Modified Contrast Enhancement Algorithm Using Numerical Integration Simpson's Method

Amiya Halder, Apurba Sarkar and Nikita Shah

Abstract This paper presents an efficient method to improve the contrast of low contrast images and at the same time provides satisfactory performances in terms of image contrast and sharpness. It is based on numerical cubic interpolation formula known as Simpson's $\frac{3}{8}$th rule. The neighboring pixel information is used in interpolation method, and it gives the high contrast image and also preserves fine details of the images. Different quality-based parameters are used to measure the improvement of the contrast of the images. The obtained results are compared with various standard existing methods for further validation.

Keywords Contrast enhancement · Numerical integration · Similarity index measure · Simpson's $\frac{3}{8}$th rule.

1 Introduction

Image enhancement is done to increase the quality of images by removing unwanted distortion and noise present in the image due to deterioration of contrast, improper intensity saturation, etc. Basic enhancement operations on an image can be the reduction of the noise present, removal of blurriness, increasing contrast of dark images [1], etc. However, the most crucial image enhancement operation is contrast enhancement. Contrast refers to the brightness of the image, and it can be defined as the difference of the highest and lowest pixel intensities of the image [2]. The contrast of an image is very important characteristics by which the quality of an image can be judged as good or poor [3]. Over the years, several different contrast enhancement approaches have been proposed. Most of the published algorithms depend on

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histogram equalization or modified histogram equalization. Genetic algorithms (GA)-based image enhancement [4] method, adaptive histogram equalization(AHE) [5], local histogram equalization based-technique [6], CLAHE [7] have been develope-d which improve the contrast of the images. In later years, more efficient contrast enhancement methods are published like dynamic histogram equalization (HE) [8], and it increase the quality of the image without any loss of details, but in dominating portions, it does not work properly. To improve this limitation, modified adaptive histogram equalization(AHE) [9], WTHEISMF [10] and image histogram adjustmen-t(IM) [11] have been proposed. The method AGCWD [12] automatically enhances the brightness of dimmed images using gamma correction. Recently, to improve the low contrast images, efficient methods like histogram equalization(HE) [13, 14] have been proposed which is the most commonly used spatial domain method with uniform distribution of pixel intensities, but it gives unnatural effects such as intensity saturation, over-enhancement, and noise amplification. Also, other methods have been proposed like sigmoid function [2]-based contrast enhancement, sub-image histogram equalization [15] that depends on mean and variance, the global and local mean method (GLM) [16], piecewise-constant interpolation (InterPWC) and local max-min interpolation methods (InterMaxMin) [17, 18].

In this paper, we present a contrast enhancement algorithms, which is based on the mathematical formula of integration known as Simpson's $\frac{3}{8}$th rule. Based on equally spaces arguments, Simpson's $\frac{3}{8}$th rule is a four points formula in the specified interval with degree of precision as three. The proposed method is an extension of previously published paper based on Simpson's $\frac{1}{3}$rd rule [19] to enhance the contrast of images. To evaluate the performance of the proposed method using different quantitative measures, the proposed method is demonstrated on a set of different standard images.

2 Background

In present study, the contrast enhancement method is modeled by mathematical formula of Simpson's $\frac{3}{8}$th rule [19]. We consider $f(\tau)$ to be a set of $(\rho + 1)$ points (pixel positions) $\tau_0, \tau_1, \ldots, \tau_n$. $P(x)$ is an interpolating polynomial of degree less than or equal to ρ, which replaces $f(\tau)$ on the set of interpolating points τ_j [20, 21]. We can evaluate definite integration using Simpson's $\frac{3}{8}$th rule. It is the one of the statistical methods to get the value of definite integration using subinterval between endpoints a and b.

$$f(\tau_j) = P(\tau_j)(j = 0, 1, 2, \ldots \rho) \tag{1}$$

we can write Eq. 1 as given in Eq. 2,

$$f(\tau) = P(\tau) + R_{\rho+1}(\tau) \tag{2}$$

Now, integrating both sides within the limits a and b, we get Eq. 3,

$$\int_a^b f(\tau)d\tau = \int_a^b P(\tau)d\tau + \int_a^b R_{\rho+1}(\tau)d\tau = I + \epsilon_{\rho+1} \tag{3}$$

Taking the polynomial $P(x)$ as the Lagrange's form in Eq. 3, we get Eq. 4,

$$I = \int_a^b \sum_{r=0}^{\rho} \frac{\Omega(\tau) f(\tau_r)}{(\tau - \tau_r)\Omega^{/}(\tau_r)} d\tau = \sum_{r=0}^{\rho} f(\tau_r) H_r^{(\rho)} \tag{4}$$

We consider the $(\rho + 1)$ interpolating points $\tau_j (j = 0, 1, 2, \dots \rho)$ equispaced and such that $a = \tau_0$, $\tau_j = \tau_0 + jh$, $\tau_\rho = b$, $(j = 0, 1, 2, \dots, \rho)$ and $h = (b - a)/\rho =$ the interval of difference.

From Lagrange's Formula in 3 where

$$H_r^{(\rho)} = \int_a^b \frac{\Omega(\tau) f(\tau_r) d\tau}{(\tau - \tau_r)\Omega^{/}(\tau_r)} = \int_{\tau_0}^{\tau_0 + \rho h} \frac{\Omega(\tau) f(\tau_r) d\tau}{(\tau - \tau_r)\Omega^{/}(\tau_r)} \tag{5}$$

To evaluate $H_r^(\rho)$ in Eq. 5, we set $\tau = \tau_0 + ht$ to get the conditions in Eq. 6,

$$\tau - \tau_r = \tau_0 + ht - (\tau_0 + rh) = (t - r)h \tag{6}$$

and $d\tau = hdt$. Along with conditions in Eq. 6, we come to the following conditions as in Eq. 7

$$\tau = \tau_0 = a, t = 0 \quad and \ \tau = \tau_\rho = b, t = \rho \tag{7}$$

Now, $\Omega(\tau) = (\tau - \tau_0)(\tau - \tau_1)(\tau - \tau_2)\dots(\tau - \tau_\rho) = h^{(\rho+1)} t(t-1)(t-2)\dots(t - \rho)$ and $\Omega'(\tau_r) = (\tau_r - \tau_0)(\tau_r - \tau_1)\dots(\tau_r - \tau_{r-1})(\tau_r - \tau_{r+1})\dots(\tau_r - \tau_{\rho-1})(\tau_r - \tau_\rho) = h^\rho r!(\rho - r)!(-1)^{\rho - r}$.

From Eq. 5, we get Eqs. 8 and 9,

$$H_r^{(\rho)} = \int_0^\rho \frac{h^{\rho+1} t(t-1)(t-2)(t-3)\dots(t-\rho) hdt}{h^\rho . r!(n-r)!(-1)^{\rho-r} . h . (t-r)} = \rho h \mu_r^{(\rho)} \tag{8}$$

where

$$\mu_r^{(\rho)} = \frac{(-1)^{\rho-r}}{\rho . r!(\rho - r)!} \int_0^\rho \frac{t(t-1)(t-2)\dots(t-\rho)dt}{(t-r)} \tag{9}$$

Since $\frac{(b-a)}{\rho} = h$, thus finally we obtain

$$\int_a^b f(\tau)d\tau = (b-a)\sum_{r=0}^{\rho} f(\tau_r)\mu_r^{(\rho)} \tag{10}$$

Simpson's Rule is a four points Newton–Cotes' formula in the interval $[a, b]$. Considering $\rho = 3$ and $h = \frac{(b-a)}{3}$ in Eq. 10, we get Eqs. 11 and 12 as:

$$I = (b-a)\sum_{r=0}^{3} f(\tau_r)\mu_r^{(3)} \tag{11}$$

$$I = (b-a)[f(\tau_0)\mu_0^{(3)} + f(\tau_1)\mu_1^{(3)} + f(\tau_2)\mu_2^{(3)} + f(\tau_3)\mu_2^{(3)}] \tag{12}$$

where $\mu_0^{(3)} = \frac{1}{3.3!}\int_0^t (t-1)(t-2)(t-3)dt = \frac{1}{8}$ Similarly, $\mu_3^{(3)} = \frac{1}{8}$, $\mu_1^{(3)} = \mu_2^{(3)} = \frac{3}{8}$. So, from Eq. 12,

$$I = (b-a)[\frac{1}{8}f(\tau_0) + \frac{3}{8}f(\tau_1) + \frac{3}{8}f(\tau_2) + \frac{1}{8}f(\tau_3)] \tag{13}$$

Thus, in Eq. 13, $f(\tau_0) = \gamma_0, f(\tau_1) = \gamma_1, f(\tau_2) = \gamma_2, f(\tau_3) = \gamma_3$ to obtain Eq. 14,

$$\begin{aligned} I &= (b-a)[\frac{1}{8}\gamma_0 + \frac{3}{8}\gamma_1 + \frac{3}{8}\gamma_2 + \frac{1}{8}\gamma_3] = \frac{(b-a)}{8}[\gamma_0 + 3\gamma_1 + 3\gamma_2 + \gamma_3] \\ &= \frac{3*h}{8}[\gamma_0 + 3\gamma_1 + 3\gamma_2 + \gamma_3] \end{aligned} \tag{14}$$

The final formula as depicted in Eq. 14 is used to interpolate along the pixel positions. $\gamma_0, \gamma_1, \gamma_2, \gamma_3$ are the consecutive pixel intensities of the low contrast image. The process is repeated for each and every pixel in the spatial domain so as to get the new contrast improved intensity values, respectively.

3 Proposed Method

The proposed method is a unique algorithm to improve the contrast of images based on mathematical integration. Simpson's $\frac{3}{8}$th rule is four points Newton–Cotes' formula in the interval [a, b] [22]. The proposed method is obtained based on the above mathematics. The above groundwork is used in the algorithm (Algorithm 1) below to improve the contrast of low-quality dark images.

4 Result Analysis

The proposed algorithm is compared with several standard existing methods like AGCWD, GLM, GA, AHE, HE, Imadjust, WTHEISMF, InterPWC, InterMaxMin, Simpson's $\frac{1}{3}$rd rule. In the paper, compare the contrast and performance of the different techniques with the proposed method using quantitative measure parameters

Algorithm 1: SIMPSON'S $\frac{3}{8}$TH RULE BASED CONTRAST ENHANCEMENT ALGORITHM

Input: Low contrast image $\alpha(P, Q)$
Output: High contrast image $\beta(P, Q)$

for $\sigma = 1..P - 1$ **do**
  for $\theta = 1..Q - 1$ **do**
    $\eta(\sigma, \theta) = \frac{3*h}{8}[\alpha(\sigma - 1, \theta - 1) + 3 * \alpha(\sigma, \theta) + 3 * \alpha(\gamma + 1, \theta + 1) + \alpha(\gamma + 2, \theta + 2)]$
    if $\eta(\sigma, \theta) < 0$ **then**
      $\beta(\sigma, \theta) = 0$
    else if $\eta(\sigma, \theta) > 255$ **then**
      $\beta(\sigma, \theta) = 255$
    else
      $\beta(\sigma, \theta) = \eta(\sigma, \theta)$

Table 1 RMSE values of the various low contrast images using GA, AHE, WTHEISMF, IM, AGCWD, InterPWC, InterMaxMin, GLM, HE, *Simpson*$\frac{1}{3}$, and the proposed methods

Methods	Lena	Barbara	Pepper	Donald	Cameraman	Mandril	Baloon
GA	54.31	54.91	56.23	63.84	65.84	74.31	55.76
AHE	24.09	32.81	39.23	47.83	64.42	31.40	44.18
WTHEISMF	46.21	29.56	24.37	39.41	31.00	43.27	42.43
IM	23.80	16.21	23.62	34.81	21.45	27.75	32.63
AGCWD	53.09	45.65	47.01	31.96	44.54	58.66	58.74
InterPWC	48.14	25.52	24.40	45.09	31.09	34.34	40.80
InterMaxMin	45.12	28.21	30.02	44.40	37.50	136.27	43.02
GLM	78.85	96.41	100.97	58.73	101.39	95.73	90.45
HE	45.44	24.76	23.97	39.30	27.81	35.08	39.98
Simpson$\frac{1}{3}$	15.15	13.58	20.17	13.88	18.92	19.21	17.93
Proposed method	11.60	12.86	16.83	13.52	14.59	17.98	14.73

RMSE, PSNR, and SSIM are used [19]. Table 1 shows the RMSE values of the standard images Mandril, Baloon, Cameraman, Barbara, Donald, Pepper, and Lena for different methods. The lower the RMSE value represents better the image contrast. Table 2 shows the PSNR values for the standard images for a variety of exiting methods with the proposed algorithm. Greater PSNR value represents that the quality of the image is high, and from Table 2, it is noted that the proposed method has the maximum PSNR values. SSIM value close to 1 means better quality image which is justified from the results obtained as shown in Table 3.

Table 2 PSNR values of the various low contrast images using GA, AHE, WTHEISMF, IM, AGCWD, InterPWC, InterMaxMin, GLM, HE, *Simpson* $\frac{1}{3}$, and the proposed methods

Methods	Lena	Barbara	Pepper	Donald	Cameraman	Mandril	Baloon
GA	13.43	13.34	13.13	12.02	11.76	10.71	13.20
AHE	20.49	17.81	16.26	14.53	11.95	18.19	15.23
WTHEISMF	14.83	18.71	20.39	16.21	18.30	15.40	15.57
IM	20.59	23.93	20.66	17.29	21.50	19.26	17.85
AGCWD	13.63	14.94	14.68	18.03	15.15	12.76	12.75
InterPWC	14.48	19.99	20.38	15.04	18.27	17.41	15.91
InterMaxMin	15.04	19.12	18.57	15.18	16.64	5.44	15.45
GLM	10.19	8.44	8.04	12.75	8.01	8.50	9.0
HE	14.98	20.25	20.54	16.24	19.25	17.23	16.09
Simpson $\frac{1}{3}$	24.51	25.46	22.03	25.27	22.59	22.45	23.05
Proposed method	26.83	25.94	23.60	25.51	24.84	23.03	24.76

Table 3 SSIM values of the various low contrast images using GA, AHE, WTHEISMF, IM, AGCWD, InterPWC, InterMaxMin, GLM, HE, *Simpson* $\frac{1}{3}$, and the proposed methods

Methods	Lena	Barbara	Pepper	Donald	Cameraman	Mandril	Baloon
GA	0.6878	0.6684	0.7178	0.8212	0.6882	0.6007	0.7493
AHE	0.8961	0.8428	0.8675	0.8110	0.7286	0.8852	0.8428
WTHEISMF	0.7178	0.7210	0.8276	0.5698	0.7108	0.5258	0.7002
IM	0.8365	0.9129	0.948	00.8149	0.9279	0.9085	0.7710
AGCWD	0.8371	0.8914	0.9007	0.9086	0.9298	0.8696	0.8353
InterPWC	0.8015	0.9071	0.8827	0.5444	0.7667	0.8110	0.7496
InterMaxMin	0.6748	0.7343	0.7444	0.4935	0.6216	0.4712	0.6786
GLM	0.4035	0.3688	0.3691	0.7904	0.4447	0.4385	0.4027
HE	0.7789	0.8814	0.8579	0.5450	0.7729	0.7949	0.7118
Simpson $\frac{1}{3}$	0.9497	0.9602	0.9614	0.9790	0.9412	0.8626	0.9695
Proposed method	0.9762	0.9804	0.9800	0.9844	0.9710	0.8995	0.9982

Figures 1 and 2 show the original, low contrast, and output images using different methods: AGCWD, GLM, GA, AHE, HE, Imadjust, WTHEISMF, InterPWC, InterMaxMin, Simpson's $\frac{1}{3}$rd rule for the `Lena` and `cameraman` images. From the above experimental results, it is noted that the proposed method gives better performance than the other existing contrast enhancement methods.

Fig. 1 Output images for low contrast `Cameraman` image using (p3) GLM, (p4) AGCWD, (p5) GA, (p6) AHE, (p7) HE, (p8) IM, (p9) WTHEISMF, (p10) InterPWC, (p11) InterMaxMin, (p12) $Simpson\frac{1}{3}rd$ rule, and (p13) the proposed method, (p1) original, and (p2) low contrast images

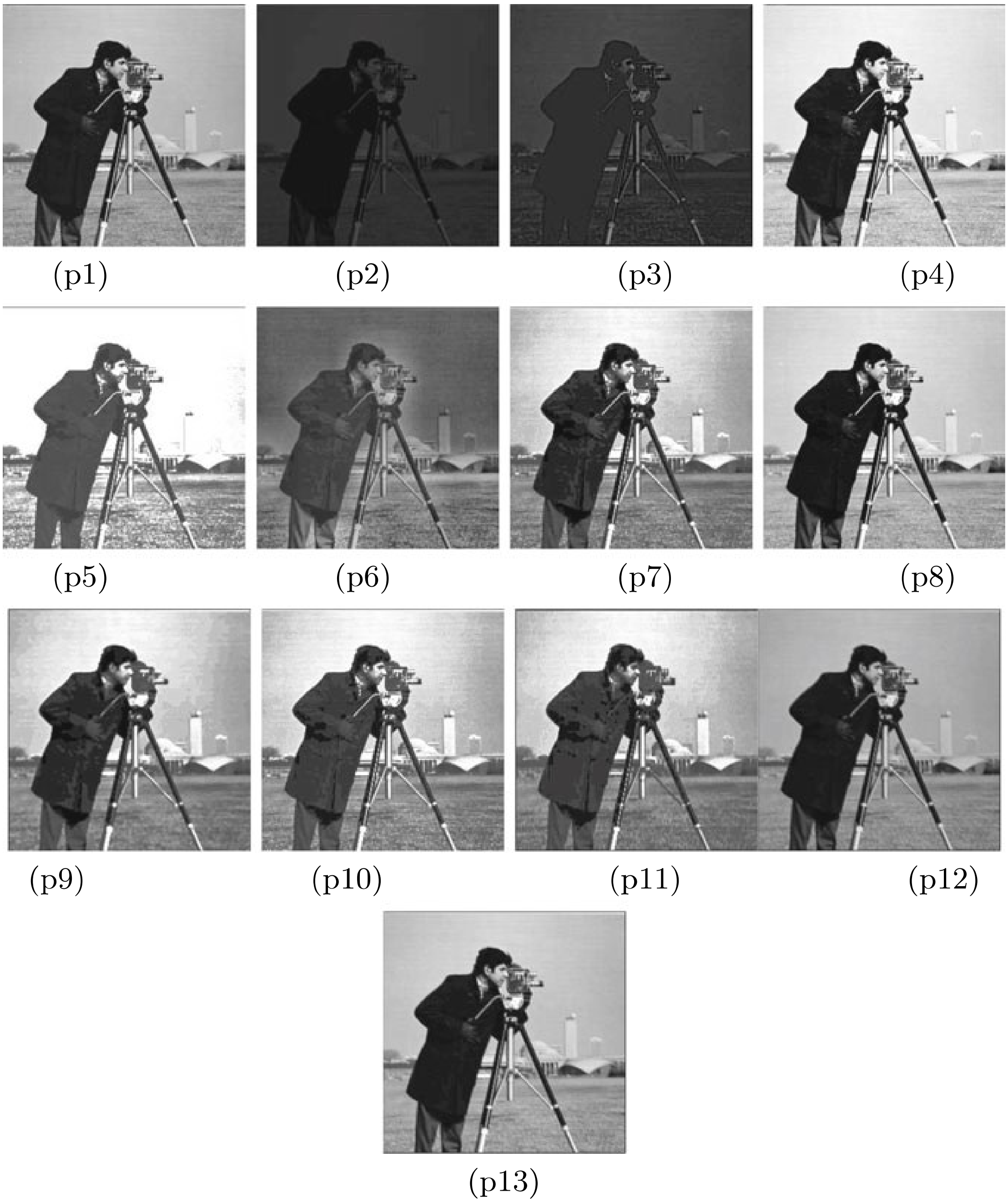

Fig. 2 Output images for low contrast `Cameraman` image using (p3) GLM , (p4) AGCWD, (p5) GA, (p6) AHE, (p7) HE, (p8) IM, (p9) WTHEISMF, (p10) InterPWC, (p11) InterMaxMin, (p12) $Simpson\frac{1}{3}rd$ rule, and (p13) the proposed method, (p1) original, and (p2) low contrast images

5 Conclusion

The proposed method presents an efficient algorithm for improving the contrast of dark low contrast images using simple mathematical integration in the spatial domain. Simpson's $\frac{3}{8}$th rule has several advantages as compared to the standard existing methods as it uses the neighborhood pixel information for the calculation of high contrast images. It assumes lesser approximation and gives the most accurate calculations. Comparatively the complexity of the proposed work is better than the other methods.

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