CS 736 : Assignment Image Denoising with MRFs

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Maximum Marks: 100 (5 marks for abiding by the submission format)

1. (55 marks) Denoising a Magnetic Resonance (MR) Image of the Brain.

   The "data" folder contains 4 MR images, including one ground truth (noiseless) image and images corrupted with three different levels of noise (low, medium, and high). Implement a maximum-a-posteriori Bayesian image-denoising algorithm that uses a noise model coupled with a MRF prior that uses a 4-neighbor neighborhood system (each pixel has 4 neighbors: left, right, up, down; the neighborhood wraps around at image boundaries) that has cliques of size *no* more than 2.

   Use gradient ascent (or descent) optimization with dynamic step size. Ensure that the value of the objective function (i.e., the log posterior or its negative) at each iteration increases (or decreases if using gradient descent). Use the noisy image as the initial solution.

   Use 3 different MRF priors where the potential functions $V(x_i, x_j) := g(x_i - x_j)$ underlying the MRF penalize the difference between the neighboring voxel values x_i, x_j as follows (see class notes for details).

   Introduce a real-valued parameter α to control the weighting between the prior (weight α) and the likelihood. Manually tune the parameters to give the best denoised image that, based on your judgment, gives the right tradeoff between noise removal and edge preservation. Specifically, implement the following functionality as part of the denoising algorithm:

   (a) (10 marks) gradient descent/ascent based image denoising algorithm with Gaussian likelihood term.

   (b) (5 marks) MRF prior: Quadratic function: $g_1(u) := |u|^2$.

   (c) (5 marks) MRF prior: Discontinuity-adaptive Huber function: $g_2(u) := 0.5|u|^2$, when $|u| \leq \gamma$ and $g(u) := \gamma|u| - 0.5\gamma^2$, when $|u| > \gamma$, where $0 < \gamma < \infty$ is a constant.

   (d) (5 marks) MRF prior: Discontinuity-adaptive function: $g_3(u) := \gamma|u| - \gamma^2 \log(1 + |u|/\gamma)$, where $0 < \gamma < \infty$ is a constant.

   The RRMSE for 2 images A and B is defined as :
   RRMSE$(A, B) = \sqrt{\sum_p (|A(p)| - |B(p)|)^2} / \sqrt{\sum_p |A(p)|^2}$, where the summation is over all pixels p. Always use the noiseless image as A.

   For each MRF prior, manually tune the parameters α and γ (where applicable) to denoising the noisy image in order to achieve the least possible relative root-mean-squared error (RRMSE).

   For each of the three denoising algorithms, report the following:

   (a) (0 marks) the RRMSE between the noisy and noiseless images.

(b) (3 $\times$ 3 = 9 marks) the optimal values of the parameters and the corresponding RRMSEs. For each optimal parameter value reported, give evidence of the optimality of the reported values by reporting the RRMSE values for two nearby parameter values (around the optimal) at plus/minus 20% of the optimal value. That is, if a^*, b^* are the optimal parameter values, then report:
$a^*, b^*,$ RRMSE(a^*, b^*),
RRMSE$(1.2a^*, b^*)$, RRMSE$(0.8a^*, b^*)$,
RRMSE$(a^*, 1.2b^*)$, RRMSE$(a^*, 0.8b^*)$.

(c) (5 $\times$ 3 = 15 marks) the following 5 images (at each pixel, show the magnitude of the pixel value) in the report using exactly the same colormap (i) Noiseless image, (ii) Noisy image, (iii) Image denoised using quadratic prior $g_1(\cdot)$ and optimal parameter tuning, (iv) Image denoised using Huber prior $g_1(\cdot)$ and optimal parameter tuning, (v) Image denoised using discontinuity-adaptive prior $g_3(\cdot)$ and optimal parameter tuning.

(d) (2 $\times$ 3 = 6 marks) the plots of the objective-function values (on the vertical axis) versus iteration (on the horizontal axis) corresponding to each of the 3 denoised results.