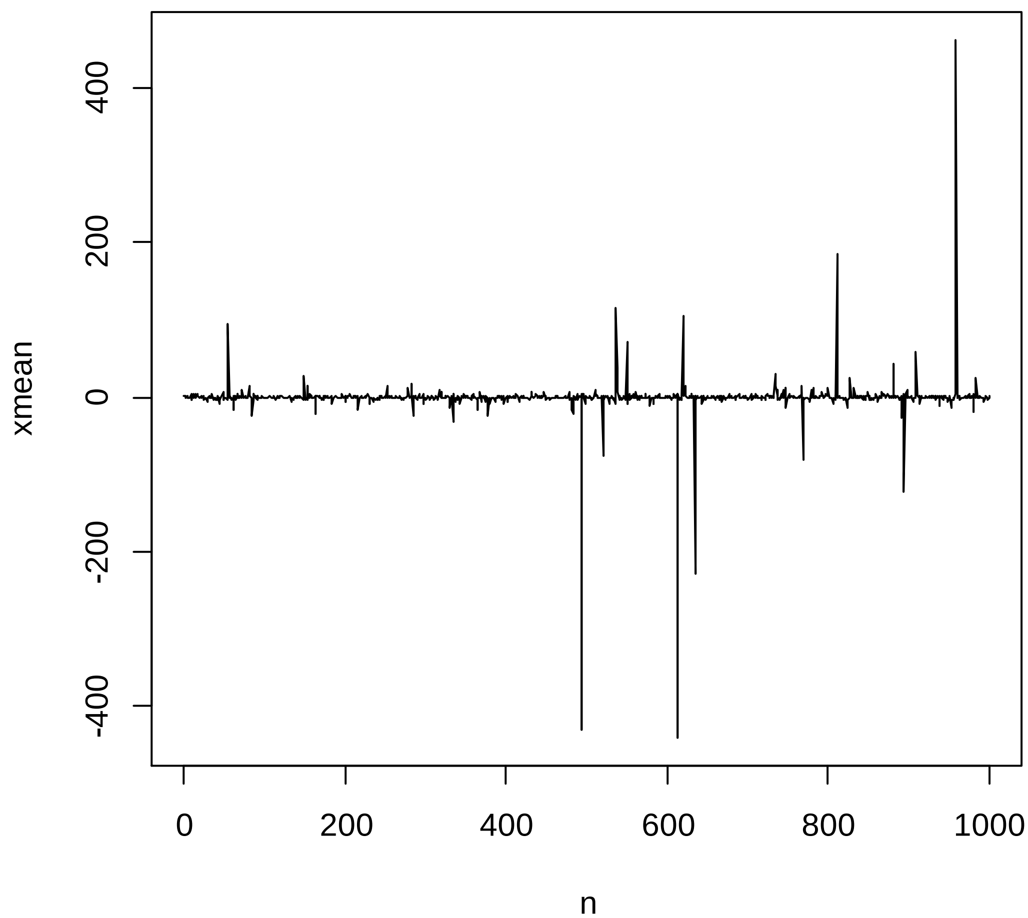
CAUCHY DISTRIBUTION

Here we will find guess limits –

The term "guess limit" is used to represent that particular value to which the statistics tends to converge as the sample size n tends to infinity. Here we are trying to find guess limits for sample mean , median , X(n) , X(1) from the population C(0,1) , C(1,1) , C(1,2) to make a well defined comparison between the statistical behaviour of the statistics

A.MEAN-

i) C(0,1)

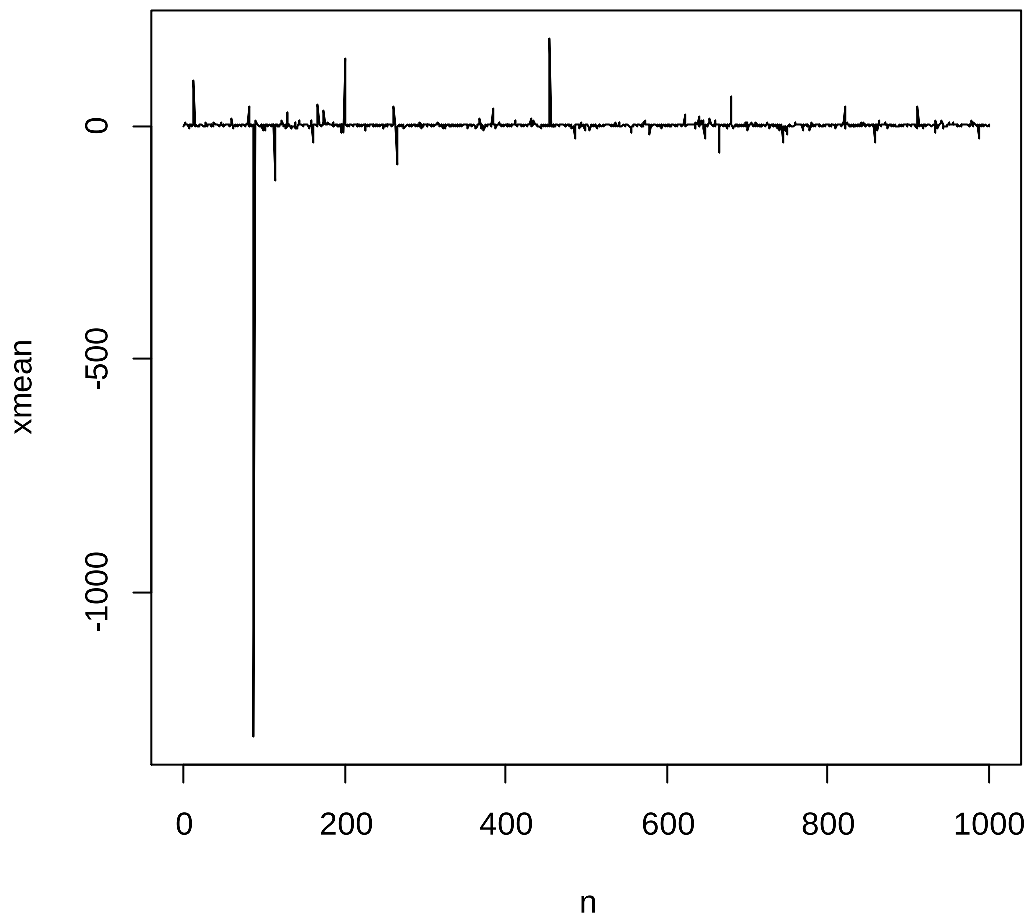


Observations-

1) ***For C(0,1), the sample mean Xbar tends to converge to its location parameter '0'.***

2) As the sample size is increased, the convergence becomes clearer.

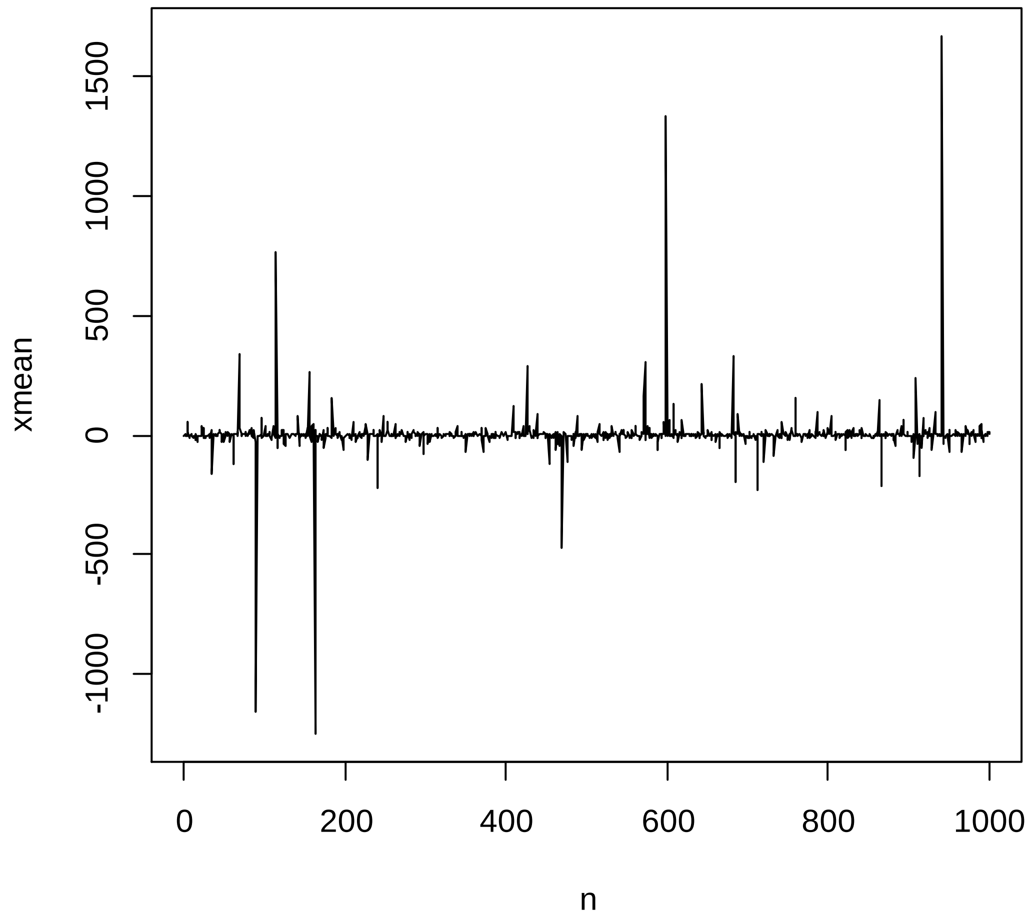
ii) C(1,1)-



Observations-

1) **As the sample size n tends to infinity, the sample mean tends to '0', which is not the location parameter of C(1,1) distribution**
2) **With the increment of the sample size, the convergence becomes more obvious.**

iii) C(1,5)-



Observations-

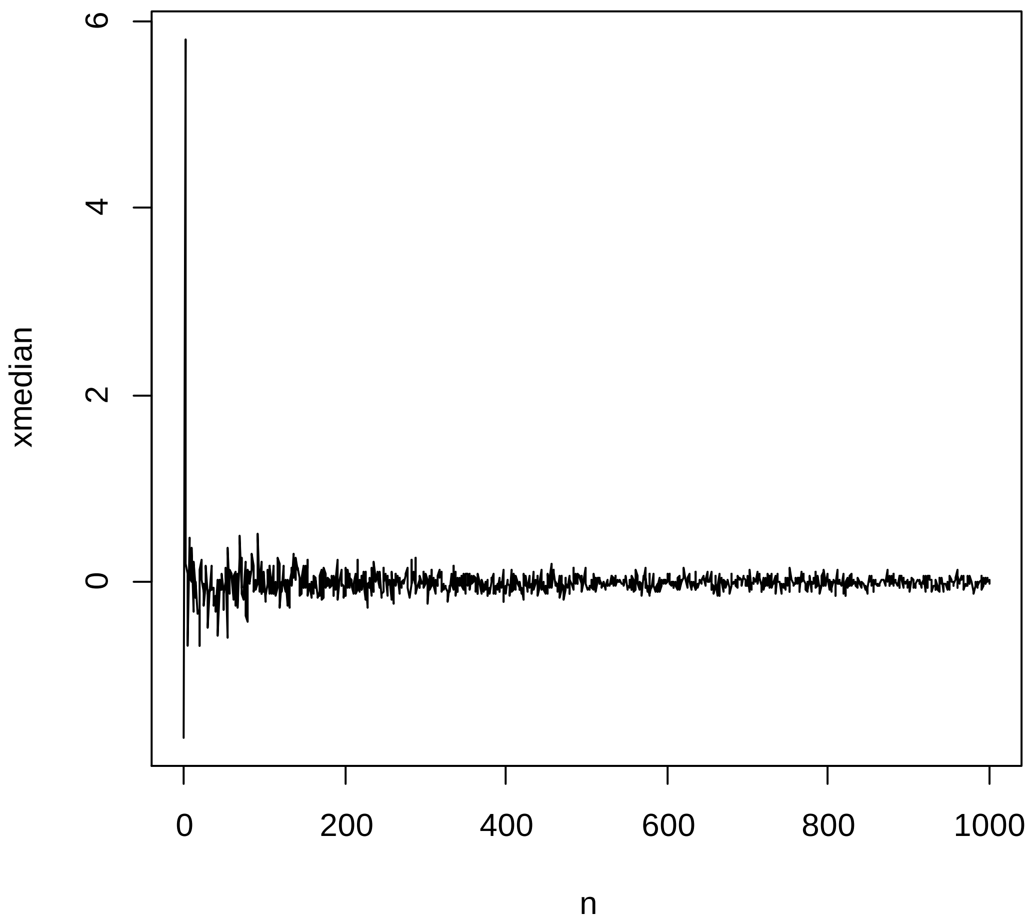
1) **As the sample size n tends to infinity, the sample mean tends to '0', which is not the location parameter of C(1,5) distribution.**
2) **With the increment of the sample size, the convergence becomes more obvious.**
3) **As the scale parameter is increased , keeping the location parameter fixed, the fluctuation of Xbar or Xmean values around 0 is increased.**

Conclusions-

1) Irrespective of the location and parameter of Cauchy distn, as the sample size n is increased, the sample mean tends '0',which may not be the location parameter always.
2) **With the increment of the sample size, the convergence becomes more obvious.**
3) As the scale parameter is increased, keeping the location parameter fixed, the fluctuation of Xmean values around '0', increases.

B.MEDIAN-

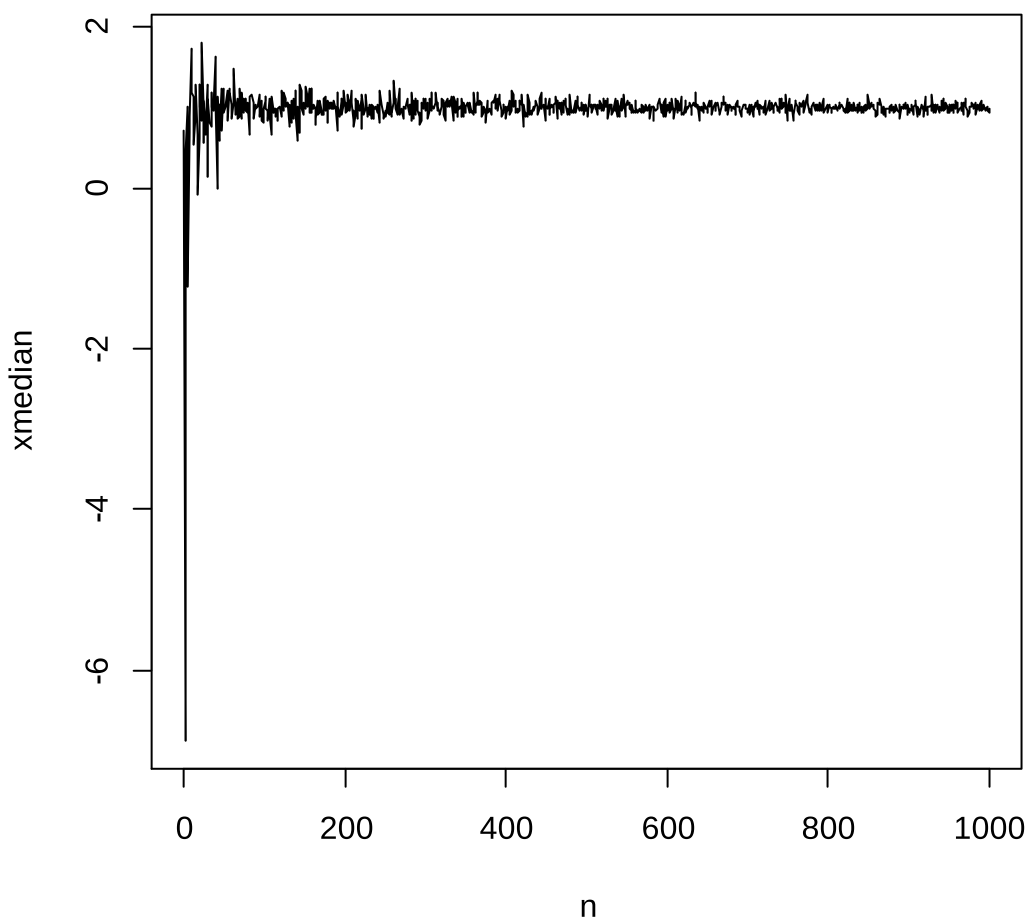
i) C(0,1)-



Observations-

1) **As n tends to infinity, the sample Xmedian values tends to ‘0’, which is the location parameter of C(0,1).**
2) **With the increment in sample size n, the convergence becomes more rapid and fast.**

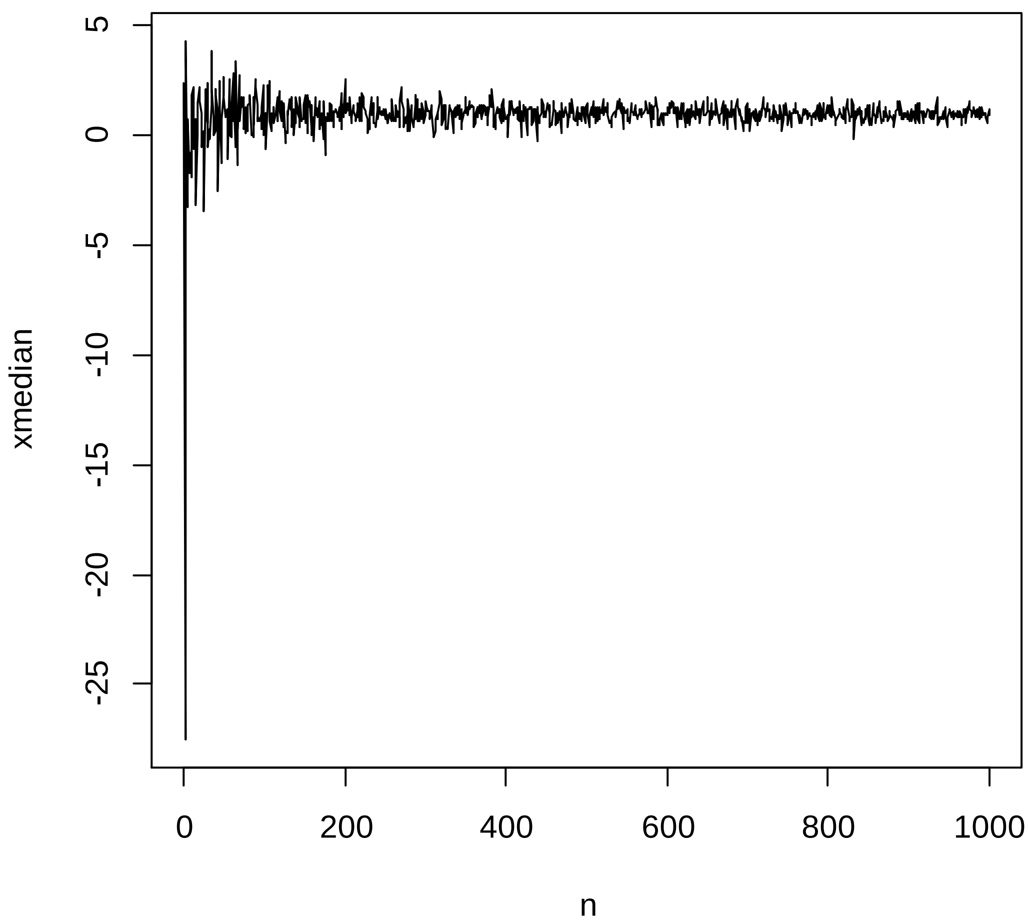
ii) C(1,1)-



Observations-

1) **As n tends to infinity, the sample Xmedian values tends to '1', which is the location parameter of C(1,1).**
2) **With the increment in sample size n, the convergence becomes more rapid and fast.**

iii) C(1,5)-



Observations-

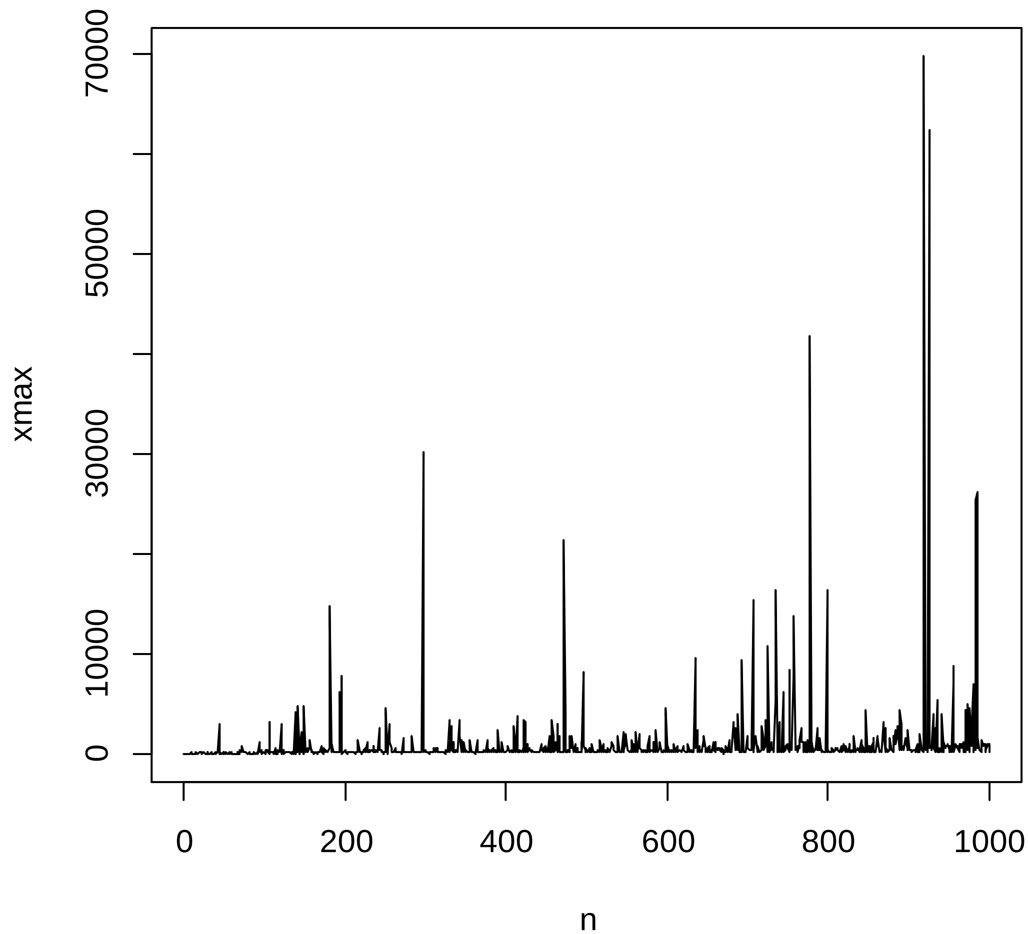
1) **As n tends to infinity, the sample Xmedian values tends to '0', which is the location parameter of C(0,1).**
2) **With the increment in sample size n, the convergence becomes more rapid and fast.**
3) **Though during transformation from C(1,1) to C(1,5), scale parameter is increased keeping location parameter fixed, there is no such significant effect6 on the rate of convergence of Xmedian values.**

Conclusions-

1)As n goes on increasing irrespective of the location and scale parameter of a Cauchy distn, (,say C(u , A) , u belongs to real line and A>0)), the sample Xmedian value tends to 'u', which is the location parameter of the parent population.
2) With the increment in sample size, the convergence becomes more fast and rapid.
3) Scale parameter has no such significant effect on convergence rate of Xmedian.

C.MAXIMUM-

i) C(0,1)-

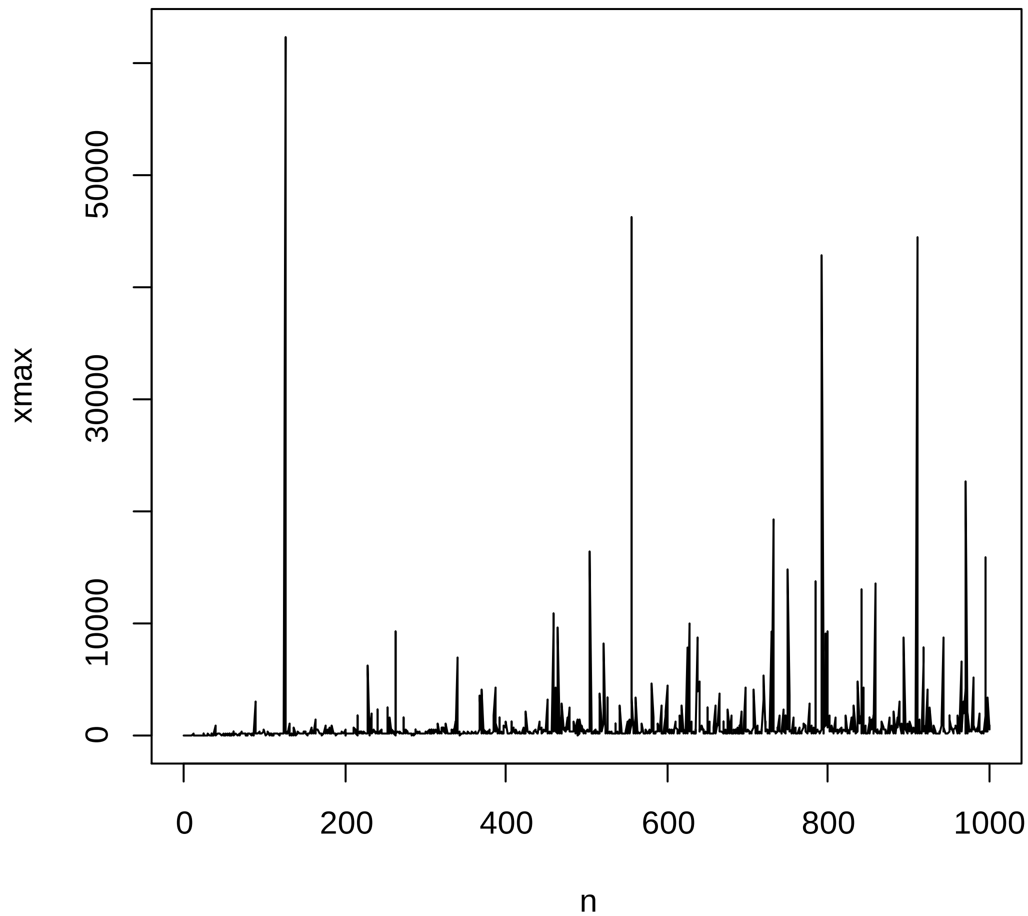


Observations-

1) ***As the sample size is increasing, Xmax values does not seem to converge to any particular value***

2) ***Though most of the values are clustering around 0, still, there are some values which are largely deviated from '0' and the clustering may occur around different values for different samples.***

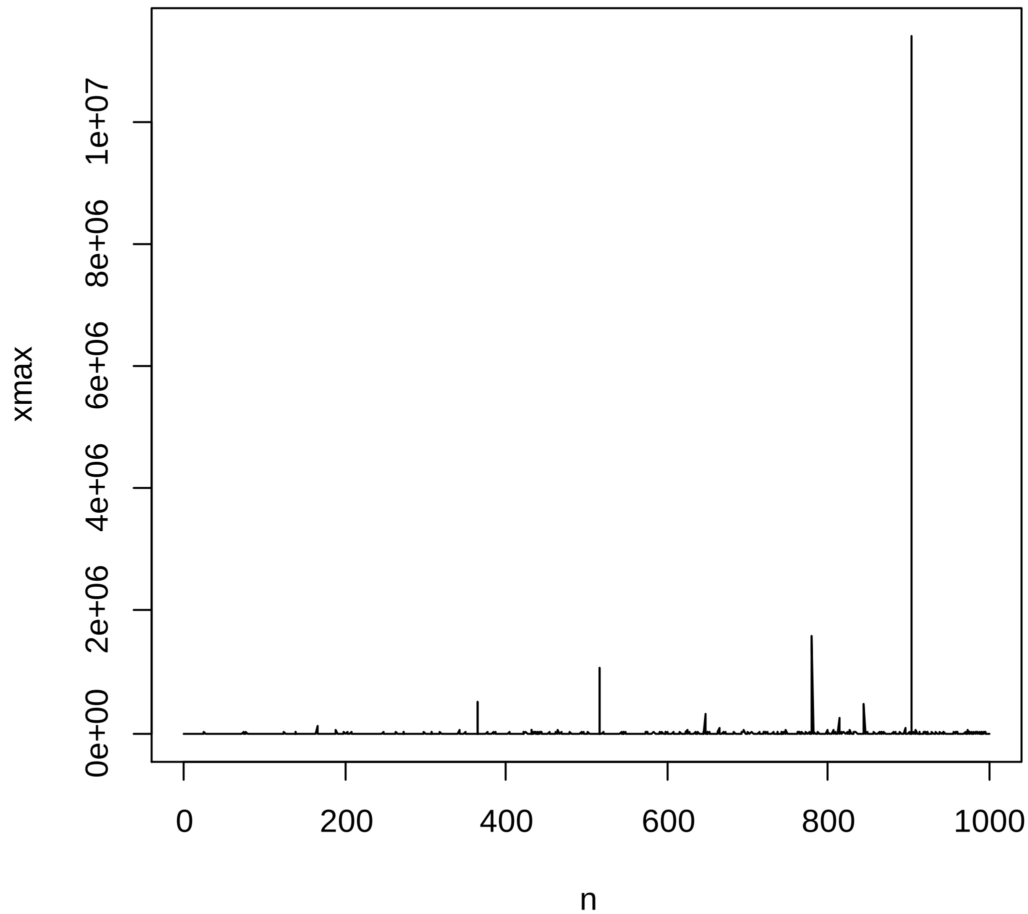
ii) C(1,1)-



Observations-

1) ***As the sample size is increasing, Xmax values does not seem to converge to any particular value.***
2) ***Though most of the values are clustering around 0, still, there are some values which are largely deviated from '0' and the clustering may occur around different values for different samples.***

iii) C(1,5)-



Observations-

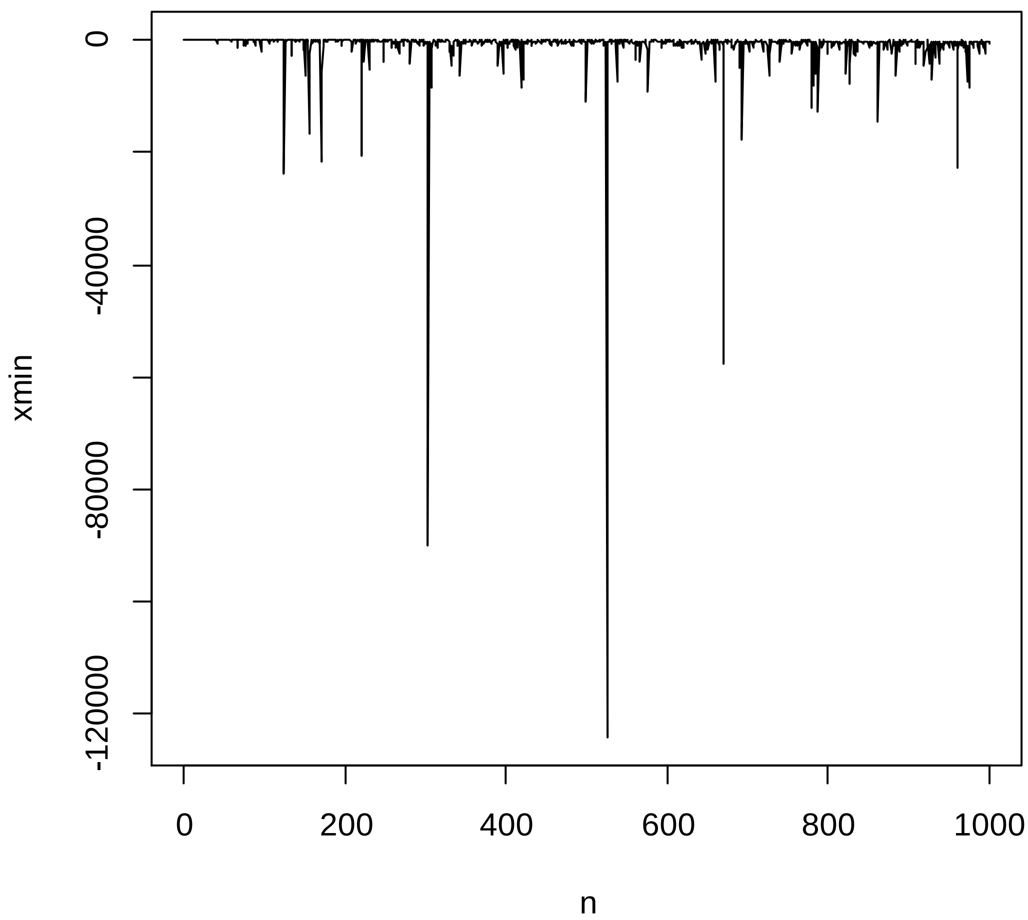
1) ***Here as the sample size is small enough , the Xmax values seem to tend to '0',but as n is increasing, some values are fluctuating from '0'.***
2) ***On an average the Xmax values are less oscillating around 0 in the case of C(1,5) than that of C(1,1) or C(0,1).***

Conclusions-

1) ***As the sample size is increasing, Xmax values does not seem to converge to any particular value except for Cauchy distn with larger variance.***
2) ***For Cauchy distn with larger variance Xmax values are more or less converging to '0'.***

D.MINIMUM-

i) C(0,1)-

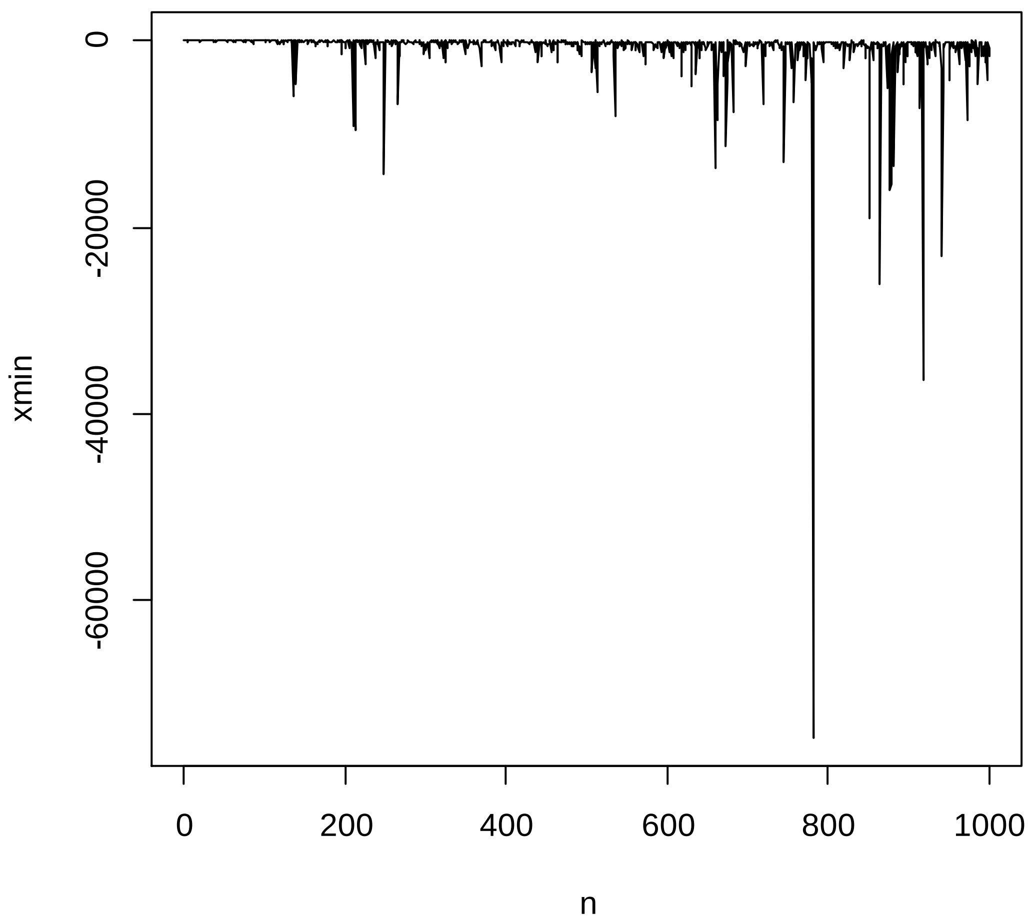


Observations-

1) ***As the sample size is increasing, Xmin values does not seem to converge to any particular value.***

> *2) Though most of the values are clustering around 0, still, there are some values which are largely deviated from '0' and the clustering may occur around different values for different samples.*

ii) C(1,1)-

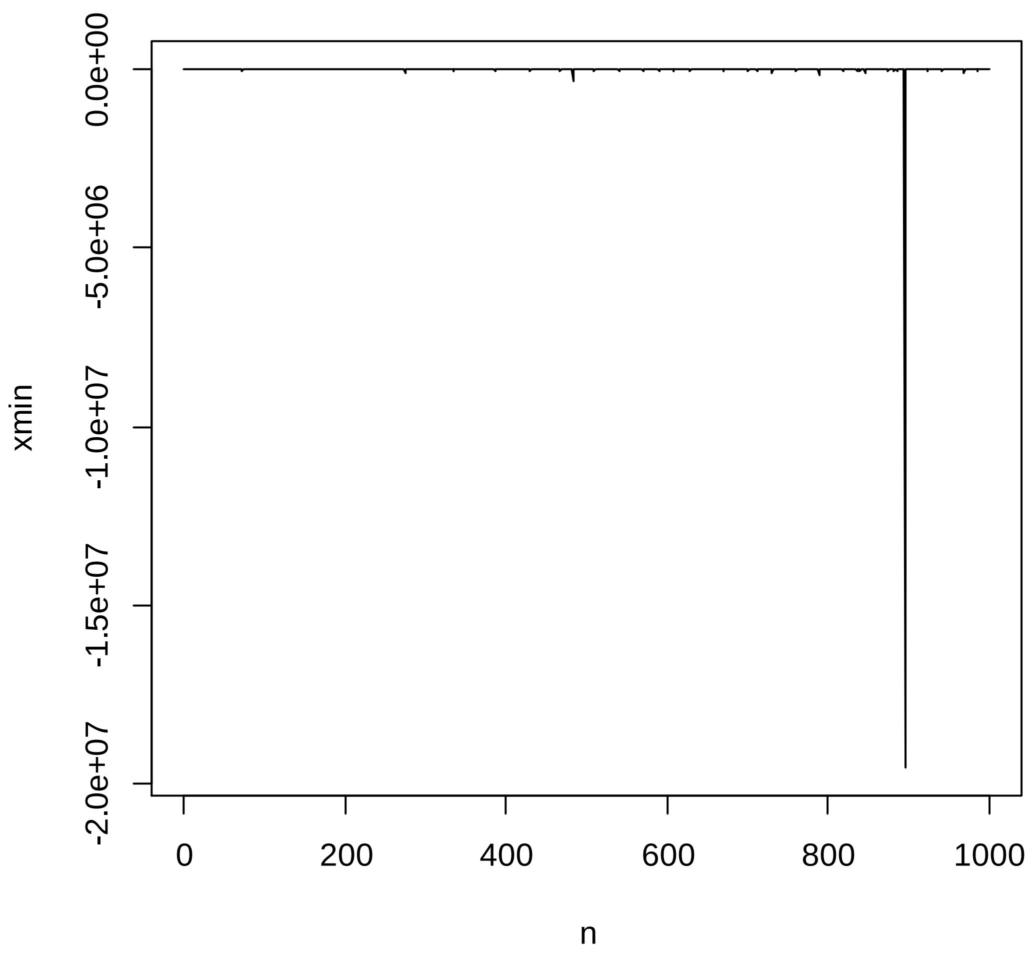


Observations-

> *1) As the sample size is increasing, Xmax values does not seem to converge to any particular value.*

2) ***Though most of the values are clustering around 0, still, there are some values which are largely deviated from '0' and the clustering may occur around different values for different samples.***

iii) C(1,5)-



Observations-

1) ***Here as the sample size is small enough , the Xmin values seem to tend to '0',but as n is increasing, some values are fluctuating from '0'.***

2) ***On an average the Xmin values are less oscillating around 0 in the case of C(1,5) than that of C(1,1) or C(0,1).***

Conclusions-

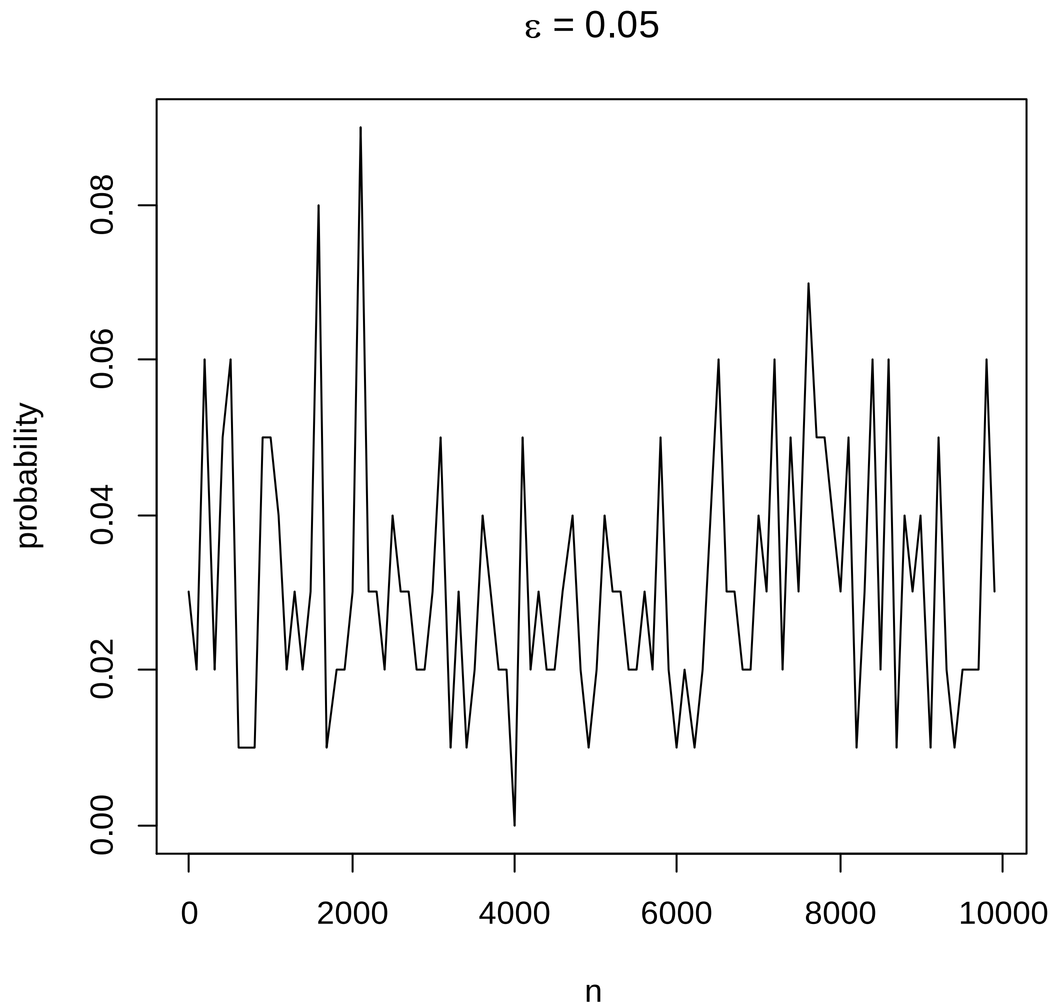
1) ***As the sample size is increasing, Xmin values does not seem to converge to any particular value except for Cauchy distn with larger variance.***
2) ***For Cauchy distn with larger variance Xmin values are more or less converging to '0'.***

NOW WE SHALL CHECK FOR CONVERGENCE IN PROBABILITY-
FOR R=100,n=100,200,300,……..,10000.

A.MEAN-

BY TAKING THE GUESS LIMIT AS 0.

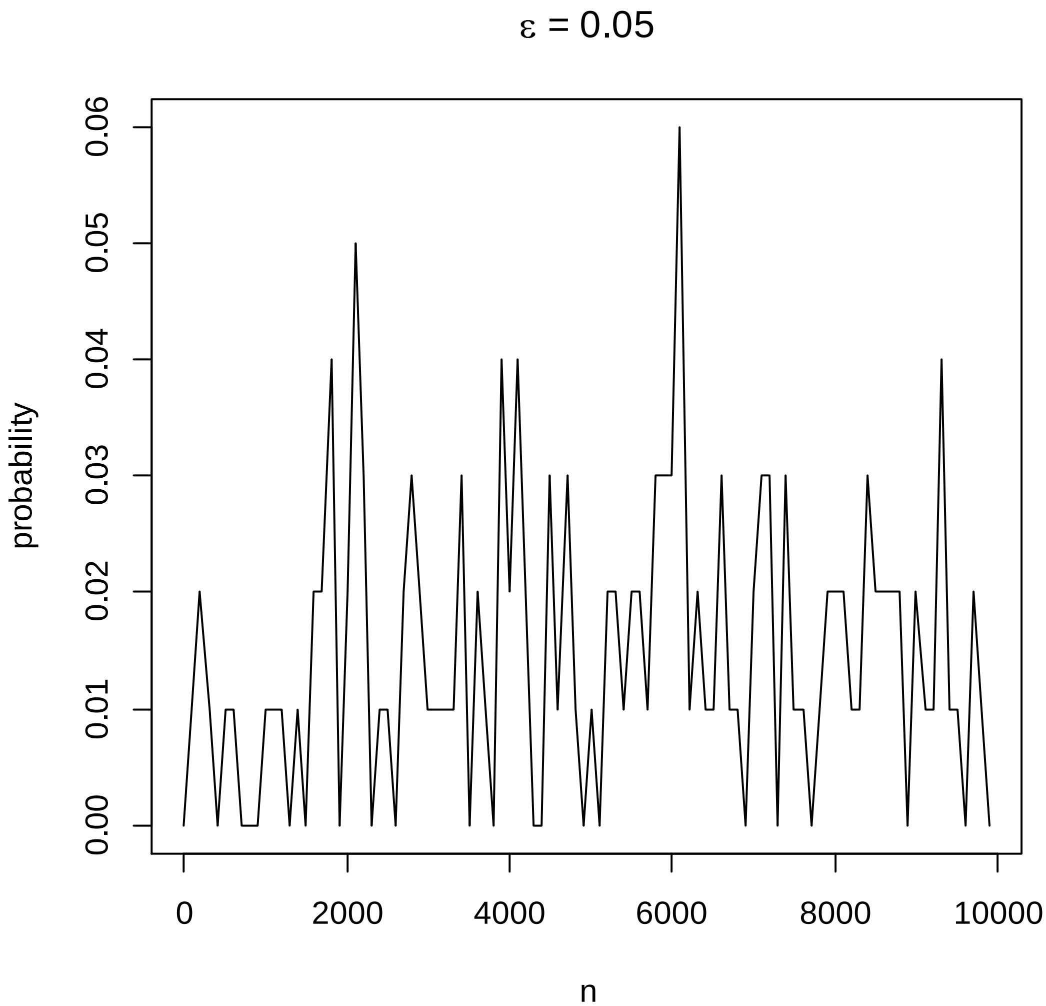
i) C(0,1)-



Observations-

1) ***We have obtained the guess limit as '0'.***
2) ***As n tends to infinity, P[|Xbar-0|<eps=0.05] does not tend to 1, hence Xbar does converge in probability to 0***
3) ***In other words Xbar is not consistent for '0'.***

ii) C(1,1)-



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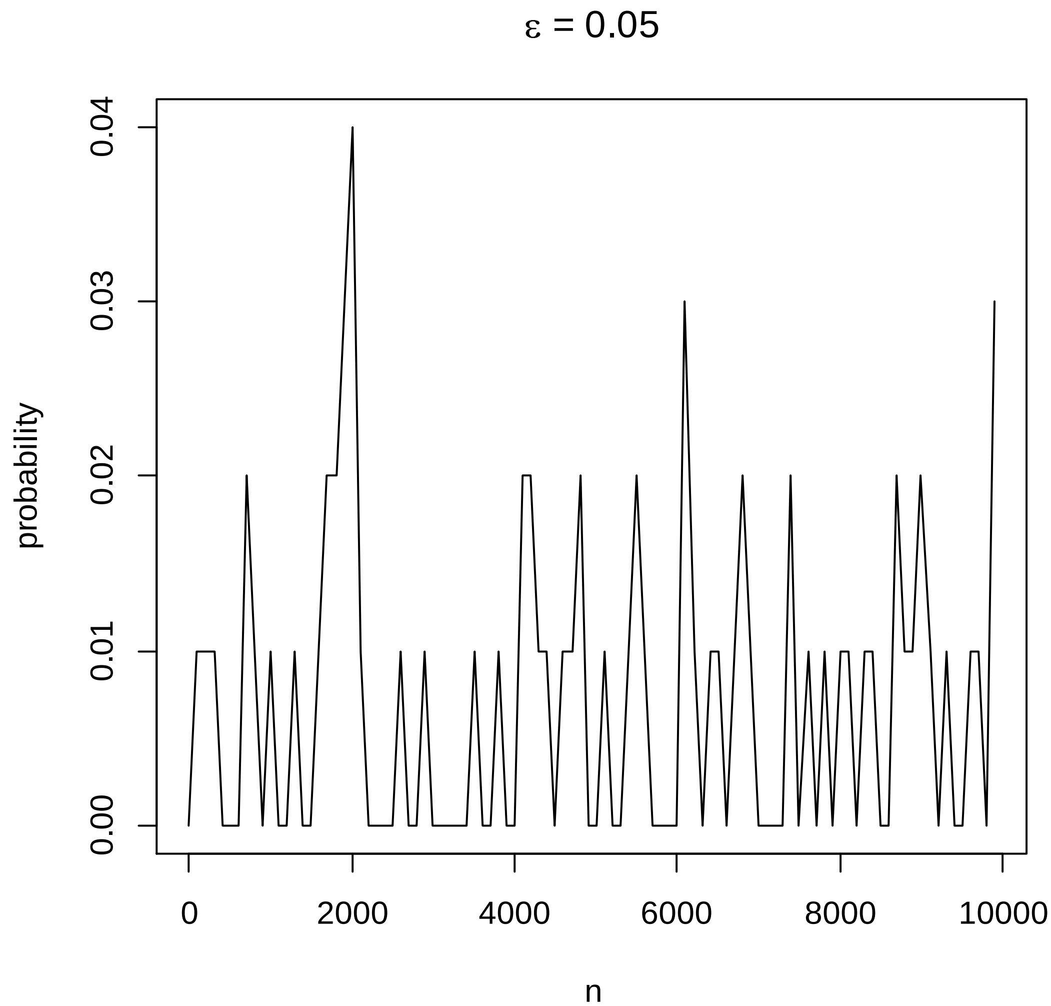
iii) C(1,5)-



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1) ***We have obtained the guess limit as '0'.***
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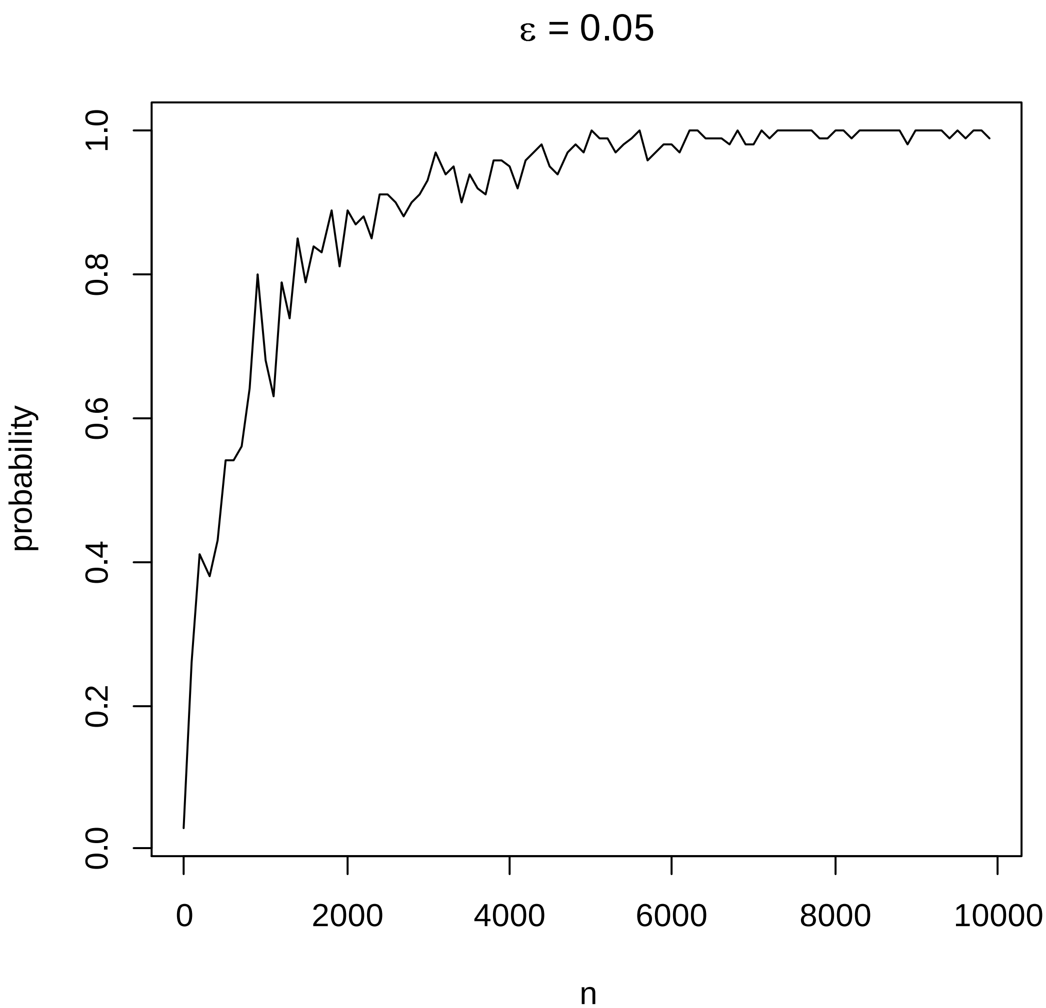
B.MEDIAN-

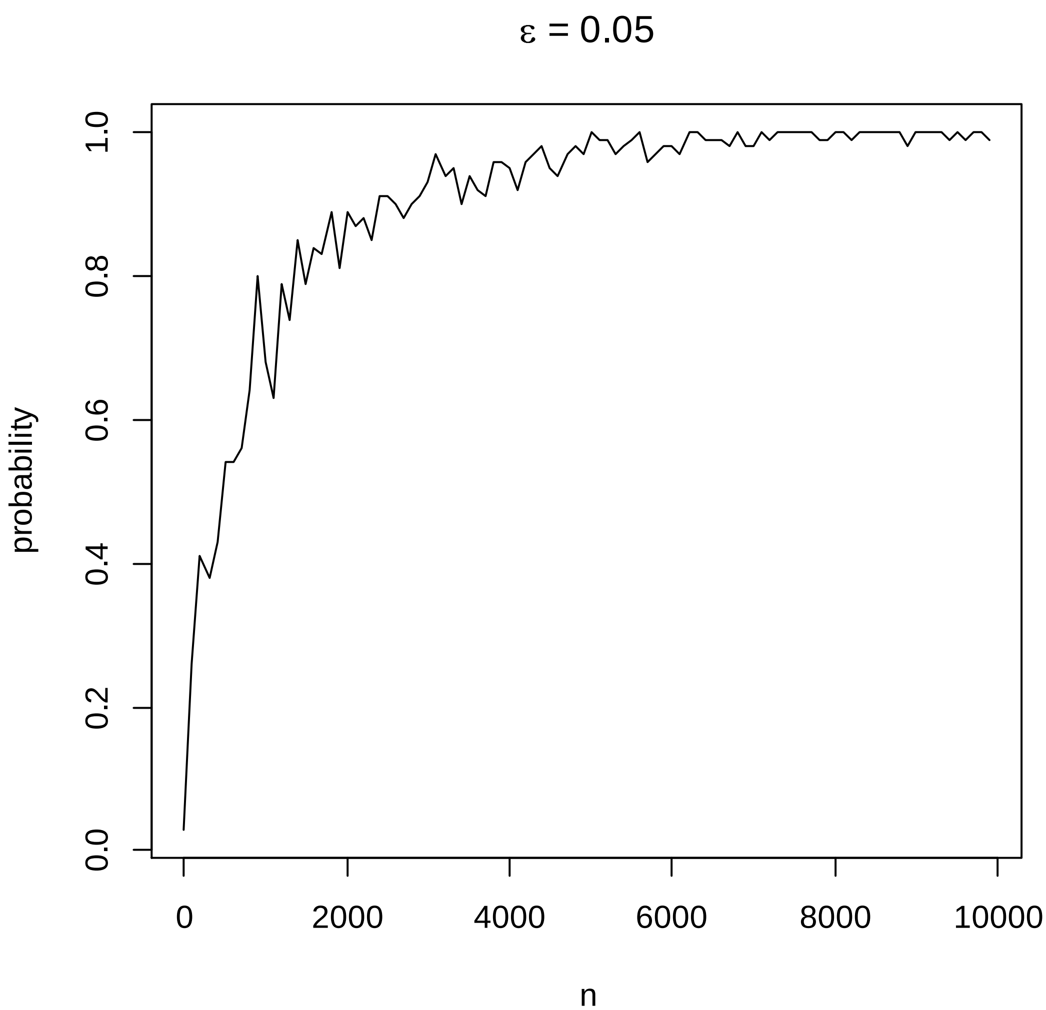
i) C(0,1)-



Observations-

1) ***Here for C(0,1) we have got the guess limit as '0',which is the population location parameter.***
2) ***As n tends to infinity, P[|Xmed-0|<eps=0.05]→1, hence , Xmed converges in probability to 0 .***
3) ***In other words Xmed is consistent for location parameter u=0.***

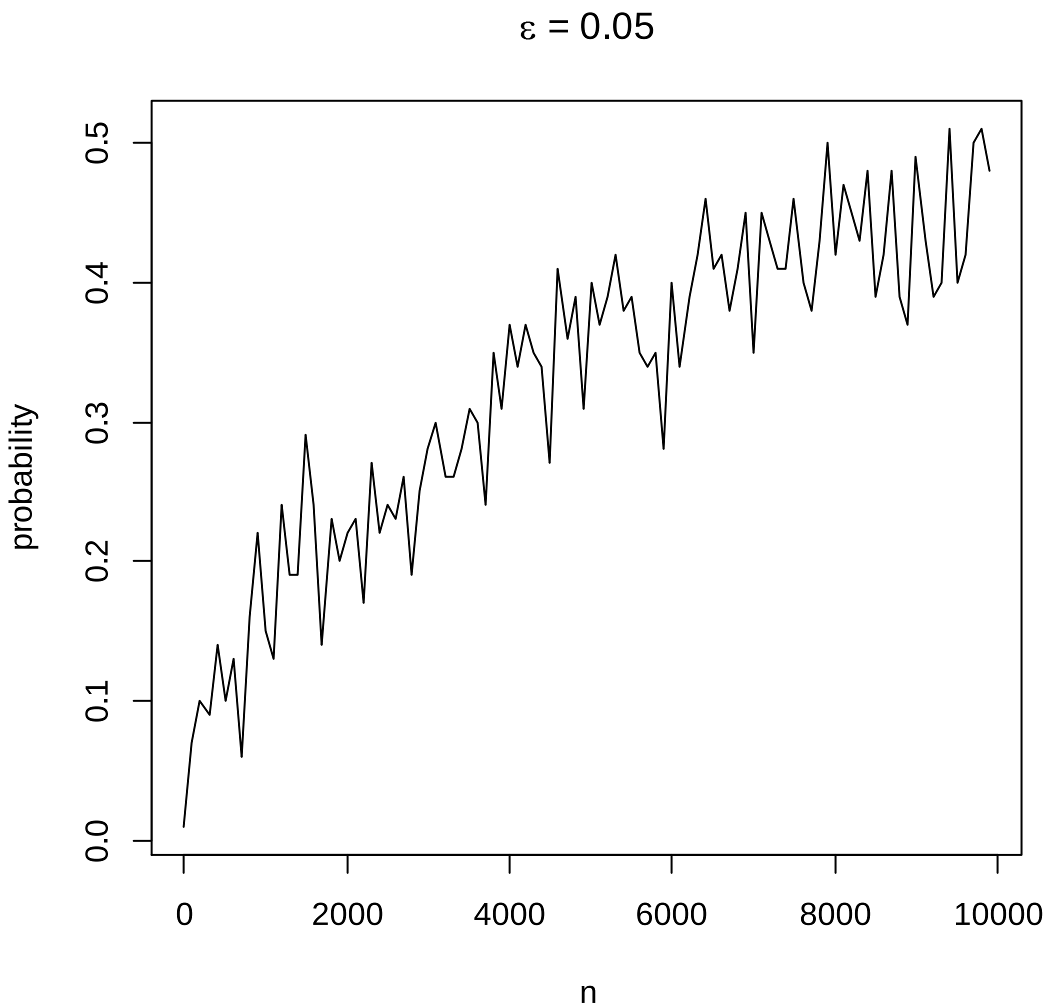
ii) C(1,1)-



Observations-

1) ***Here for C(1,1) we have got the guess limit as '1',which is the population location parameter.***
2) ***As n tends to infinity, P[|Xmed-1|<eps=0.05]→1, hence , Xmed converges in probability to 1 .***
3) ***In other words Xmed is consistent for location parameter u=1.***
4) ***The convergence rate is almost same as C(0,1).***

iii) C(1,5)-



Observations-

1) ***Here for C(1,1) we have got the guess limit as '1',which is the population location parameter.***
2) ***As n tends to infinity, P[|Xmed-1|<eps=0.05]→1, hence , Xmed converges in probability to 1 .***
3) ***In other words Xmed is consistent for location parameter u=1.***
4) ***The convergence rate is very slow, much slower than C(0,1) or C(1,1) distn.***

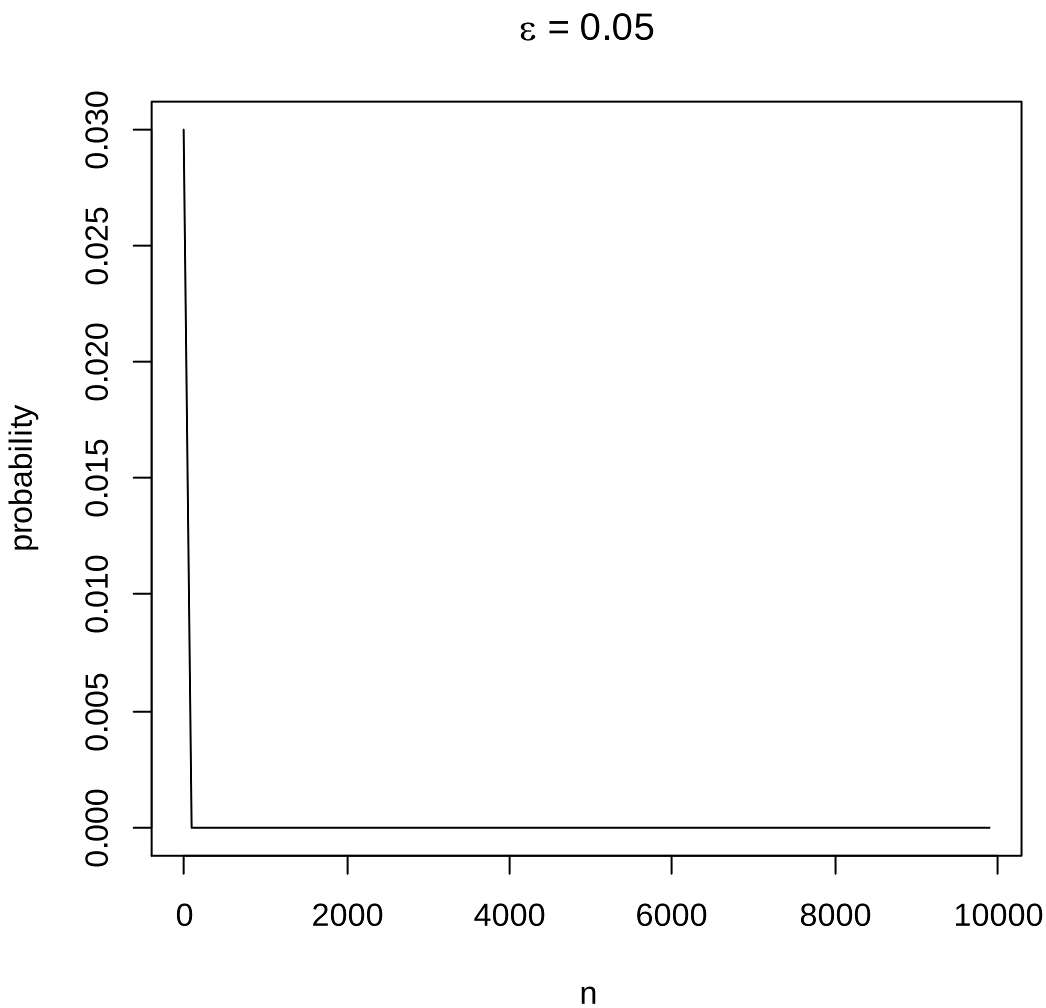
Conclusions-

1) ***If the population is C(u, A), u belongs to real line and A>0, then sample Xmed is converges in probability to 'u'.***
2) ***In other words Xmed is consistent for 'u'.***

3) ***The location parameter has no such significant effect on the rate of convergence, but the scale parameter has a lot of effect on the convergence rate , increment in A results in decrement of the rate of convergence.***

C. MAXIMUM (X(n))-

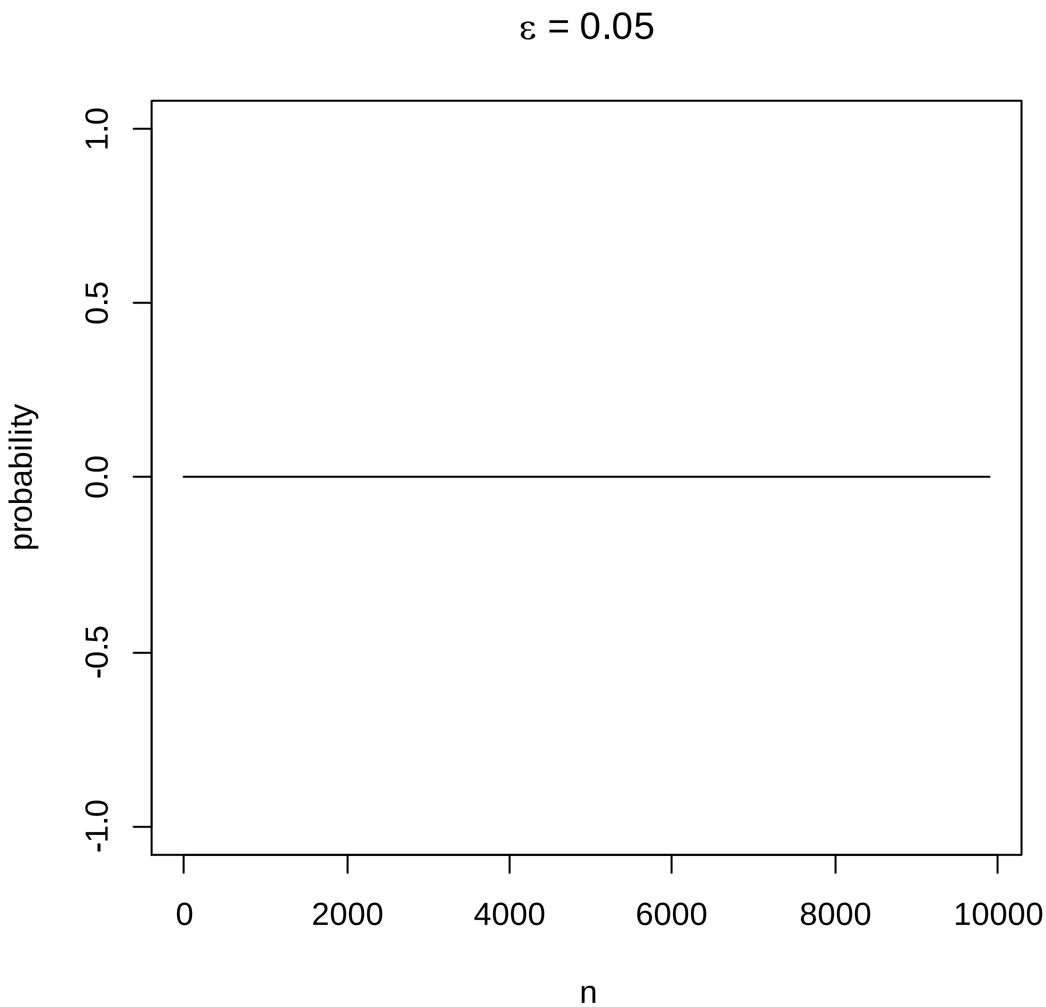
i) C(0,1)-



Observations-

1) ***Although we have failed to find any guess limit for X(n), still we will check the probability convergence by taking the guess limit as '0'.***
2) ***Clearly P[|X(n)-0|<eps=0.05] does not tend to 1 as n tends to infinity, rather it is going to '0' ,hence X(n) does not converge in probability to '0'.***
3) ***In other words X(n) is not consistent for '0'.***
4) ***We can take any other guess limit say 'l' and show that P[|X(n)-l|<eps=0.05]does not tend 1 as n tends to infinity.***

ii) C(1,1)-



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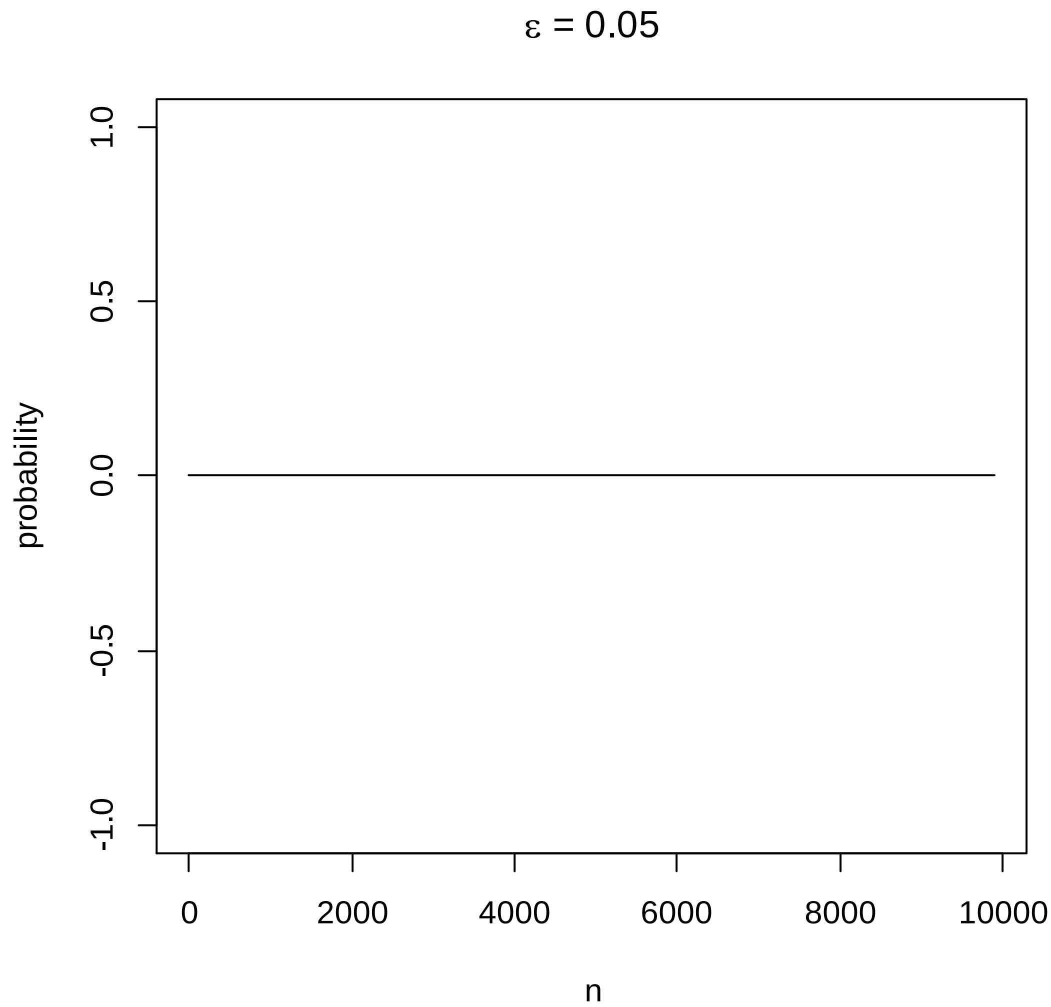
iii) C(1,5)-



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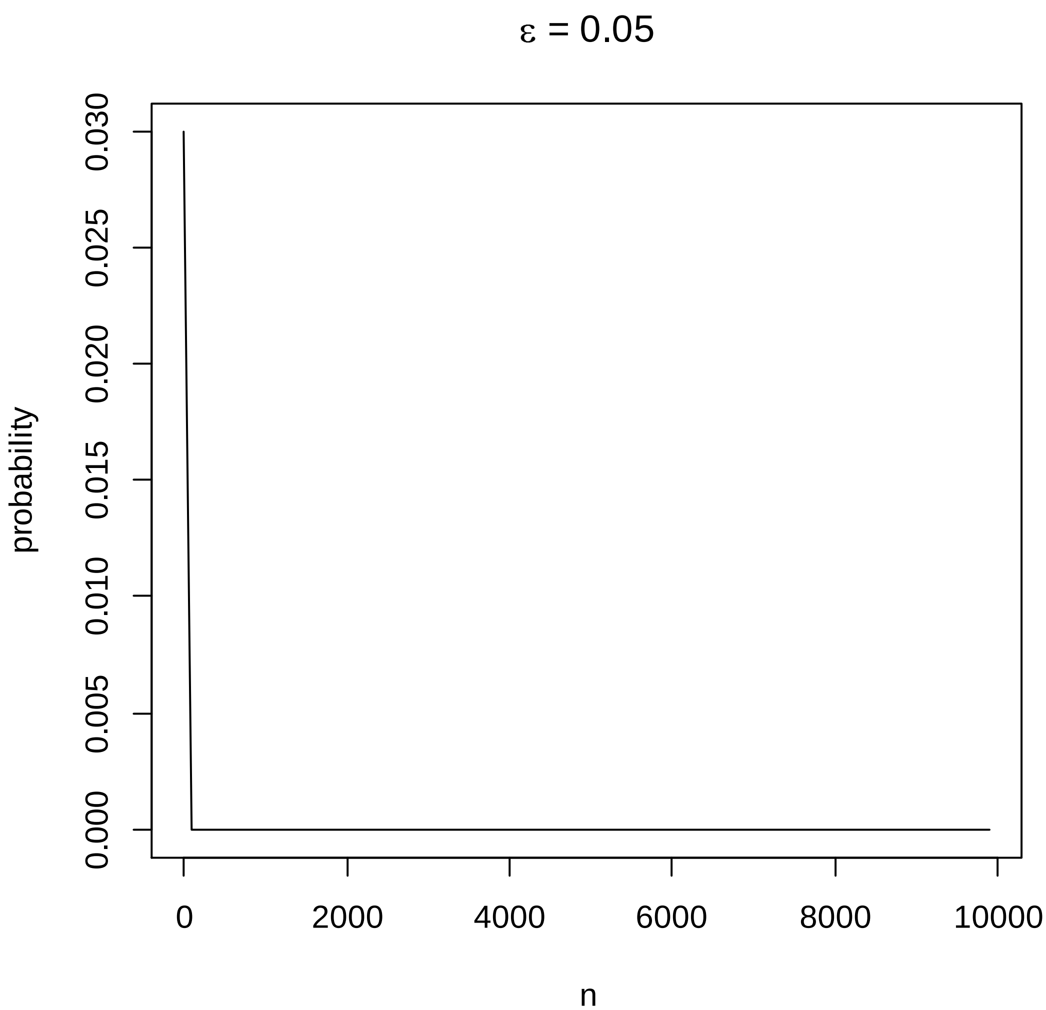
Conclusions-

1) ***Irrespective of the location and scale parameter of the Cauchy population, Although we have failed to find any guess limit for X(n), still we will check the probability convergence by taking the guess limit as '0'.***
2) ***Clearly P[|X(n)-0|<eps=0.05] does not tend to 1 as n tends to infinity, rather it is going to '0' ,hence X(n) does not converge in probability to '0'.***

3) *In other words X(n) is not consistent for '0'.*
4) *We can take any other guess limit say 'l' and show that P[|X(n)-l|<eps=0.05]does not tend 1 as n tends to infinity.*

D. MINIMUM X(1)-

i) C(0,1)-



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1) *Although we have failed to find any guess limit for X(1), still we will check the probability convergence by taking the guess limit as '0'.*

2) ***Clearly P[|X(1)-0|<eps=0.05] does not tend to 1 as n tends to infinity, rather it is going to '0' ,hence X(1) does not converge in probability to '0'.***
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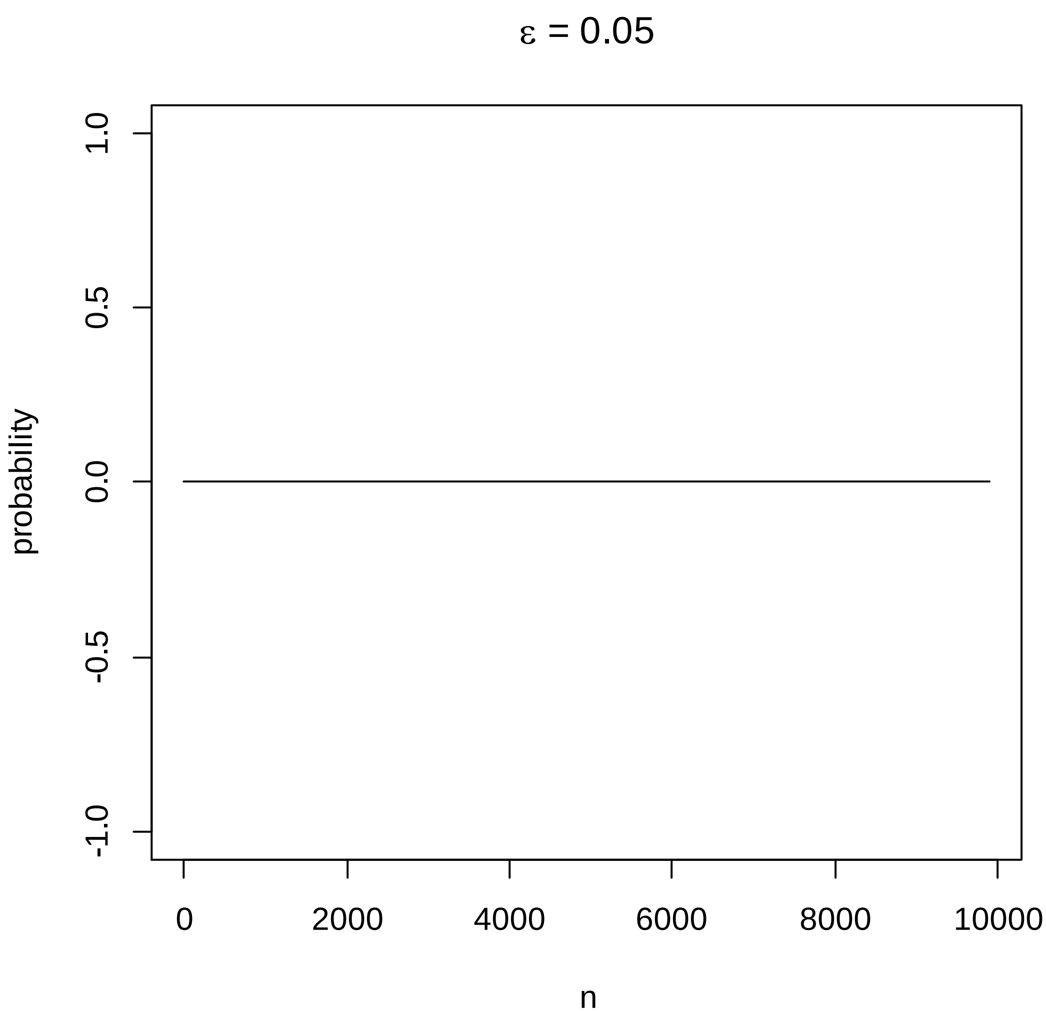
ii) C(1,1)-



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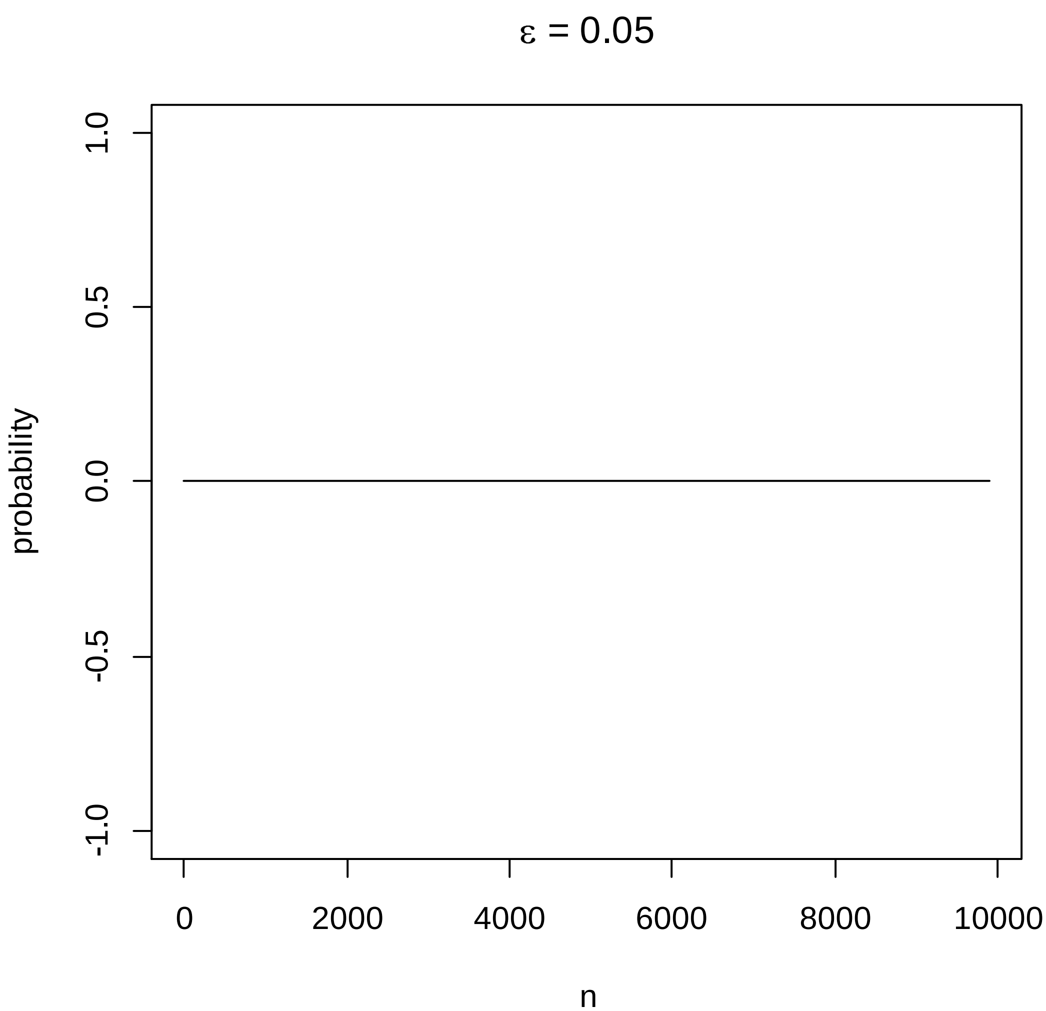
iii) C(1,5)-



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Conclusions-

1) *Irrespective of the location and scale parameter of the Cauchy population, Although we have failed to find any guess limit for X(1), still we will check the probability convergence by taking the guess limit as ‘0’.*
2) *Clearly P[|X(1)-0|<eps=0.05] does not tend to 1 as n tends to infinity, rather it is going to ‘0’ ,hence X(1) does not converge in probability to ‘0’.*
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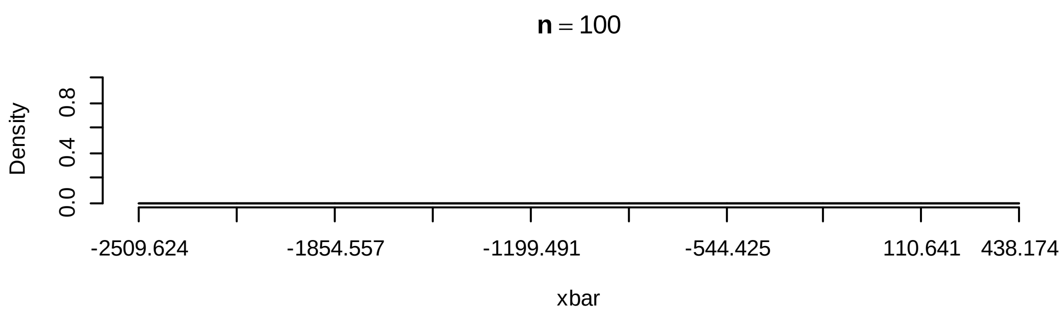
4) *We can take any other guess limit say ‘l’ and show that P[|X(1)-l|<eps=0.05]does not tend 1 as n tends to infinity.*

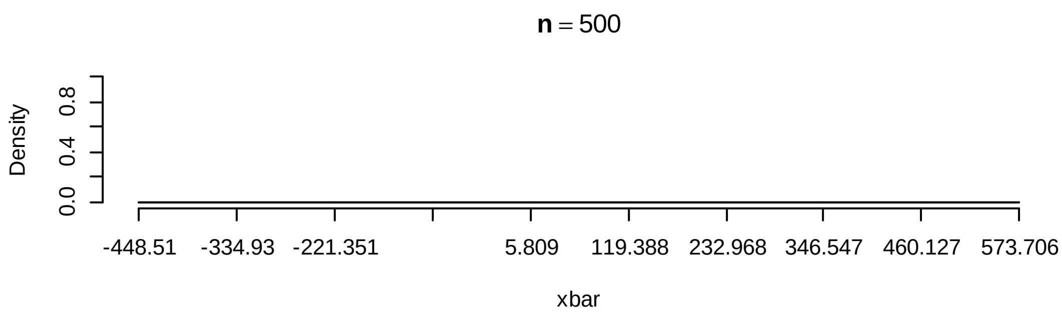
Now here we will check convergence in distribution for fixed R=1000 and varying n= 100, 500, 1000.

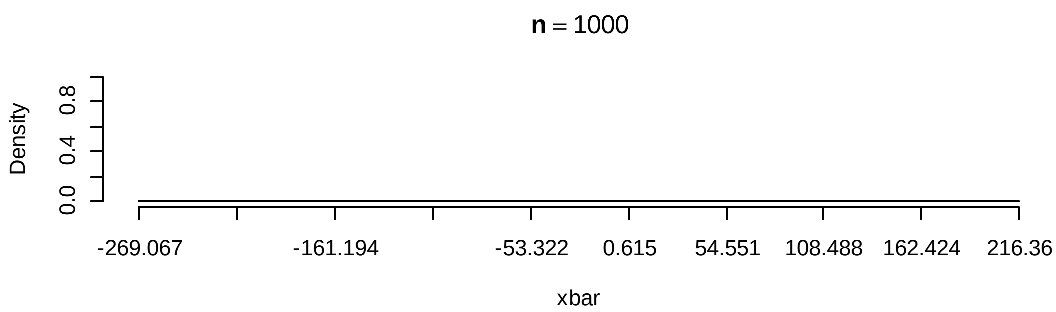
Here we will check the convergence in distribution for some well known statistics like sample mean , median , minimum , maximum , midrange etc for Cauchy population with certain location and scale parameter and make a significant comparison between their large sample behaviour and their dependency on parameters for fixed repetition number R=1000 and n=100,500,1000 .

A. MEAN-

i) C(0,1)-


n = 100
Density
0.0
0.4
0.8
-2509.624
-1854.557
-1199.491
-544.425
110.641
438.174
xbar

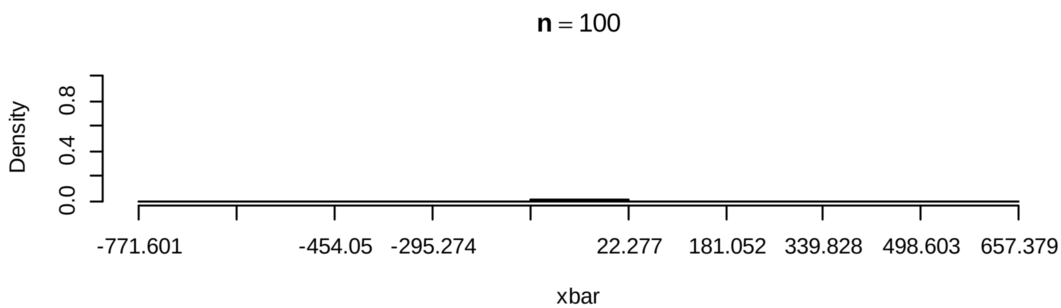

n = 500
Density
0.0
0.4
0.8
-448.51
-334.93
-221.351
5.809
119.388
232.968
346.547
460.127
573.706
xbar

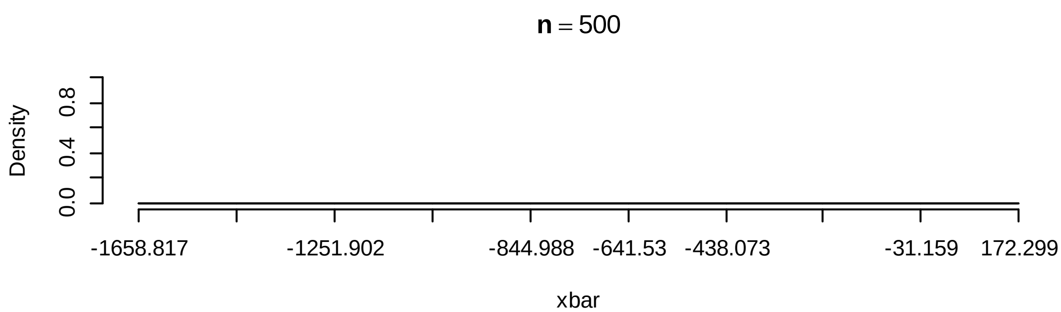

n = 1000
Density
0.0
0.4
0.8
-269.067
-161.194
-53.322
0.615
54.551
108.488
162.424
216.36
xbar

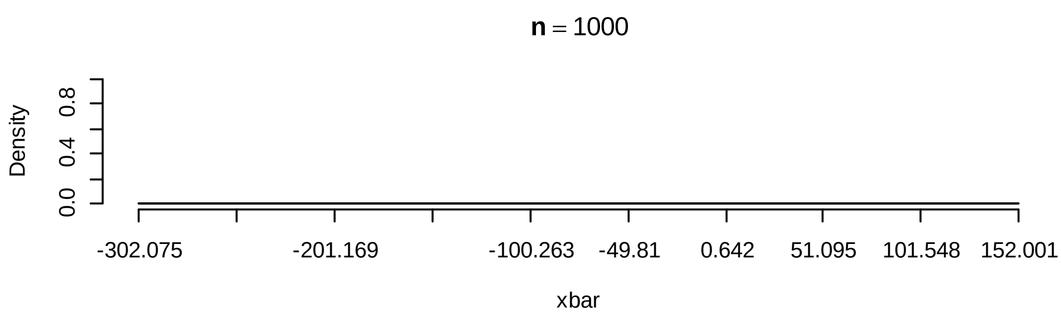
Observations-

1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

ii) C(1,1)-







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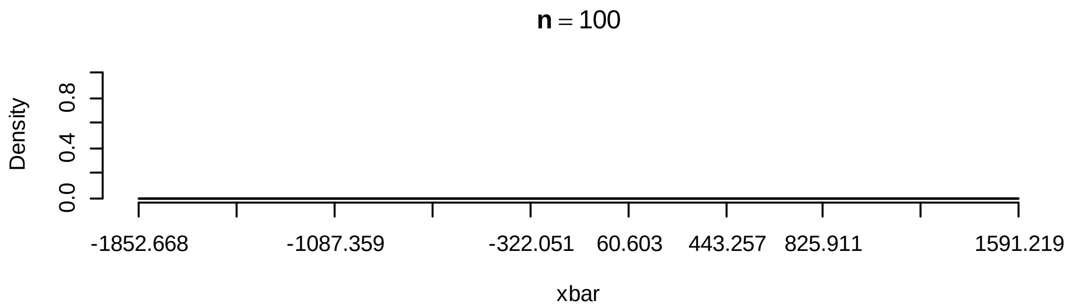
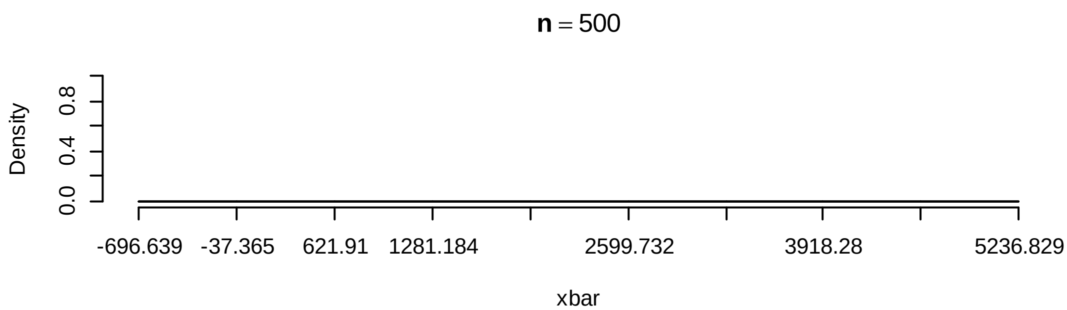
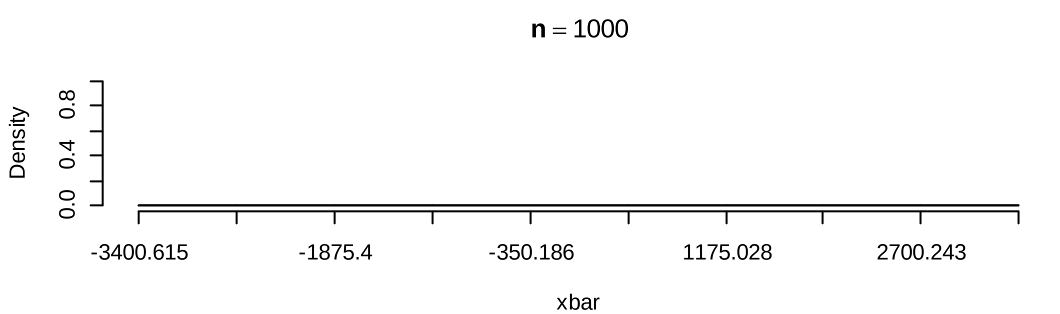
1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

iii) C(1,5)-







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1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

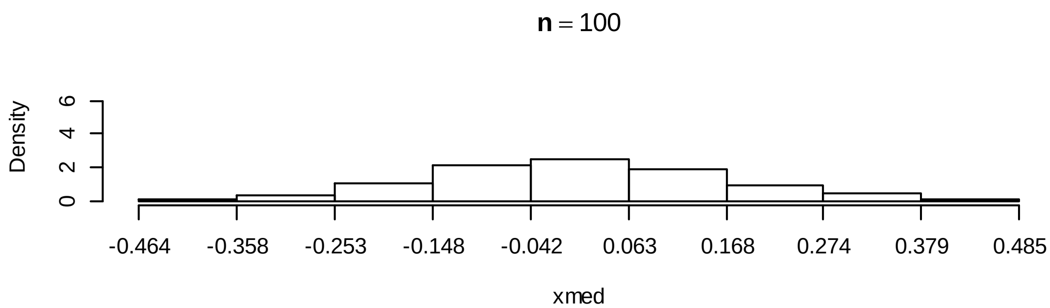
Conclusions-

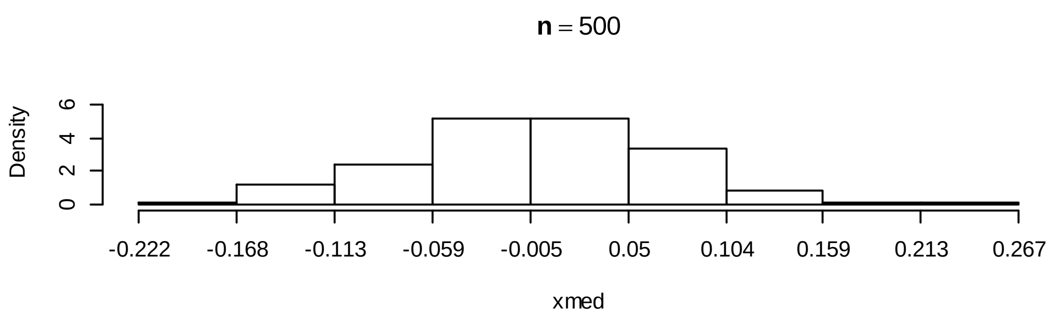
1) ***Irrespective of location and scale parameter, For relatively small sample size, (n=100), there is no such frequency density histogram.***

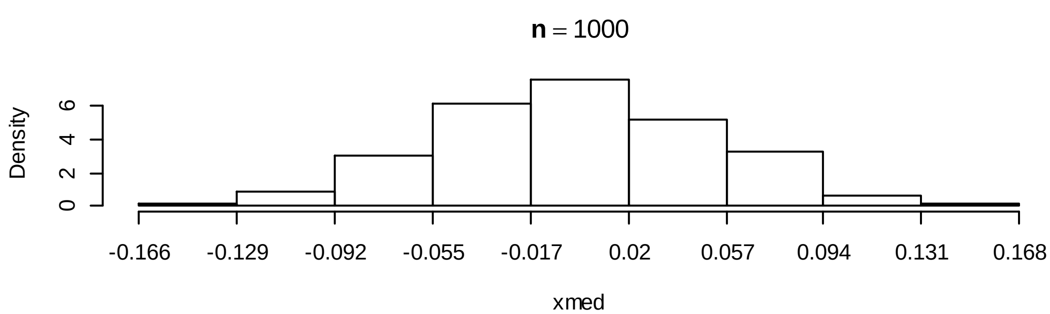
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***
3) ***The scale and location parameter has no such significant effect on the diagram.***

B. MEDIAN-

i) C(0,1)-



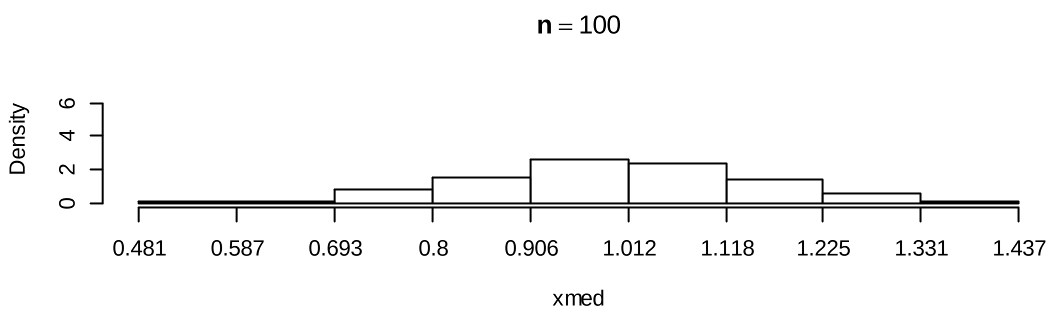


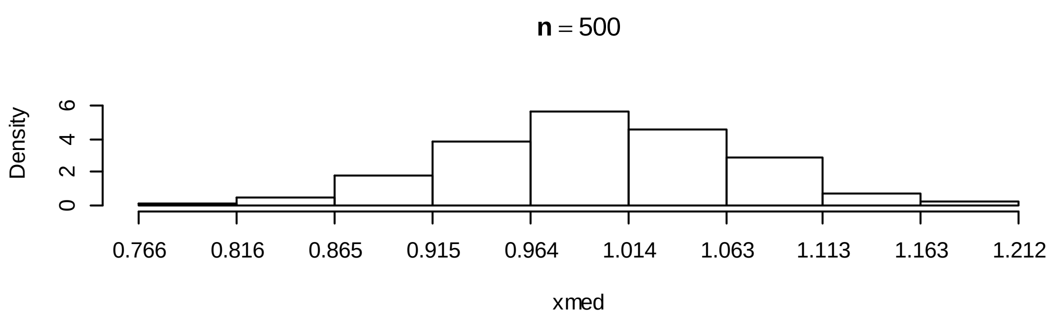


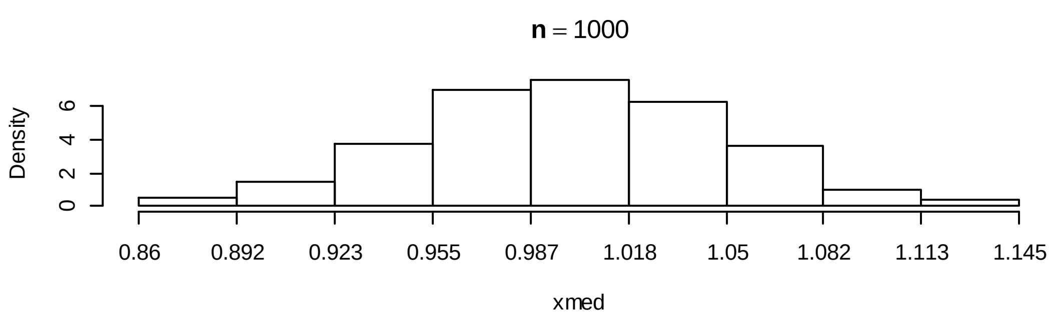
Observations-

1) ***For comparatively small sample size, n=100, the frequency density histogram seems to be symmetric.***
2) ***As we increase the sample size, (say n=500 or 1000), the asymptotic nature of Xmed is becoming more obvious .***
3) ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xmed might have an asymptotic normal distribution.***
4) ***For fixed location and scale parameter of the Cauchy distribution, the height of the histograms are increasing as we are increasing the sample size, indicating the asymptotic variance of Xmed is decreasing, i.e., it is inversely proportionate to sample size.***

ii) C(1,1)-







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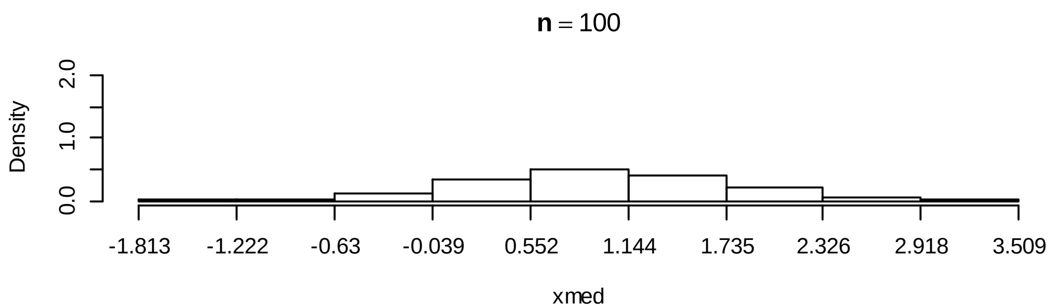
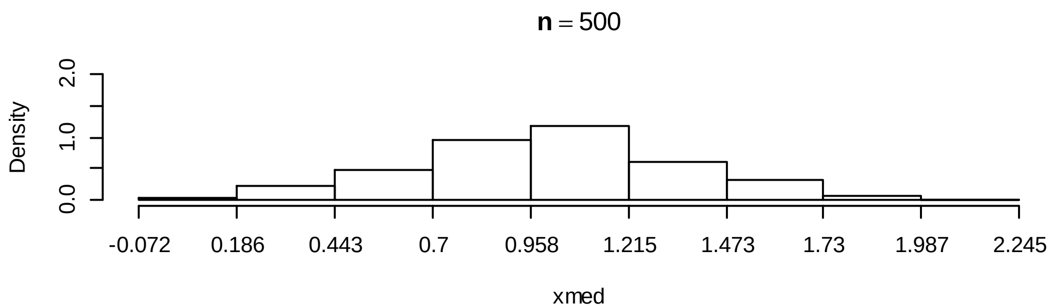
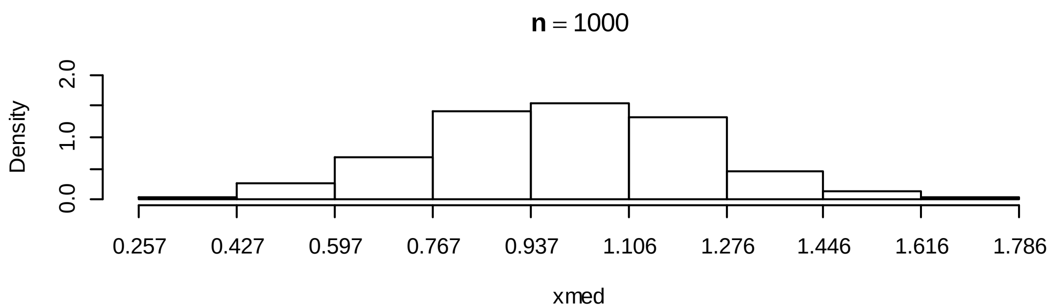
iii) C(1,5)-







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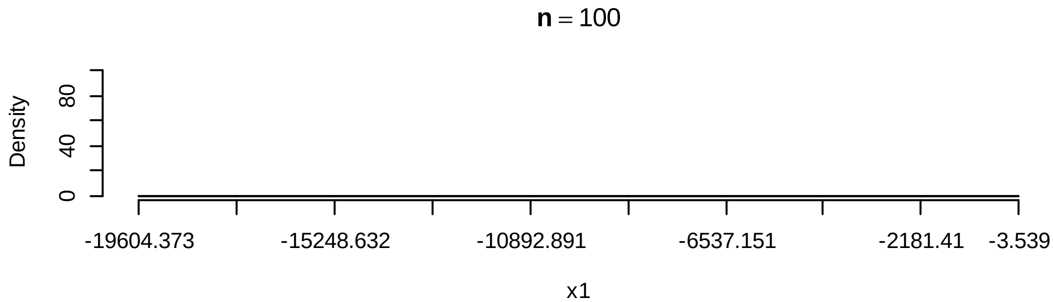
3) ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xmed might have an asymptotic normal distribution.***
4) ***For fixed location and scale parameter of the Cauchy distribution, the height of the histograms are increasing as we are increasing the sample size, indicating the asymptotic variance of Xmed is decreasing, i.e., it is inversely proportionate to sample size.***
5) ***As scale parameter is increased keeping the location parameter fixed, the average height of the histogram is increased in C(1,5) case than C(0,1) or C(1,1) case.***

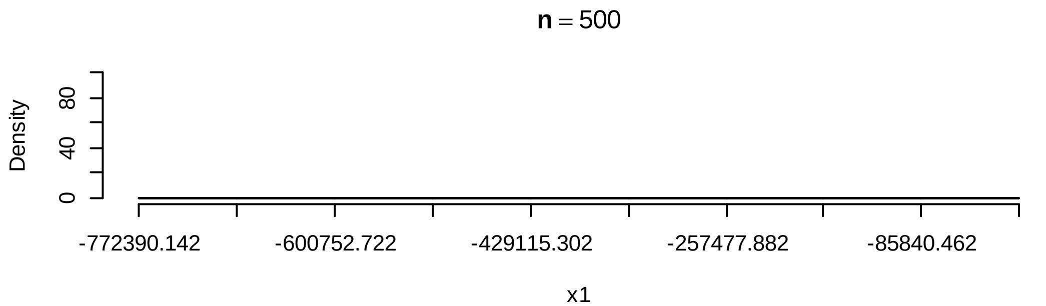
Conclusions-

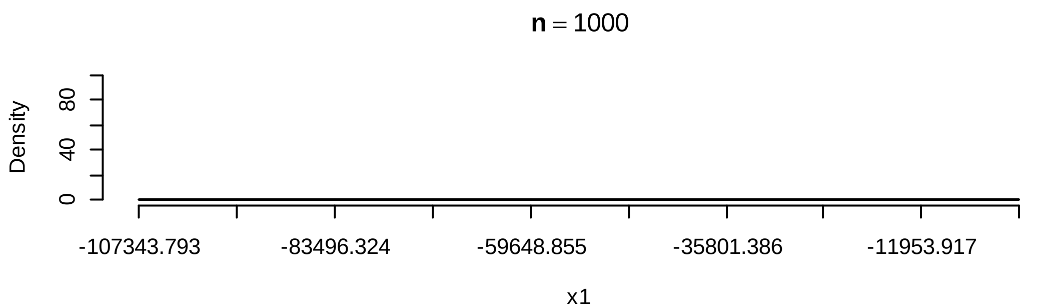
1) ***Irrespective of the location and scale parameter of Cauchy distn for comparatively small sample size, n=100, the frequency density histogram seems to be symmetric.***
2) ***As we increase the sample size, (say n=500 or 1000), the asymptotic nature of Xmed is becoming more obvious .***
3) ***As the frequency density histogram is attaining symmetric nature, we can interpret that sample Xmed might have an asymptotic normal distribution.***
4) ***For fixed location and scale parameter of the Cauchy distribution, the height of the histograms are increasing as we are increasing the sample size, indicating the asymptotic variance of Xmed is decreasing, i.e., it is inversely proportionate to sample size.***
5) ***The scale parameter has better effect on the height of the histogram than that of the location parameter.***

C. MINIMUM X(1)-

i) C(0,1)-



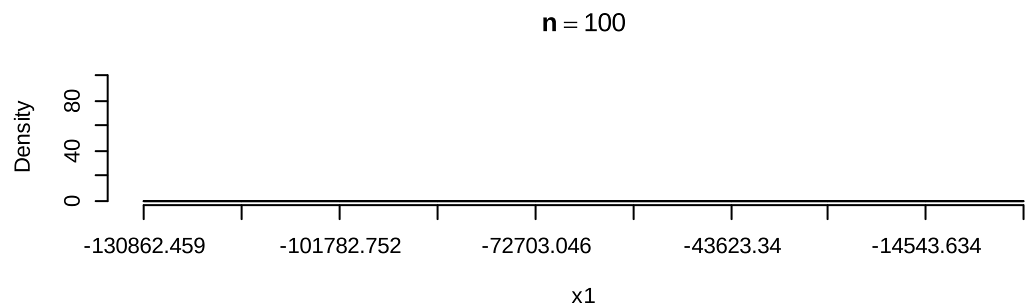


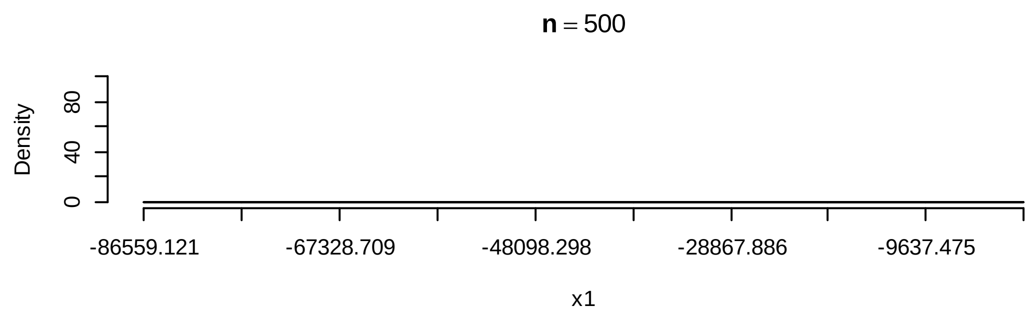


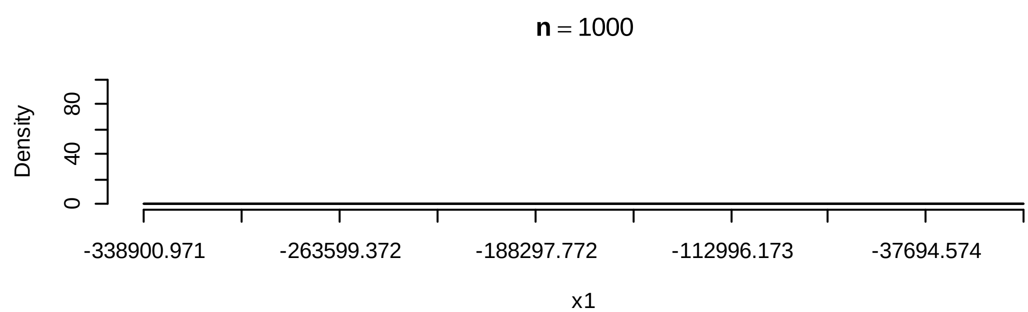
Observations-

1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

ii) C(1,1)-



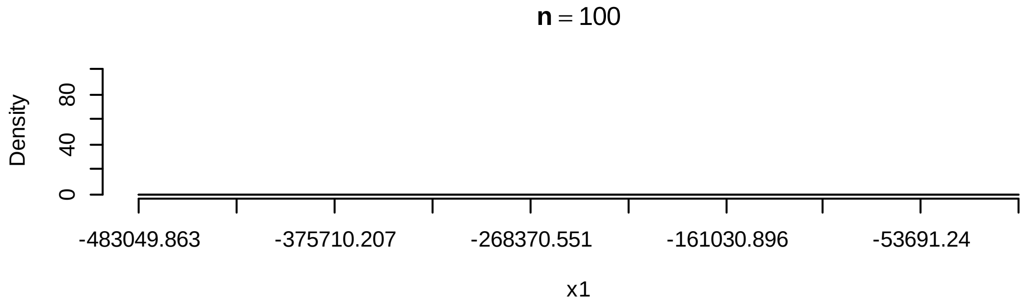


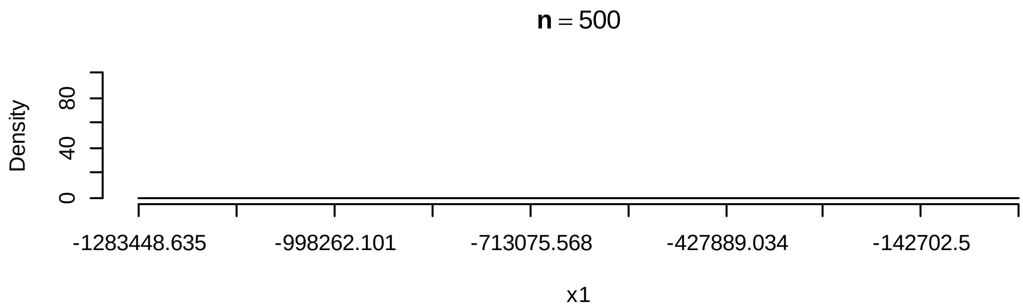


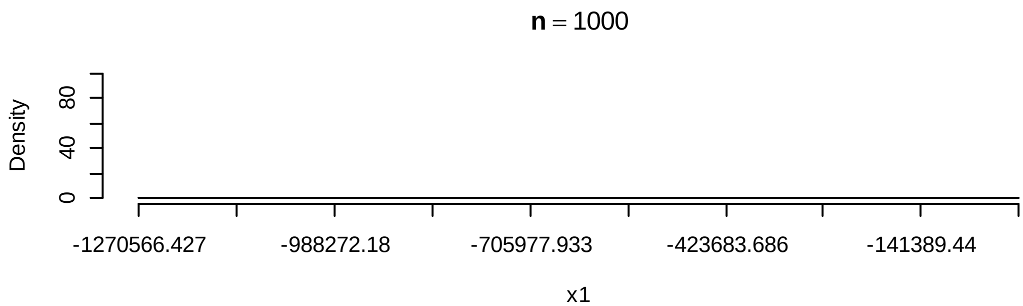
Observations-

1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

iii) C(1,5)-







Observations-

1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

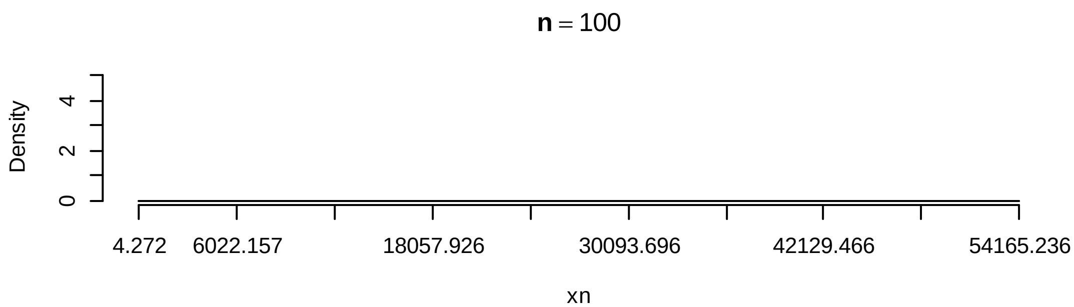
Conclusions-

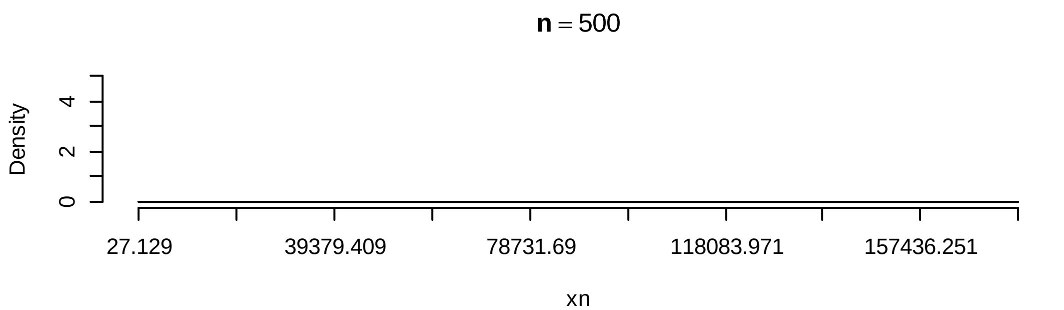
1) ***Irrespective of location and scale parameter, For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

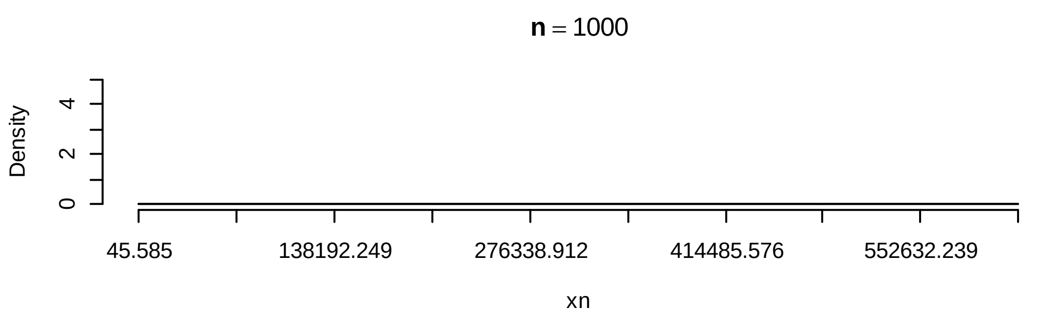
3) ***The scale and location parameter has no such significant effect on the diagram.***

D. MAXIMUM X(n)-

i) C(0,1)-



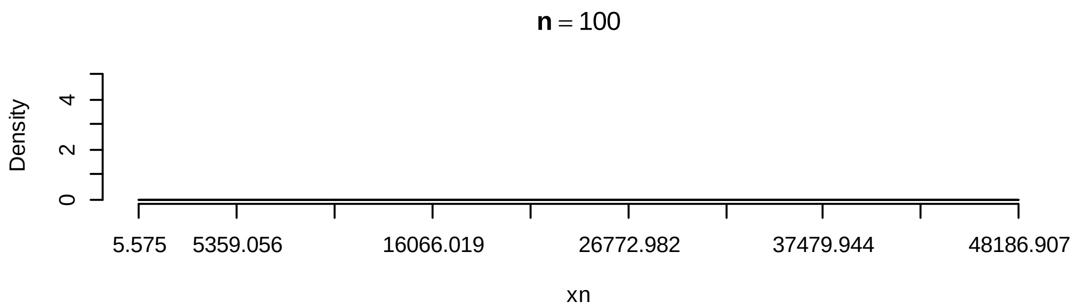


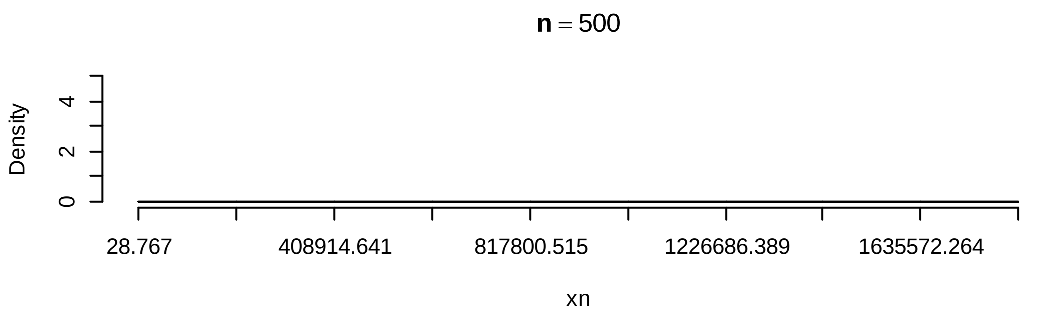


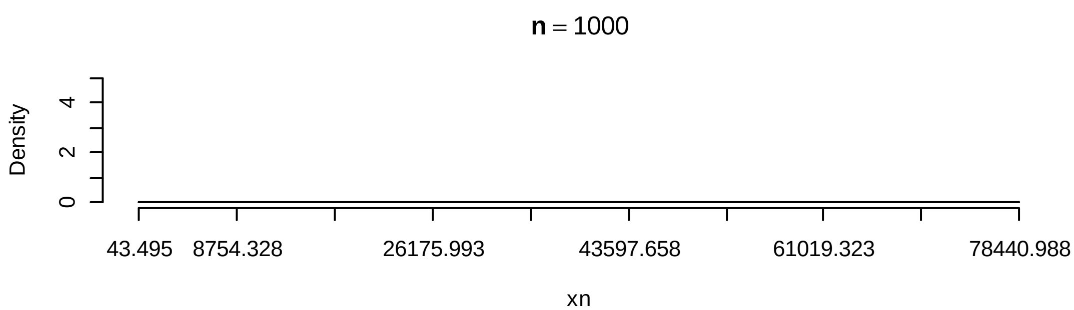
Observations-

1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

ii) C(1,1)-



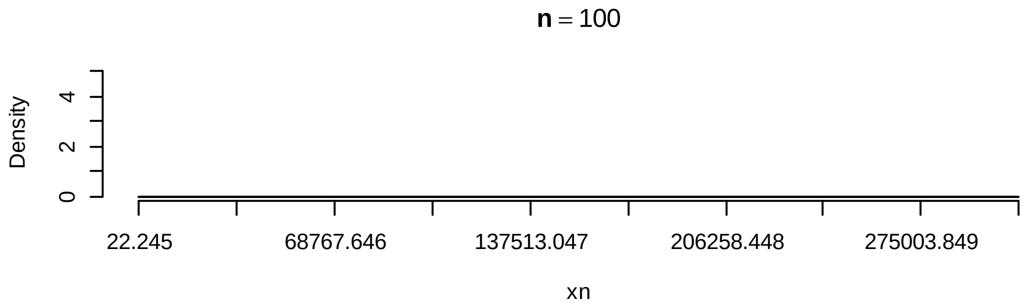


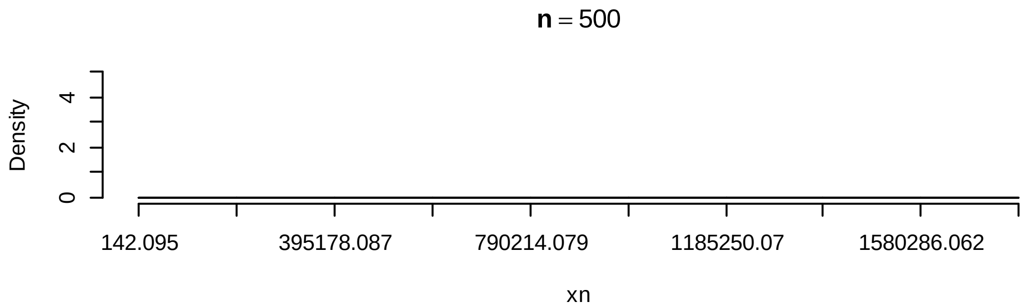


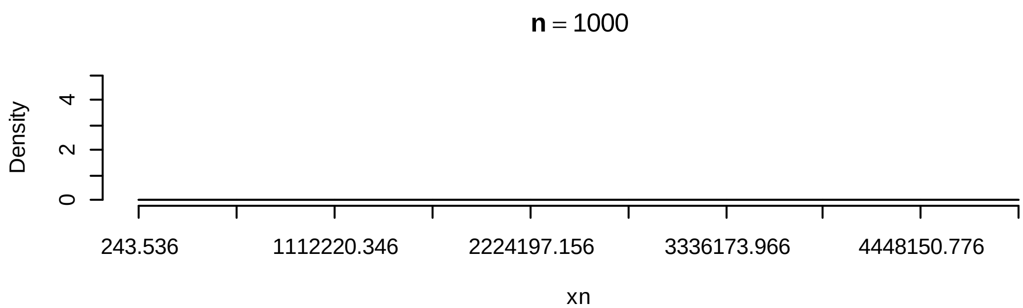
Observations-

1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

iii) C(1,5)-







Observations-

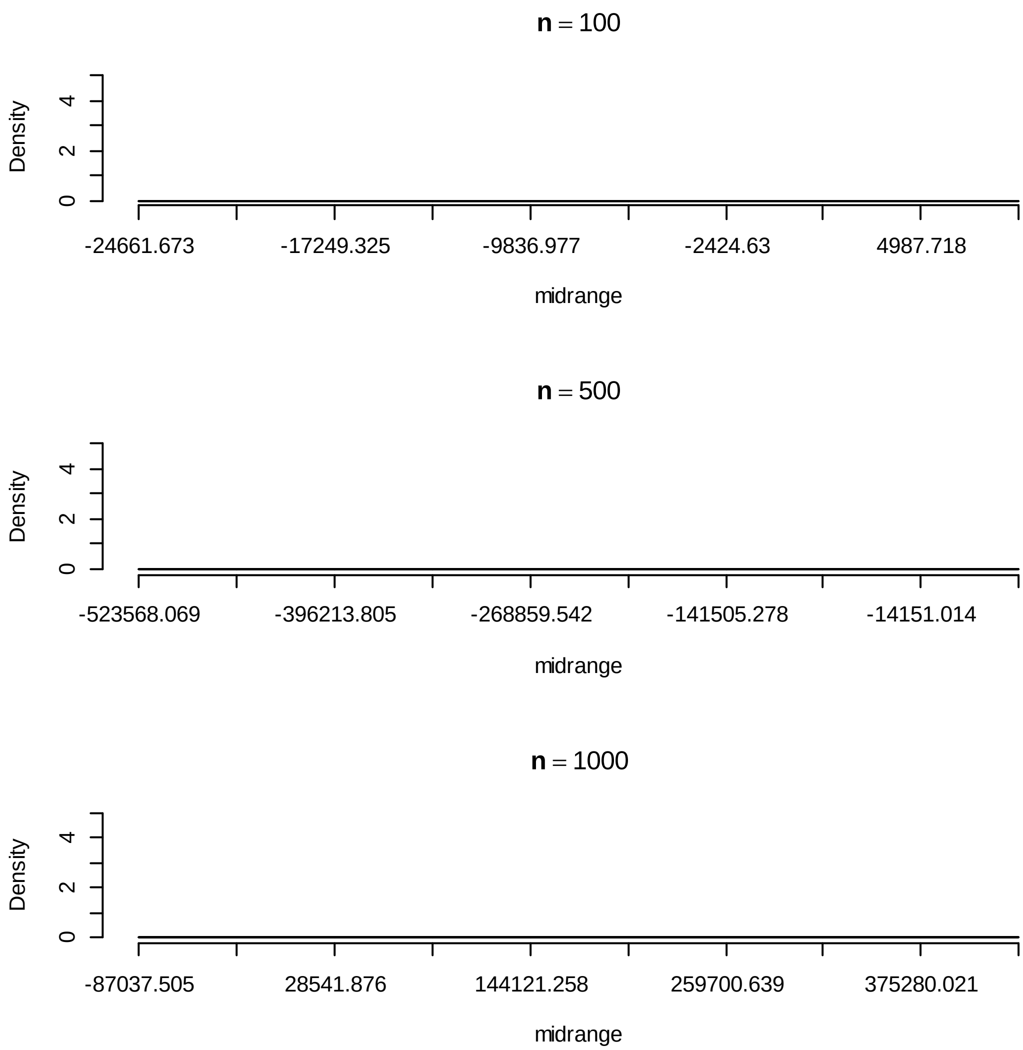
1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

Conclusions-

1) ***Irrespective of location and scale parameter, For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***
3) ***The scale and location parameter has no such significant effect on the diagram***

E. MIDRANGE-

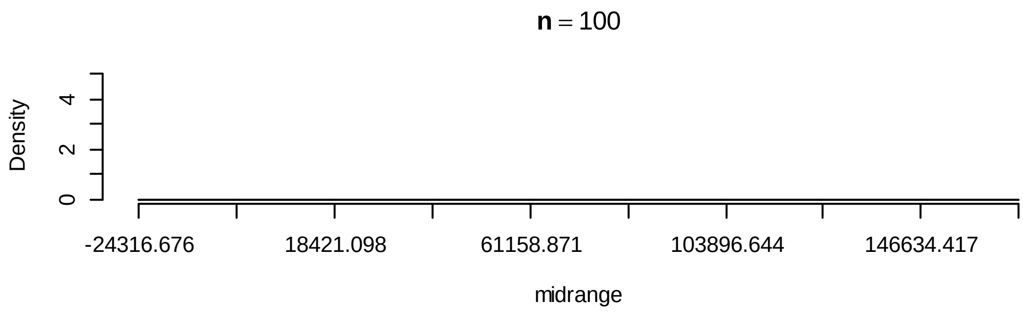
i) C(0,1)-

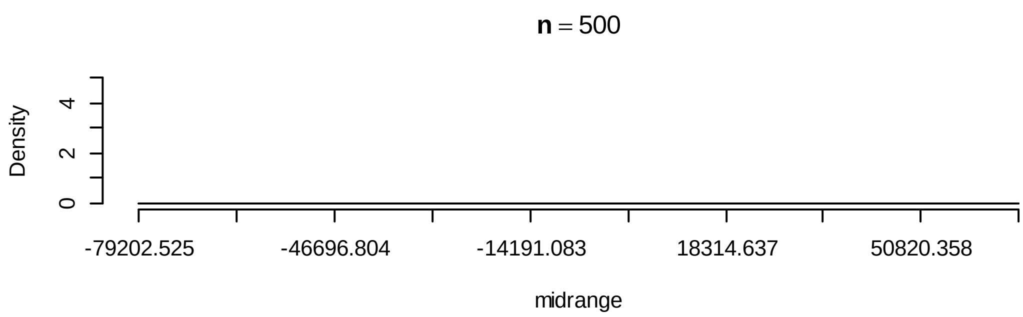


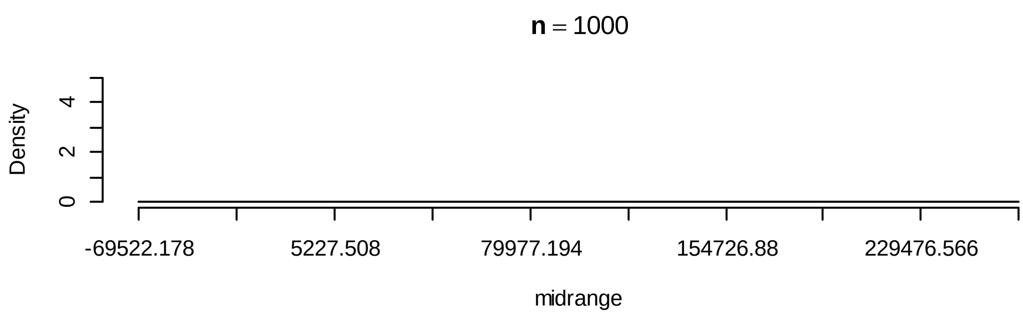
Observations-

1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

ii) C(1,1)-



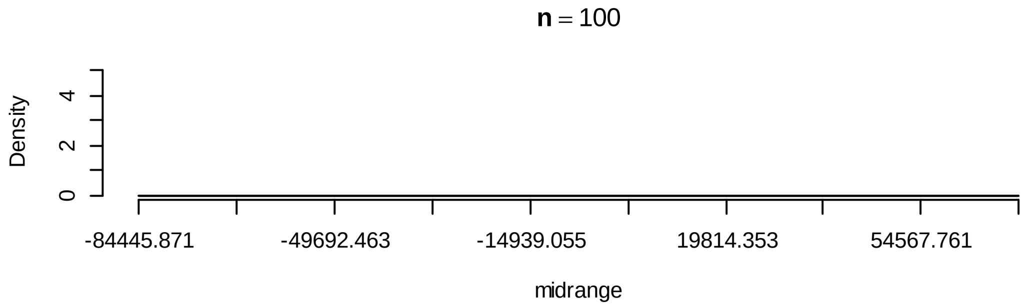


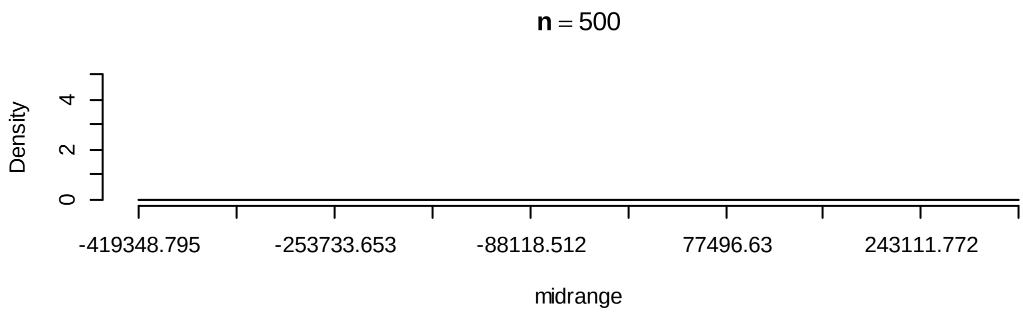


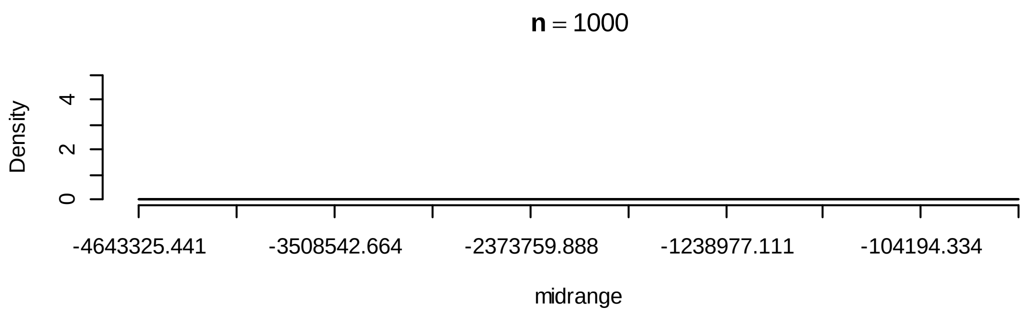
Observations-

1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

iii) C(1,5)-







Observations-

1) ***For relatively small sample size, (n=100), there is no such frequency density histogram.***
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***

Conclusions-

1) ***Irrespective of location and scale parameter, For relatively small sample size, (n=100), there is no such frequency density histogram.***

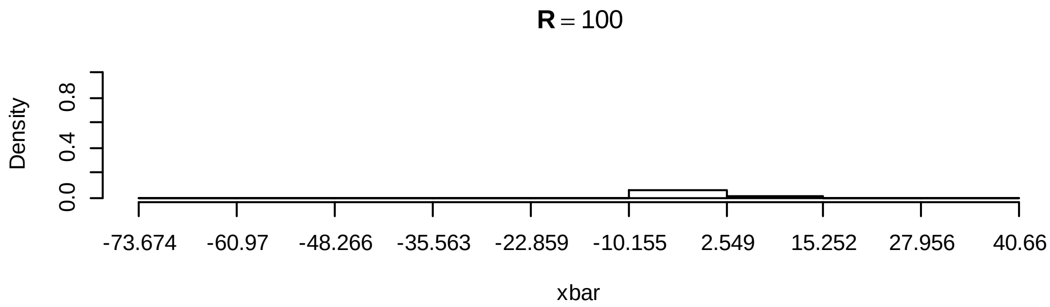
2) ***As we increase the sample size, keeping the repetition number fixed, then also, there is no such change in the behaviour of the diagram.***
3) ***The scale and location parameter has no such significant effect on the diagram.***

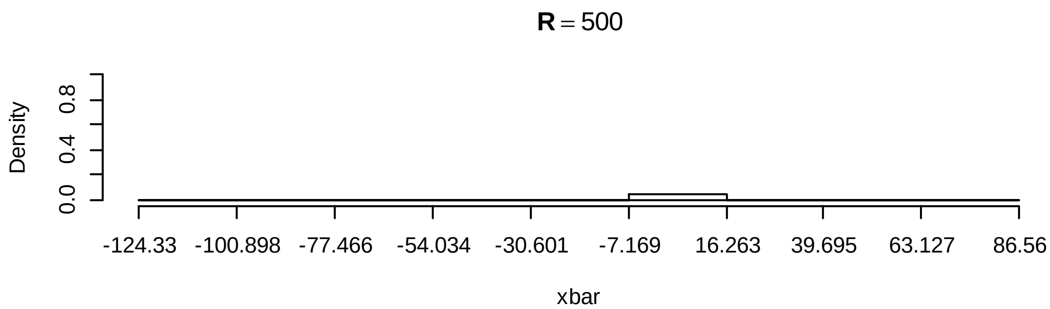
Now here we will check convergence in distribution for fixed n=1000 and varying R=100,500,1000.

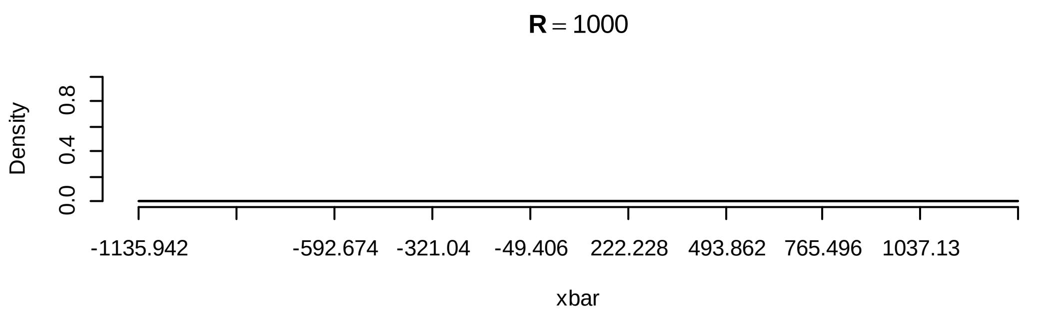
Here we will check the convergence in distribution for some well known statistics like sample mean , median , minimum , maximum , midrange etc for Cauchy population with certain location and scale parameter and make a significant comparison between their large sample behaviour and their dependency on parameters for fixed sample size n=1000 and repetition numbers R=100,500,1000.

A. MEAN-

i) C(0,1)-



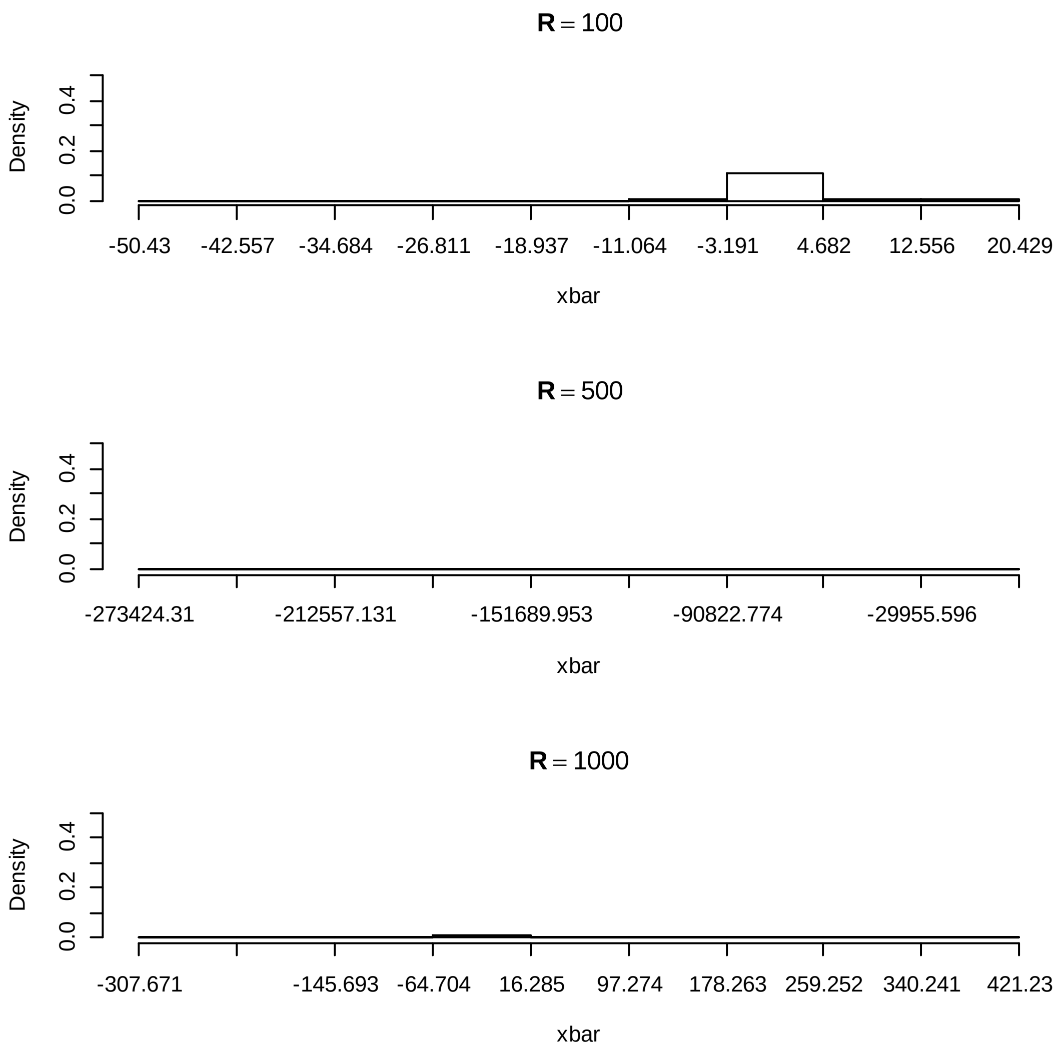




Observations-

1) ***For relatively small repetition number, (R=100), there is no such frequency density histogram.***
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***

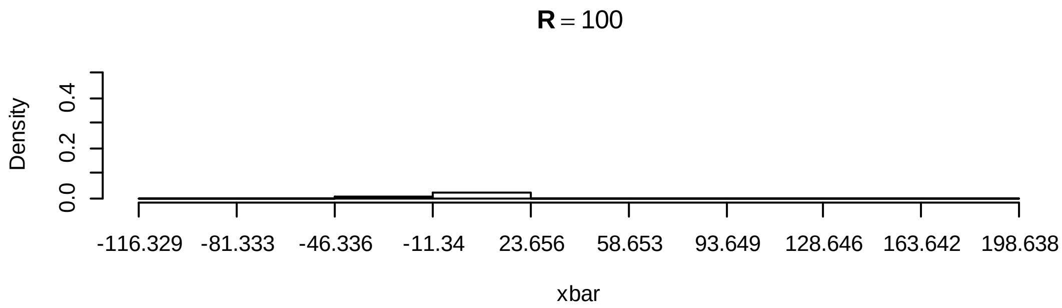
ii) C(1,1)-

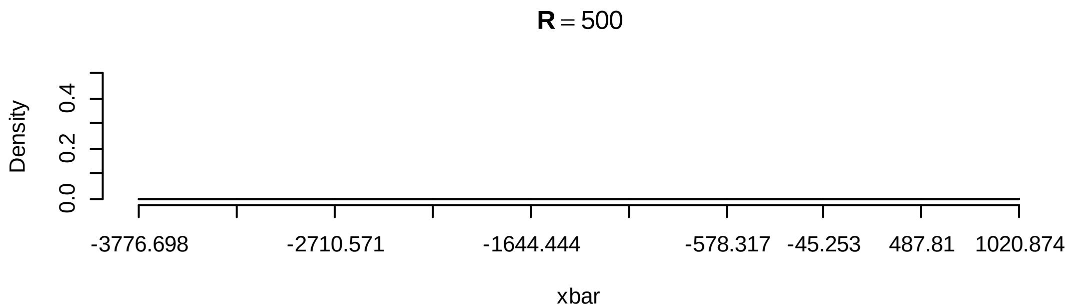


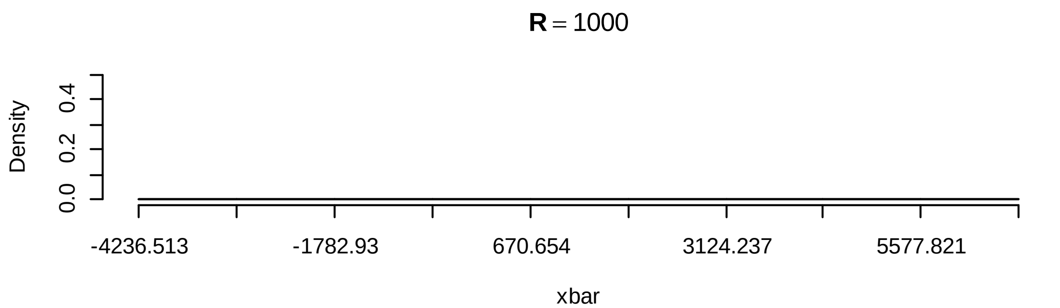
Observations-

1) ***For relatively small repetition number, (R=100), there is no such frequency density histogram.***
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***

iii) C(1,5)-







Observations-

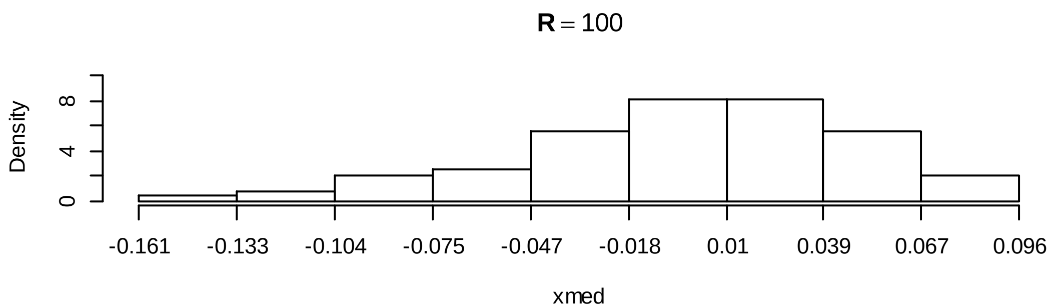
1) ***For relatively small repetition number, (R=100), there is no such frequency density histogram.***
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***

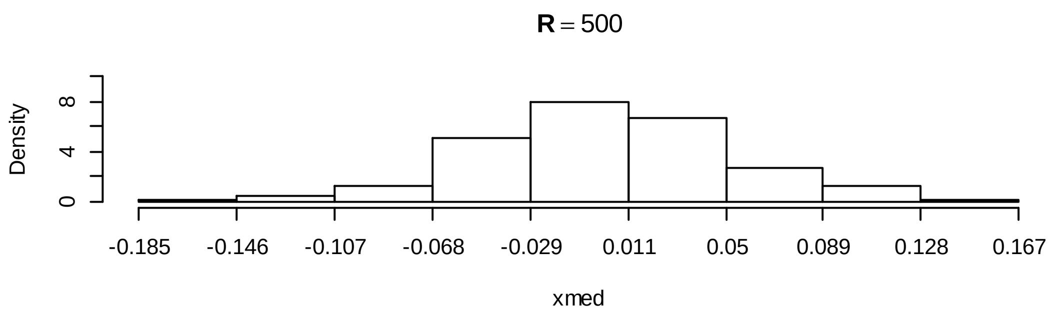
Conclusions-

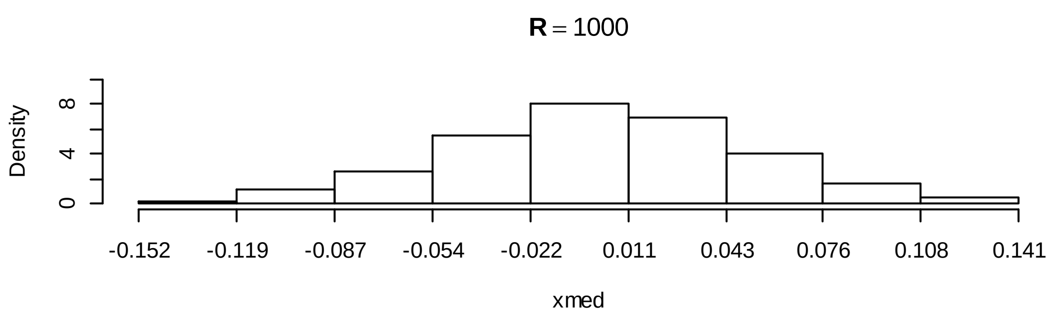
1) ***Irrespective of location and scale parameter for relatively small repetition number, (R=100), there is no such frequency density histogram.***
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***
3) ***The scale and location parameter has no such significant effect on the diagram.***

B. MEDIAN-

i) C(0,1)-



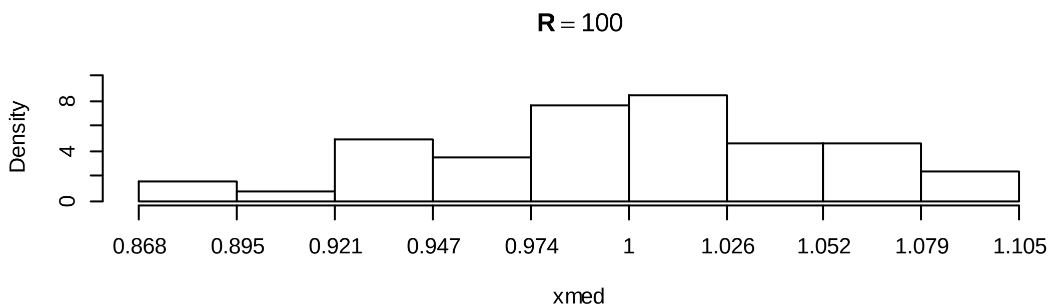


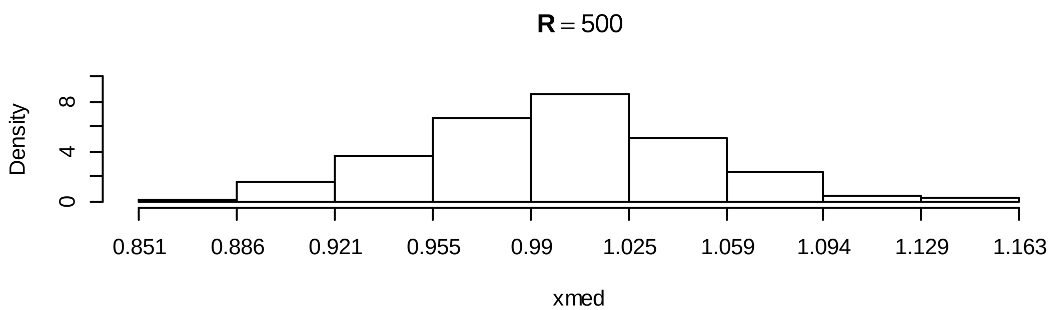


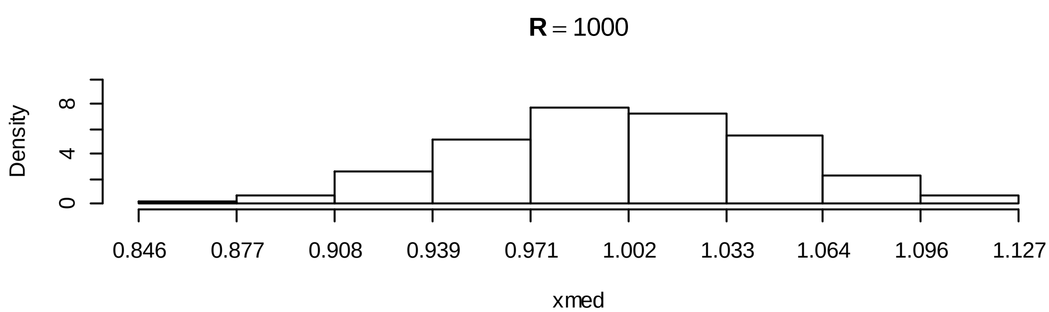
Observations-

1) ***For relatively lesser replication number, say R=100, and fixed sample size, the frequency density histogram is slightly negatively skewed.***
2) ***As the repetition number is increased, the histogram becomes more or less symmetric.***
3) ***The symmetric nature of the frequency density histogram for large repetition number and fixed n , indicates asymptotic normal behaviour of Xmed.***
4) ***Here keeping sample size fixed, as the replication number is increased, there is no such change in average height of the histograms, this implies that asymptotic variance of Xmed does not depend on R(unlike sample size n).***

ii) C(1,1)-



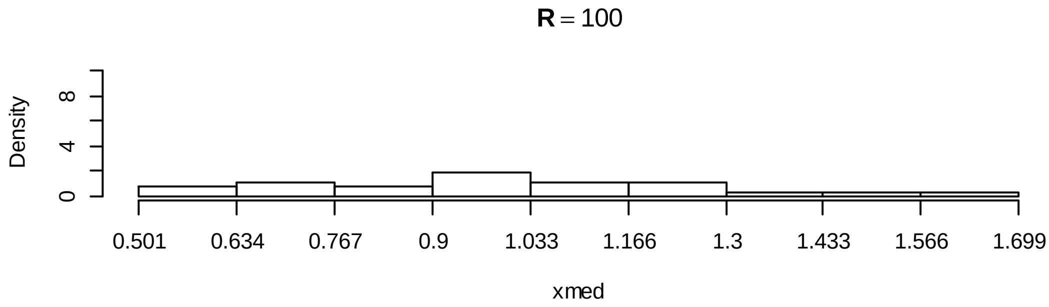


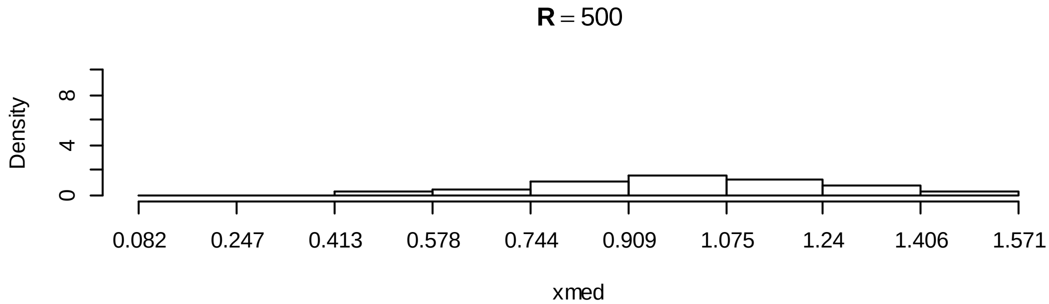


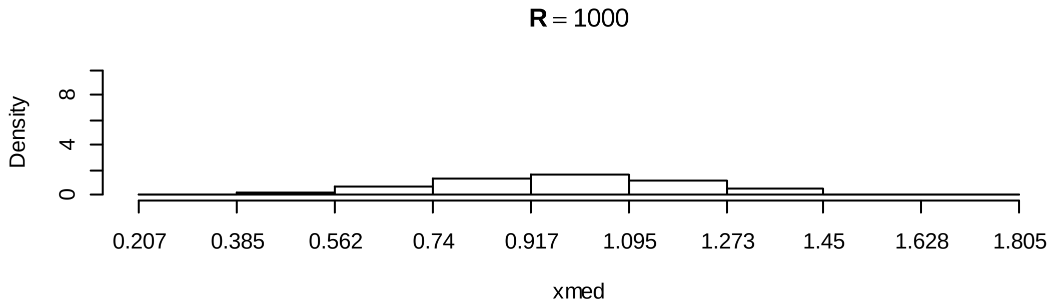
Observations-

1) ***For relatively lesser replication number, say R=100, and fixed sample size, the frequency density histogram is slightly negatively skewed, but the skewness is lesser than that of R=100 in C(0,1) distn.***
2) ***As the repetition number is increased, the histogram becomes more or less symmetric.***
3) ***The symmetric nature of the frequency density histogram for large repetition number and fixed n , indicates asymptotic normal behaviour of Xmed.***
4) ***Here keeping sample size fixed, as the replication number is increased, there is no such change in average height of the histograms, this implies that asymptotic variance of Xmed does not depend on R(unlike sample size n).***

iii) C(1,5)-







Observations-

1) ***For relatively lesser replication number, say R=100, and fixed sample size, the frequency density histogram is slightly positively skewed.***
2) ***As the repetition number is increased, the histogram becomes more or less symmetric.***

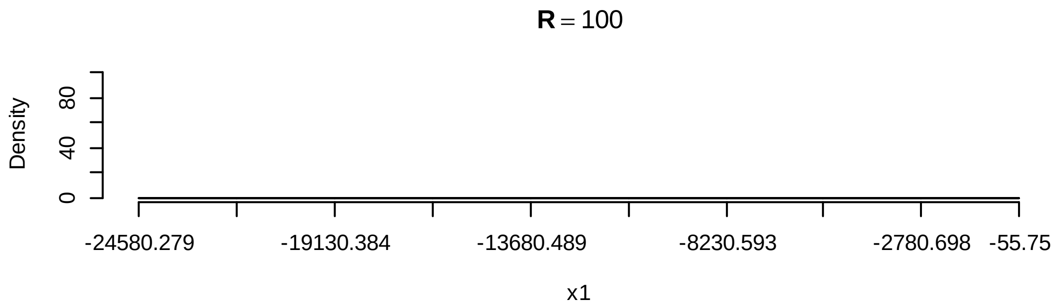
3) ***The symmetric nature of the frequency density histogram for large repetition number and fixed n , indicates asymptotic normal behaviour of Xmed.***
4) ***Here keeping sample size fixed, as the replication number is increased, there is no such change in average height of the histograms, this implies that asymptotic variance of Xmed does not depend on R(unlike sample size n).***
5) ***As keeping the location parameter fixed , the scale parameter is increased, as we transform from C(1,1) to C(1,5) , the average height of the histograms are decreased.***

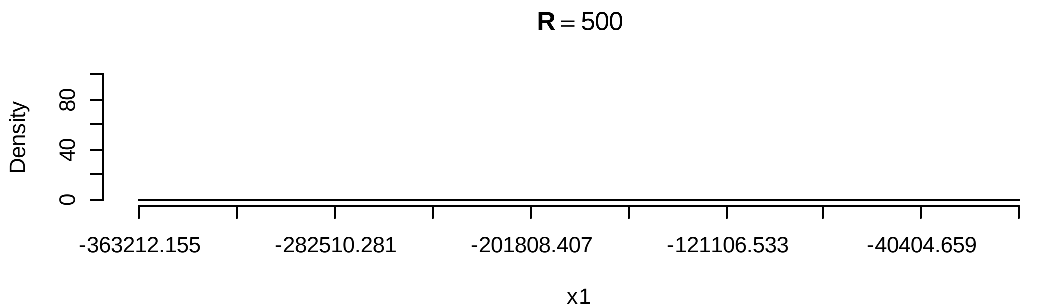
Conclusions-

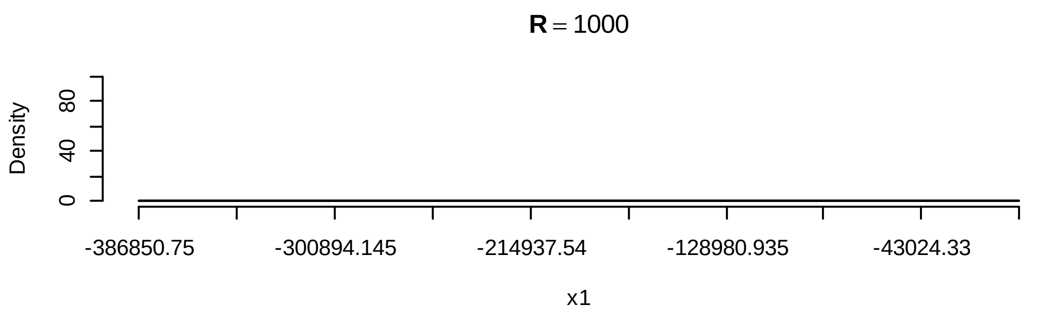
1) ***Irrespective of the location and scale parameter, of the Cauchy distribution, for small repetition number and fixed sample size n, there is no fixed shape of the histogram.***
2) ***As the repetition number is increased, the histogram becomes more or less symmetric.***
3) ***The symmetric nature of the frequency density histogram for large repetition number and fixed n , indicates asymptotic normal behaviour of Xmed.***
4) ***Here keeping sample size fixed, as the replication number is increased, there is no such change in average height of the histograms, this implies that asymptotic variance of Xmed does not depend on R(unlike sample size n).***
5) ***Clearly the scale parameter has more impact on the average height of the histograms than that of the location parameter.***

C. MINIMUM X(1)-

i) C(0,1)-



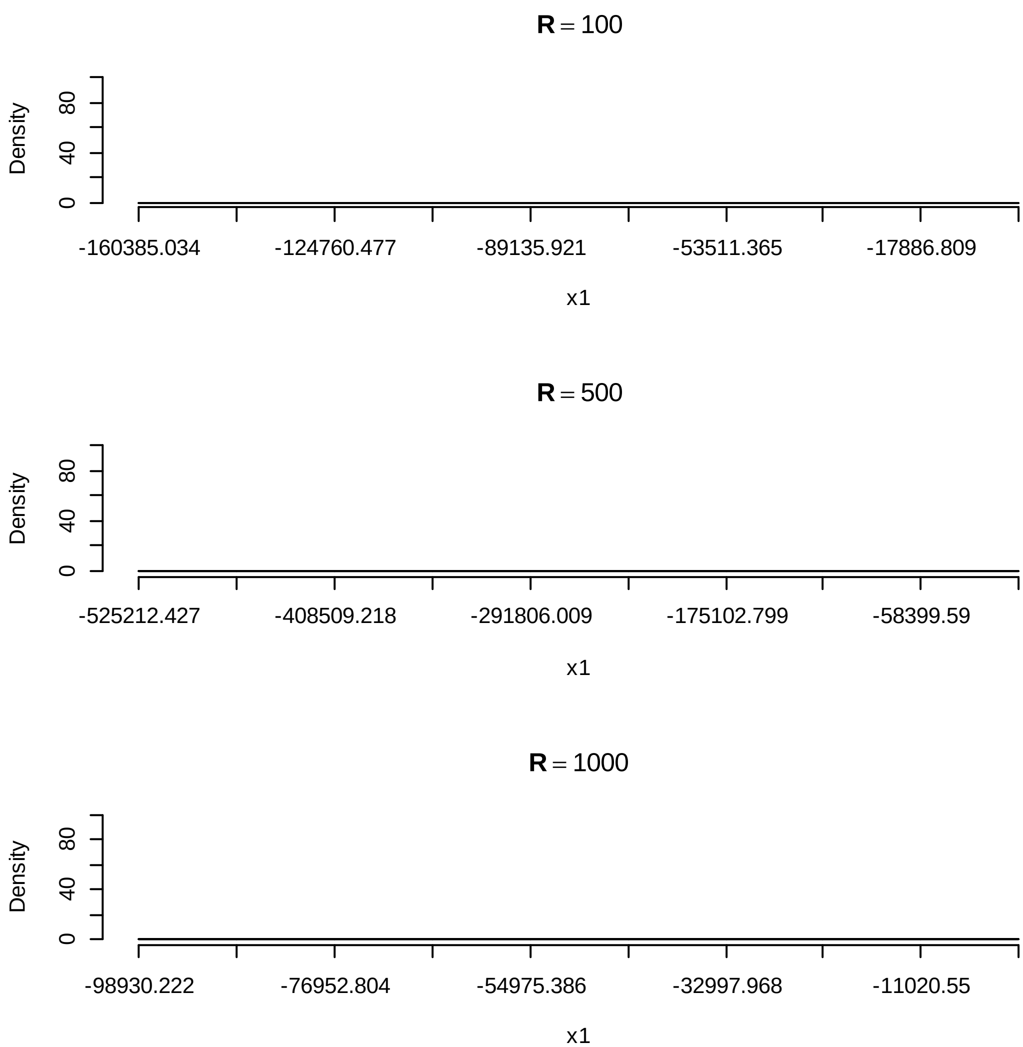




Observations-

1) ***For relatively small repetition number, (R=100), there is no such frequency density histogram.***
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***

ii) C(1,1)-

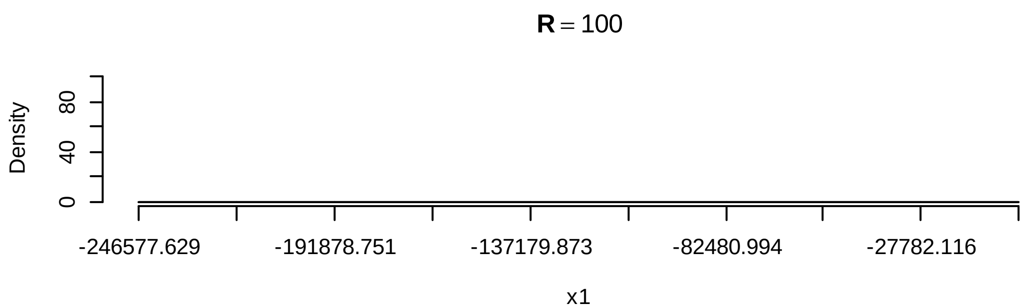


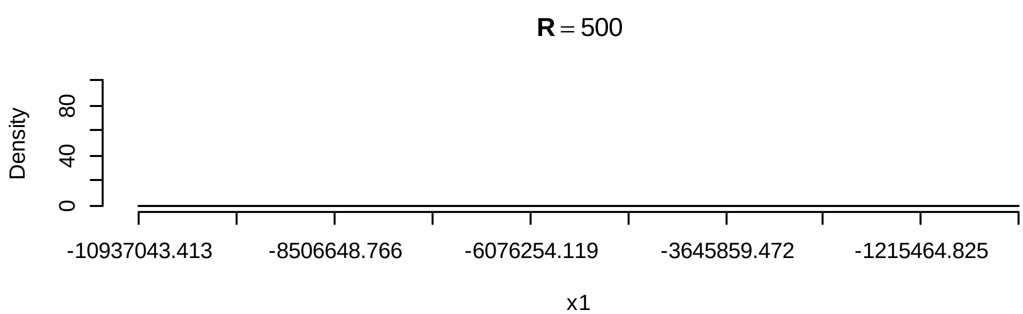
Observations-

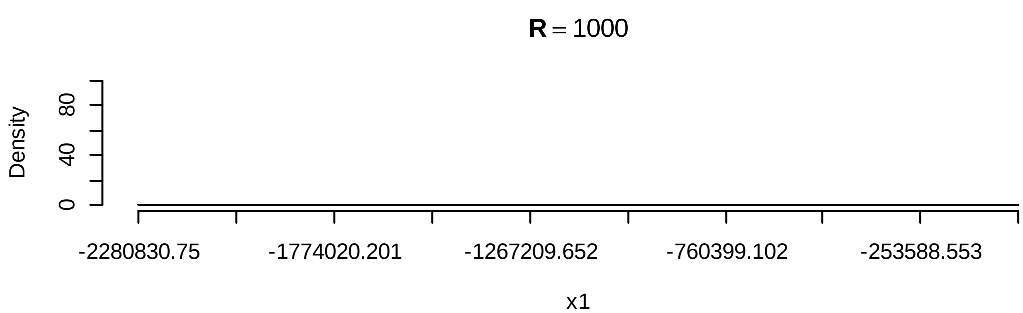
1) ***For relatively small repetition number, (R=100), there is no such frequency density histogram.***

2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***

iii) C(1,5)-







Observations-

1) ***For relatively small repetition number, (R=100), there is no such frequency density histogram.***

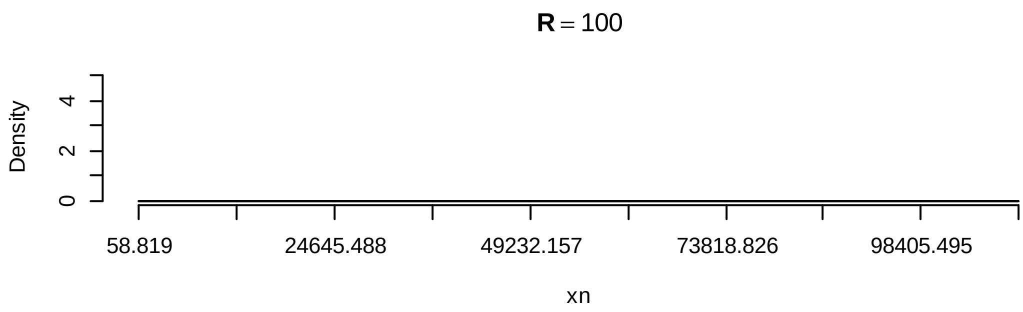
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***

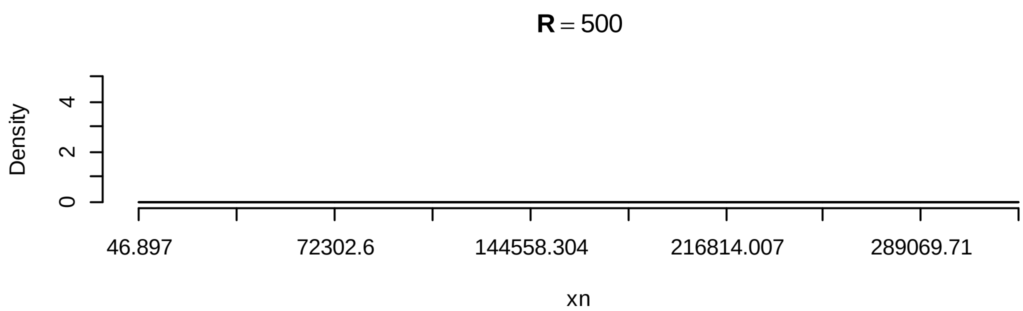
Conclusions-

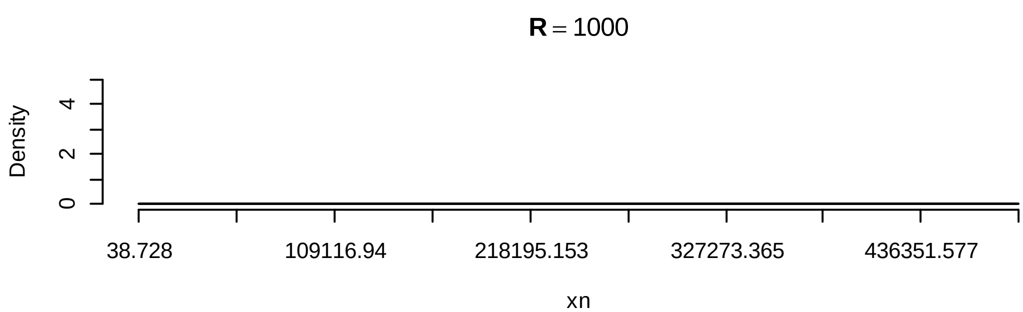
1) ***Irrespective of location and scale parameter for relatively small repetition number, (R=100), there is no such frequency density histogram.***
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***
3) ***The scale and location parameter has no such significant effect on the diagram.***

D. MAXIMUM X(n)-

i) C(0,1)-



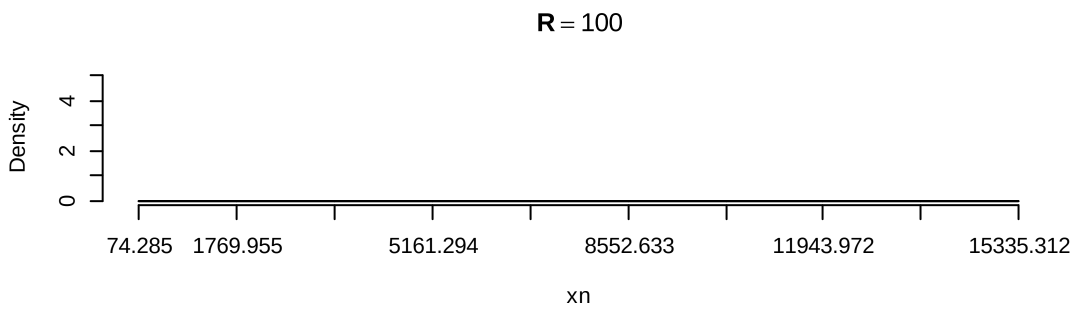


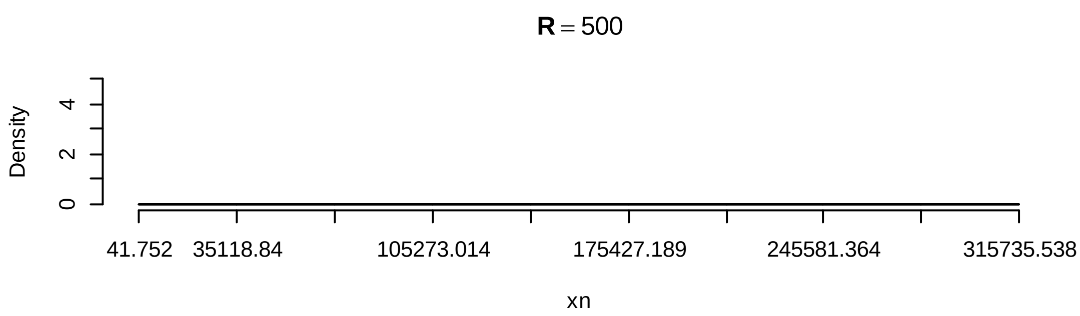


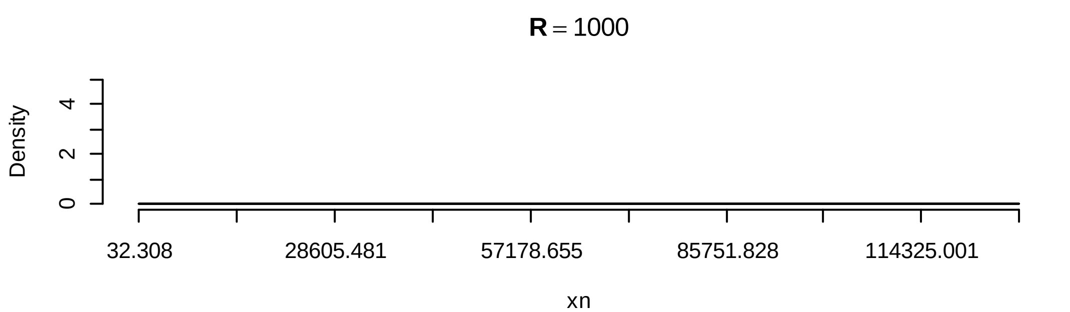
Observations-

1) ***For relatively small repetition number, (R=100), there is no such frequency density histogram.***
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***

ii) C(1,1)-



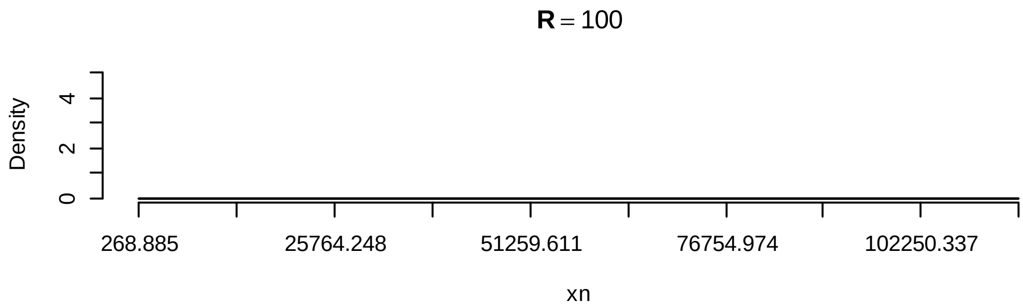


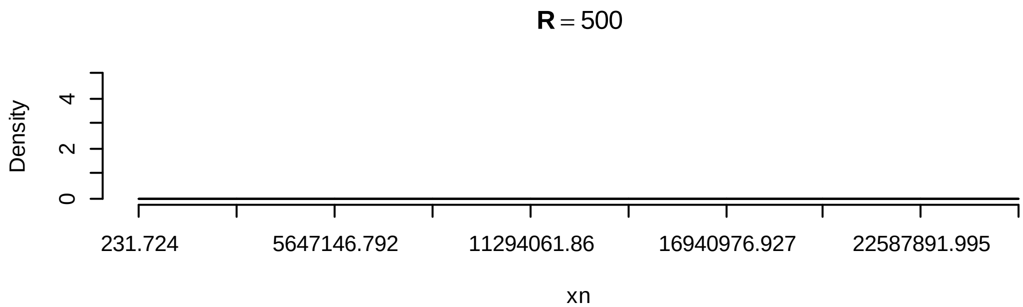


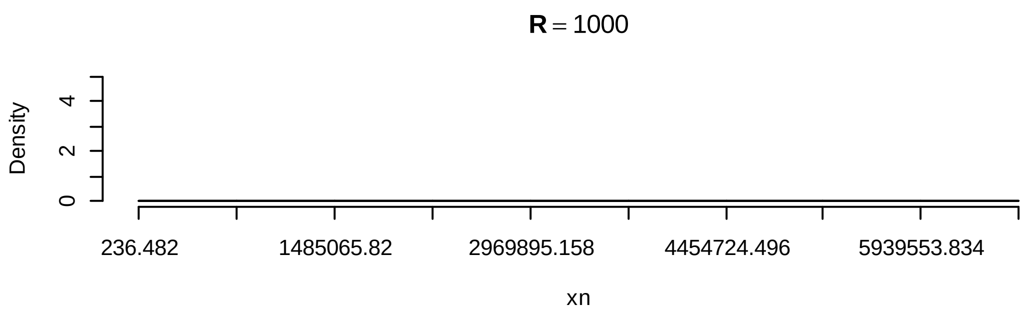
Observations-

1) ***For relatively small repetition number, (R=100), there is no such frequency density histogram.***
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***

iii) C(1,5)-







Observations-

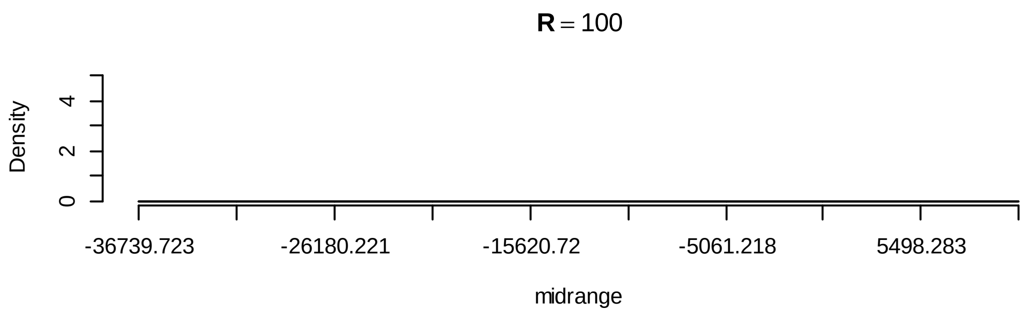
1) ***For relatively small repetition number, (R=100), there is no such frequency density histogram.***
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***

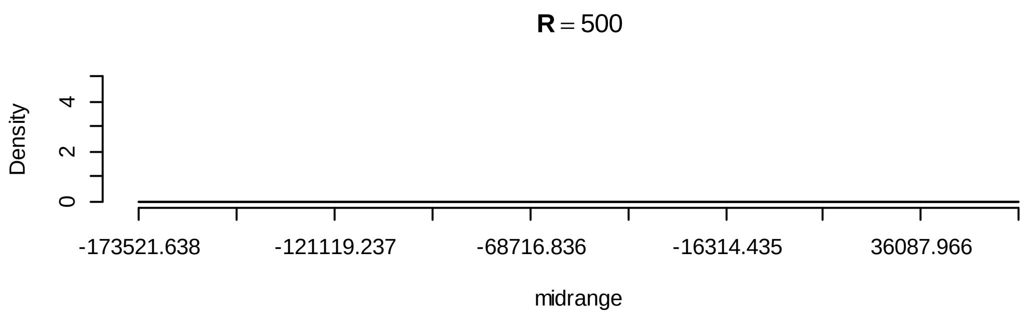
Conclusions-

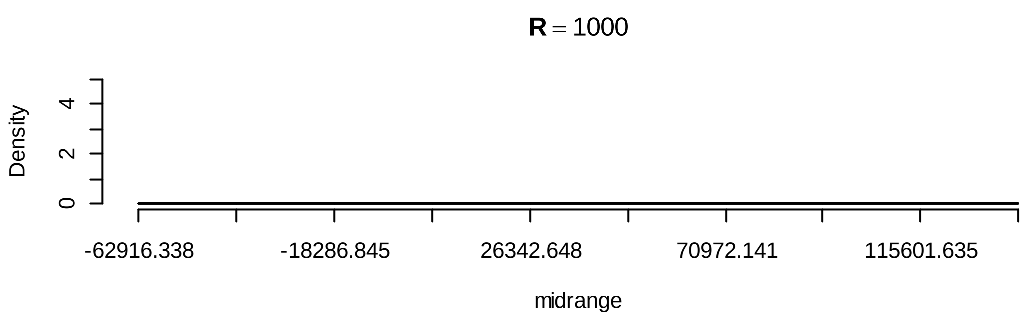
1) ***Irrespective of location and scale parameter for relatively small repetition number, (R=100), there is no such frequency density histogram.***
2) ***As we increase the repetition number, keeping the sample size fixed, then also, there is no such change in the behaviour of the diagram.***
3) ***The scale and location parameter has no such significant effect on the diagram.***

E. MIDRANGE-

i) C(0,1)-







Observations-

1) ***For relatively small repetition number, (R=100), there is no such frequency density histogram.***
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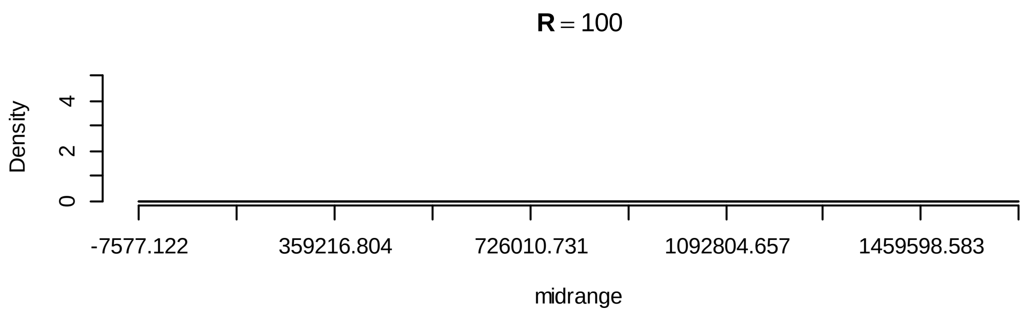
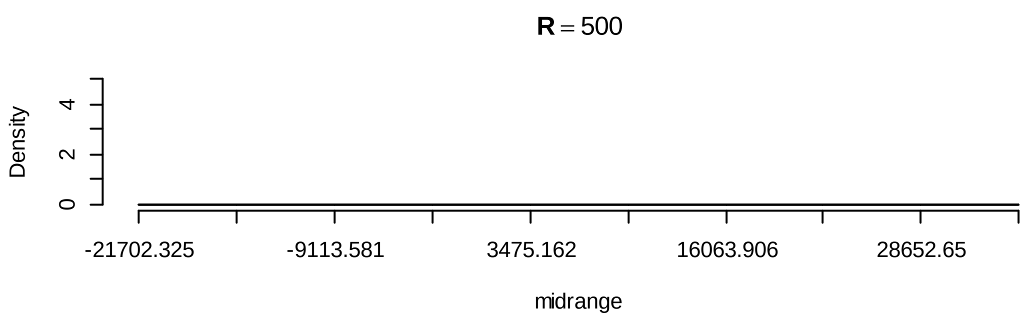
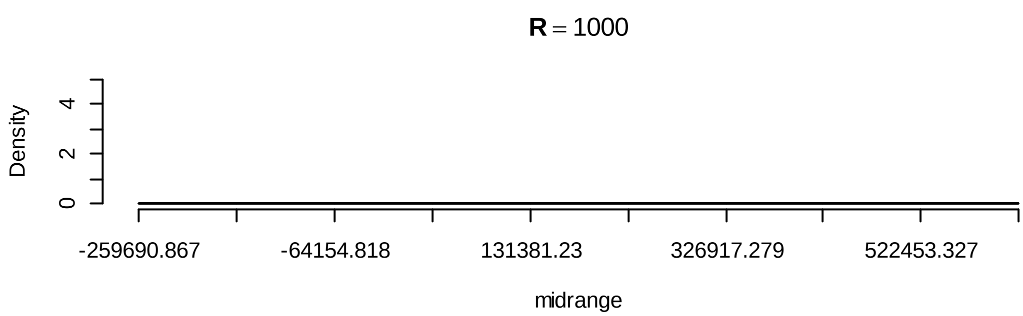
ii) C(1,1)-







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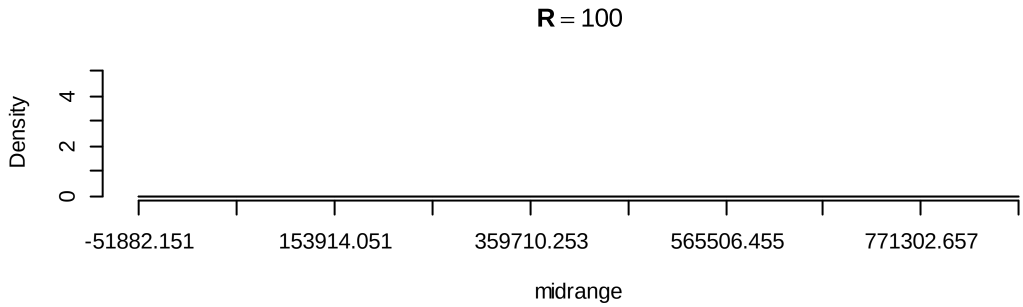

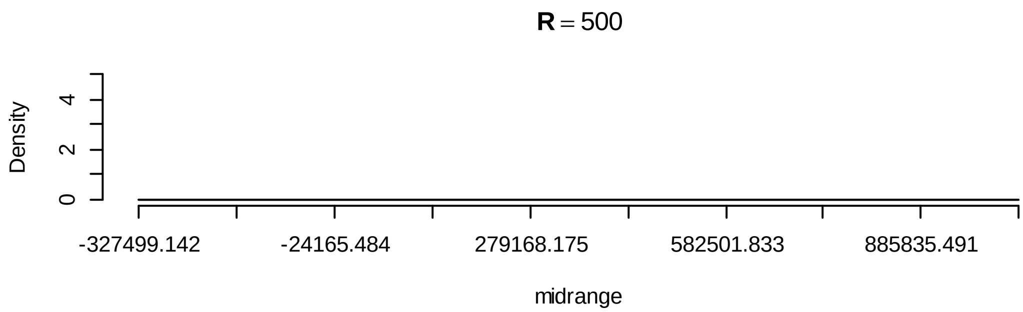

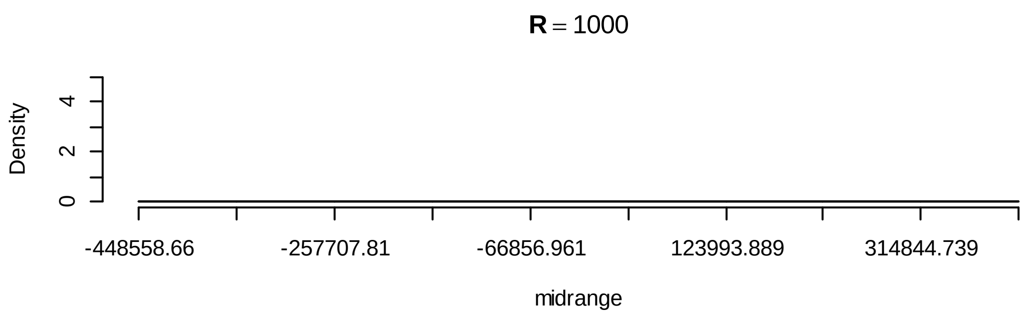

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Conclusions-

1) ***Irrespective of location and scale parameter for relatively small repetition number, (R=100), there is no such frequency density histogram.***

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3) ***The scale and location parameter has no such significant effect on the diagram.***

FINAL CONCLUSIONS-

Normal distribution-

1. ***Irrespective of the parameters of the Normal population, the sample mean tends to converge to its population mean , i.e, the location parameter.***
2. ***The sample median also tends to converge to its location parameter.***
3. ***Where as the sample minimum and maximum does not tend to converge to any particular limit.***
4. ***For fixed R=100 and varying n=100,200,…100, the sample mean and median converges in probability to its location parameter, though there is no such significant effect of the location parameter on the convergence rate, the scale parameter has a negative impact on the rate.***
5. ***For fixed R=1000 and varying n=100,500 or 1000, the frequency histogram of X(1) is negatively skewed, X(n) is positively skewed, but sample mean, median and midrange are more or less symmetric .This symmetric nature indicates asymptotic behaviour of sample mean , median and midrange.***
6. ***For fixed n=1000, and R=100, 500, 1000, the frequency density histogram of sample X(1) is negatively skewed and X(n) is positively skewed but that of sample mean median midrange are symmetric, indicating their asymptotic normal behaviour.***

7. For sample mean ,median and midrange t he average height of the histograms remain fixed in case of varying R and fixed n, where as the sample size n is increased, keeping R fixed, the average height decreases.

8. For both fixed R varying n and varying R and fixed n, V(midrange)>V(Xmed)>V(Xbar).

9. The asymptotic variance of sample mean, median, midrange is independent of R but depends on n inversely.

Exponential distribution-

1. ***Irrespective of the population parameter, the sample mean tends to converge in probability to the population mean say, u and the sample median converges in probability to uln(2), for fixed R=100 and vsarying n=100, 200, 300,…..,1000.***
2. ***We can not find any guess limit for sample X(n),further we can show that X(n) does not converge in probability to any value.***
3. ***Also , we can find the guess limit for X(1) as '0', and show that X(1) is consistent for '0'.***
4. ***For fixed n=1000 and varying R, sample mean and median tends to normality but for relatively small repetition number , say R=100 the frequency density histogram is unbounded for X(1) and X(n), i.e., for some x(1) and x(n) values density is undefined.***

5. ***For fixed R=1000 and varying n, , sample mean and median tends to normality but for relatively small sample size , say R=100 the frequency density histogram is unbounded for X(1) and X(n), i.e., for some x(1) and x(n) values density is undefined.***
6. ***The asymptotic variance of Xbar and Xmed is independent of R but depends on n invetsely.***

Cauchy Distribution-

1. ***Irrespective of the location and scale parameter of the Cauchy distribution the sample mean tends to converge to '0', which is not always the location parameter, but sample mean does not converge to '0', in probability.***
2. ***The sample median converges in probability to the location parameter 'u'. But the location parameter has no such impact on the rate of convergence, the scale parameter has a negative impact on the rate of convergence.***
3. ***But we can not find any guess limit for X(1) or X(n), rather we can show that they do not converge in probability to any particular limit.***
4. ***For fixed R=1000 and varying n , only sample median has symmetric frequency density histogram which indicates asymptotic normal behaviour of the sample median. Other statistics like sample mean, X(1),X(n) or midrange has no such frequency density histogram.***
5. ***For fixed n=1000 and varying R, only sample median has symmetric frequency density histogram which indicates asymptotic normal behaviour of the sample median. Other statistics like sample mean, X(1),X(n) or midrange has no such frequency density histogram***

6. The asymptotic variance of Xmed is independent of R but depends on n inversely.

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Again I would like to thank my supervisor professor Mrs. Surabhi Dasgupta, for the blessing, help and guidance given by her from time to time , which encouraged me to devote my self into this project work.

Thank You all.

APPENDIX

Here we will discuss the required R codes in brief-

A. Code to find guess limits-

```
rm(list=ls())
F=function(n)
{

a=c(rnorm(n,5,2))
a
b=mean(a)
return(b)
}
xbar=array(dim=1)
n=seq(1:1000)
n
 for(i in n)
{
 xbar[i]=F(i)

}
 plot(n,xbar,'l')
```

***** It is just an example to find the guess limit for sample mean of N(5,2) distribution, we can easily modify this code to draw sample from Cauchy or Exponential population with different parameters and find the guess limits of different statistics.***

B. Code for Checking convergence in probability-

```
wlln<-function(n,eps,k = 100)
{
  x = rexp(n*k,rate = 1)
  M = matrix(x,nr = k)
  X = apply(M,1,'median')
  t = sum(abs(X - log(2)) < eps)/k
```

```
  return(t)
}

#This is a quiet slow code because traking
#convergence in probability is in itself difficult
set.seed(50)

#epsilon = 0.05
prob2=NULL
ind = seq(1,10000,100)
for (n in 1:length(ind))
  prob2[n]=wlln(n = ind[n],eps = 0.05,k = 100)
plot(ind,prob2,type="l",xlab="n",ylab="probability",main=expression(epsilon~
"= 0.05"))
```

***** This is a code to check convergence in probability for sample median of Exp(mean=1) distribution. We can simply modify the code to check convergence in probability for other statistics like sample mean, maximum or minimum of other distributions like Cauchy or Normal with different parameters.***

C. Code for convergence in distribution for fixed R=1000 and n=100,500 ,1000-

```
rm(list=ls())
f=function(n,R)
{
 x1=array(dim=1)
 xn=array(dim=1)
 midrange=array(dim=1)
 xmed=array(0)
 xbar=array(0)
 b=array(0)


 for(i in 1:R )
{b=rnorm(n,0,1)
 x1[i]=min(b)
 xn[i]=max(b)
 midrange[i]=(xn[i]+x1[i])/2
 xmed[i]=median(b)
 xbar[i]=mean(b)
}
print(max(x1))
#br1=seq(min(x1),max(x1),length=10)
```

```
#h1=hist(x1,breaks=br1,freq=FALSE,xaxt="n",ylim=c(0,100),main=bquote(bold(n==.
(n))))
#axis(1,at=br1,round(br1,3))
#br2=seq(min(xn),max(xn),length=10)
#h2=hist(xn,breaks=br2,freq=FALSE,xaxt="n",ylim=c(0,5),main=bquote(bold(n==.(n
))))
#axis(1,at=br2,round(br2,3))
#br3=seq(min(midrange),max(midrange),length=10)
#h3=hist(midrange,breaks=br3,freq=FALSE,xaxt="n",ylim=c(0,5),main=bquote(bold(
n==.(n))))
#axis(1,at=br3,round(br3,3))
#br4=seq(min(xmed),max(xmed),length=10)
#h4=hist(xmed,breaks=br4,freq=FALSE,xaxt="n",ylim=c(0,150),main=bquote(bold(n
==.(n))))
#axis(1,at=br4,round(br4,3))
#curve(dnorm(x),from=0,to=3,add=T)
br5=seq(min(xbar),max(xbar),length=10)
h5=hist(xbar,breaks=br5,freq=FALSE,xaxt="n",ylim=c(0,15),main=bquote(bold(n==.
(n))))
axis(1,at=br5,round(br5,3))

}

 par(mfrow=c(3,1))
f(100,1000)
f(500,1000)
f(1000,1000)
```

***** Note that this is a code to check convergence in distribution of sample mean from N(0,1) distribution , for fixed R=1000 and varying n=100,500,1000.We can modify the code to check convergence in distribution for other statistics like sample median, midrange, maximum, minimum for different distributions like Cauchy or Exponential with different parameters.***

D. ***Code for convergence in distribution for fixed n=1000 and varying R=100,500,1000.***

```
rm(list=ls())
f=function(n,R)
{
 x1=array(dim=1)
 xn=array(dim=1)
```

```
 midrange=array(dim=1)
 xmed=array(0)
 xbar=array(0)
 b=array(0)


 for(i in 1:R )
{b=rexp(n,rate=1)
 x1[i]=min(b)
 xn[i]=max(b)
 midrange[i]=(xn[i]+x1[i])/2
 xmed[i]=median(b)
 xbar[i]=mean(b)
}
print(max(x1))
#br1=seq(min(x1),max(x1),length=10)
#h1=hist(x1,breaks=br1,freq=FALSE,xaxt="n",ylim=c(0,100),main=bquote(bold(
R==.(R))))
#axis(1,at=br1,round(br1,3))
#br2=seq(min(xn),max(xn),length=10)
#h2=hist(xn,breaks=br2,freq=FALSE,xaxt="n",ylim=c(0,5),main=bquote(bold(n=
=.(n))))
#axis(1,at=br2,round(br2,3))
br3=seq(min(midrange),max(midrange),length=10)
h3=hist(midrange,breaks=br3,freq=FALSE,xaxt="n",ylim=c(0,1),main=bquote(bo
ld(n==.(n))))
axis(1,at=br3,round(br3,3))
#br4=seq(min(xmed),max(xmed),length=10)
#h4=hist(xmed,breaks=br4,freq=FALSE,xaxt="n",ylim=c(0,150),main=bquote(bol
d(n==.(n))))
#axis(1,at=br4,round(br4,3))
#curve(dnorm(x),from=0,to=3,add=T)
#br5=seq(min(xbar),max(xbar),length=10)
#h5=hist(xbar,breaks=br5,freq=FALSE,xaxt="n",ylim=c(0,15),main=bquote(bold(
n==.(n))))
#axis(1,at=br5,round(br5,3))

}

 par(mfrow=c(3,1))
f(1000,100)
f(1000,500)
f(1000,1000)
```

***** Note that this is a code to check convergence in distribution of sample median from C(0,1) distribution , for fixed n=1000 and varying R=100,500,1000.We can modify the code to check convergence in distribution for other statistics like sample mean, midrange, maximum, minimum for different distributions like Normal or Exponential with different parameters.***

-------------------THE END---------------

THANK YOU.