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1| Explain closure properties of regular language.

=> The closure properties of regular language are:

i| Closure under union: If L & M are regular languages, so is L∪M. Let L & M be the languages of regular ~~languag~~ expressions R & S respectively. Then R+S is a regular expression whose language is L∪M.

ii| Closure under Intersection: If L & M are regular languages, so is L∩M. Let L & M be the languages of regular expressions R & S respectively then R∩S is a regular expression whose language is L∩M.

iii| Closure under concatenation: If L & M are regular languages so is L.M. If L & M be the languages of regular expressions R & S respectively, then R.S. is a regular expression whose language is L.M.

iv| Closure under Kleen closure: If L is the regular language of regular expression R then R* is a regular expression R then R* is a regular expression whose language is L*.

v| Closure under complement: The complement of a language L (with respect to an alphabet Σ such that Σ* contains L) is Σ* - L. Since Σ* is surely regular by the property of closure under kleen closure, the complement of a regular language is always regular.

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2) State & prove pumping lemma for regular language.

- Pumping lemma is used to prove that a language is not regular.

Statement: If A is a regular language, then A has a pumping length 'p' such that any string 'S' where $|S| \geq P$ may be divided into 3 parts $S = xyz$ such that the following conditions must be true:

i) $xy^iz \in A$ for every $i \geq 0$

ii) $|y| > 0$

iii) $|xy| \leq P$

Prove: $A = \{a^n b^n \mid n \geq 0\}$ is not regular.

Proof:

Assume that A is regular language

Pumping length = P

Now,

$S = a^P b^P$

Now, we need to divide S into three parts x, y, z.
For that let we assume pumping length $P = 7$. Then, we can write the string S as:

$S = aaaaaaabbbbbbb$

Now, let us see all the possible ways in which we can divide this S into three parts x, y, z for that let we take cases as follows:

Case I: The y is in the 'a' part.

i.e. $\underbrace{aa}_{x}\ \underbrace{aaaa}_{y}\ \underbrace{abbbbbbb}_{z}$

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Case II : The y is in the 'b' part.

i.e. $\underbrace{aaaaaaaabb}_{x}\ \underbrace{bbbb}_{y}\ \underbrace{b}_{z}$

Case III : The y is in the 'a' & 'b' part.

i.e. $\underbrace{aaaaaa}_{x}\ \underbrace{aabb}_{y}\ \underbrace{bbbbb}_{z}$

For case I :

No. of a's = 11 | No. of a's ≠ No. of b's
No. of b's = 7 | ~~No~~ So $xy^2z \notin A$

$\underbrace{aa}_{x}\ \underbrace{aaaa}_{y}\underbrace{aaaa}_{y}\ \underbrace{abbbbbbb}_{z}$

For case II :

aaaaaaabbbbbbbbbbb

No. of a's = 7 | No. of a's ≠ No. of b's
No. of b's = 11 | ~~No~~. So, $xy^2z \notin A$

For case II :

aaaaaaabbaabb bbbbb

Since this string does not follow the pattern $a^n b^n$ as given by question. So, this doesn't lie in our language.

So, we proved that xy^iz on taking $i=2$ all cases doesn't lie in our language. This means s cannot be pumped which leads 'to contradiction'. Hence, the language is not regular.