Indian Association for the Cultivation of Science
(Deemed to be University under the *de novo* category)

Master's/Integrated Master's-PhD Program/Integrated Bachelor's-Master's Program/PhD Course

Mid-Semester Examination-Autumn 2023

Subject: Introductory Classical and Quantum Mechanics ***Subject Code(s): PHS1101***
Full marks: 25 ***Time allotted: 2 hr***

Answer *all* questions

1. (a) Show that for any vector field $\vec{A}$

$$\vec{\nabla} \cdot \left(\vec{\nabla} \times \vec{A} \right) = 0 \, .$$

(3 marks)

(b) Show that,

$$\nabla^2 \left(\frac{1}{|\vec{r} - \vec{r}'|} \right) = 0 \, ,$$

for, $|\vec{r} - \vec{r}'| \neq 0$. **(2 marks)**

2. (a) Find the expression for radial and cross-radial acceleration for motion in a plane. **(2 marks)**

(b) For motion in a central inverse square force $\vec{F} = (k/r^2)\hat{r}$, show that (i) angular momentum is conserved, and (ii) the path of a point mass m is given by,

$$\frac{1}{r} = -\frac{k}{h^2} + A \cos(\theta - \theta_0) \, .$$

Here, $\hat{r}$ is the unit vector along the radial direction, h is angular momentum per unit mass, and A, θ_0 are constants. **(3 marks)**

3. (a) A particle of mass m moving in one dimension x, experiences restoring force and damping force $k_1 x$ and $k_2(dx/dt)$ respectively. Find the conditions on k_1 and k_2, when the motion will be (i) damped oscillation, (ii) critically damped. **(4 marks)**

(b) Plot x versus t for the above two cases. **(1 mark)**

4. (a) Define centre of mass for a n-particle system with mass m_1, m_2, $\cdots$, m_n with coordinates $\vec{r}_1$, $\vec{r}_2$, $\cdots$, $\vec{r}_n$, with respect to a given origin 'O'. Show that the position of the centre of mass is independent of the choice of origin. **(3 marks)**

(b) If the above masses follows Newton's laws of motion and exert forces on each other, such that $\vec{f}_{ij}$ is the force on the ith particle by the jth particle, then show that the centre of mass remains static, though the particles will move due to inter-particle forces. (No F_{ext} on system) **(2 marks)**

5. (a) What is meant by the work done by a force $\vec{F}$ on a mass m? If the work is path independent, then show that $\vec{\nabla} \times \vec{F} = 0$. **(3 marks)**

(b) From the above define the potential of a force. **(2 marks)**