SICP - Exercise 1.19

Pratyush Nalam

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The original transformation, T_{pq} is given as

$$T_{pq}(a,b) = \begin{cases} a \leftarrow a(p+q) + bq \\ b \leftarrow bp + aq \end{cases}$$

The new transformation $T_{p'q'}$ is defined as $T_{pq}(T_{pq})(a,b)$ – basically the transformation is applied twice. Substituting this in the expression for T_{pq}, we get the following:

$$T_{p'q'}(a,b) = \begin{cases} a \leftarrow (a(p+q) + bq)(p+q) + (bp + aq)q \\ b \leftarrow (bp + aq)p + (a(p+q) + bq)q \end{cases}$$

We expand the expression and simplify it to the following

$$T_{p'q'}(a,b) = \begin{cases} a \leftarrow a(p^2 + q^2 + 2pq + q^2) + b(2pq + q^2) \\ b \leftarrow b(p^2 + q^2) + a(2pq + q^2) \end{cases}$$

Clearly, this means that

$$p' = p^2 + q^2$$
$$q' = 2pq + q^2$$

This is the required solution and we use it to finish the given program.