

Indian Association for Cultivation of Science
B. S. - First Year
Mid-Semester Examination 2023, Semester I
Subject: Calculus of Single Variables
Subject Code: MAT 1101 A
Full Marks:—25 Time Allotted:—2 hours

1. The paper carries 60 Marks. You can answer as many questions as you wish. If you score X , your final score will be $\frac{\min\{X, 50\}}{2}$.

2. You are free to use any theorem that is taught to you by me. However you must state them at least once in your answer-script because they carry credits.

3. \mathbb{R} will denote the set of all real numbers and \mathbb{Q} the set of all rational numbers.

4. Partwise scores for each question is shown at the end of the question.

(1) (a) Compute (giving a brief justification)

$$\lim_{x \rightarrow \infty} \left(1 - \frac{5}{x} + \frac{2}{x^2} - \frac{6}{x^3}\right).$$

(b) Show (giving brief justifications) that

$$\lim_{x \rightarrow \infty} (x^3 - 5x^2 + 2x - 6) = \infty$$

and

$$\lim_{x \rightarrow -\infty} (x^3 - 5x^2 + 2x - 6) = -\infty$$

[8 + (5 + 5)]

(2) Let $\emptyset \neq A \subset \mathbb{R}$ be a bounded set. Show that

$$-\infty < \liminf(A) \leq \limsup(A) < \infty.$$

[10]

(Caution. As per convention, $\sup \emptyset = -\infty$ and $\inf \emptyset = \infty$.)

(3) Let a_1, \dots, a_n and b_1, \dots, b_n be positive real numbers. Show the following.

(a) For every real number x

$$\sum_{i=1}^n a_i^2 - 2\left(\sum_{i=1}^n a_i b_i\right)x + \left(\sum_{i=1}^n b_i^2\right)x^2 \geq 0.$$

(b)

$$\sum_{i=1}^n (a_i \cdot b_i) \leq \sqrt{\sum_{i=1}^n a_i^2} \cdot \sqrt{\sum_{i=1}^n b_i^2}.$$

[6 + 10]

(4) (a) If a and b are real numbers, show that

$$a^2 < b^2 \Leftrightarrow |a| < |b|.$$

(b) Set

$$A = \{x \in \mathbb{Q} : x^2 < 2023\}$$

and

$$B = \{x \in \mathbb{R} : x^2 < 2023\}.$$

Show that both A and B are non-empty and bounded above, and

$$\sup(B) = \sup(A).$$

[5 + (2 + 9)]