Principles of Communication Systems Lab

Lab 2, 20th August 2019

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Answer to Q1

Note: For question 1 the signal u(t) = 2I[1,3](t) – 3I[2,4](t) and umf = u(-t),

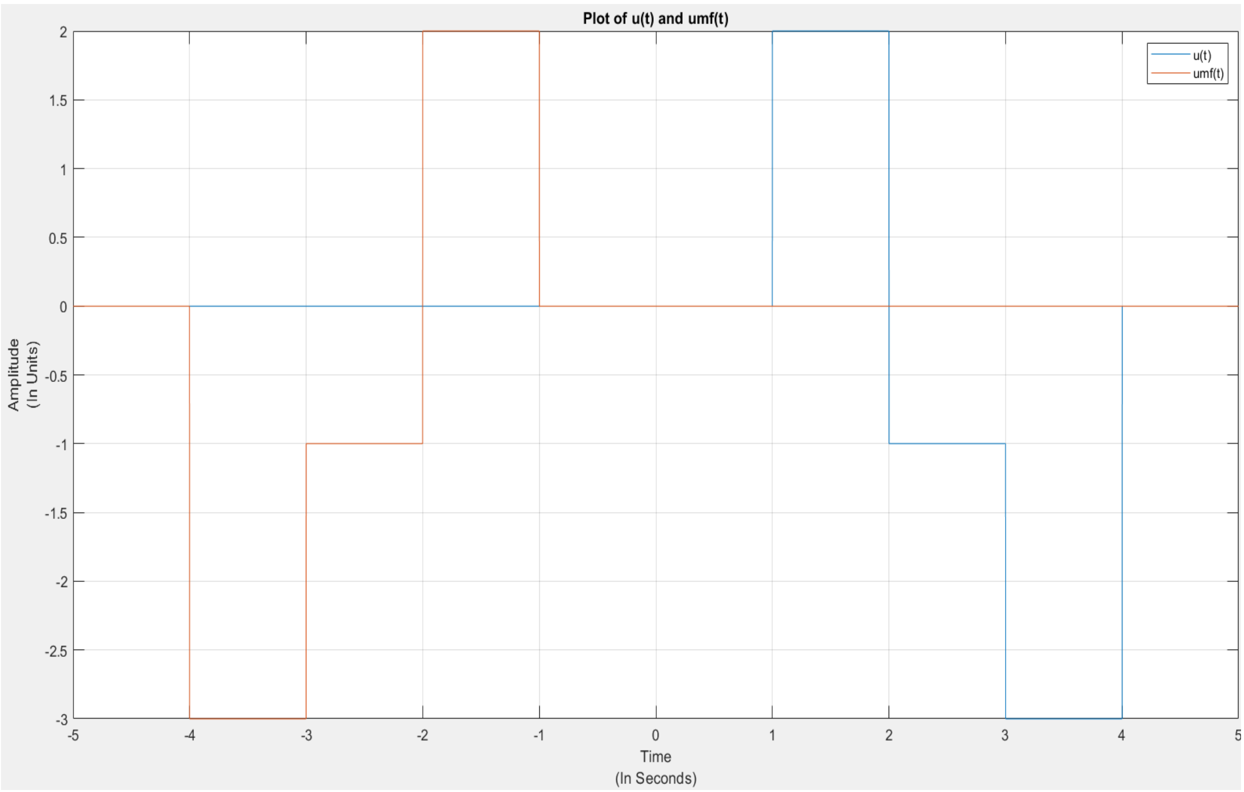
s(t) = u(t) + j v(t) where v(t) = I[-1,2](t) + 2I[0,1](t) and smf = s*(-t),

s1(t) = s(t – to)x(ejθ) here to = 2 and θ = (pi/4),

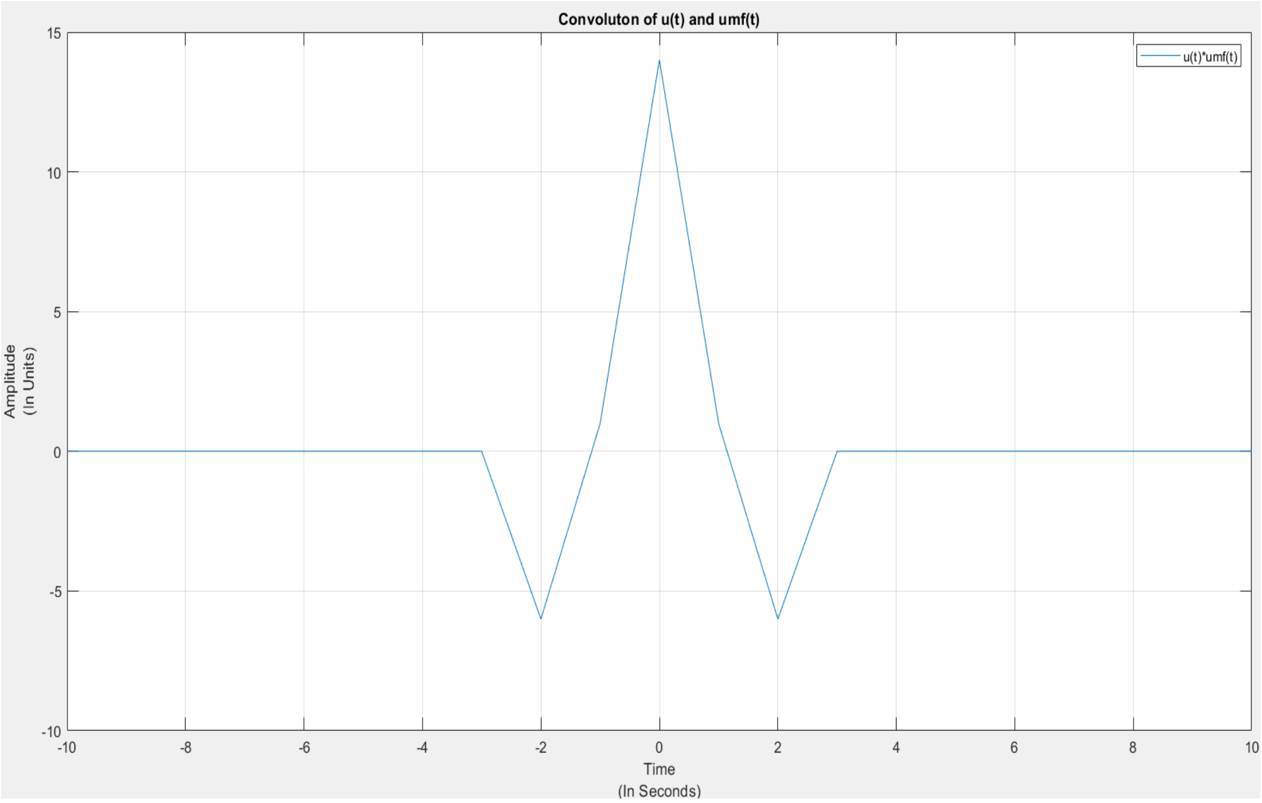
'*' here means conjugate.

Unit time is in seconds and sampled at 1KHz frequency.

1(a):

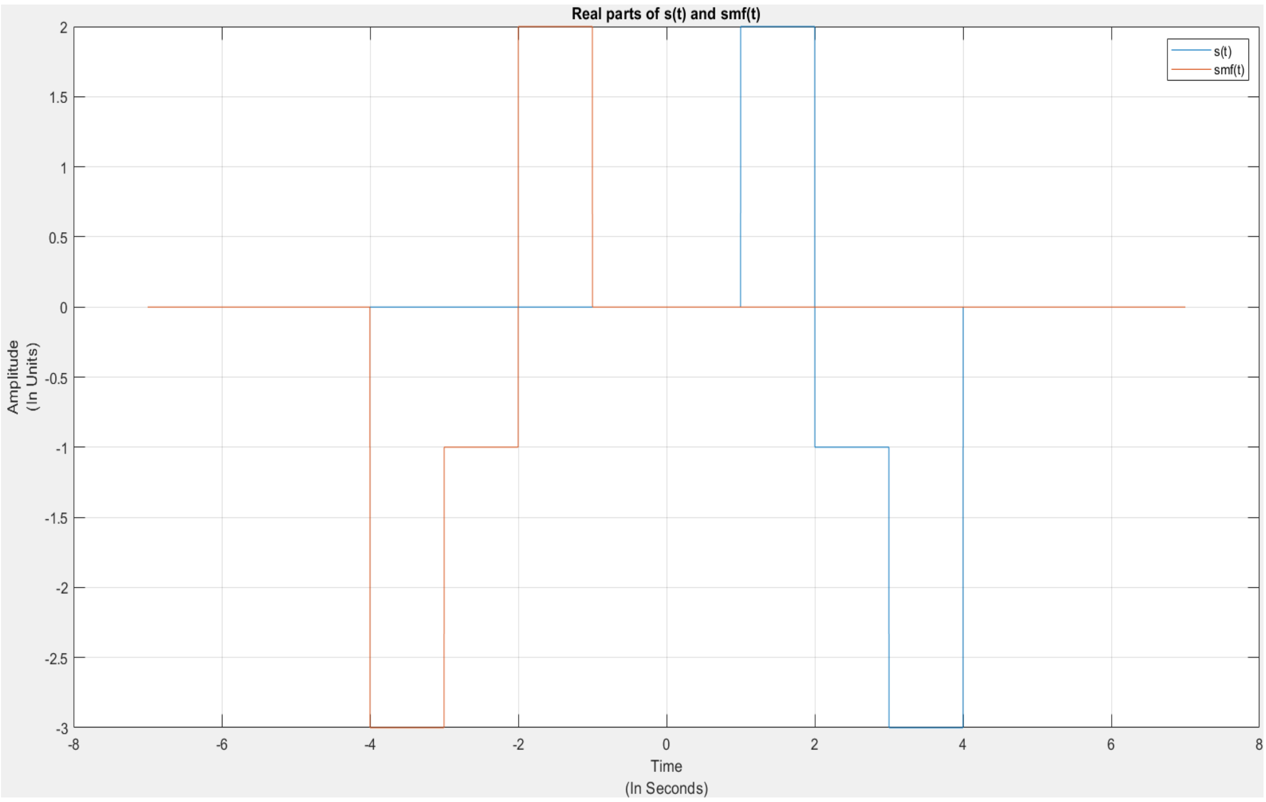


1(b):

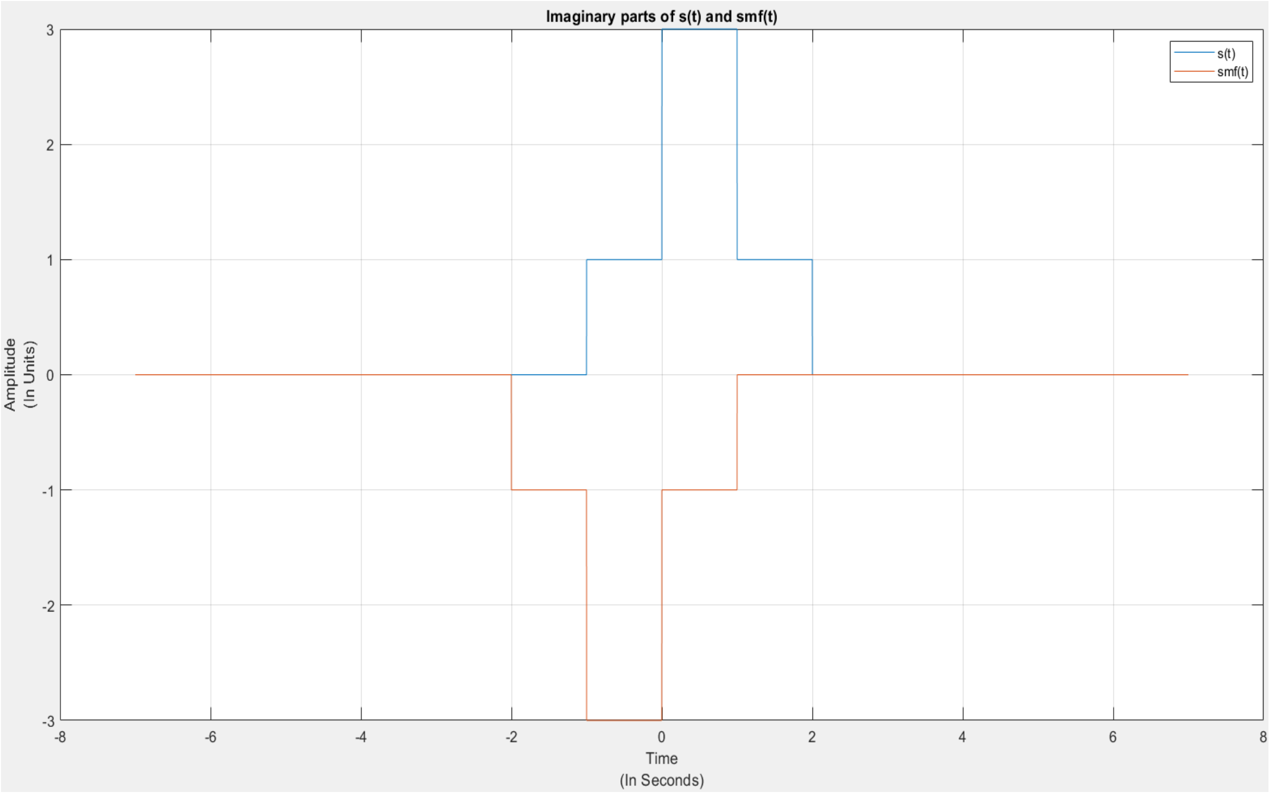


Ans: Peak can be seen at t = 0s in the plot.

1(c): (i)

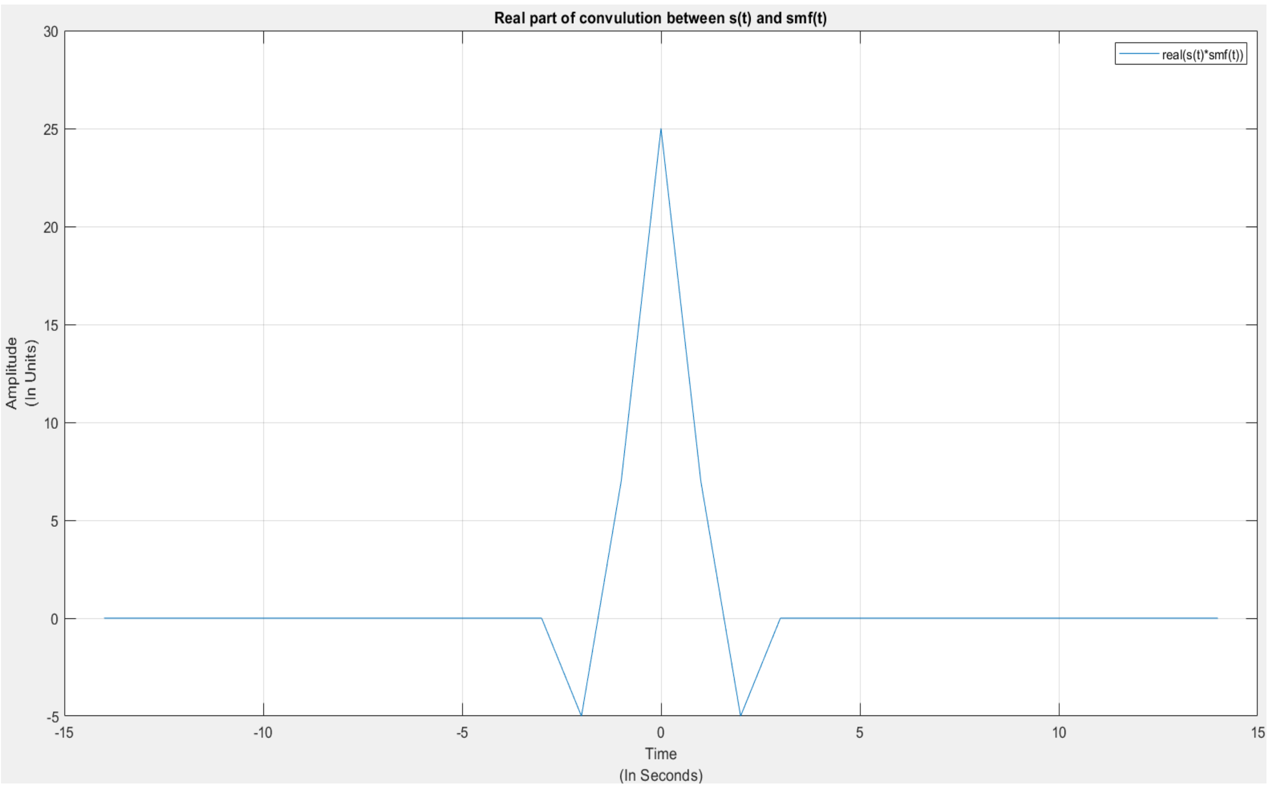


1(c): (ii)



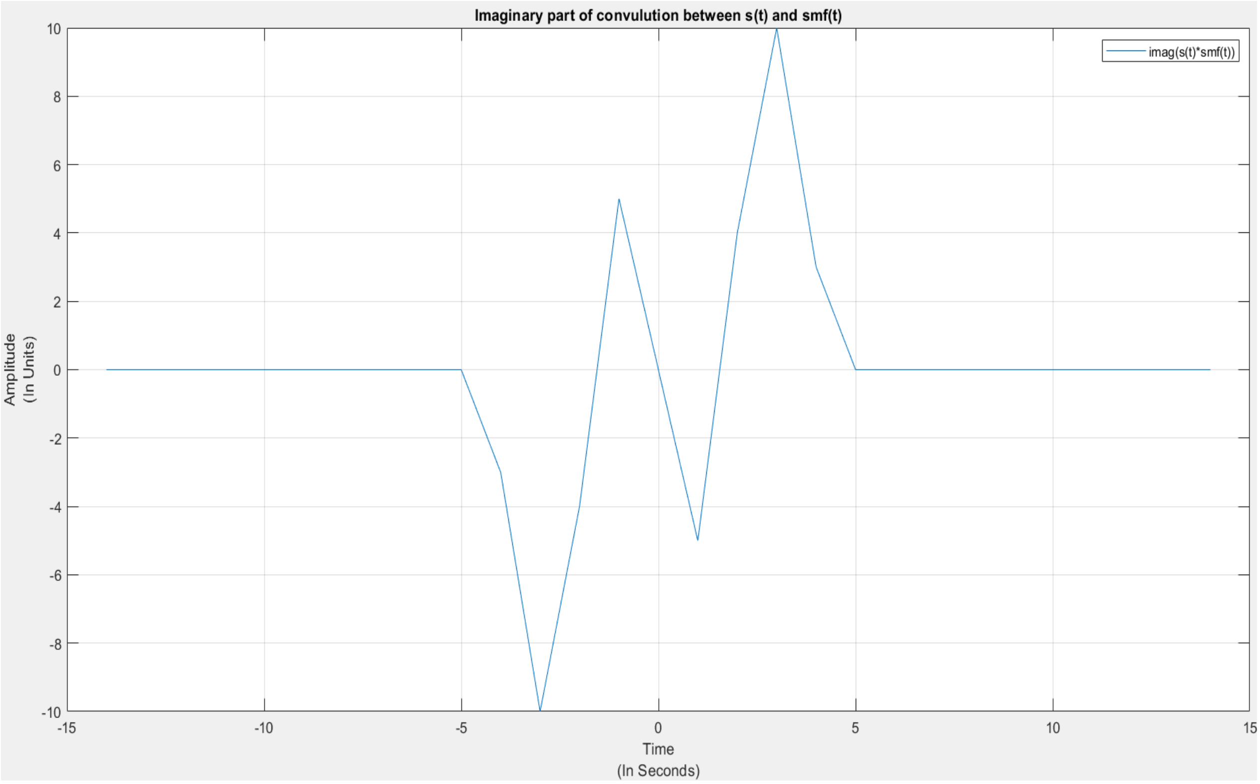
Imaginary parts of s(t) and smf(t)
s(t)
smf(t)
Amplitude
(In Units)
Time
(In Seconds)
3
2
1
0
-1
-2
-3
-8
-6
-4
-2
0
2
4
6
8

1(d): (i)



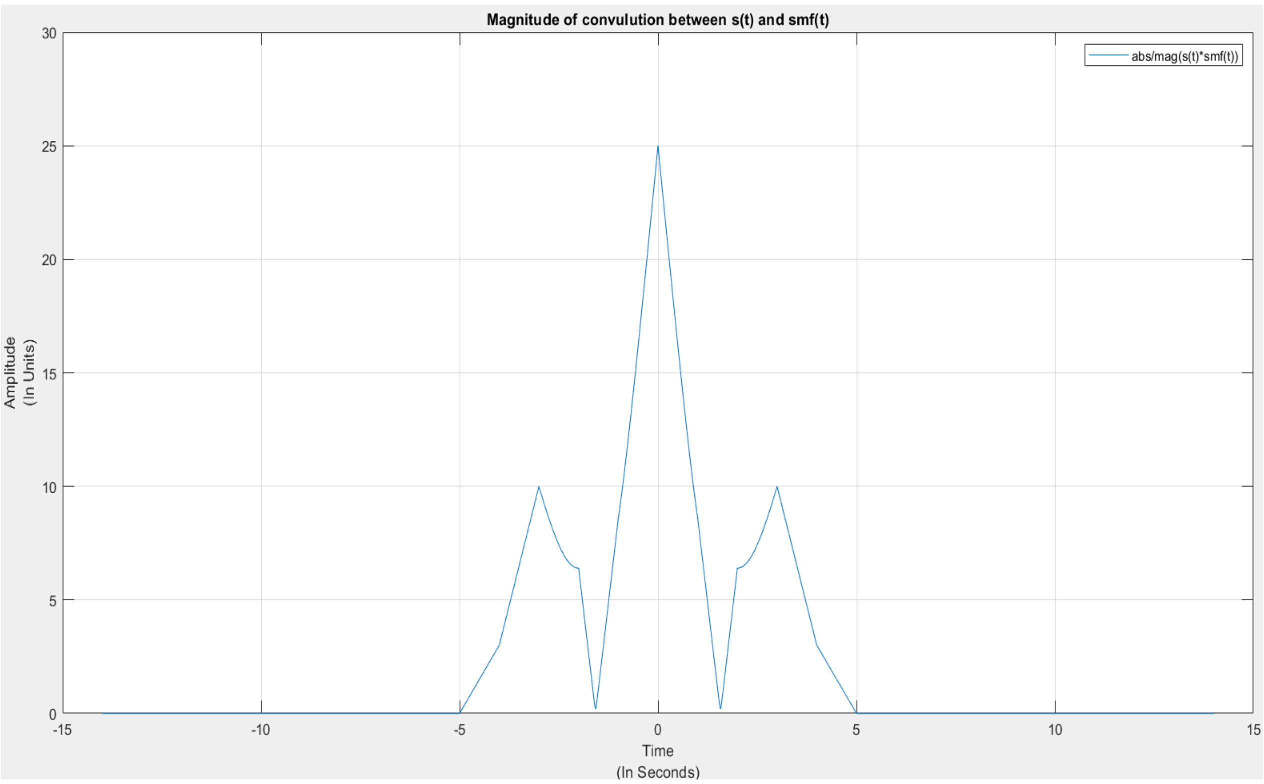
Real part of convulution between s(t) and smf(t)
real(s(t)*smf(t))
Amplitude
(In Units)
Time
(In Seconds)
30
25
20
15
10
5
0
-5
-15
-10
-5
0
5
10
15

1(d): (ii)



Imaginary part of convulution between s(t) and smf(t)
imag(s(t)*smf(t))
Amplitude
(In Units)
Time
(In Seconds)

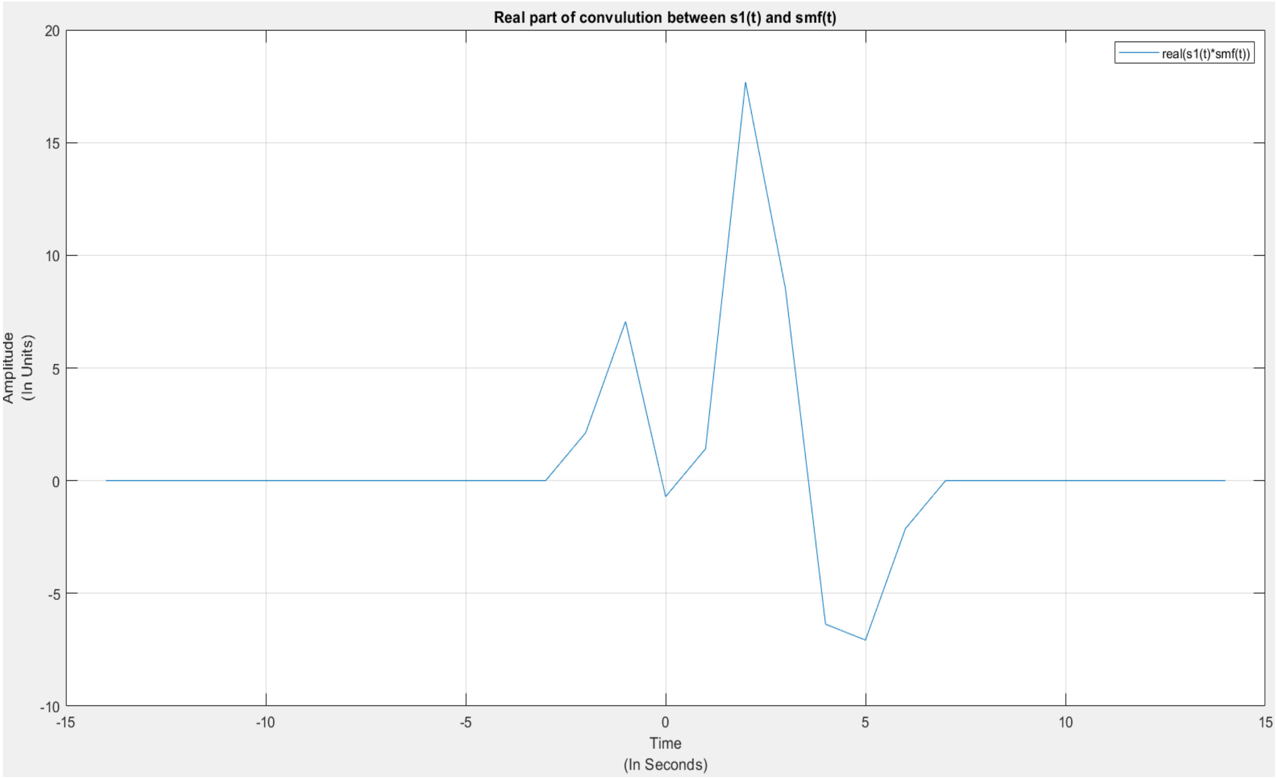
1(d): (iii)



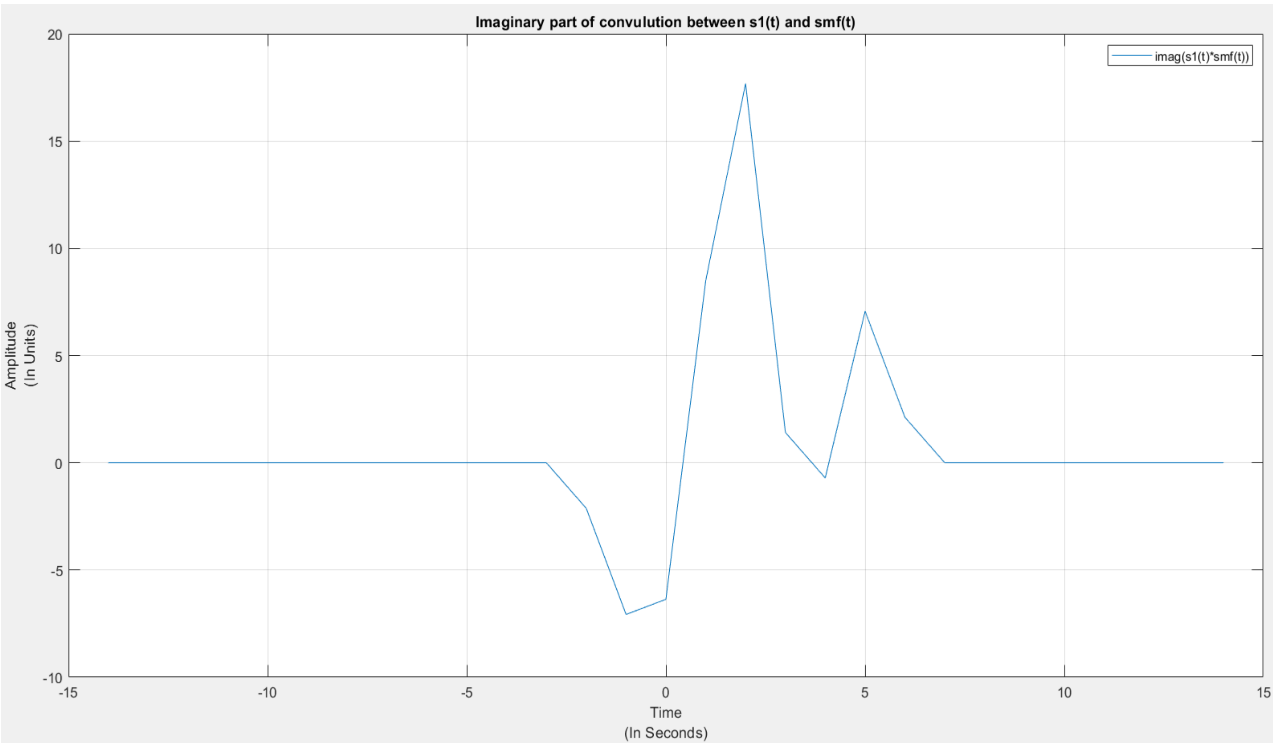
Magnitude of convulution between s(t) and smf(t)
abs/mag(s(t)*smf(t))
Amplitude
(In Units)
Time
(In Seconds)

Ans: Peak can be seen in plot-7 at t = 0s.

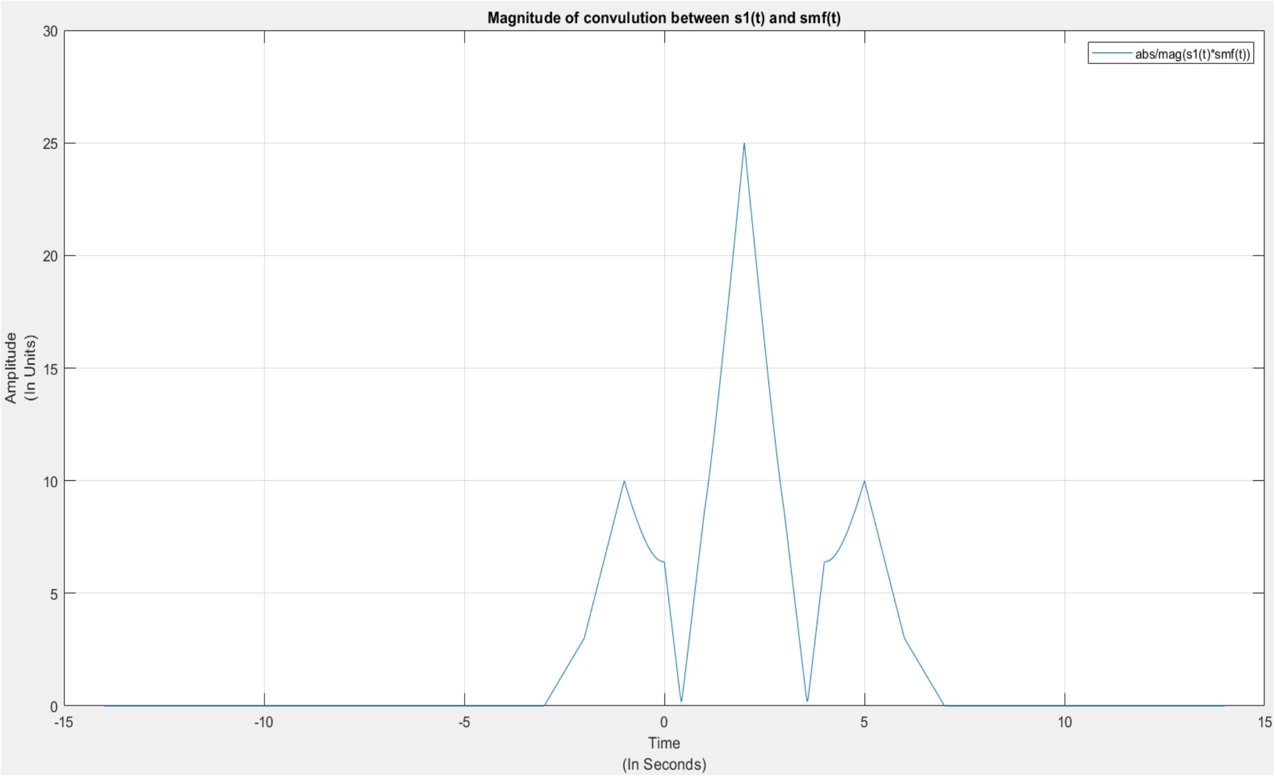
1(e): (i)



1(e): (ii)



1(e): (iii)



Ans: Peak can be seen in plot-10 at t = 2s.

1(f):

Let $y(t) = s(t) * smf(t)$. (Here $y(t)$ is output of convolution and '*' here means convolution)

$s1(t) * smf(t) = \{ei\theta \{s(t - to)\} * smf(t)$ Eq(1)

We know that,

There is a shift in the convolution of s1(t) and smf(t)(Plot-10 shifted by 2 unit compared to Plot-7) that shift corresponds to the shift in s1(t-to). In our case to = 2.

Eq(1):

$= [y(t - t0)]x[ei(\theta)]$

$= [y(t - t0)]x[\cos(\theta) + i \sin(\theta)]$ (here $ei(\theta) = \cos(\theta) + i \sin(\theta)$)

$= \{[real(y(t-to])]x[\cos(\theta)] - [imag(y(t-to))]x[\sin(\theta)]\} + i \{[real(y(t-to))]x[\sin(\theta)] + [imag(y(t-to))]x[\cos(\theta)]\} \}$

Now, in the imaginary graph of y(t) find t' where the imag(y(t')) = 0. Substitute t = t' + t0 in above equation. And in the real graph of the equation get the value at t = t' + t0. Divide this value with the value of the real graph of y(t) at t = t'. The ratio will be equal to cos(θ).

Similarly, we get the time t'' in the real graph of y(t) where the value is 0. We substitute t = t'' + t0 in the above equation and divide it with real graph of y(t) at t = t''. This gives us the -sin(θ).

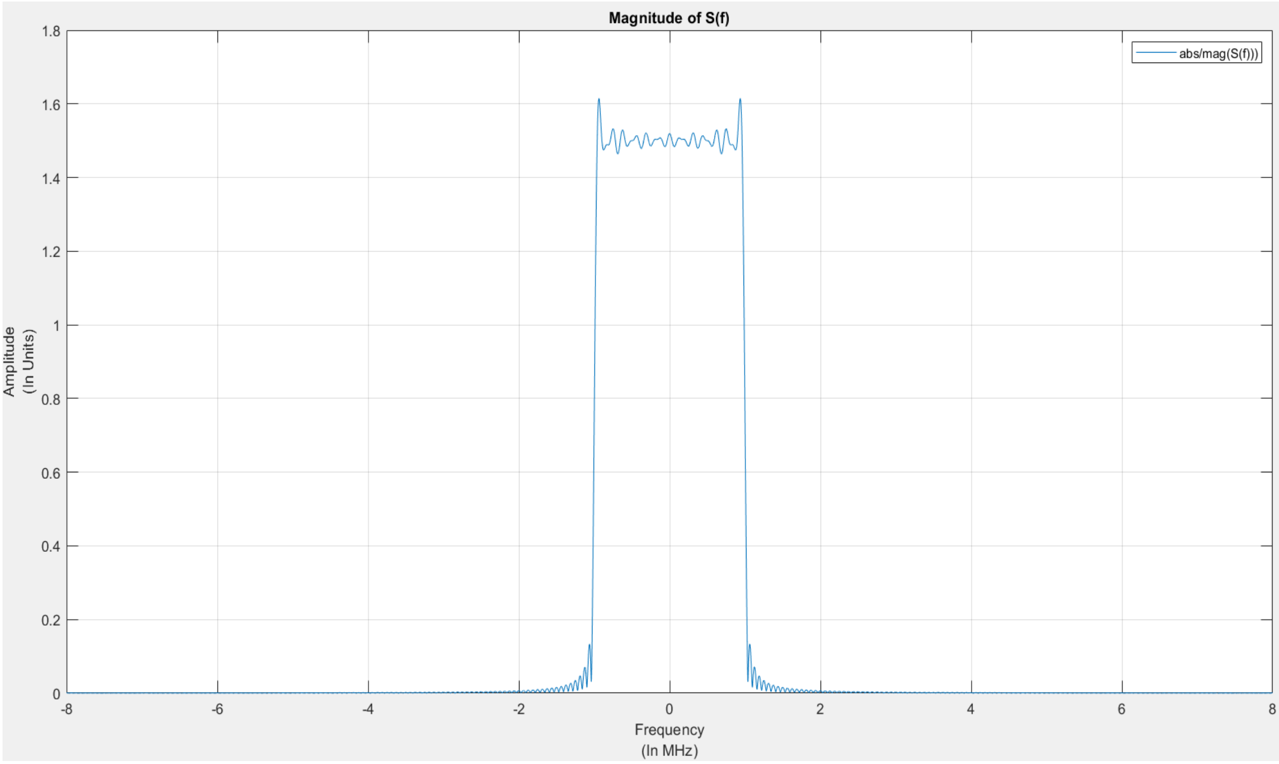
Final Conclusion:

$$\theta = \tan^{-1}(\sin(\theta)/\cos(\theta))$$

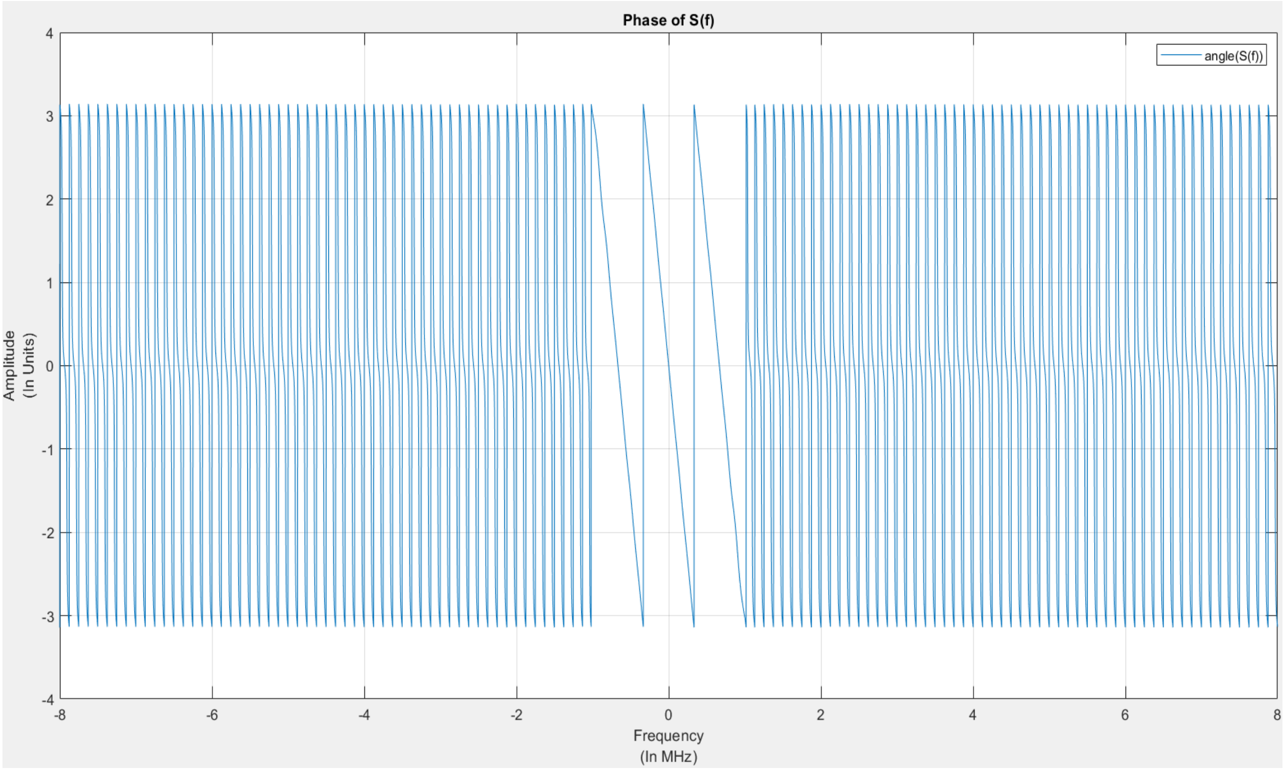
Answer to Q2

Note: For question 2 $s(t) = 3sinc(2t - 3)$, unit time is microsecond and signal is sampled at 16MHz and is truncated to the range [-8,8] and desired frequency is 1 MHz. Here S(f) is fourier transform of s(t)

2(b):



2(c):



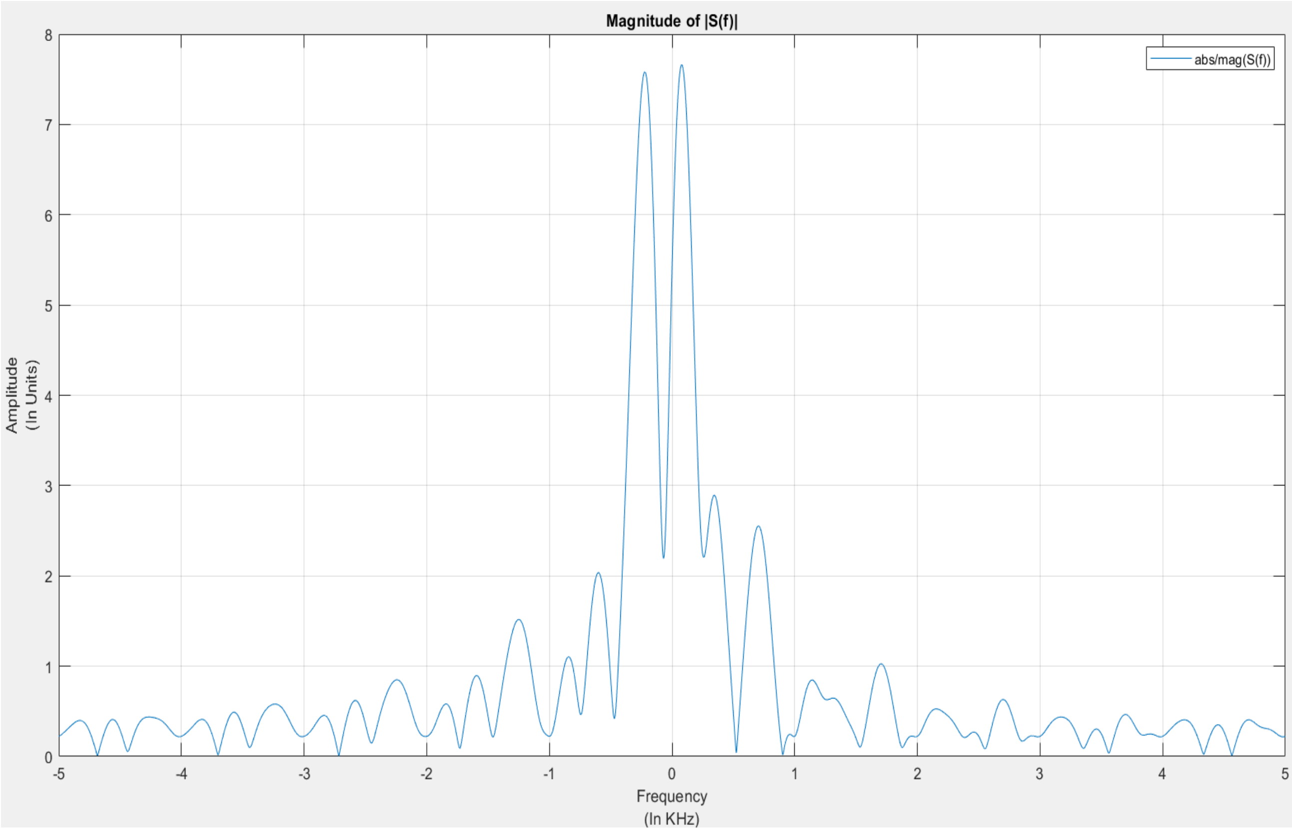
Ans: The range of frequencies over which phase plot has meaning is[-1,1] MHz.

Answer to Q3

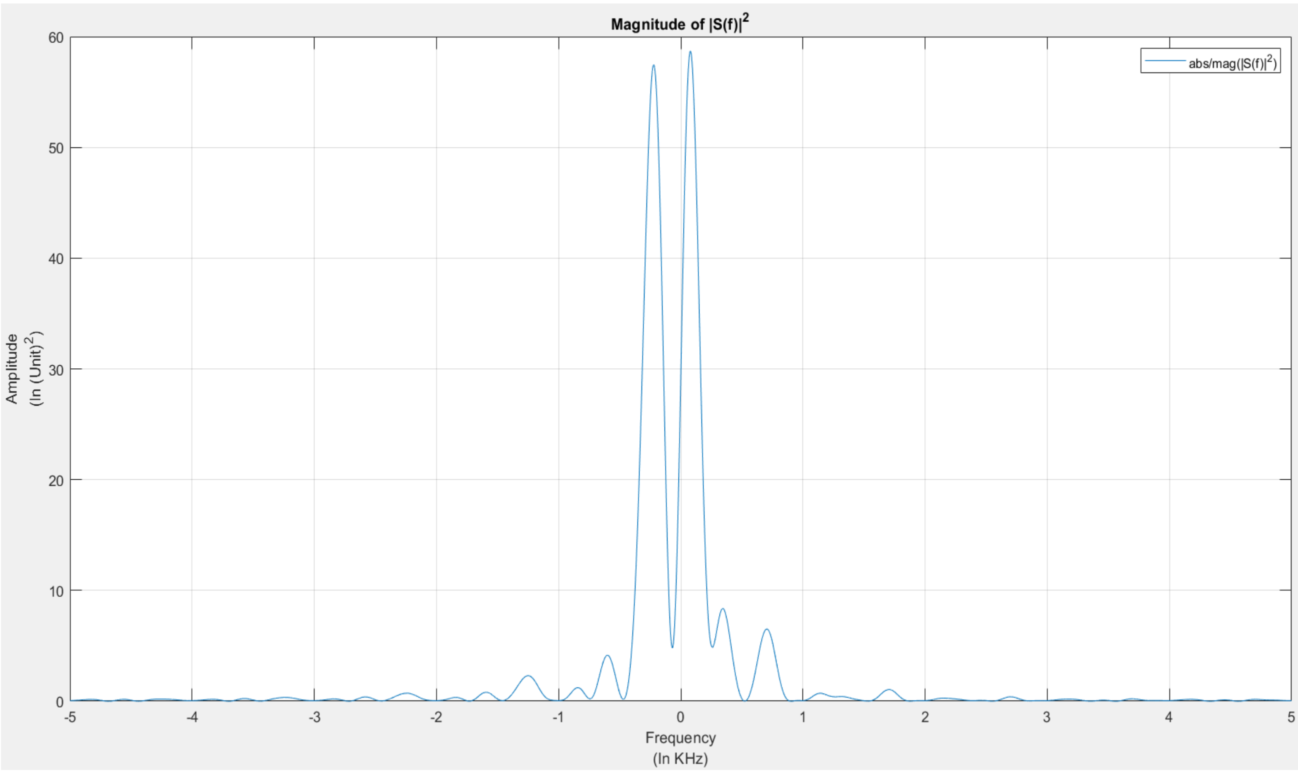
Note: For question 3 we will consider s(t) from question 1 ie s(t) = u(t) + j v(t) where u(t) = 2I[1,3](t) – 3I[2,4](t), v(t) = I[-1,2](t) + 2I[0,1](t) and smf = s*(-t),

'*' here means conjugate. Here unit time is millisecond and is sampled at 10MHz

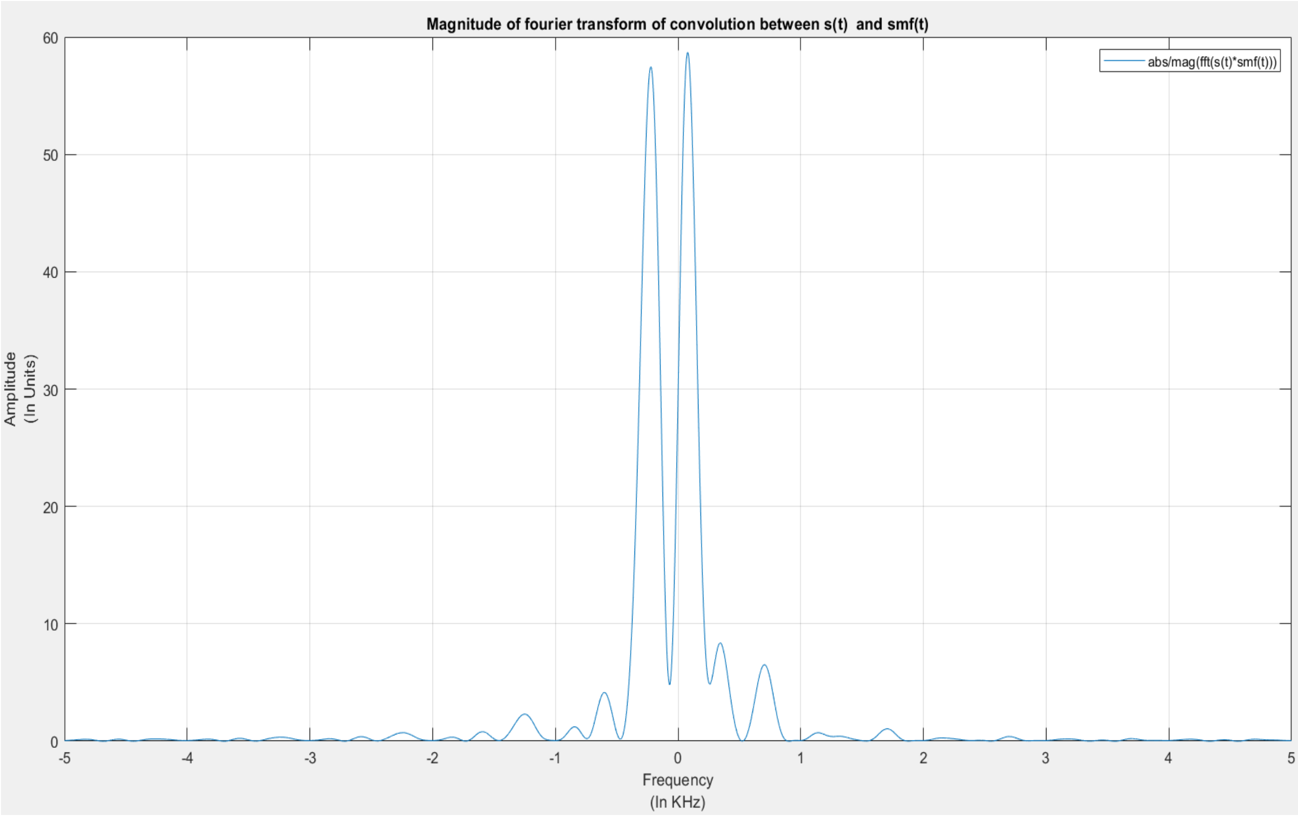
3(a):



3(a2): The 3(b) Plot ie Plot-14 matches with this plot that means magnitude of convolution and |S(f)^2| is same.

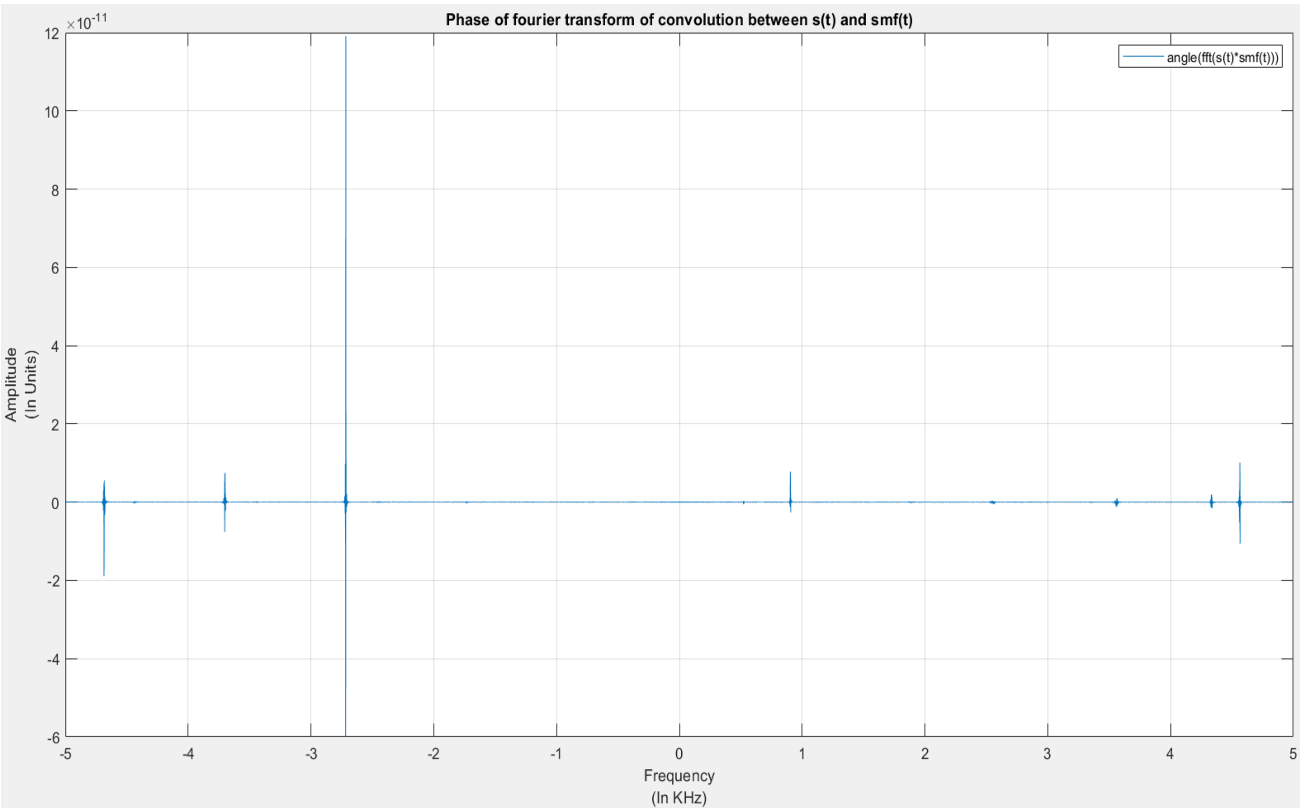


3(b):



Magnitude of fourier transform of convolution between s(t) and smf(t)
abs/mag(fft(s(t)*smf(t)))
Amplitude
(In Units)
Frequency
(In KHz)
60
50
40
30
20
10
0
-5
-4
-3
-2
-1
0
1
2
3
4
5

3(c):



Phase of fourier transform of convolution between s(t) and smf(t)
angle(fft(s(t)*smf(t)))
×10^-11
Amplitude
(In Units)
Frequency
(In KHz)
12
10
8
6
4
2
0
-2
-4
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5

Ans: The convolution in time domain is nothing but multiplication in frequency domain so s(t)*smf(t) →(fft) gives us S(f)xS(-f) (because smf(t) = s*(-t) and we know that s*(-t) →(fft) is S(-f)) so we can see that S(f) and S(-f) have opposite in sign and equal in magnitude phase so we get all the phase cancelled and phase plot is almost zero in amplitude.

Appendix:

Code for Q1 :

```
dt = 0.001;
t = -5:dt:5;
u = signalx(t);
umf = signalx(-t);
figure(1);
plot(t,u);
hold on;
plot(t,umf);
hold off;
xlabel({'Time','(In Seconds)'});ylabel({'Amplitude','(In Units)'});
title('Plot of u(t) and umf(t)');
legend('u(t)','umf(t)');
grid on;
[y,t1] = contconv(double(u),double(umf),t(1),t(1),dt);
figure(2);
plot(t1,y);
xlabel({'Time','(In Seconds)'});ylabel({'Amplitude','(In Units)'});
title('Convoluton of u(t) and umf(t)');
legend('u(t)*umf(t)');
grid on;
function u = signalx(t)
syms x;
y = piecewise(1 <= x <= 2, 2, 2 <= x <= 3, -1, 3 <= x <= 4, -3, 0);
u = subs(y,x,t);
end
function [y,t] = contconv(x1,x2,s1,s2,dt)
y = conv(x1,x2)*dt;
s1_2 = s1 + (length(x1)-1)*dt;
s2_2 = s2 + (length(x2)-1)*dt;
t1 = s1+ s2;
t2 = s2_2 + s1_2;
t = t1:dt:t2;
end


dt = 0.001;
to = 2;
theta = pi/4;
t = -7:dt:7;
u = signalx(t);
v = signalx1(t);
s = u + 1i*v;
sc = signalx(-t) - 1i*signalx1(-t);
figure(3);
plot(t,double(real(s)));
hold on;
```

```
plot(t,double(real(sc)));
hold off;
xlabel({'Time','(In Seconds)'});ylabel({'Amplitude','(In Units)'});
title('Real parts of s(t) and smf(t)');
legend('s(t)','smf(t)');
grid on;
figure(4);
plot(t,imag(s));
hold on;
plot(t,imag(sc));
hold off;
xlabel({'Time','(In Seconds)'});ylabel({'Amplitude','(In Units)'});
title('Imaginary parts of s(t) and smf(t)');
legend('s(t)','smf(t)');
grid on;
[s_c,t1] = contconv(double(s),double(sc),t(1),t(1),dt);
figure(5);
plot(t1,real(s_c));
xlabel({'Time','(In Seconds)'});ylabel({'Amplitude','(In Units)'});
title('Real part of convulution between s(t) and smf(t)');
legend('real(s(t)*smf(t))');
grid on;
figure(6);
plot(t1,imag(s_c));
xlabel({'Time','(In Seconds)'});ylabel({'Amplitude','(In Units)'});
title('Imaginary part of convulution between s(t) and smf(t)');
legend('imag(s(t)*smf(t))');
grid on;
figure(7);
plot(t1,abs(s_c));
xlabel({'Time','(In Seconds)'});ylabel({'Amplitude','(In Units)'});
title('Magnitude of convulution between s(t) and smf(t)');
legend('abs/mag(s(t)*smf(t))');
grid on;
s1 = (signalx(t-2) + 1i*signalx1(t-2))*exp(1i*theta);
[s1_c,t3] = contconv(double(s1),double(sc),t(1),t(1),dt);
figure(8);
plot(t3,real(s1_c));
xlabel({'Time','(In Seconds)'});ylabel({'Amplitude','(In Units)'});
title('Real part of convulution between s1(t) and smf(t)');
legend('real(s1(t)*smf(t))');
grid on;
figure(9);
plot(t3,imag(s1_c));
xlabel({'Time','(In Seconds)'});ylabel({'Amplitude','(In Units)'});
title('Imaginary part of convulution between s1(t) and smf(t)');
legend('imag(s1(t)*smf(t))');
grid on;
figure(10);
plot(t3,abs(s1_c));
xlabel({'Time','(In Seconds)'});ylabel({'Amplitude','(In Units)'});
title('Magnitude of convulution between s1(t) and smf(t)');
legend('abs/mag(s1(t)*smf(t))');
xt = get(gca, 'XTick');
set(gca, 'XTick',xt, 'XTickLabel',xt/1)
grid on;
grid minor;
function u = signalx(t)
syms x;
y = piecewise(1 <= x <= 2, 2, 2 <= x <= 3, -1, 3 <= x <= 4, -3, 0);
u = subs(y,x,t);
```

```
end
function v = signalx1(t)
syms x;
y = piecewise(-1 <= x <= 0, 1, 0 <= x <= 1, 3, 1 <= x <= 2, 1, 0);
v = subs(y,x,t);
end
function [y,t] = contconv(x1,x2,s1,s2,dt)
y = conv(x1,x2)*dt;
s1_2 = s1 + (length(x1)-1)*dt;
s2_2 = s2 + (length(x2)-1)*dt;
t1 = s1+ s2;
t2 = s2_2 + s1_2;
t = t1:dt:t2;
end
```

Code for Q2 and Q3 :

```
function two
q_2();
q_3();
end
function q_2
dt = (1/16);
t = -8:dt:8;
s = 3*sinc(2*t - 3);
[Y,f,df] = contFT(s,t(1),dt,10^(-3));
figure(11);
plot(f,abs(Y));
xlabel({'Frequency','(In MHz)'});ylabel({'Amplitude','(In Units)'});
title('Magnitude of S(f)');
legend('abs/mag(S(f)))');
grid on;
figure(12);
plot(f,angle(Y));
xlabel({'Frequency','(In MHz)'});ylabel({'Amplitude','(In Units)'});
title('Phase of S(f)');
legend('angle(S(f))');
grid on;
grid minor;
end
function q_3
dt = 0.1;
to = 2;
theta = pi/4;
t = -7:dt:7;
u = signalx(t);
v = signalx1(t);
s = u + 1i*v;
sc = signalx(-t) - 1i*signalx1(-t);
[s_c,t1] = contconv(double(s),double(sc),t(1),t(1),dt);
[S,f,df] = contFT(double(s),t(1),dt,10^(-3));
[S_C,F,DF] = contFT(double(s_c),t1(1),dt,10^(-3));
figure(13);
plot(f,abs(S));
xlabel({'Frequency','(In KHz)'});ylabel({'Amplitude','(In Units)'});
title('Magnitude of |S(f)|');
legend('abs/mag(S(f))');
grid on;
figure(14);
```

```
plot(F,abs(S_C));
xlabel({'Frequency','(In KHz)'});ylabel({'Amplitude','(In Units)'});
title('Magnitude of fourier transform of convolution between s(t)  and 
smf(t)');
legend('abs/mag(fft(s(t)*smf(t)))');
grid on;
figure(15);
plot(F,angle(S_C));
xlabel({'Frequency','(In KHz)'});ylabel({'Amplitude','(In Units)'});
title('Phase of fourier transform of convolution between s(t) and smf(t)');
legend('angle(fft(s(t)*smf(t)))');
grid on;
end
function [y,t] = contconv(x1,x2,s1,s2,dt)
y = conv(x1,x2)*dt;
s1_2 = s1 + (length(x1)-1)*dt;
s2_2 = s2 + (length(x2)-1)*dt;
t1 = s1+ s2;
t2 = s2_2 + s1_2;
t = t1:dt:t2;
end
function u = signalx(t)
syms x;
y = piecewise(1 <= x <= 2, 2, 2 <= x <= 3, -1, 3 <= x <= 4, -3, 0);
u = subs(y,x,t);
end
function v = signalx1(t)
syms x;
y = piecewise(-1 <= x <= 0, 1, 0 <= x <= 1, 3, 1 <= x <= 2, 1, 0);
v = subs(y,x,t);
end
function [X,f,df] = contFT(x,tstart,dt,df_desired)
%Use Matlab DFT for approximate computation of continuous time Fourier 
transform
%INPUTS
%x = vector of time domain samples, assumed uniformly spaced %tstart= time 
at which first sample is taken
%dt = spacing between samples
%df_desired = desired frequency resolution
%OUTPUTS
% X=vector of samples of Fourier transform
%f=corresponding vector of frequencies at which samples are obtained
%df=freq resolution attained (redundant--already available from %difference 
of consecutive entries of f
%%%%%%%%%
%minimum FFT size determined by desired freq res or length of x
Nmin=max(ceil(1/(df_desired*dt)),length(x));
%choose FFT size to be the next power of 2
Nfft = 2^(nextpow2(Nmin));
%compute Fourier transform, centering around DC
X=dt*fftshift(fft(x,Nfft));
%achieved frequency resolution
df=1/(Nfft*dt);
%range of frequencies covered
f = ((0:Nfft-1)-Nfft/2)*df;
%same as f=-1/(2*dt):df:1/(2*dt) - df %phase shift associated with start 
time
X=X.*exp(-1i*2*pi*f*tstart);
end
```