MTH 100 - Math 1 - Monsoon Semester 2016-17

[Section A: 18/08/2016
Section B: 22/08/2016

Notes for VIT :-

Column Form of a Matrix :-

Given an $m \times n$ - matrix, we can regard it as consisting of n columns, each of which is an m-vector, i.e. given

$$B = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \\ b_{m1} & \cdots & b_{mn} \end{bmatrix}$$ we can write it in

the form $B = [\bar{v}_1 \; \cdots \; \bar{v}_n]$ where $\bar{v}_1 = \begin{bmatrix} b_{11} \\ \vdots \\ b_{1m} \end{bmatrix}$,

$\bar{v}_2 = \begin{bmatrix} b_{12} \\ \vdots \\ b_{m2} \end{bmatrix}$, ---, etc.

Similarly, given an ordered list of n vectors, $\bar{v}_1, \ldots, \bar{v}_n$, not necessarily distinct, we can ~~treat them~~ construct a matrix by taking these as the columns, i.e. $B = [\bar{v}_1 \; \bar{v}_2 \; \cdots \; \bar{v}_n]$

Matrix Product in Column Form: If now A is a $k \times n$ - matrix, so that the product $C = AB$ is well-defined, then C can be easily expressed in column form as follows:

$$C = AB = A[\bar{v}_1 \; \ldots, \; \bar{v}_n] = [A\bar{v}_1 \; \cdots \; A\bar{v}_n]$$

i.e. C is the matrix whose columns are $A\bar{v}_1, A\bar{v}_2, \ldots, A\bar{v}_n$.

(PTO)

Proof of (a) $\iff$ (d)

[After Cor 1.2 ~~3~~ and 1.3]

VIT – Proof Completed

(a) $\implies$ (d)

Suppose A is invertible and $\bar{b} \in \mathbb{R}^m$.

Consider the vector $\bar{u} = A^{-1}\bar{b} \in \mathbb{R}^m$

Then
$$A\bar{u} = A(A^{-1}\bar{b})$$
$$= I\bar{b}$$
$$= \bar{b}.$$

$\therefore$ the system $A\bar{x} = \bar{b}$ has $\bar{u}$ as a solution.

(d) $\implies$ (a)

Suppose the system $A\bar{x} = \bar{b}$ has a solution for every $\bar{b} \in \mathbb{R}^m$.

Let $\bar{u}_i$ be a solution of the system $A\bar{x} = \bar{e}_i$ for $i = 1, 2, \cdots, m$.

Let B be the matrix whose columns are the $\bar{u}_i$, i.e. $B = [\bar{u}_1 \ \bar{u}_2 \cdots \bar{u}_m]$

matrix whose columns are $\bar{u}_i$

Then
$$AB = A[\bar{u}_1 \ \bar{u}_2 \cdots \bar{u}_m]$$
$$= [A\bar{u}_1, A\bar{u}_2 \cdots A\bar{u}_m] = [\bar{e}_1 \ \bar{e}_2 \cdots \bar{e}_m]$$
$$= I$$

Since A has a right inverse, by Cor 1.2, it is invertible.