1. Which of the following describe equivalence relations? For those that are not equivalence relations, specify which of (R), (S), and (T) fail, and illustrate the failures with examples.

   (a) $L_1 \parallel L_2$ for straight lines in the plane if L_1 and L_2 are the same or are parallel.

   (b) $L_1 \perp L_2$ for straight lines in the plane if L_1 and L_2 are perpendicular.

   (c) $p_1 \sim p_2$ for Americans if p_1 and p_2 live in the same state.

   (d) $p_1 \approx p_2$ for Americans if p_1 and p_2 live in the same state or in neighboring states.

   (e) $p_1 \approx p_2$ for people if p_1 and p_2 have a parent in common.

   (f) $p_1 \cong p_2$ for people if p_1 and p_2 have the same mother.

2. For each example of an equivalence relation in Exercise 1, describe the members of some equivalence class.

3. Let S be a set. Is equality, i.e., "=", an equivalence relation?

4. Define the relation $\equiv$ on $\mathbb{Z}$ by $m \equiv n$ in case $m - n$ is even. Is $\equiv$ an equivalence relation? Explain.

5. If G and H are both graphs with vertex set $\{1, 2, \ldots, n\}$, we say that G is **isomorphic** to H, and write $G \simeq H$, in case there is a way to label the vertices of G so that it becomes H. For example, the graphs in Figure 3, with vertex set $\{1, 2, 3\}$, are isomorphic by relabeling $f(1) = 2$, $f(2) = 3$, and $f(3) = 1$.

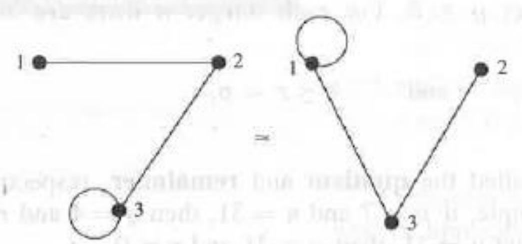

Figure 3 ▲

   (a) Give a picture of another graph isomorphic to these two.

   (b) Find a graph with vertex set $\{1, 2, 3\}$ that is not isomorphic to the graphs in Figure 3, yet has three edges, exactly one of which is a loop.

   (c) Find another example as in part (b) that isn't isomorphic to the one you found in part (b) [or the ones in Figure 3].

   (d) Show that $\simeq$ is an equivalence relation on the set of all graphs with vertex set $\{1, 2, \ldots, n\}$.

6. Can you think of situations in life where you'd use the term "equivalent" and where a natural equivalence relation is involved?

7. Define the relation $\approx$ on $\mathbb{Z}$ by $m \approx n$ in case $m^2 = n^2$.

   (a) Show that $\approx$ is an equivalence relation on $\mathbb{Z}$.

   (b) Describe the equivalence classes for $\approx$. How many are there?

8. (a) For $m, n \in \mathbb{Z}$, define $m \sim n$ in case $m - n$ is odd. Is the relation $\sim$ reflexive? symmetric? transitive? Is $\sim$ an equivalence relation?

   (b) For a and b in $\mathbb{R}$, define $a \sim b$ in case $|a - b| \le 1$. One could say that $a \sim b$ in case a and b are "close enough" or "approximately equal." Answer the questions in part (a).

9. Consider the functions g and h mapping $\mathbb{Z}$ into $\mathbb{N}$ defined by $g(n) = |n|$ and $h(n) = 1 + (-1)^n$.

   (a) Describe the sets in the partition $\{g^{\leftarrow}(k) : k \text{ is in the codomain of } g\}$ of $\mathbb{Z}$. How many sets are there?

   (b) Describe the sets in the partition $\{h^{\leftarrow}(k) : k \text{ is in the codomain of } h\}$ of $\mathbb{Z}$. How many sets are there?

10. On the set $\mathbb{N} \times \mathbb{N}$ define $(m, n) \sim (k, l)$ if $m + l = n + k$.

   (a) Show that $\sim$ is an equivalence relation on $\mathbb{N} \times \mathbb{N}$.

   (b) Draw a sketch of $\mathbb{N} \times \mathbb{N}$ that shows several equivalence classes.

11. Let Σ be an alphabet, and for w_1 and w_2 in Σ^* define $w_1 \sim w_2$ if $\text{length}(w_1) = \text{length}(w_2)$. Explain why $\sim$ is an equivalence relation, and describe the equivalence classes.

12. Let P be a set of computer programs, and regard programs p_1 and p_2 as equivalent if they always produce the same outputs for given inputs. Is this an equivalence relation on P? Explain.