MTH100 MATHS I Monsoon Semester – 2016-17

Tutorial exercise for week of Monday 22nd August 2016.

1. Given an m×n matrix A and an n×k matrix B, the product AB = [A$\mathbf{v}_1$ A$\mathbf{v}_2$ ..... A$\mathbf{v}_k$] in column form where B = [$\mathbf{v}_1$ $\mathbf{v}_2$ ..... $\mathbf{v}_k$] in column form. Construct an example to illustrate this rule. The matrix A in your example should be at least 3×3 and B should be at least 3×2.
2. Suppose AB = AC, where B and C are n×k matrices and A is invertible. Show that B = C. Is this true, in general, when A is not invertible ? Justify your answer (proof if true, counter-example if false).
3. Verify the properties of a vector space for the space R^∞ of real sequences using the field of real numbers as the underlying field of scalars.
4. Verify the properties of a vector space for the space C[0,1] of continuous real-valued functions defined on the closed interval [0,1] using the field of real numbers as the underlying field of scalars. (*NB: note that we can use any closed interval [a,b] as the domain for the continuous functions under consideration.*)
5. Verify the properties of a vector space for the space $R^{m\times n}$ of m×n matrices with real entries using the field of real numbers as the underlying field of scalars.

SOLUTION

Q 1. An example was done in class; students should be able to construct their own.

Q 2. Suppose that A is invertible.

Then: $AB = AC$ (given)

Multiply on the left by A^{-1} :-

$$A^{-1}(AB) = A^{-1}(AC)$$

or $(A^{-1}A)B = (A^{-1}A)C$

or $IB = IC$

or $B = C$

~~PTO for~~ (PTO)

→ This doesn't hold in general if A is not invertible.

For example:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Then: $AB = AC = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$,

but $B \neq C$