Numerical Integration Based Contrast Enhancement Using Simpson's Method

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Abstract This paper is based on mathematical numerical integration method and provides an efficient algorithm for improving the contrast of an image. The aim of this unique method is to use neighboring pixel information and process them in accordance with mathematics (Simpson's $\frac{1}{3}$rd rule) to improve contrast of images. Simpson's rule is a numerical integration method for the accurate approximation of definite integrals. It gives the exact results for polynomials of degree three or less. We use the neighboring pixel information, interpolate among them, and produce the enhanced pixel information. Various parameters like Root Mean Square Error (RMSE), Peak-Signal-to-Noise-Ratio (PSNR) and Structural Similarity Index (SSIM) are used to measure the quality of the images. The experimental results of the proposed method are compared with some existing methods for further validation. Also, the computational time of the proposed method is much less as compared to the different existing methods.

Keywords Image enhancement · Contrast enhancement · Numerical integration · Simpson's $\frac{1}{3}$rd rule

1 Introduction

The perception of information in images for human viewers and to provide "better" input for other automated image processing techniques is termed as Image enhancement [9]. Reducing the noise, removing blurring effect, and increasing contrast of images are examples of enhancement operations. Contrast enhancement is one of

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the most crucial problems in image enhancement. Actually, contrast refers to the striking difference in the pixel intensities or difference in luminance component of images. Vital information cannot be retrieved, if the images are too dark or low contrast images, in most of the cases. Thus, efficient algorithms are needed for the improvement of visual quality of images for further processing in computer vision, pattern recognition, the processing of digital images [6], etc.

The classical techniques of Histogram Equalization (HE) [16, 17] for image enhancement have undergone several modifications but still fail to produce desirable results when the contrast characteristics keep varying across the image. Histogram-based enhancement techniques does not succeed in preserving the brightness and lead to loss of information in case of medical images [2]. For improving the contrast in digital images, HE is the most commonly used method but it results in over-enhancement and noise amplification [4]. Also, several modifications have been made to HE such as Adaptive Histogram Equalization (AHE) [12, 18] which uses statistical information in the neighborhood pixel for equalization but the major problem with this is over enhancement, creating objects that are not visible in the original image. Sub image histogram equalization has also been implemented by dividing the input image into segments based on the mean and variance of each region [20]. In global histogram equalization method which is simple and fast, but its contrast-enhancement power is relatively low. On the other hand, the local histogram equalization, enhances contrast more effectively, but increases the complexity of computation due to its fully overlapped sub-blocks [8]. An adaptation of HE is Contrast Limited Adaptive Histogram Equalization (CLAHE) [15] which gives a more localized enhancement as it divides the input image into equal-sized blocks and performs HE locally, but it is computationally very intensive as compared to the classical method. Dynamic HE controls the effect of traditional HE and performs the enhancement without any loss of details in it but this is achieved by partitioning and repartitioning and it fails to give desired results when dominating portions appear in the image [1].

The Genetic Algorithm (GA) [14] approach makes use of several different parameters on the mating pool where convergence becomes very crucial. Image histogram adjustment (IM) [11] method is also based on histogram equalization where image histogram is adjusted automatically. In Adaptive Gamma Correction With Weighting Distribution (AGCWD) [5] method, where gamma correction and probability distribution for luminance pixels were used, is not desirable when the input image lacks bright pixels. The sigmoid function [10] method is done by a sigmoid function as contrast enhancer and then adaptive histogram equalization where unnatural intensity saturation occurs.

In proposed method, we try to develop a contrast enhancement method using mathematical function Simpson's $\frac{1}{3}$rd rule to improve the contrast of images. It makes use of the midpoint sum and the trapezoidal sum on each pixel intensity such that the midpoint sum is used twice. Simpson's $\frac{1}{3}$rd rule is a part of Newton-Cotes numerical integration method which is based on equally spaced arguments. The $\frac{1}{3}$rd rule is a three points formula in the interval specified, where the degree of precision is three. Moreover, the computational time of the proposed method is much less than the existing methods.

2 Proposed Method

To improve the contrast of images, different existing methods have been proposed. Most of these methods are based on gamma correction, sigmoid function, and several different approaches of histogram equalization. The proposed method uses mathematical operations to increase the contrast of different types of images. Simpson's $\frac{1}{3}$rd rule is used for integration calculation. To calculate the formula, $(p+1)$ interpolating points or data points $\Phi_j (j = 0, 1, 2, 3, .., p)$ are equi-spaced and the interval $h = \frac{b-a}{p}$, where $a = \Phi_0$, $\Phi_j = \Phi_0 + jh$ and $b = \Phi_p$ [3, 13]. From Lagrange's formula, we have the following Eq. 1 using Eq. 2:

$$\int_a^b \pi(\Phi)d\Phi \simeq I = \sum_{r=0}^{p} \pi(\Phi_r)H_r^{(p)} \tag{1}$$

where,

$$H_r^{(p)} = \int_a^b \frac{\Omega(\Phi)d\Phi}{(\Phi - \Phi_r)\Omega'(\Phi_r)} = \int_{\Phi_0}^{\Phi_0+ph} \frac{\Omega(\Phi)d\Phi}{(\Phi - \Phi_r)\Omega'(\Phi_r)} \tag{2}$$

To evaluate $H_r^{(p)}$, set $\Phi = \Phi_0 + ht$ and then the result is Eq. 3:

$$\Phi - \Phi_r = \Phi_0 + ht - (\Phi_0 + rh) = (t - r)h \tag{3}$$

and $d\Phi = hdt$. Further, obtained the following conditions as $\Phi = \Phi_0 = a, t = 0, \Phi = \Phi_p = b, t = p$ Thus, $\Omega(\Phi) = (\Phi - \Phi_0)(\Phi - \Phi_1)(\Phi - \Phi_2)...(\Phi - \Phi_p) = h^{(p+1)}t(t-1)(t-2)....(t-p)$ and $\Omega'(\Phi_r) = (\Phi_r - \Phi_0)(\Phi_r - \Phi_1)..(\Phi_r - \Phi_{r-1})(\Phi_r - \Phi_{r+1})..(\Phi_r - \Phi_{p-1})(\Phi_r - \Phi_p) = h^p r!(p-r)!(-1)^{p-r}$.

Hence, obtained Eq. 4 using Eq. 5:

$$H_r^{(p)} = \int_0^p \frac{h^{p+1}t(t-1)(t-2)(t-3)..(t-p)hdt}{h^p.r!(n-r)!(-1)^{p-r}.h.(t-r)} = ph\mu_r^{(p)} \tag{4}$$

Where

$$\mu_r^{(p)} = \frac{(-1)^{p-r}}{p.r!(p-r)!} \int_0^p \frac{t(t-1)(t-2)..(t-p)dt}{(t-r)} \tag{5}$$

Since $\frac{(b-a)}{p} = h$, thus the following equation is obtained:

$$\int_a^b \pi(\Phi)d\Phi \simeq I = (b-a)\sum_{r=0}^{p} \pi(\Phi_r)\mu_r^{(p)} \tag{6}$$

Simpson's rule is a three points Newton-Cotes' formula in the interval $[a, b]$. Considering $p = 2$ and $h = \frac{(b-a)}{2}$ in Eq. 6, we get Eq. 7 using Eqs. 8, 9 and 10

$$I = (b-a)\sum_{r=0}^{2} \pi(\Phi_r)\mu_r^{(2)} = (b-a)[\pi(\Phi_0)\mu_0^{(2)} + \pi(\Phi_1)\mu_1^{(2)} + \pi(\Phi_2)\mu_2^{(2)}] \tag{7}$$

Where

$$\mu_0^{(2)} = \frac{1}{2.2!}\int_0^2 (t-1)(t-2)dt = \frac{1}{6} \tag{8}$$

$$\mu_1^{(2)} = \frac{1}{2.2.1!}\int_0^2 t(t-1)dt = \frac{2}{3} \tag{9}$$

$$\mu_2^{(2)} = \mu_{2-2}^{(2)} = \mu_0^{(2)} = \frac{1}{6} \tag{10}$$

Thus on substituting above conditions and Eq. 11, we reach Eq. 12

$$I = (b-a)[\frac{1}{6}\pi(\Phi_0) + \frac{2}{3}\pi(\Phi_1) + \frac{1}{6}\pi(\Phi_2)] \tag{11}$$

$$= (b-a)[\frac{1}{6}\alpha_0 + \frac{4}{6}\alpha_1 + \frac{1}{6}\alpha_2] \tag{12}$$

where $\alpha_0 = \pi(\phi_0)$, $\alpha_1 = \pi(\phi_1)$ and $\alpha_2 = \pi(\phi_2)$.

$$= \frac{b-a}{6}[\alpha_0 + 4\alpha_1 + \alpha_2] \tag{13}$$

$$= \frac{h}{3}[\alpha_0 + 4\alpha_1 + \alpha_2] \tag{14}$$

This mathematics is used for the foundation of proposed method. We consider the three neighboring pixels intensities $\alpha_0, \alpha_1, \alpha_2$ of the low contrast input image and apply Eq. 14 using Eq. 13 on them. This is done for each pixel position and the new approximated value is used to generate the contrast improved output image. The proposed technique is shown in Algorithm 1.

Algorithm 1: CONTRAST ENHANCEMENT USING SIMPSON'S RULE

Input: Low contrast image $\Omega(M, N)$
Output: High contrast image $\Theta(M, N)$

```
for θ = 1..M − 1 do
    for ψ = 1..N − 1 do
        η(θ, ψ) = (h/3)[Ω(θ−1,ψ−1) + 4 ∗ Ω(θ,ψ) + Ω(θ+1,ψ+1)]
        if η(θ, ψ) < 0 then
            Θ(θ, ψ) = 0
        else if η(θ, ψ) > 255 then
            Θ(θ, ψ) = 255
        else
            Θ(θ, ψ) = η(θ, ψ)
```

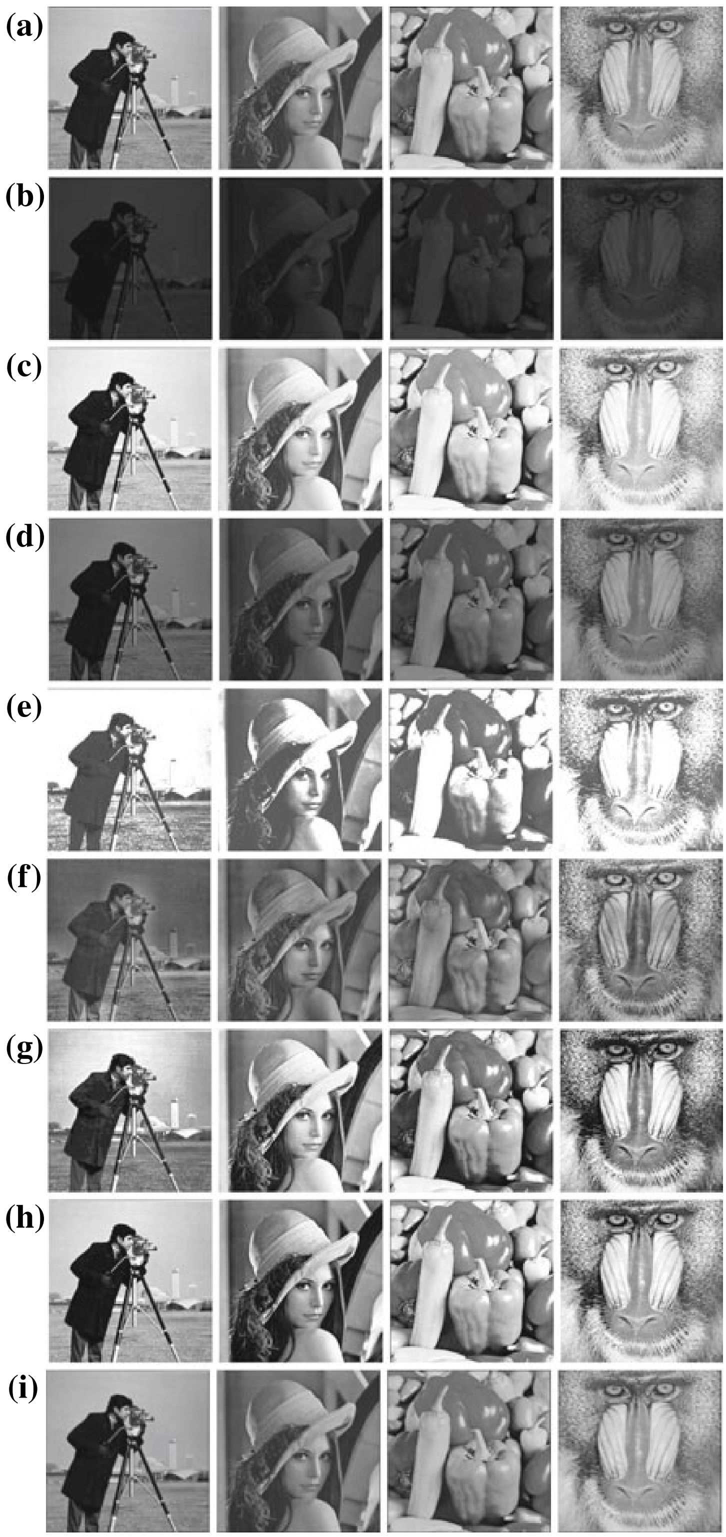

Fig. 1 Comparison of the output contrast images `Cameraman`, `Lena`, `Pepper` and `Mandril` using **c** AGCWD, **d** Sigmoid, **e** GA based, **f** AHE approach, **g** HE approach, **h** IM method and **i** PM, **a** is the original images and **b** is the low contrast images

Table 1 Compare RMSE values for different images using several existing methods and proposed method

Methods	Baloon	Barbara	Cameraman	Donald	Lena	Mandril	Pepper	Sania
Sigmoid	119.41	129.20	59.33	17.72	46.87	40.55	59.12	64.04
AGCWD	58.74	45.65	44.54	31.96	53.09	58.66	47.01	33.61
GA	55.76	54.91	65.84	63.84	54.31	74.31	56.23	36.40
AHE	44.18	32.81	64.42	47.83	24.09	31.40	39.23	64.23
HE	39.98	24.76	27.81	39.30	45.44	35.08	23.97	19.83
IM	32.63	16.21	21.45	34.81	23.80	27.75	23.62	12.58
PM	17.93	13.58	18.92	13.88	15.15	19.21	20.17	12.21

Table 2 Compare PSNR values for different images using several existing methods and proposed method

Methods	Baloon	Barbara	Cameraman	Donald	Lena	Mandril	Pepper	Sania
Sigmoid	6.58	5.90	12.66	23.15	14.71	15.96	12.69	12.00
AGCWD	12.75	14.94	15.15	18.03	13.63	12.76	14.68	17.60
GA	13.20	13.34	11.76	12.02	13.43	10.71	13.13	16.90
AHE	15.23	17.81	11.95	14.53	20.49	18.19	16.26	11.97
HE	16.09	20.25	19.25	16.24	14.98	17.23	20.54	22.18
IM	17.85	23.93	21.50	17.29	20.59	19.26	20.66	26.13
PM	23.05	25.46	22.59	25.27	24.51	22.45	22.03	26.39

Table 3 Compare SSIM values for different images using several existing methods and proposed method

Methods	Baloon	Barbara	Cameraman	Donald	Lena	Mandril	Pepper	Sania
Sigmoid	0.8017	0.7620	0.7688	0.9802	0.7766	0.8301	0.7761	0.7889
AGCWD	0.8353	0.8914	0.9298	0.9086	0.8371	0.8696	0.9007	0.9137
GA	0.7493	0.6684	0.6882	0.8212	0.6878	0.6007	0.7178	0.8007
AHE	0.8428	0.8428	0.7286	0.8110	0.8961	0.8852	0.8675	0.8255
HE	0.7118	0.8814	0.7729	0.5450	0.7789	0.7949	0.8579	0.8122
IM	0.7710	0.9129	0.9279	0.8149	0.8365	0.9085	0.9480	0.9601
PM	0.9695	0.9602	0.9412	0.9790	0.9497	0.8626	0.9614	0.9760

3 Experimental Results

For performance comparison, the proposed algorithm is compared with several existing methods such as Sigmoid function, AGCWD, GA, AHE, HE, and IM using three different parameters applied on several low contrast images namely `Baloon`, `Barbara`, `Cameraman`, `Donald`, `Lena`, `Mandril`, `Pepper` and, `Sania`, etc.

The qualitative parameters RMSE, PSNR [7] and SSIM [19] are used to measure and analyze the contrast of images. RMSE, PSNR, and SSIM values of the proposed method (PM) in comparison with different existing methods are shown in Table 1, Table 2, and Table 3, respectively. From the results, it shows that the proposed algorithm gives better outputs as compared to the other existing methods which is shown in Fig. 1. In proposed algorithm, the parameter h (the subinterval length) varies from 1.5 to 2.5 for better quality images. The parameters RMSE, PSNR, and SSIM are given in following Eq. 15, Eq. 16, and Eq. 17, respectively.

$$RMSE = \sqrt{\frac{1}{M \times N} \sum_{\theta=0}^{M-1} \sum_{\psi=0}^{N-1} |\Omega(\theta, \psi) - \Theta(\theta, \psi)|^2} \tag{15}$$

$$PSNR = 20 * log_{10} \frac{255}{RMSE} \tag{16}$$

Here $M \times N$ is the size of the images. $\Omega(i, j)$ represents the original input image whereas $\Theta(i, j)$ denotes the contrast improved output image.

$$SSIM(i, j) = \frac{(2\upsilon_i \upsilon_j + t_1)(2\beta_{ij} + t_2)}{(\upsilon_i{}^2 + \upsilon_j{}^2 + t_1)(\beta_i{}^2 + \beta_j{}^2 + t_2)} \tag{17}$$

where υ_i, υ_j are the mean values of the two windows i and j. β_i, β_j are the standard deviations of the two windows. β_{ij} is the covariance of i and j. t_1 and t_2 are the constants and the value of $t_1 = (0.01 \times 255)^2$ and $t_2 = (0.03 \times 255)^2$. From Fig. 1 and Tables 1, 2 and 3, it is noticed that visually as well as theoretically, the proposed algorithm gives the better results than other existing methods.

4 Conclusions

The proposed work presents a unique efficient method for enhancing the contrast of images based on numerical integration Simpson's $\frac{1}{3}$rd rule. Simpson's rule has distinct advantages over previously existing methods as it simply depends on the neighboring pixel intensities. This makes it useful and can be used for various other fields. However, further improvements are always possible for better contrast results.

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