Optimal strategy for students in a college semester

Use Case Report

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Outline

Students have always wanted to perform at the peak of their capabilities to maximize their academic performance along with having sufficient amount of relaxation and fun in their daily lives. This use case takes a look into a common university scenario and tries to figure out the optimal strategy a student can follow by applying game theoretic principles on the problem statement.

Contents

1

Introduction

For the better or worse grades have become synonymous with capability in the modern education system. But every student might not be able to perform to his peak capability to maximize his academic performance. This might be due to a lot of reasons like distractions, inefficient planning, improper time division amongst the subjects etc.
In this paper, we analyse and recommend an optimal strategy for each student so that they can get the best possible grades as per their capability along with having the maximum fun they can have with their remaining time.

1.1 Problem Statement

We model the scenario as follows:

1. We consider a semester where all students are taking the same set of courses.

2. A student is evaluated continuously throughout the semester and the performance of the student in these examinations is directly proportional to the effort, in particular the number of hours put by the student towards studying the course.

3. We assume that a student does not use any kind of unfair means to cheat in the examinations and other evaluations i.e. there exist mechanisms to ensure that if a student cheats he/she is detected and will receive an F grade for the course. Thus it is not in a student's best interest to cheat in exams.

4. Each student has some learning rate that represents how quick he/she is able to learn the concepts of the course. This is different for each student for each course.

5. We also provide group study as a good mechanism of learning. 2 students can study together in which case the learning rate for the better learner decreases and the learning rate for the weaker student increases.

6. We will consider every student to have the same amount of hours per day that he/she can utilize for studies or other activities. These activities will not include sleeping, eating and other necessary daily activities i.e. every student is supposed to have same free hours but the hours he/she commits towards studying will depend on his/her value for grades and other activities.

7. The marks scored in a course by a student will be given by the learning rate for the course multiplied by the number of hours put by the student in the course. The students are then ordered on the basis of their marks. The grading scheme is relative but with a minimum marks(33) required to pass the course. Each course has equal weightage and the final Semester grade point average (SGPA) is the mean of the grade points received in each course. The relative scheme is described as follows:

8. Each student has a grade valuation constant which when multiplied by the SGPA gives the utility of the student for getting that SGPA. Similarly each student has a fun valuation constant which when multiplied by the number of hours spent in engaging in these other activities gives us the utility of the student for participating in them.

Student Category	Grade	Grade Point
Top 5%	A	10
Next 10%	A-	9
Next 20%	B	8
Next 30%	B-	7
Next 20%	C	6
Next 10%	C-	5
Next 5%	D	4
Scores below 33%	F	0

Table 1: Relative grading table

The sum of both these utilities is what a student wants to maximize.

9. Learning rates of each student are common knowledge. This can be derived either from the student's previous history of similar courses or can be found out within a small amount of time during group study - in which case it is quickly shared with everyone else and becomes common knowledge. This small amount of time will be negligible in the long run of 4 months is why we are making this approximation.

1.2 Formal Statement

The system we modelled has n students denoted by P = $\{P_1, P_2, P_3, P_4....P_n\}$. Each student takes m courses denoted by C = $\{c_1, c_2, c_3, ...c_m\}$.
The total number of hours available to each student throughout the semester of 4 months is H.

For each student P_i :

1. the learning rate corresponding to course c_j given as l_{ij}.
2. the grade valuation constant is given by g_i
3. the fun valuation constant is given by f_i.
4. the amount of hours spent on studying for course c_j is given by h_{ij} and the remaining $H - \sum_{j=1}^{m} h_{ij}$ hours is spent on other activities.
5. the students with student P_i groups up for course c_j is given by γ_{ij} and $\gamma_{ij} = i$ if P_i does not group up with anyone.
6. the grade points received for course c_j is given as G_{ij}.

If 2 students P_i and P_j decide to study together for a course c_k then
if $l_{ik} >= l_{jk}$

$$newl_{ik} = (3 \cdot l_{ik} + l_{jk})/4 \quad (1)$$
$$newl_{jk} = (l_{ik} + l_{jk})/2 \quad (2)$$

if $l_{ik} < l_{jk}$

$$newl_{ik} = (l_{ik} + l_{jk})/2 \quad (3)$$
$$newl_{jk} = (l_{ik} + 3 \cdot l_{jk})/4 \quad (4)$$

Each student P_i wants to maximize his utility given by:

$$u_i = (\sum_{j=1}^{m} G_{ij}/m) * g_i + (H - \sum_{j=1}^{m} h_{ij}) * f_i \quad (5)$$

2

Proposed Solution

The proposed solution is to formulate the above model as an N-player game. We start with a simpler setting.

2.1 Absolute Grading Without Collaboration

In this setting we assume that:

1. Collaboration i.e. group study is not allowed.
2. The grading is absolute and learning rate * number of hours put in the course is the marks obtained by the student. This is upper bounded by 100.
3. The number of hours that student can give to a course is a continuous variable.

Formalizing the assumptions we get:

$$h_{ij} \in [0, H] \quad (6)$$
$$\sum_{j=1}^{m} h_{ij} \leq H \quad (7)$$
$$G_{ij} = min(100, h_{ij} * l_{ij}) \quad (8)$$
$$u_i = (\sum_{j=1}^{m} min(100, h_{ij} * l_{ij})/m) * g_i + (H - \sum_{j=1}^{m} h_{ij}) * f_i \quad (9)$$

Every agent wants to maximize their own utility u_i. As we see from equation 9 u_i depends only on h_{ij}, l_{ij}, m, g_i, f_i and H. None of these variables depend on the actions and choices of other agents in the game. Thus the problem statement in this case reduces to an optimisation problem for each agent.

Rewriting equation 9, we get:

$$u_i = f_i * H + \frac{1}{m} \sum_{j=1}^{m} (min(100, h_{ij} * l_{ij}) * g_i - h_{ij} * f_i * m) \quad (10)$$

The solution to this optimisation is obtained by using linear programming. As we can see from equation 10, investing hours in a particular course increases utility till d. After that investing any amount of hours into it will just be a waste. Leaving those hours for fun time or investing them in any other course will increase the utility. Thus we can assume that the agent will not make this sub-optimal decision. This allows us to convert the min function and introduce it as a constraint in the linear program. This gives us the following linear program formulation:

$$\text{maximize } u_i = f_i * H + \frac{1}{m}\sum_{j=1}^{m} h_{ij} * (l_{ij} * g_i - f_i * m) \quad (11)$$

subject to

$$\begin{aligned} & h_{ij} * l_{ij} \leq 100 \quad \forall j \\ & h_{ij} >= 0 \quad \forall j \\ & h_{ij} <= H \quad \forall j \\ & \sum_{j=1}^{m} h_{ij} \leq H \end{aligned}$$

This is a classic linear program formulation that can be solved by finding the corners of the feasible region defined by the above constraints. This can be solved by any open source linear program solvers.

2.2 Relative Grading With Discreet Space

We move on to our initial problem model of relative grading but we analyze this in the space where the number of hours can only be discrete integer values. Putting it formally:

$$h_{ij} \in \{0, 1, 2, 3...H\} \quad (12)$$

We model this as an N-player game with collaboration. The strategic form game representation of a game is given by a tuple $\Gamma = \langle N, (S_i)_{i\epsilon N}, (u_i)_{i\epsilon N}\rangle$ where N = {1,2,...,n} is a finite set of players; S_1,S_2, · · · ,S_n are the strategy sets of the players; and $u_i : S_1 \times S_2 \times \cdot\cdot\cdot \times S_n \to R$ for i=1,2,...,n are utility functions[1].

For our problem statement we add contracts to this formulation. When 2 players sign a contract for a course j, they agree to be each other's group partner for the course.

Now if only 1 player signs the contract, then there is no change in this player's learning rate. If both players sign the contract then the learning rate will be modified as given by equations 1, 2, 3 and 4. This learning rate is effective for the common time they study together. If the number of hours they decide to spend on the course is distinct, then the player with the higher hours will have his own learning rate for the rest of the time when he/she is not studying with his/her partner.

Formally speaking, the strategic form game representation will be as follows:

1. $N = P$ as all the students are agents or players of the game.

2. If $\gamma_i = (\gamma_{i1}, \gamma_{i2}...\gamma_{im})$ and $h_i = (h_{i1}, h_{i2}...h_{im})$ and s_i denotes a possible strategy for player i, $s_i = (\gamma_i, h_i)$. All such possible s_i's form the strategy space S_i for player i.

3. The way we define γ_{ij} by allowing it to be i if i is not partnering up and the way we have set up our contracts means that the strategy space S_i for each player will be the same. Formally:

$$S_1 = S_2 = S_3 = ... = S_n \quad (13)$$
$$S = S_1^n = S_2^n = S_3^n = ... = S_n^n \quad (14)$$

4. u_i is defined by equation 5 which indirectly depends on the strategy chosen by each player.

Grade Calculation
Grade calculation is done by first calculating the marks for each student. Let the marks for student i in course j be given by m_{ij}:

$$m_{ij} = h_{ij} * l_{ij} \quad (15)$$

Now for a given course j we collect all the marks scored by the students in the course and order the students based on the marks. Then we use the grading table to calculate the grade G_{ij} for each student.

Now that the game is defined in it's strategic form representation we calculate all the Pure Strategy Nash Equilibriums for the game. We then propose the maximal of those equilibriums as our final equilibrium.

2.2.1 Code

The simulator that we wrote for this has been made public on github[2]. The readme explains all the notations and formats along with instructions on how to run the code.

2.2.2 Complexity

During the above simulation

$$|S_i| = \binom{H+m-1}{m} * n^m \quad (16)$$
$$|S| = |S_I|^n \quad (17)$$

The complexity of finding the PSNEs is given by $O(n * |S|)$. This values is very high making the computations unfeasible beyond some basic examples.

2.2.3 Example 1

P = $\{P_0, P_1, P_2\}$, C = $\{c_0, c_1\}$, H = 5 hours, |S| = 6751269

	P_0	P_1	P_2
g	2.5	3	1
f	0.5	1	0

	c_0	c_1
P_0	20	30
P_1	32	38
P_2	14	34

	c_0		c_1		u_i
	γ_{ij}	h_{ij}	γ_{ij}	h_{ij}	
P_0	0	0	0	3	25.0
P_1	0	1	0	3	31.5
P_2	0	2	2	3	5.0

2.2.4 Example 2

P = $\{P_0, P_1, P_2\}$, C = $\{c_0, c_1\}$, H = 5 hours, |S| = 6751269

	P_0	P_1	P_2
g	13.5	3	2
f	5	0.5	1

	c_0	c_1
P_0	410	30
P_1	312	38
P_2	114	14

	c_0		c_1		u_i
	γ_{ij}	h_{ij}	γ_{ij}	h_{ij}	
P_0	0	1	0	0	150.0
P_1	0	1	0	1	31.5
P_2	2	1	1	1	9.0

2.2.5 Observations

We ran some simulations using the above code and gained a few insights. These insights were limited by the high complexity of the solution.

1. If a player has high learning rate in all the courses he does not collaborate with anyone.

2. At equilibrium collaborations become mutually beneficial. If a student loses by collaborating with a lower learning rate partner, he/she also gets some benefits by collaborating with a higher learning rate partner in some other course.

3. Because we are looking for maximal PNE, if a student's learning rate is too high that collaboration without receiving rewards in return does not affect the grades in that course, the player still does it.

4. If learning rate is same for all individuals, collaboration makes no difference.

3

Additional Remarks

If there are no Pure Strategy Nash Equilibrium for the above game then one can look for Mixed Strategy Nash Equilibriums and randomize the player's strategy accordingly.

4

Conclusion and Future Work

Using game theoretic principles, we attempted to create and analyse the academic discipline of students in the current report. There were several limitations in this paper that can be tackled as future work in this field:

1. We currently only allow groups of two students to form, but in real life, any number of students can form groups

2. We ran many simulations, but the games were mostly small with single digit players due to a lack of efficient algorithm to find Nash equilibrium for general n player games. Future work in this space can be an efficient way to calculate PSNEs.

3. The above algorithms are constrained to discrete spaces while time is continuous. There can be work done to calculate PSNEs for continous time.

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References

[1] Y. Narahari. *Game theory and mechanism design*, volume 4. World Scientific Pub. Co., 222 Rosewood Drive, 2014.

[2] Pratyanshu Pandey. *Semester Simulator.* https://github.com/pratyanshupandey/Semester-simulator, IIIT Hyderabad, 2022.