Computational Methods And Optimization Project Abstract

Group 11: ANAGHA VASISTA, CHAITANYA MODI, MANAN CHAWLA, PRATHAM ARORA

Capacitated Vehicle Routing Problem (CVRP) With TIME Constraints

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OVERVIEW

In the Vehicle Routing Problem (VRP), the goal is to find optimal routes for multiple vehicles visiting a set of locations. (When there's only one vehicle, it reduces to the Traveling Salesperson Problem.)

The capacitated vehicle routing problem (CVRP) is a VRP in which vehicles with limited carrying capacity need to pick up or deliver items at various locations. The items have a quantity, such as weight or volume, and the vehicles have a maximum capacity that they can carry.
Time constraints refer to the requirement that vehicle visits to customers should occur only during specific time windows when the customers are available.

We will be focusing on CVRPTW, where the vehicles will be delivering items at various locations within specific time windows, all originating from one central location. In this scenario, vehicles are dispatched from a single point and make deliveries to multiple locations while adhering to the specified time windows.

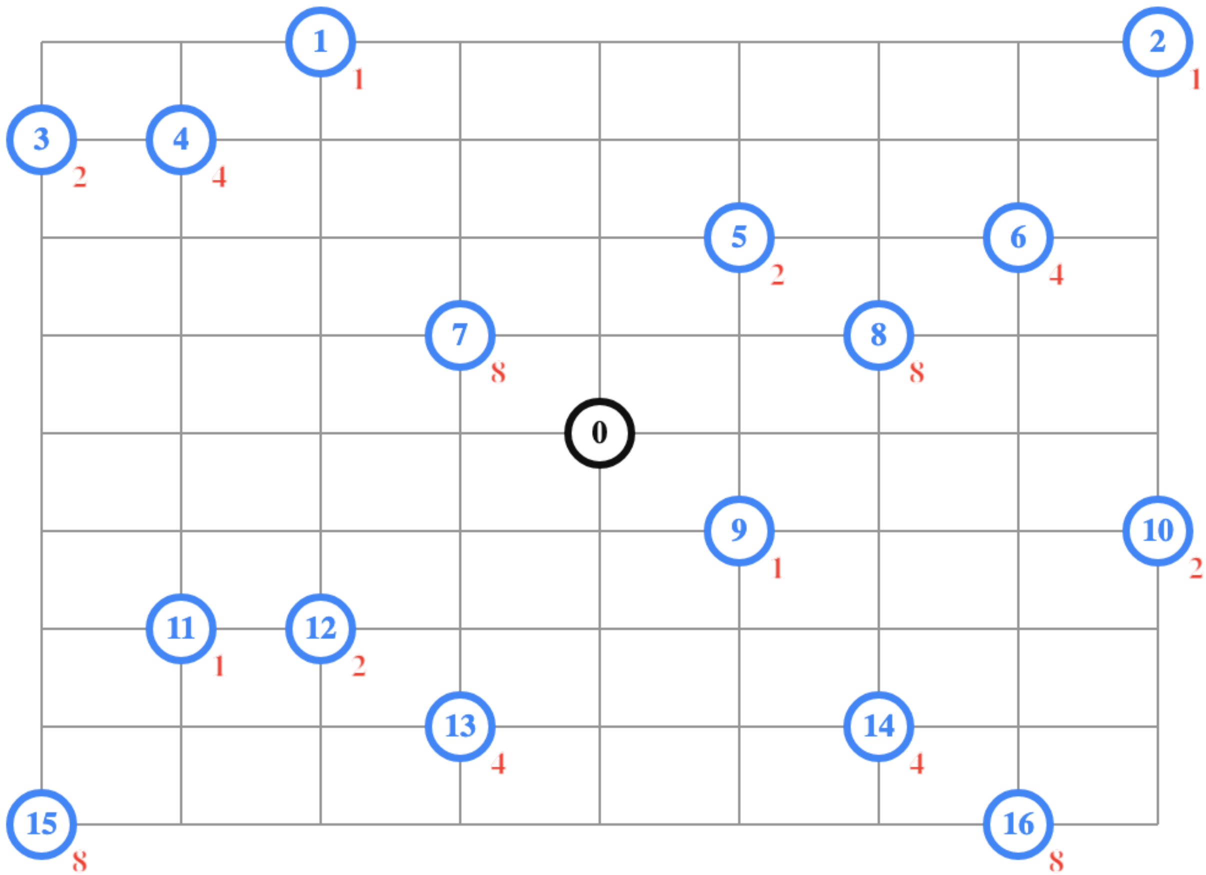
MATHEMATICS INVOLVED

Capacitated Vehicle Routing Problem (CVRP) with time constraints can be mathematically modeled as a mixed-integer linear program (MILP). The following mathematical formulation represents the CVRP with time windows and vehicle capacity constraints:

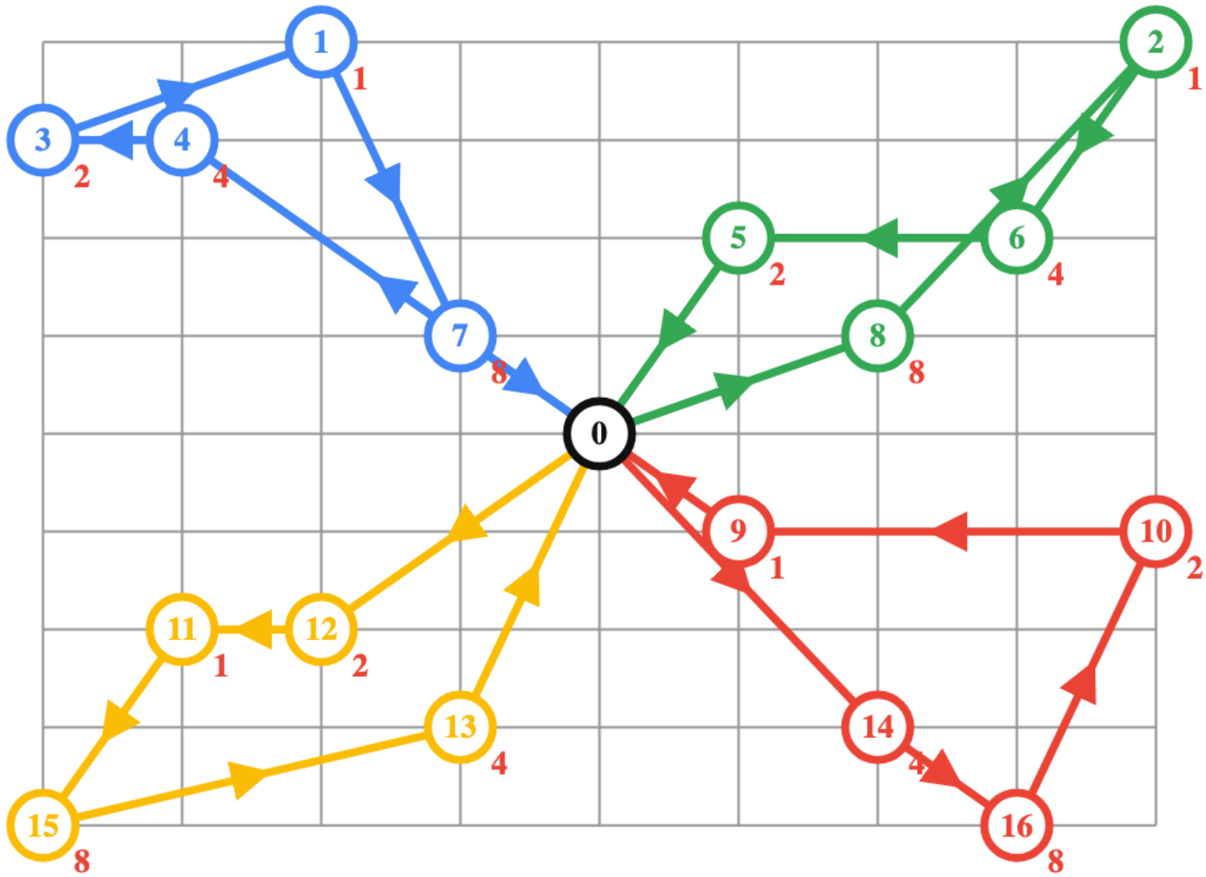
Consider a set V of nodes (demands and depot) and a set K of vehicles. We will use lowercase i and j to indicate node indexes and lowercase k to indicate vehicle indexes. As this model is valid for the asymmetric case, let us assume the nodes are part of a complete directed graph G(V, A) with arcs A. In this problem, there is a single depot node indexed by 0 and all vehicles have the same capacity Q. Consider two groups of decision variables:

x {i, j, k}: Is a binary variable that indicates an active arc from node i to node j performed by vehicle k.

y {i, k}: Is a binary variable that indicates that the demand from node i is fulfilled by vehicle k.



In the image, origin is the depot node from where all vehicles would originate. In blue are various locations where deliveries are to be made. In red are the quantity that each node (in blue) is expecting.



Our objective is to minimize the cost value (Total duration) associated with active arcs. The cost of traversing the arc i, j is c_{ij}. The objective function can be stated as the following.

$$\min \quad \sum_{k \in K} \sum_{i \in V} \sum_{j \in V} c_{i,j} x_{i,j,k}$$

We also need to include constraints that ensure:

- Each customer i is visited once, therefore has one active arc which starts from it and one that arrives on it.
- If any arc variable indexed by vehicle k goes into one node i or out of it, the demand q of this node is assigned to vehicle k.
- The total demand assigned to a vehicle must not exceed its capacity Q.
- Exactly |K| nodes start at the depot and arrive at the depot.
- There are no subtours.

$$\text{s.t} \quad \sum_{k\in K}\sum_{j\in V} x_{i,j,k} = \sum_{k\in K}\sum_{j\in V} x_{j,i,k} = 1 \qquad \forall\, i \in V \setminus \{0\}$$

$$\sum_{j\in V} x_{i,j,k} = \sum_{j\in V} x_{j,i,k} = y_{i,k} \qquad \forall\, i \in V \setminus \{0\}\ k \in K$$

$$\sum_{i\in V} q_i y_{i,k} \leq Q \qquad \forall\, k \in K$$

$$\sum_{k\in K}\sum_{j\in V} x_{0,j,k} = \sum_{k\in K}\sum_{i\in V} x_{i,0,k} = |K|$$

$$\sum_{i\in S}\sum_{j\notin S} x_{i,j,k} \geq y_{h,k} \qquad \forall\, S \subseteq V \setminus \{0\}\ h \in S\ k \in K$$

$$x_{i,j,k} \in \{0,1\} \qquad \forall\, i \in V\ j \in V\ k \in K$$

$$y_{i,k} \in \{0,1\} \qquad \forall\, i \in V\ k \in K$$

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