IPM forumalations for thesis

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1 Original Problem

1.i Primal

$$
\begin{aligned}
\min_{\mathbf{x}} \quad & \mathbf{c}^T\mathbf{x} \\
\text{s.t.} \quad & \mathbf{A}\mathbf{x} = \mathbf{b}, \\
& \mathbf{x} \leq \boldsymbol{u}, \\
& \mathbf{x} \in \mathbb{R}^m_{\geq 0}
\end{aligned}
\tag{1}
$$

1.ii Primal Standard Form

$$
\begin{aligned}
\min_{\mathbf{x}} \quad & \mathbf{c}^T\mathbf{x} \\
\text{s.t.} \quad & \mathbf{A}\mathbf{x} = \mathbf{b}, \\
& \mathbf{x} + \mathbf{x}_u = \boldsymbol{u}, \\
& \mathbf{x}, \mathbf{x}_u \in \mathbb{R}^m_{\geq 0}, \mathbb{R}^m_{\geq 0}
\end{aligned}
\tag{2}
$$

1.iii Dual

$$
\begin{aligned}
\max_{\mathbf{y}, \mathbf{y}_u} \quad & \mathbf{b}^T\mathbf{y} - \mathbf{u}^T\mathbf{y}_u \\
\text{s.t.} \quad & \mathbf{A}^T\mathbf{y} - \mathbf{y}_u \leq \mathbf{c}, \\
& \mathbf{y}, \mathbf{y}_u \in \mathbb{R}^n, \mathbb{R}^m_{\geq 0}
\end{aligned}
\tag{3}
$$

1.iv Dual Standard form

$$
\begin{aligned}
\max_{\mathbf{y}, \mathbf{y}_u} \quad & \mathbf{b}^T\mathbf{y} - \mathbf{u}^T\mathbf{y}_u \\
\text{s.t.} \quad & \mathbf{A}^T\mathbf{y} + \mathbf{y}_u + \mathbf{z} = \mathbf{c}, \\
& \mathbf{y}, \mathbf{y}_u, \mathbf{z} \in \mathbb{R}^n, \mathbb{R}^m_{\geq 0}, \mathbb{R}^m_{\geq 0}
\end{aligned}
\tag{4}
$$

But if we want $\mathbf{y}$ variables to be free, then dualize the standard from, then standardize again:

$$
\begin{aligned}
\max_{\mathbf{y}, \mathbf{y}_u} \quad & \mathbf{b}^T\mathbf{y} + \mathbf{u}^T\mathbf{y}_u \\
\text{s.t.} \quad & \mathbf{A}^T\mathbf{y} + \mathbf{y}_u + \mathbf{z}_1 = \mathbf{c}, \\
& \mathbf{y}_u + \mathbf{z}_2 = \mathbf{0}, \\
& \mathbf{y}, \mathbf{y}_u, \mathbf{z}_1, \mathbf{z}_2 \in \mathbb{R}^n, \mathbb{R}^m, \mathbb{R}^m_{\geq 0}, \mathbb{R}^m_{\geq 0}
\end{aligned}
\tag{5}
$$

2 Solver Forms

2.i Long Step Path Following Method

Ref: Numerical Optimization (Alg. 14.2)

Due to the formulation, we have to work with the standard form.

2.i.1 Primal

$$\begin{aligned} \min \quad & c^T x \\ \text{s.t.} \quad & Ax = b, \\ & x \in \mathbb{R}^n_{\geq 0} \end{aligned} \tag{6}$$

$$A = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{I} & \mathbf{I} \end{bmatrix} \quad | \quad x = \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_u \end{bmatrix} \quad | \quad b = \begin{bmatrix} \mathbf{b} \\ \mathbf{u} \end{bmatrix} \quad | \quad c = \begin{bmatrix} \mathbf{c} \\ \mathbf{0} \end{bmatrix}$$

2.i.2 Dual

$$\begin{aligned} \max \quad & b^T y \\ \text{s.t.} \quad & A^T \lambda + s = c, \\ & \lambda, s \in \mathbb{R}^m, \mathbb{R}^n_{\geq 0} \end{aligned} \tag{7}$$

$$A = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{I} & \mathbf{I} \end{bmatrix} \quad | \quad \lambda = \begin{bmatrix} \mathbf{y} \\ \mathbf{y}_u \end{bmatrix} \quad | \quad b = \begin{bmatrix} \mathbf{b} \\ \mathbf{u} \end{bmatrix} \quad | \quad c = \begin{bmatrix} \mathbf{c} \\ \mathbf{0} \end{bmatrix} \quad | \quad s = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \end{bmatrix}$$