MTH100 **Monsoon Semester – 2016-17** **Section A**

Submission for Tuesday 6th September 2016. Time: 15 minutes. Max Marks: 5

Consider the vector space V = R [t] of polynomials with real coefficients over the base field R of real numbers. Let X be the set of all polynomials which have only even order terms, i.e. if $p(t) \in X$, with $p(t) = a_0 + a_1t + a_2t^2 + ... + a_nt^n$, then $a_i = 0$ for all odd indices i. Prove or disprove: X is a subspace of V. *(Note: You must clearly state PROVE or DISPROVE at the top of your answer sheet. If you fail to do so, you will get zero marks.)*

(5 marks)

SOLUTION

PROVE

We will apply Proposition 8.

1. The zero polynomial $\bar{0}(t) = 0 \in X$ (all odd coefficients are zero).

2. Closure under addition:
Suppose $p(t) = a_0 + a_1t + \cdots + a_nt^n \in X$
and $q(t) = b_0 + b_1t + \cdots + b_mt^m \in X$.

WOLOG, $n \geq m$.

Hence $p(t) + q(t) = (a_0+b_0) + (a_1+b_1)t + \cdots + (a_n+b_n)t^n$,

where $b_i = 0$ for $i > m$.

For any odd term t^k, the coefficient of t^k
$= (a_k + b_k) = 0 + 0 = 0$ since $p(t), q(t) \in X$.

$\therefore (p+q)(t) = p(t) + q(t) \in X$.

3. Closure under scalar multiplication. If $c \in \mathbb{R}$ and $p(t) \in X$, then

$(cp)t = ca_0 + ca_1t + \cdots + ca_nt^n$, and for any odd term t^k, its coefficient is $ca_k = c.0 = 0$.
Result follows.

MTH100 **Monsoon Semester – 2016-17** **Section B**

Submission for Friday 9th September 2016. Time: 15 minutes. Max Marks: 5

Consider the set X = $C^1[0,1]$ of all real-valued functions defined on the closed interval [0,1] which have a continuous first derivative on [0,1], i.e. X = {f(x): [0,1] → R : f '(x) is continuous on [0,1]}. **Prove or disprove**: X is a subspace of C[0,1] . *(Note: You must clearly state PROVE or DISPROVE at the top of your answer sheet. If you fail to do so, you will get zero marks.)*

(5 marks)

SOLUTION

PROVE.

Step 1: We need to prove that ~~X ⊆~~ is a subset of C[0,1], i.e. $X \subseteq C[0,1]$.

However, if $f(x)$ is differentiable at x_0, then it is continuous at x_0. Hence, $X = C^1[0,1] \subseteq C[0,1]$

Step 2: To show X is a subspace, we apply Proposition 8.

1. The zero function ~~$f(x) =$~~ $\bar{0}(x) = 0$ ~~is~~ has a continuous derivative, $\bar{0}'(x) = 0$; hence $\bar{0} \in$ ~~E~~ X

2. ~~Sp~~ Suppose $f(x), g(x) \in X$
   ∴ $(f+g)(x)$ is a differentiable function, and $(f+g)'(x) = f'(x) + g'(x)$.
   But $f'(x)$ and $g'(x)$ are both continuous, and the sum of two continuous functions is continuous. ∴ $(f+g)'(x)$ is continuous, i.e. $f+g \in X$.
   This proves closure under addition.

3. If now $c \in \mathbb{R}$ and $f \in X$, then cf is differentiable and $(cf)'(x) = cf'(x)$. Since $f'(x)$ is continuous, so is $(cf)'$, i.e. $cf \in X$. This proves closure under scalar multiplication.