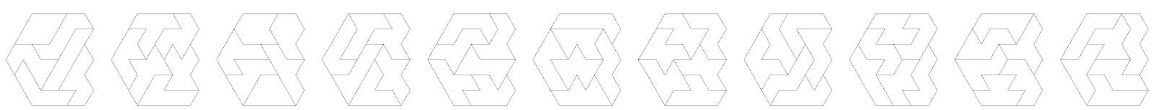
Christopher Monckton's Eternity and other new polyforms

by Ed Pegg Jr

While I was young, Martin Gardner didn't provide an answer for the 3×20 pentomino problem, so I had to solve it myself. My first introduction to the powerful concept of parity was through his hexomino rectangle impossibility proof. Ever since, I've been fascinated by complete polyform sets, and what can be done with them. Iamonds, for example, are based on equilateral triangles. The 4 pentiamonds can cover an icosahedron. The 12 hexiamonds can make a side-6 diamond. The 24 heptiamonds can make a 3×28 rhombus. The 66 octiamonds can make a 4×66 rhombus. All of these are nice sets. Combined, the hexiamonds and octiamonds make a perfect side-10 hexagon (beautifully realized by Kadon Enterprises). Michael Dowle sent me the below solution:

After studying octiamonds, and believing my money would be safe, I made a $100 challenge: make 11 identical shapes with the full set of octiamonds -- a "perfect" solution. Patrick Hamlyn, a programmer from Perth, Australia, solved it [15]:

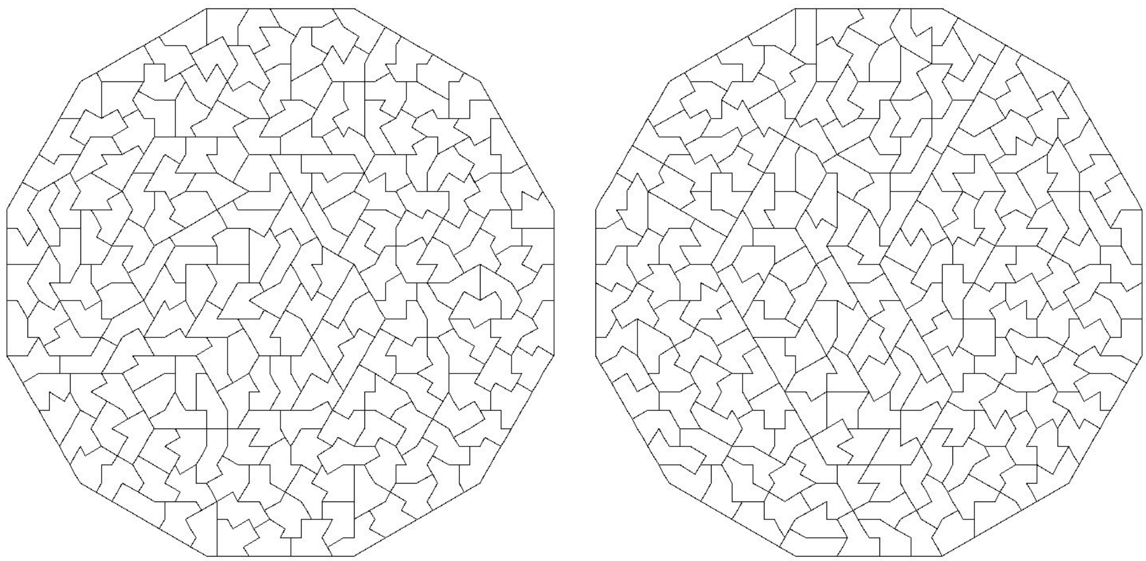


While he's out windsurfing, Hamlyn's computer is usually solving some nasty polyform problem or another. This particular problem above fell to a divide, sort, and conquer algorithm. He found all of the solutions to this particular shape, then sorted by the hardest pieces. Through this problem and others, a friendly polyform group had been developed, mostly led by Andrew Clarke and Hamlyn. Something unexpected then happened.

In early 1999, scottish lord Christopher Monckton debuted a new polyform puzzle -- Eternity -- with a million pound prize for the first solver. A former member of Margaret Thatcher's staff, Mr. Monckton designed it during a bout with ill-health. The 209-piece Eternity puzzle was so hard, no-one could solve it. In theory.

His challenge did not escape our attention. A study of a publicity photo revealed the pieces were polydrafters -- polyforms made from 30-60-90 triangles. The polydrafters were *really hard* -- a trapezoid could be made with the 14 tridrafters, but in only 2 ways. Undeterred, the top polyformists formed the Eternity mailing list. Monckton launched the puzzle to great fanfair in Europe, but it never caught on in the United States.

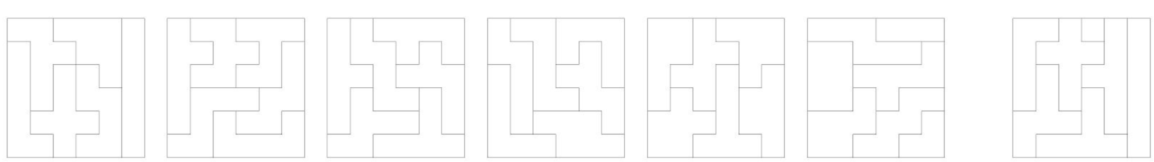
The science of polyform solving increased vastly over the coming months. The Eternity mailing list, the breadth-first search algorithm, and many clever computer programs cracked the difficult puzzle. Alex Selby was the first to find a solution, followed by mere weeks by Guenter Stertenbrink. Monckton's intended solution remains unknown.



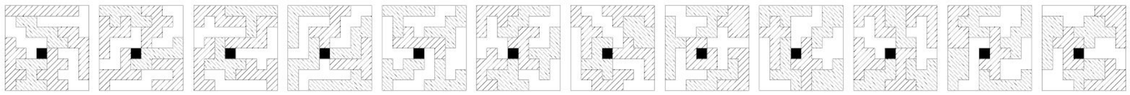
Eternity pieces copyright (c) 1999 by Christopher Monckton

Due to the lack of sales (mainly in the US), and the short amount of time needed to solve it (less than a year), Eternity wound up being a financial failure. Monckton lost his mansion and a lot of money, but he made sure that the first solver, Alex Selby, got paid. It was a fair contest. Monckton was a thoroughly good sport through it all, and helped out the cause of recreational mathematics in the process.

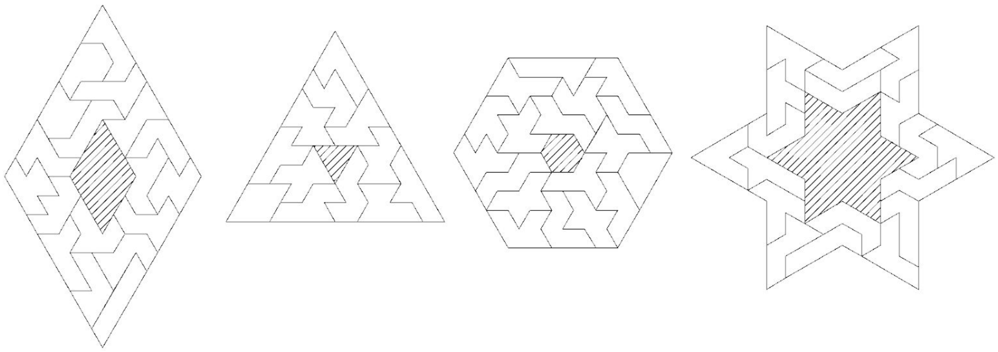
Powerful solving techniques had been created, so Hamlyn started a new mailing list dedicated to new problems. He demonstrated that the hexominoes and trominoes can make six 6×6 squares, as seen below. In a separate solution, if a monomino, a pentomino, and 4 hexominoes make the last square, the other hexominoes will make 5 more 6×6 squares in a unique way.



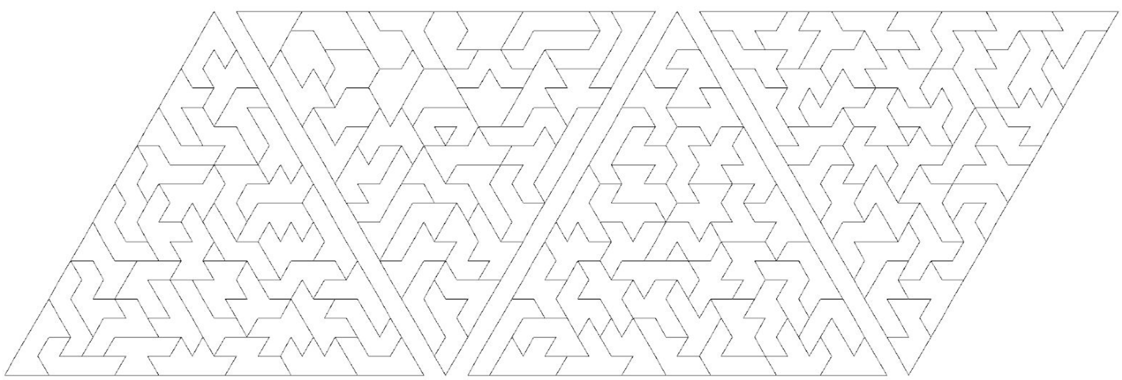
The heptominoes are the ways to put 7 squares together. Hamlyn found that 12 chessboards could be cut up to make the whole set. Note that the solution is three-colored -- three identical rectangles can be made with each color.



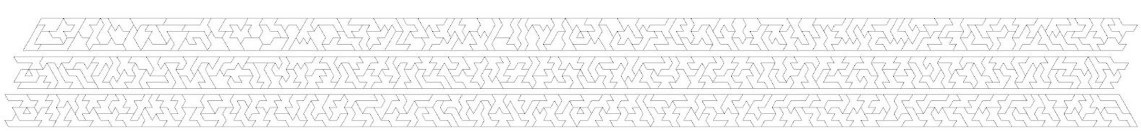
When Hamlyn designed a sailboard, he turned to the octiamonds for his design:



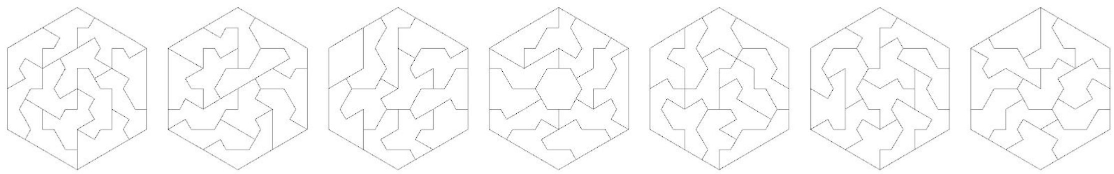
The 160 enneiamonds (9 triangles) can fit into four side-19 triangles.



The above solution was found by hand by Andrew Clarke. Curiously, the 24 heptiamonds can make a side-13 triangle with a hole in the center, and the 66 octiamonds can make a side-23 triangle with a hole at the center. Hamlyn packed the enneiamonds into five side-17 triangles, and put the 448 deciamonds in a strip.

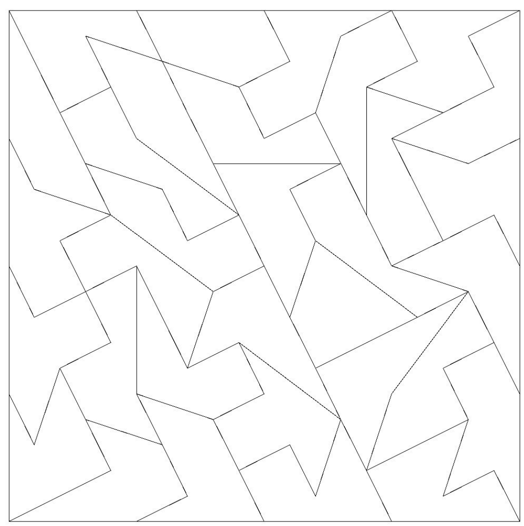


New types of polyforms were created, inspired in part by Christopher Monckton's polyforms. For example, Brendan Owen looked at a way of dividing a triangle into 3 kites. Hamlyn found that the 84 unholey hexakites could make 7 hexagons:

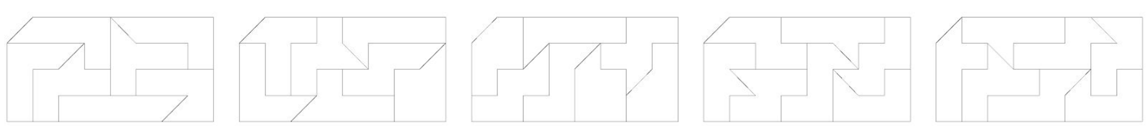


Brendan Owen then solved one of my own challenges, packing the 225 one-sided checkered octiamonds into a side-30 diamond.

A perfect 3-4-5 triangle with squares on each side is the Pythagorus figure -- what is the minimal number of identical triangles it can be divided into? Answer: 56 triangles are needed (how?). As one thing led to another, the following set of 32 pieces was developed:



Another set involves the 4.5-ominoes. The 35 pieces can make five clipped rectangles.



The polyform solutions presented here are just a small sample of what has been discovered since Christopher Monckton launched his Eternity challenge. For many more, I urge the interested reader to peruse the websites I've listed beliow.

I close with five nice problems.

1. Divide a 3×4×4 box into 8 polycubes that all share at least one face with each other.
2. There are 26 ways to put 3 dominoes together. Arrange 12 dominoes in such

a way to contain all 26 of these arrangements. (Brendan Owen showed that the 40 one-sided tridominoes will fit into five 6×8 boxes)
3. Divide a 3×3×3 cube into 4 polycubes that cannot be taken apart.
4. Divide an 11×11 square into differently sized rectangles with areas in the teens.
5. Divide an 11×11×11 cube into a single unit cube and 80 larger cubes.

References

1. Andrew Clarke's Poly Pages: http://clarkjag.idx.com.au/
2. Geometry Junkyard: http://www.ics.uci.edu/~eppstein/junkyard/polyomino.html
3. Roel Huisman: http://www.homepages.hetnet.nl/%7Eeomer/pforms/start.html
4. Kadon Enterprises: http://gamepuzzles.com/
5. Rodolfo Kurchan's Puzzle Fun:
http://www.eldar.org/~problemi/pfun/pfunmain.html
6. Alexandre Owen Muñiz: http://xprt.net/~munizao/mathrec/pentcol.html
7. Brendan Owen: http://members.optusnet.com.au/polyforms/
8. Ed Pegg Jr: http://www.mathpuzzle.com
8. Michael Reid: http://www.math.umass.edu/%7Emreid/Polyomino/index.html
9. Alex Selby: http://www.archduke.demon.co.uk/eternity/index.html
10. Torsten Sillke: http://www.mathematik.uni-bielefeld.de/~sillke/
11. Miroslav Vicher's polyforms: http://alpha.ujep.cz/~vicher/puzzle/ethernity.htm
12. Aad van de Wetering: http://home.wxs.nl/~avdw3b/aad.html
13. Livio Zucca: http://www.geocities.com/liviozuc/index.html
14. Eternity group: http://groups.yahoo.com/group/Eternity/
15. Polyforms group: http://groups.yahoo.com/group/polyforms/