MTH100 MATHS I Monsoon Semester – 2016-17

Tutorial exercise for week commencing Monday 31st October 2016

1. Given an **ordered list** $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_n}$, $n \geq 1$, of vectors in a vector space V, show that the list is linearly dependent if and only if either $\mathbf{v_1} = \mathbf{0}$ or at least one vector $\mathbf{v_k}$ is a linear combination of the preceding vectors in the list.

2. a) Verify the result of Q1 for the ordered list of vectors given below by expressing some vector $\mathbf{v_k}$ as a linear combination of the preceding vectors: $\mathbf{v_1} = (1,2,-1)$, $\mathbf{v_2} = (-1,1,-2)$, $\mathbf{v_3} = (4,11,7)$, $\mathbf{v_4} = (1,-4,5)$, $\mathbf{v_5} = (1,8,1)$

   b) Is it true that if an ordered list is linearly dependent, then every vector, other than the first in the list, can be expressed as a linear combination of the preceding vectors (YES/NO) ? Justify your answer briefly.

3. Find the eigenvalues and corresponding eigenvectors for the matrix A given below. Is A diagonalizable ? Justify your answer in at most one sentence.

$$A = \begin{bmatrix} 3 & -1 & -1 \\ -12 & 0 & 5 \\ 4 & -2 & -1 \end{bmatrix}$$

4. Show that a linear transformation $T: V \to W$, where V and W are finite-dimensional with $\dim V = \dim W$, is injective if and only if it is surjective. (*NB: This is part of Proposition 39, so you cannot use Prop 39 in your proof.)*

5. Give an example of a vector space V, and two linear transformations $T, U: V \to V$, such that T is surjective but not injective, and U is injective but not surjective.

6. A square matrix A is said to **satisfy** a polynomial $p(t) \in \mathbb{R}[t]$ if $p(A) = \mathbf{0}$, i.e. if we substitute the matrix A in the polynomial by taking powers of A (in which the constant term is multiplied by identity matrix of appropriate size), then the resultant is the zero-matrix. Show that every $n \times n$ non-zero square matrix with real entries satisfies a non-zero polynomial of degree $\leq n^2$.