Q ① Show that $m(a+bX) = a+b \times m(X)$

we know $m(a+bX) = \frac{1}{N} \sum_{i=1}^{N} (a+bx_i)$

we can use linearity of summation: $\frac{1}{N} \left(\sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i \right)$

$= \frac{1}{N}(Na) + \frac{b}{N} \sum_{i=1}^{N} = a + b \cdot m(X)$

$m(a+bX) = a + b \cdot m(X)$

② $\text{cov}(X, a+bY) = b \cdot \text{cov}(X,Y)$ $\quad\quad$ * $m(Z) = m(a+bY)$

$Z = a+bY$, then: $\text{cov}(X,Z)$

$= \frac{1}{N} \sum_{i=1}^{N} (x_i - m(X))(z_i - m(Z))$

$m(Z) = m(a+bY) = a + b \cdot m(Y)$ → we have this from part 1

$z_i - m(Z) = a + by_i - (a + b m(Y)) = b(y_i - m(Y))$

so: $\text{cov}(X, a+bY) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(X)) \cdot b(y_i - m(Y)) = b \cdot \text{cov}(X,Y)$

∴ we have shown that $\text{cov}(X, a+bY) = b \cdot \text{cov}(X,Y)$

③ $\text{cov}(a+bX, a+bX) = b^2 \cdot \text{cov}(X,X)$

$Z = a+bX$, then: $\text{cov}(Z,Z) = \text{var}(Z) = \frac{1}{N} \sum_{i=1}^{N} (z_i - m(Z))^2$

$m(Z) = a + b \cdot m(X) \Rightarrow z_i - m(Z) = b(x_i - m(X))$

$\Rightarrow (z_i - m(Z))^2 = b^2 (x_i - m(X))^2$, $\text{var}(Z) = \frac{1}{N} \sum b^2 (x_i - m(X))^2 =$

$b^2 \cdot \text{var}(X) = b^2 s^2$ $\boxed{\text{cov}(a+bX, a+bX) = b^2 s^2}$

④ Median & Non-Decreasing Transformations

Let $g(x)$ be non decreasing:

If $x_i < x_j \Rightarrow g(x_i) \leq g(x_j)$ which means the order is preserved.

$\text{median}(g(X)) = g(\text{median}(X))$. which is true for quantiles.

• IQR (Q3 − Q1) & range (max − min) also preserve order under monotonic func.

unless g is linear actual values will change.

If g is non-decreasing, $\text{median}(g(X)) = g(\text{median}(X))$

⑤ $m(g(X)) = g(m(X))$?

only if g is linear. Ex $X = [1,2,3] \Rightarrow m(X) = 2$, $g(x) = x^2$

$m(g(X)) = m([1,4,9]) = 14/3 \approx 4.67 \neq g(2) = 4$

so $m(g(X)) \neq g(m(X))$ in general (unless g is linear)