Cryptography Toolbox

So far:

stream ciphers

block ciphers

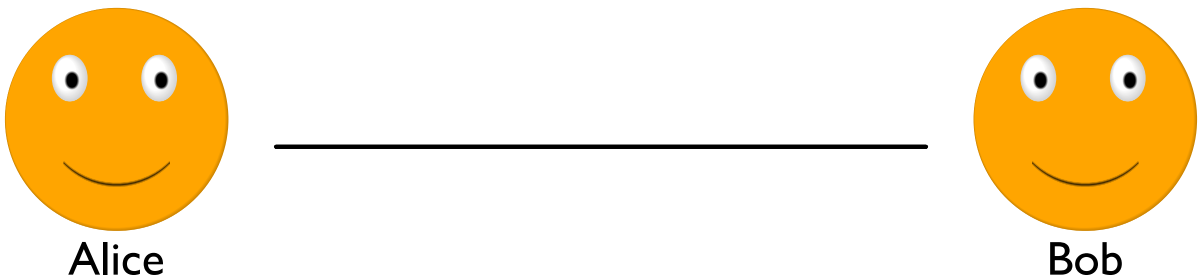
These provide **confidentiality**, but not **integrity**

Today:

cryptographic hash functions

This is a key tool for **integrity**

Message Integrity



Message Integrity

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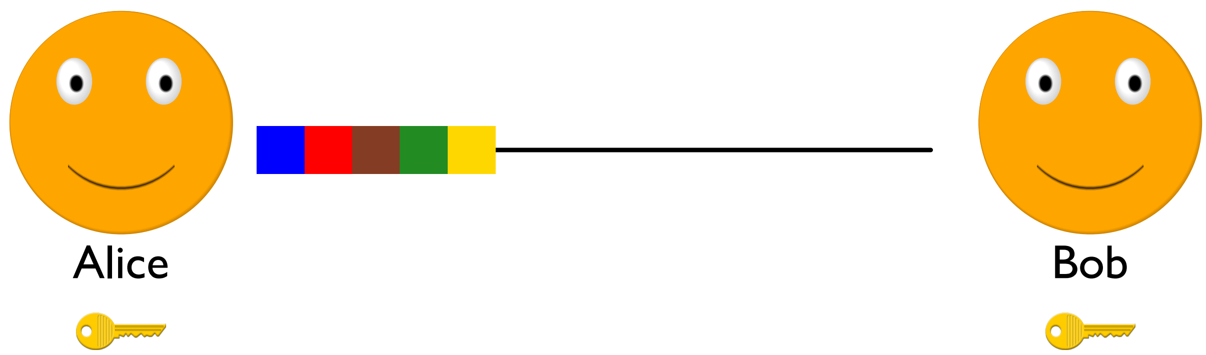

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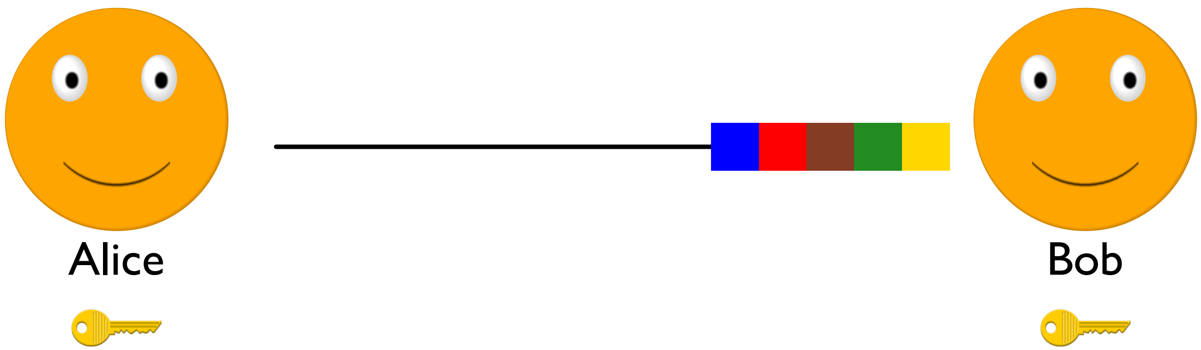

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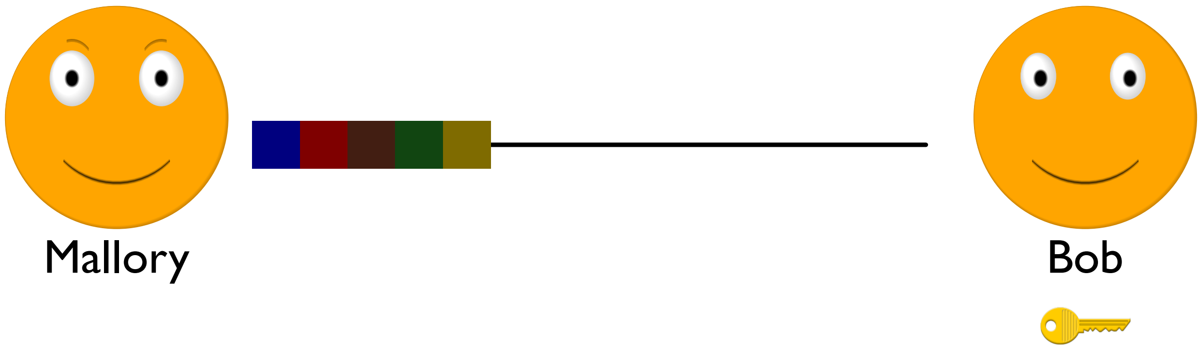

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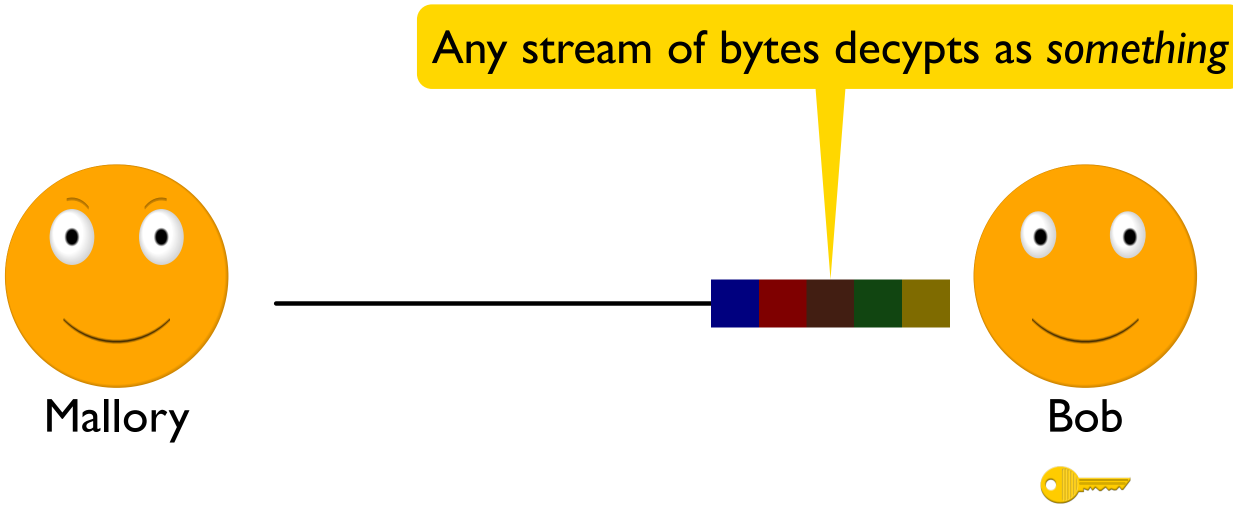

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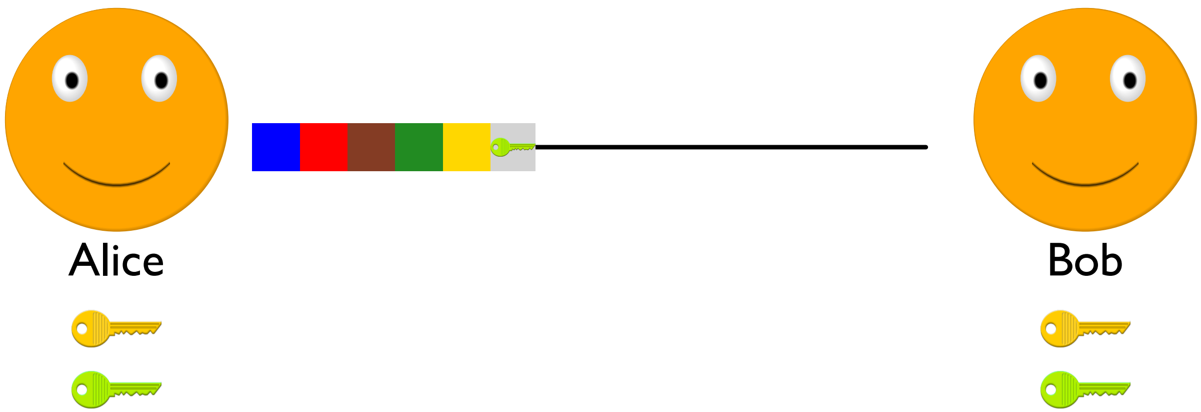

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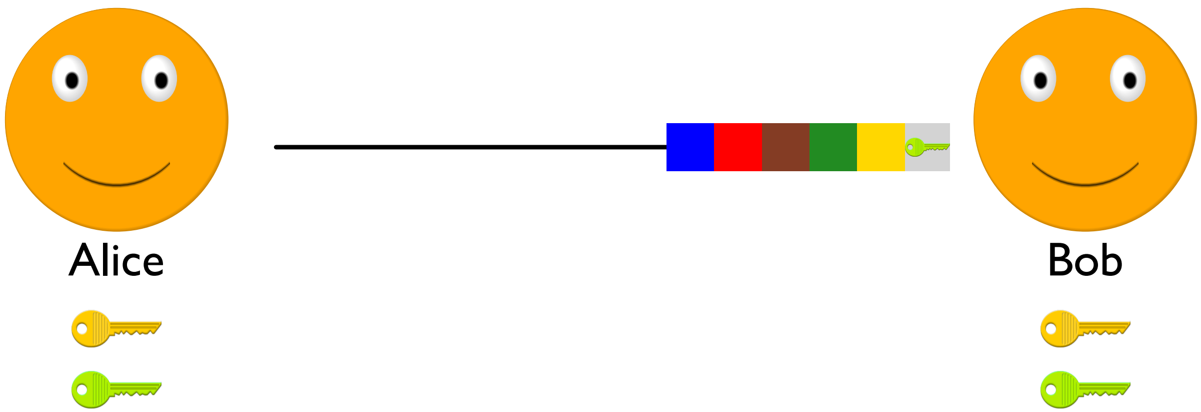
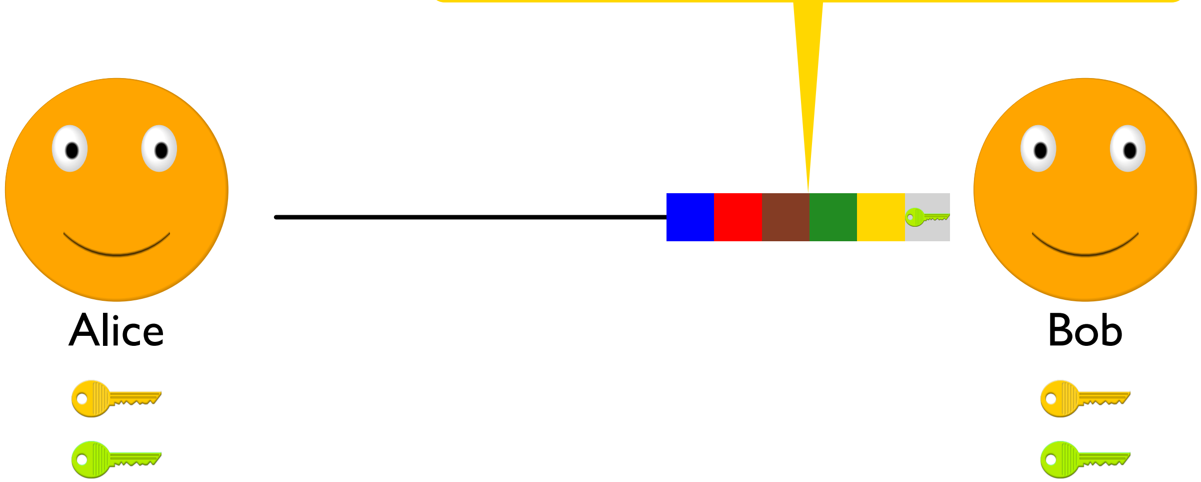


Message Integrity

Putting the key in just one place works with a **mode of operation**, but doesn't help with a stream cipher



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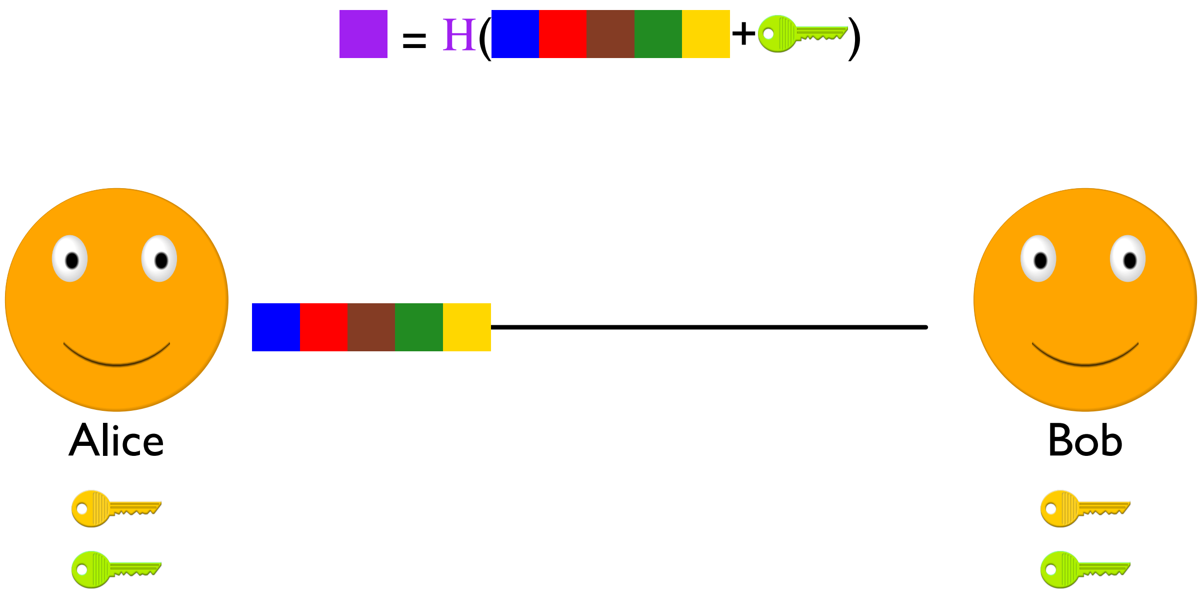
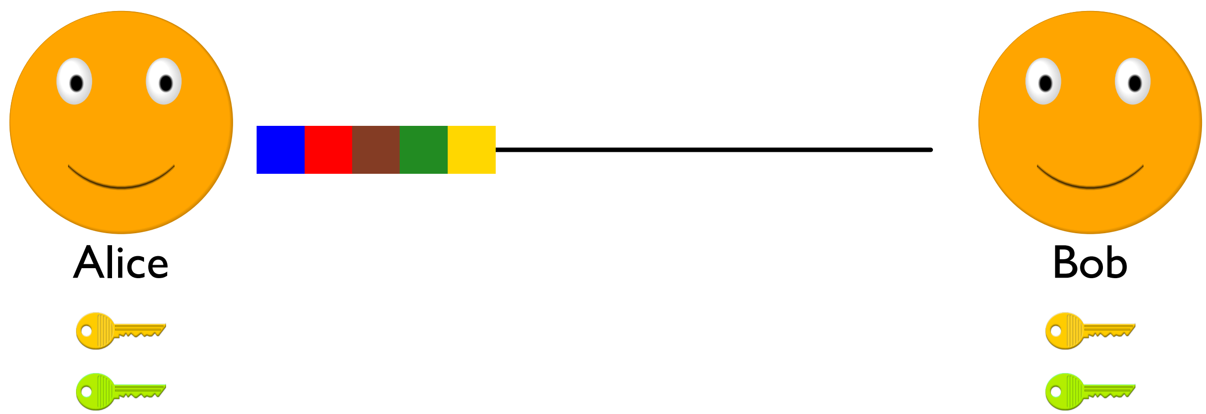


Message Integrity

hash function to summarize message and key



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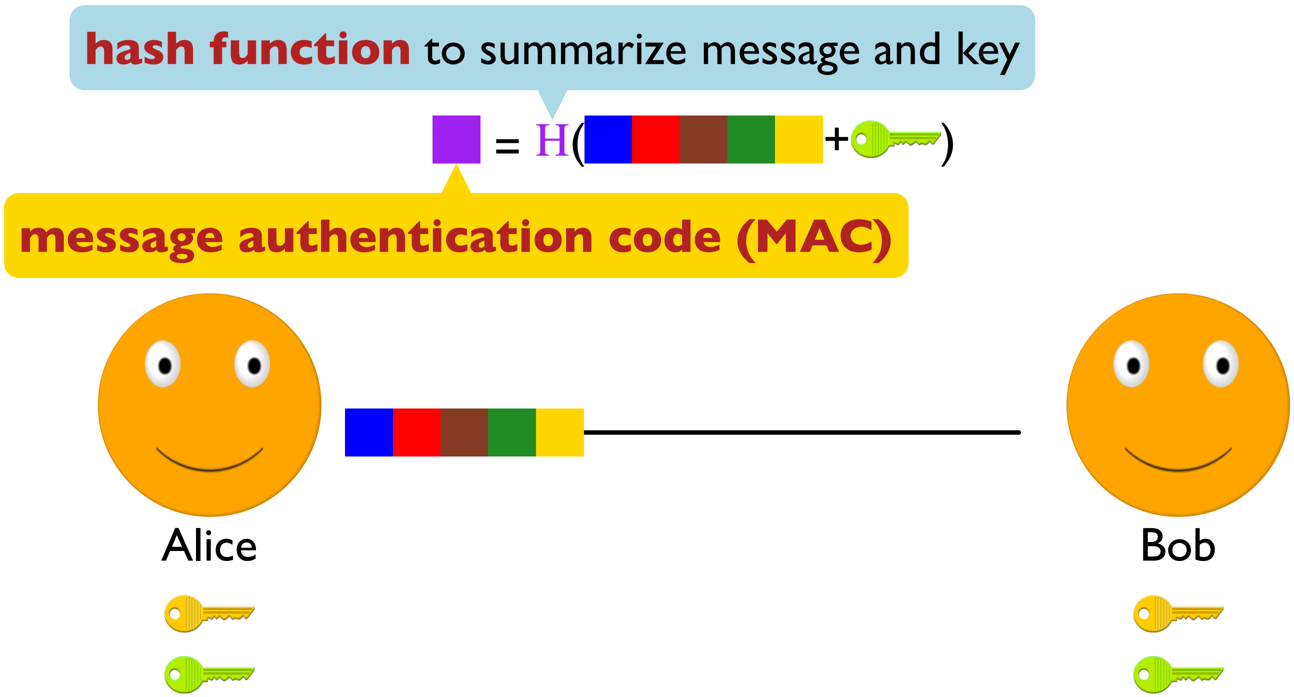

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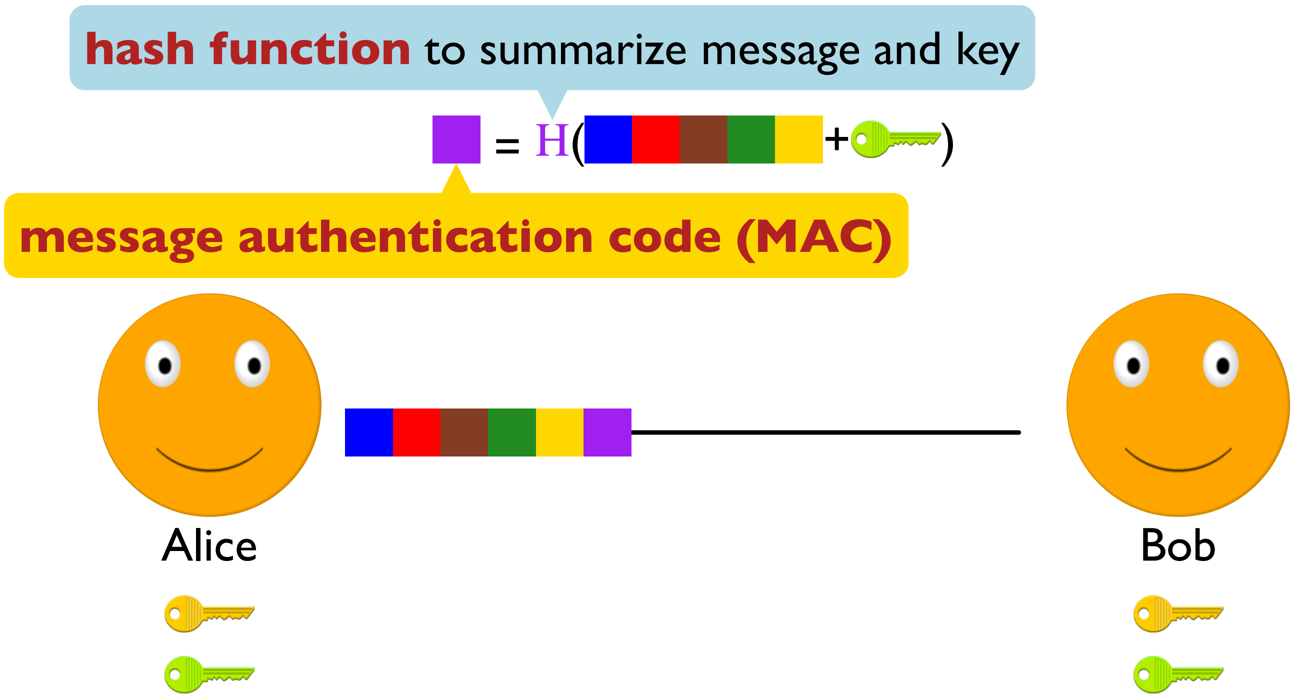

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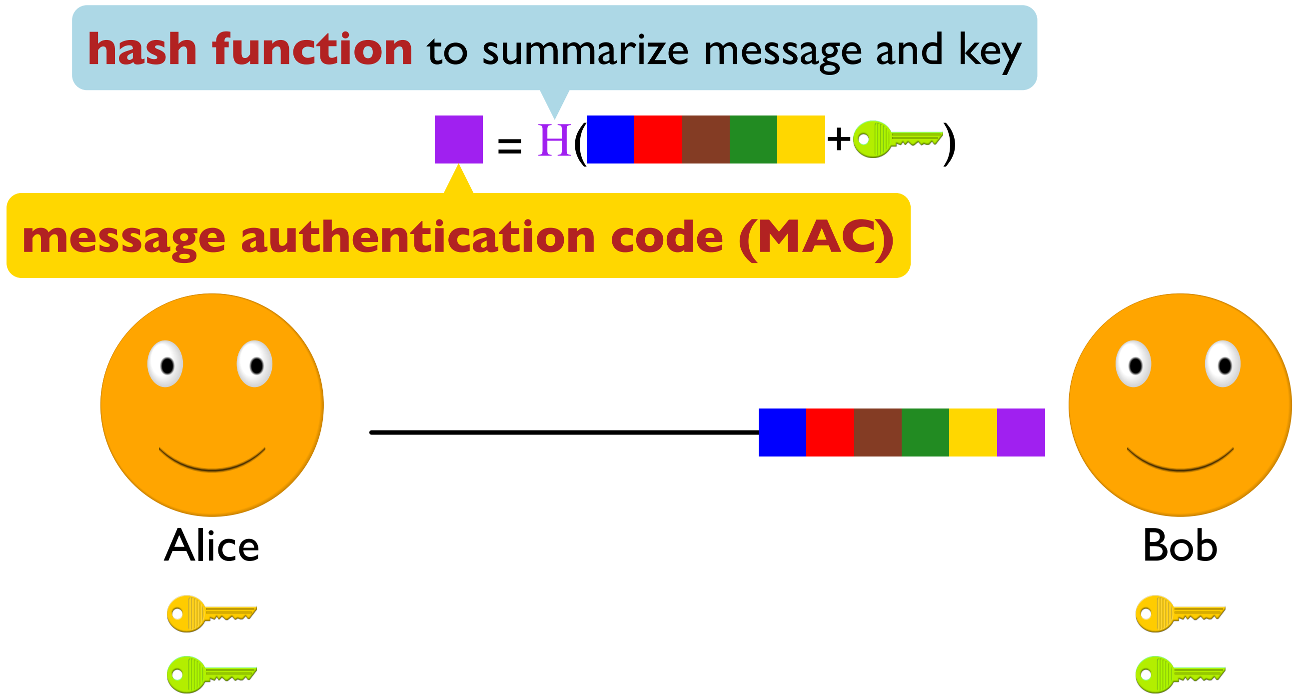

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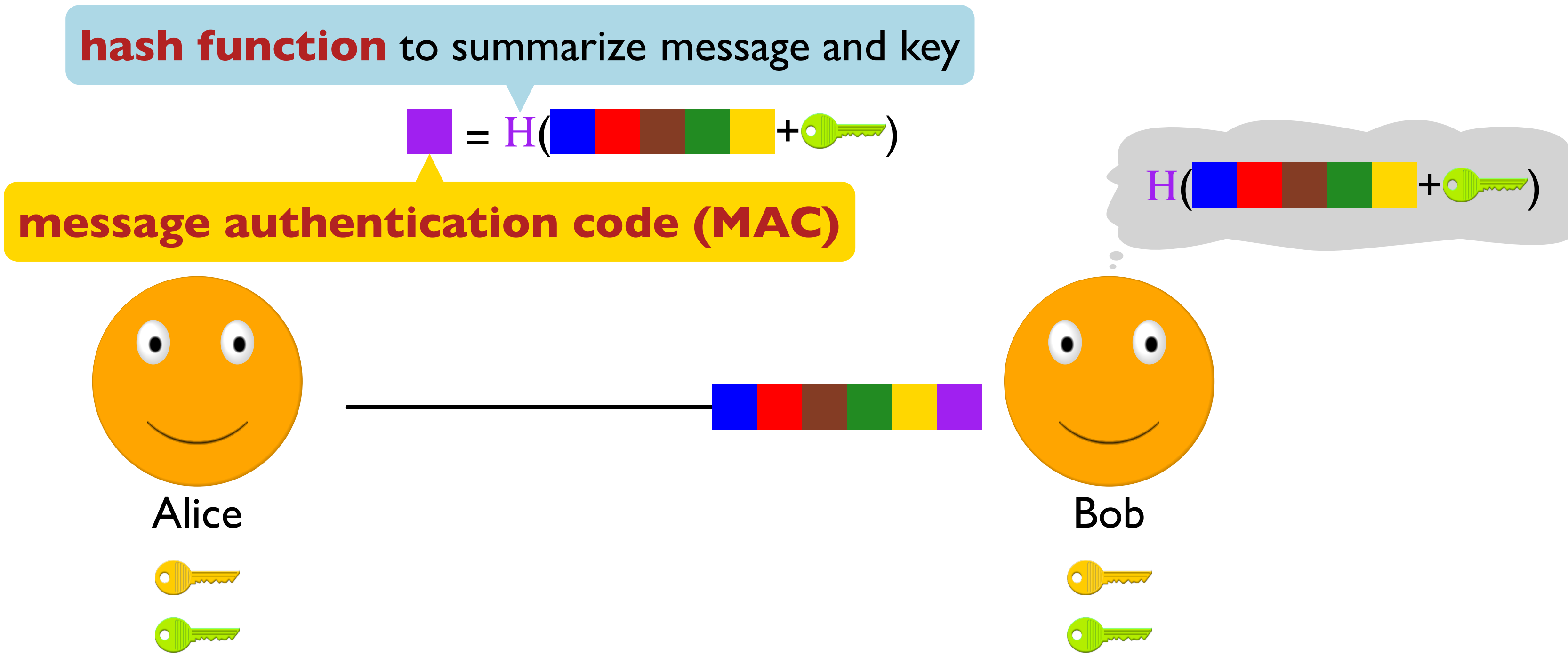

Hash Functions

A **hash function** H maps an arbitraily large value to a fixed-sized number

Data-structure usage: fast location of a value

- Use a number an an index into an array
- Collisions are inevitable

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$$H(x) = H(y) \Rightarrow x = y$$

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How is this possible?

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- Collisions should be *infeasible*

Hash Collisions

If you have a hash array of length 256 and a ideal hash function H, how many items until you expect to find a collision?

Probably that 2 items *don*'*t* collide: $\frac{255}{256} = 99.6\%$

Probably that 3 items *don*'*t* collide: $\frac{255}{256} \times \frac{254}{256} = 98.8\%$

Probably that 4 items *don*'*t* collide: $\frac{255}{256} \times \frac{254}{256} \times \frac{253}{256} = 97.6\%$

Hash Collisions

If you have a hash array of length 256 and a ideal hash function H, how many items until you expect to find a collision?

Probability of no collisions:

1	100.0%	11	80.4%	21	43.0%
2	99.6%	12	76.9%	22	39.5%
3	98.8%	13	73.3%	23	36.1%
4	97.6%	14	69.6%	24	32.8%
5	96.1%	15	65.8%	25	29.7%
6	94.2%	16	61.9%	26	26.8%
7	92.0%	17	58.0%	27	24.1%
8	89.5%	18	54.2%	28	21.6%
9	86.7%	19	50.4%	29	19.2%
10	83.6%	20	46.6%	30	17.0%

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Birthday paradox: In a room with only 23 people, probably two people in the room have the same birthday

Hash Collisions

If you have a hash array of length 2^N and a ideal hash function H, how many items until you expect to find a collision?

Probability of no collisions with k values:

$$\frac{2^N!}{2^{kN}(2^N - k)!}$$

Approximate k where probability reaches 50%:

$$2^{N/2}$$

$256 \Rightarrow N = 8 \Rightarrow k = 16$, which is in the right neighborhood

Cryptographic Hash Collisions

For cryptographic purposes, we're not allocating an array, so we can use a much larger N

Hash code bits N	Expected collsision at
128	$2^{64} = 4 \times 10^{9}$
256	$2^{128} = 3.4 \times 10^{38}$
512	$2^{256} = 1.2 \times 10^{77}$

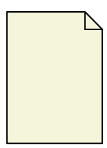
Number of atoms in the universe ≈ 10^{80}

Cryptographic Hash Assumptions

Needed for a MAC:

$$H(x) = H(y) \quad \Rightarrow \quad x = y$$

Also useful as a secure document checksum



`download_me` $\Rightarrow$ `c2c594e8d3f81db4b6a9340d5cb1903b2c9e622179ae4955d353bd54c5e3af9c`

Cryptographic Hash Assumptions

Needed for a MAC:

$$H(x) = H(y) \quad \Rightarrow \quad x = y$$

For some other purposes, we also need

given $H(x)$, cannot compute x

For example, password checks without storing passwords

Attack Modes

Known x, try to find colliding y

Example: malicious substitute for a download

Find both x and y that collide

Example: convince to accept x, later substitute y

Known $H(x)$, find x

Example: extract password from saved hash

Standardized Cryptographic Hash Functions

name	*hash bits*	*status*	*algorithm family*
MD5	128	*collisions found*	Merkle–Damgård
SHA-1	160	*some collisions found*	Merkle–Damgård
SHA-2	256 or 512	considered secure	Merkle–Damgård
SHA-3	256 or 512	considered secure	Keccak

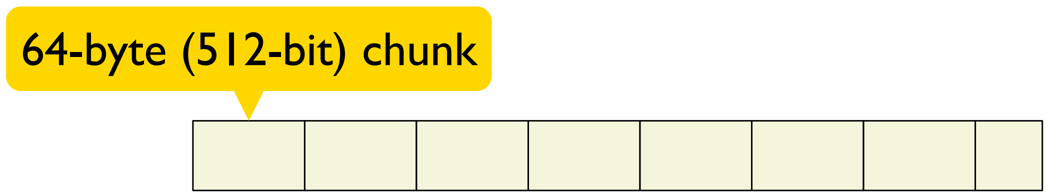
SHA-256 and SHA-512 are the 256-bit and 512-bit variants of SHA-2
SHA3-256 and SHA3-512 are the variants of SHA-3

SHA-3 is intended as a potential drop-in replacement for SHA-2 — in case a weakness in SHA-2 is discovered

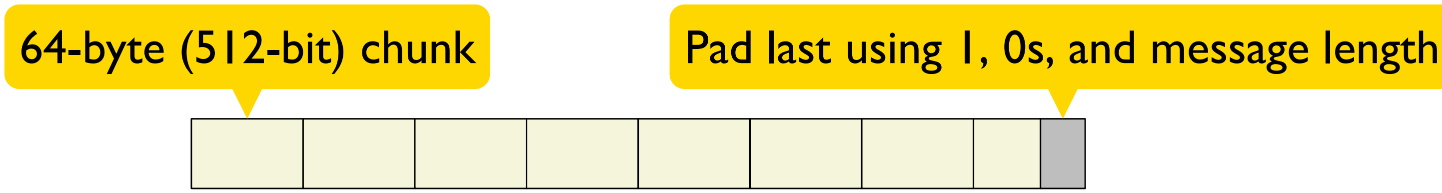
MD5

plaintext

MD5



MD5



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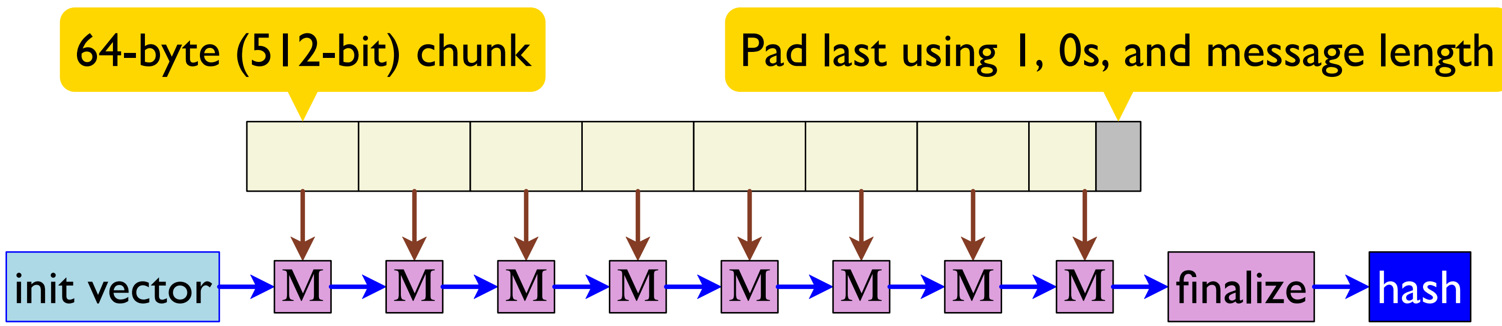
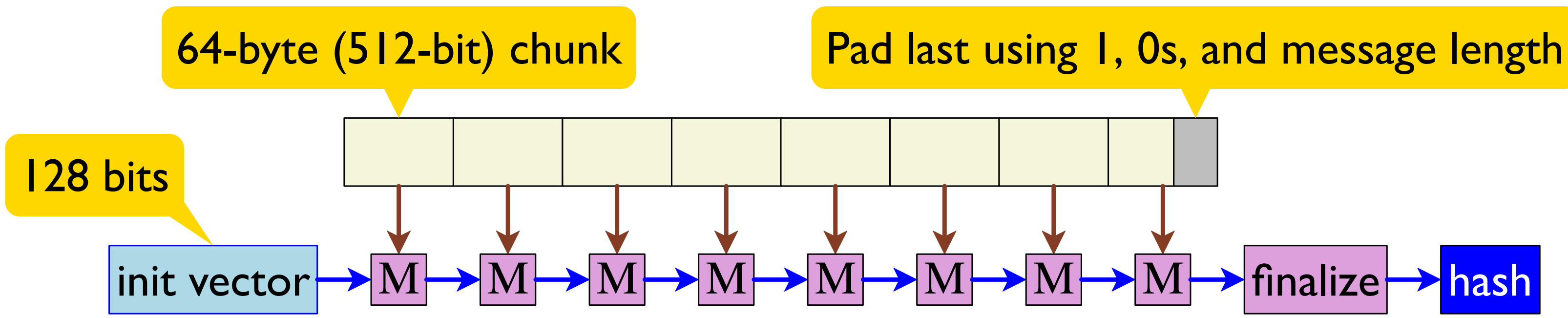
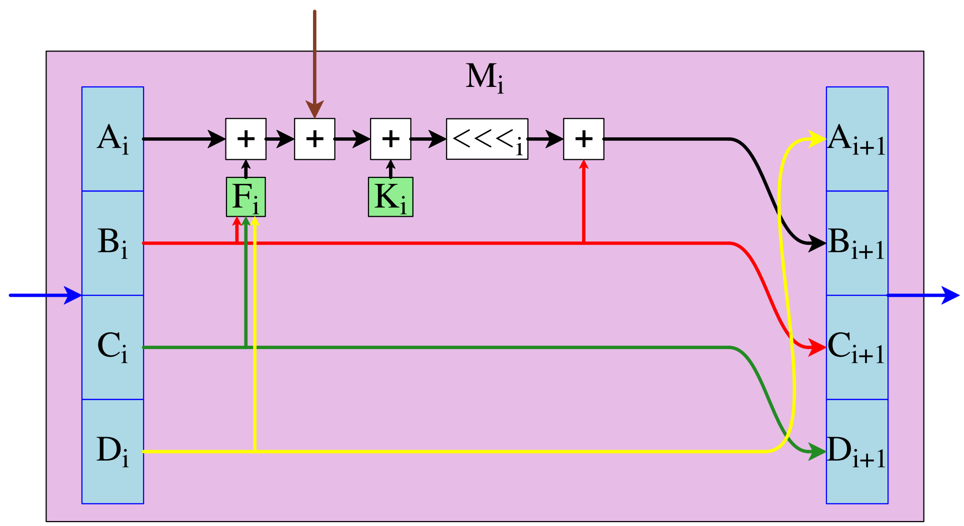

64-byte (512-bit) chunk
Pad last using 1, 0s, and message length
init vector
M
M
M
M
M
M
M
M
finalize
hash

MD5


64-byte (512-bit) chunk
Pad last using 1, 0s, and message length
128 bits
init vector
M
M
M
M
M
M
M
M
finalize
hash

M is 64 of


M_i
A_i
B_i
C_i
D_i
+
+
+
$<<<_i$
+
F_i
K_i
A_{i+1}
B_{i+1}
C_{i+1}
D_{i+1}

Hashing and Passwords

Servers don't want to know your password...

They want to know that you know it

Store a hash of a password, not the password:

user	H(password)
alice	d8ef3b7d2e6a8
bob	a6fdb8307dbc0
eve	9759a5d1558e4
carol	a6fdb8307dbc0

Server has to know password as you're logging in,
bit it only has to *store* a hash

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Cannot reconstruct `alice`'s password from hash

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Uh oh —
Can tell that `bob` and `carol` have the same password

Server has to know password as you're logging in,
bit it only has to *store* a hash

Hashing and Passwords

Don't store passwords

Don't store hashed passwords

Store a **salted hash** of a password:

user	salt	H(password+salt)
alice	adg3fee684	f3b4dd8e2e6a8
bob	992a6df99a	8307a6fbbdac0
eve	1aac7deef0	1558e49229a5d
carol	8a8721fbb1	07dbc0a99db83

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Randomly generated when password is set

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Even if `bob` and `carol` both have the password `passwd`,
H(`passwd+992a6df99a`) ≠ H(`passwd+8a8721fbb1`)

Summary

A **cryptographic hash function** is a one-way hash function that avoids collisions

Useful for ensuring message integrity

Useful for perserving evidence but forgetting details

You should use **SHA-2**

Don't manage passwords yourself, but if you do, store only **salted hashes of passwords**