

Assignment 12: TensorFlow and Keras Build various MLP architectures for MNIST dataset

Objective:

1. Building Models with 3 different architectures:
 - i) 2-Hidden layer architecture (784-472-168-10 architecture)
 - ii) 3-Hidden layer architecture (784-352-164-124-10 architecture)
 - iii) 5-Hidden layer architecture (784-216-170-136-80-38-10 architecture)
1. Train-Test error plot
1. Activation='relu'+ Adam Optimizer+Batch_Normalization +Drop_out

```
In [2]: # if you keras is not using tensorflow as backend set "KERAS_BACKEND=tensorflow" use this command
from keras.utils import np_utils
from keras.datasets import mnist
import seaborn as sns
from keras.initializers import RandomNormal
```

Using TensorFlow backend.

```
In [3]: %matplotlib inline
import matplotlib.pyplot as plt
import numpy as np
import time
# https://gist.github.com/greydanus/f6eee59eaf1d90fcb3b534a25362cea4
# https://stackoverflow.com/a/14434334
# this function is used to update the plots for each epoch and error
def plt_dynamic(x, vy, ty, ax, colors=['b']):
    ax.plot(x, vy, 'b', label="Validation Loss")
    ax.plot(x, ty, 'r', label="Train Loss")
    plt.legend()
    plt.grid()
    fig.canvas.draw()
```

```
In [4]: # the data, shuffled and split between train and test sets
(X_train, y_train), (X_test, y_test) = mnist.load_data()
```

Downloading data from <https://s3.amazonaws.com/img-datasets/mnist.npz>
11493376/11490434 [=====] - 4s 0us/step

```
In [5]: print("Number of training examples :", X_train.shape[0], "and each image is of shape (%d, %d)"%(X_train.shape[1], X_train.shape[2]))
print("Number of testing examples :", X_test.shape[0], "and each image is of shape (%d, %d)"%(X_test.shape[1], X_test.shape[2]))
```

Number of training examples : 60000 and each image is of shape (28, 28)
Number of testing examples : 10000 and each image is of shape (28, 28)

```
In [6]: # Each image we have is a (28*28) vector
# Let's convert the (28*28) vector into single dimensional vector of 1 * 784

X_train = X_train.reshape(X_train.shape[0], X_train.shape[1]*X_train.shape[2])
X_test = X_test.reshape(X_test.shape[0], X_test.shape[1]*X_test.shape[2])
```

```
In [7]: # after converting the input images from 3d to 2d vectors

print("Number of training examples :", X_train.shape[0], "and each image is of shape (%d)"%(X_train.shape[1]))
print("Number of testing examples :", X_test.shape[0], "and each image is of shape (%d)"%(X_test.shape[1]))
```

Number of training examples : 60000 and each image is of shape (784)
Number of testing examples : 10000 and each image is of shape (784)

[0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
247	127	0	0	0	0	0	0	0	3	18	18	18	126	136	175	26	166	255
170	253	253	253	253	253	225	172	253	242	195	64	0	0	0	30	36	94	154
	0	0	0	0	0	49	238	253	253	253	253	253	253	253	253	251	93	82
82	56	39	0	0	0	0	0	0	0	0	0	0	0	0	0	18	219	253
253	253	253	253	198	182	247	241	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	80	156	107	253	253	205	11	0	43	154
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	14	1	154	253	90	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	139	253	190	2	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	11	190	253	70	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	35	241
225	160	108	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	81	240	253	253	119	25	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	45	186	253	253	150	27	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	16	93	252	253	187
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	249	253	249	64	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	46	130	183	253
253	207	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	39	148	229	253	253	250	182	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	24	114	221	253	253	253	253
253	201	78	0	0	0	0												

```
X_train = X_train/255
X_test = X_test/255
```

```
In [11]: # Let's print first entry after normlizing  
print(X_train[0])
```

file:///C:/Users/pratyush.acharya/Downloads/12.Building various MLP architectures for MNIST dataset using TensorFlow and Keras .html

```

In [13]: # here we are having a class number for each image
print("Class label of first image :", y_train[0])

# Let's convert this into a 10 dimensional vector as it is needed for MLPs

y_train = np_utils.to_categorical(y_train, 10)
y_test = np_utils.to_categorical(y_test, 10)

print("After converting, class label of first image: ",y_train[0])

```

Class label of first image : 5
After converting, class label of first image: [0. 0. 0. 0. 0. 1. 0. 0. 0. 0.]

```
In [15]: # Setting model parameters

output_dim = 10
input_dim = X_train.shape[1]

batch_size = 128
nb_epoch = 20
```

1.1 MLP + ReLU + ADAM

```
In [16]: model_relu = Sequential()
model_relu.add(Dense(472, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_relu.add(Dense(168, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(output_dim, activation='softmax'))

print(model_relu.summary())

model_relu.compile(optimizer='adam',
                  loss='categorical_crossentropy',
                  metrics=['accuracy'])

history11 = model_relu.fit(X_train, y_train,
                          batch_size=batch_size,
                          epochs=nb_epoch, verbose=1,
                          validation_data=(X_test, y_test))
```

WARNING:tensorflow:From C:\Users\pratyush.acharya\AppData\Local\Continuum\anaconda3\lib\site-packages\tensorflow\python\ops\rsource_variable_ops.py:435: colocate_with (from tensorflow.python.framework.ops) is deprecated and will be removed in a future version.

Instructions for updating:
Colocations handled automatically by placer.
Model: "sequential_1"

Layer (type)	Output Shape	Param #
dense_1 (Dense)	(None, 472)	370520
dense_2 (Dense)	(None, 168)	79464
dense_3 (Dense)	(None, 10)	1690
Total params: 451,674		
Trainable params: 451,674		
Non-trainable params: 0		

None

WARNING:tensorflow:From C:\Users\pratyush.acharya\AppData\Local\Continuum\anaconda3\lib\site-packages\tensorflow\python\ops\math_ops.py:3066: to_int32 (from tensorflow.python.ops.math_ops) is deprecated and will be removed in a future version.

Instructions for updating:
Use tf.cast instead.
Train on 60000 samples, validate on 10000 samples

Epoch 1/20
60000/60000 [=====] - 8s 134us/step - loss: 0.2293 - accuracy: 0.9337 - val_loss: 0.1071 - val_accuracy: 0.9665
Epoch 2/20
60000/60000 [=====] - 8s 125us/step - loss: 0.0851 - accuracy: 0.9744 - val_loss: 0.0776 - val_accuracy: 0.9745
Epoch 3/20
60000/60000 [=====] - 8s 128us/step - loss: 0.0530 - accuracy: 0.9838 - val_loss: 0.0739 - val_accuracy: 0.9767
Epoch 4/20
60000/60000 [=====] - 7s 124us/step - loss: 0.0378 - accuracy: 0.9880 - val_loss: 0.0702 - val_accuracy: 0.9786
Epoch 5/20
60000/60000 [=====] - 9s 147us/step - loss: 0.0274 - accuracy: 0.9914 - val_loss: 0.0691 - val_accuracy: 0.9790
Epoch 6/20
60000/60000 [=====] - 8s 128us/step - loss: 0.0222 - accuracy: 0.9926 - val_loss: 0.0725 - val_accuracy: 0.9793
Epoch 7/20
60000/60000 [=====] - 8s 141us/step - loss: 0.0179 - accuracy: 0.9940 - val_loss: 0.0649 - val_accuracy: 0.9814
Epoch 8/20
60000/60000 [=====] - 8s 136us/step - loss: 0.0146 - accuracy: 0.9953 - val_loss: 0.0715 - val_accuracy: 0.9806
Epoch 9/20
60000/60000 [=====] - 9s 146us/step - loss: 0.0137 - accuracy: 0.9954 - val_loss: 0.0671 - val_accuracy: 0.9825
Epoch 10/20
60000/60000 [=====] - 8s 138us/step - loss: 0.0105 - accuracy: 0.9966 - val_loss: 0.0752 - val_accuracy: 0.9804
Epoch 11/20
60000/60000 [=====] - 8s 140us/step - loss: 0.0124 - accuracy: 0.9959 - val_loss: 0.0840 - val_accuracy: 0.9801
Epoch 12/20
60000/60000 [=====] - 8s 132us/step - loss: 0.0091 - accuracy: 0.9970 - val_loss: 0.0853 - val_accuracy: 0.9806
Epoch 13/20
60000/60000 [=====] - 8s 135us/step - loss: 0.0100 - accuracy: 0.9968 - val_loss: 0.0859 - val_accuracy: 0.9801
Epoch 14/20
60000/60000 [=====] - 9s 145us/step - loss: 0.0100 - accuracy: 0.9968 - val_loss: 0.0798 - val_accuracy: 0.9822
Epoch 15/20
60000/60000 [=====] - 8s 134us/step - loss: 0.0072 - accuracy: 0.9977 - val_loss: 0.0840 - val_accuracy: 0.9805
Epoch 16/20
60000/60000 [=====] - 8s 135us/step - loss: 0.0078 - accuracy: 0.9977 - val_loss: 0.0992 - val_accuracy: 0.9788
Epoch 17/20
60000/60000 [=====] - 8s 135us/step - loss: 0.0084 - accuracy: 0.9971 - val_loss: 0.1006 - val_accuracy: 0.9786
Epoch 18/20
60000/60000 [=====] - 8s 131us/step - loss: 0.0065 - accuracy: 0.9979 - val_loss: 0.0967 - val_accuracy: 0.9814
Epoch 19/20
60000/60000 [=====] - 8s 132us/step - loss: 0.0079 - accuracy: 0.9976 - val_loss: 0.1291 - val_accuracy: 0.9771
Epoch 20/20
60000/60000 [=====] - 8s 135us/step - loss: 0.0072 - accuracy: 0.9978 - val_loss: 0.1020 - val_accuracy: 0.9792

```

In [20]: score = model_relu.evaluate(X_test, y_test, verbose=0)
score1=score[0]
score2=score[1]
train_acc1=history11.history['accuracy']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax11 = plt.subplots(1,1)
ax11.set_xlabel('epoch') ; ax11.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

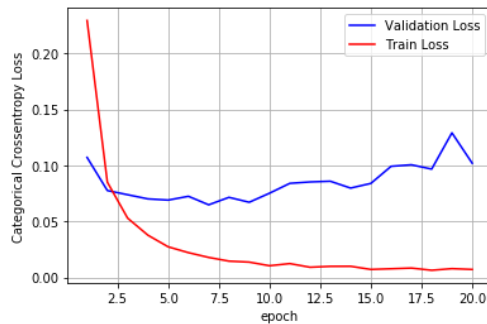
# val_loss : validation Loss
# val_acc : validation accuracy

# loss : training Loss
# acc : train accuracy

vy11 = history11.history['val_loss']
ty11 = history11.history['loss']
plt_dynamic(x, vy11, ty11, ax11)

```

Test score: 0.10199347244282822
Test accuracy: 0.979200005531311



1.2 MLP + Batch-Norm on hidden Layers + AdamOptimizer

```

In [21]: from keras.layers.normalization import BatchNormalization

model_batch = Sequential()

model_batch.add(Dense(472, activation='relu',
                      input_shape=(input_dim,),
                      kernel_initializer=he_normal(seed=None)))
model_batch.add(BatchNormalization())

model_batch.add(Dense(168, activation='relu',
                      kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(output_dim, activation='softmax'))

model_batch.summary()

```

Model: "sequential_2"

Layer (type)	Output Shape	Param #
dense_4 (Dense)	(None, 472)	370520
batch_normalization_1 (Batch Normalization)	(None, 472)	1888
dense_5 (Dense)	(None, 168)	79464
batch_normalization_2 (Batch Normalization)	(None, 168)	672
dense_6 (Dense)	(None, 10)	1690
Total params: 454,234		
Trainable params: 452,954		
Non-trainable params: 1,280		


```
In [22]: model_batch.compile(optimizer='adam', loss='categorical_crossentropy',
                             metrics=['accuracy'])

history12 = model_batch.fit(X_train, y_train,
                             batch_size=batch_size,
                             epochs=nb_epoch, verbose=1,
                             validation_data=(X_test, y_test))
```

Train on 60000 samples, validate on 10000 samples

```
Epoch 1/20
60000/60000 [=====] - 10s 166us/step - loss: 0.1890 - accuracy: 0.9433 - val_loss: 0.0932 - val_accuracy: 0.9721
Epoch 2/20
60000/60000 [=====] - 9s 158us/step - loss: 0.0712 - accuracy: 0.9788 - val_loss: 0.0890 - val_accuracy: 0.9709
Epoch 3/20
60000/60000 [=====] - 9s 158us/step - loss: 0.0453 - accuracy: 0.9858 - val_loss: 0.0792 - val_accuracy: 0.9743
Epoch 4/20
60000/60000 [=====] - 9s 155us/step - loss: 0.0324 - accuracy: 0.9900 - val_loss: 0.0865 - val_accuracy: 0.9746
Epoch 5/20
60000/60000 [=====] - 9s 150us/step - loss: 0.0268 - accuracy: 0.9917 - val_loss: 0.0709 - val_accuracy: 0.9789
Epoch 6/20
60000/60000 [=====] - 9s 149us/step - loss: 0.0216 - accuracy: 0.9930 - val_loss: 0.0910 - val_accuracy: 0.9747
Epoch 7/20
60000/60000 [=====] - 9s 147us/step - loss: 0.0200 - accuracy: 0.9936 - val_loss: 0.0737 - val_accuracy: 0.9777
Epoch 8/20
60000/60000 [=====] - 9s 150us/step - loss: 0.0164 - accuracy: 0.9948 - val_loss: 0.0727 - val_accuracy: 0.9793
Epoch 9/20
60000/60000 [=====] - 9s 156us/step - loss: 0.0124 - accuracy: 0.9959 - val_loss: 0.0760 - val_accuracy: 0.9784
Epoch 10/20
60000/60000 [=====] - 9s 158us/step - loss: 0.0134 - accuracy: 0.9957 - val_loss: 0.0797 - val_accuracy: 0.9794
Epoch 11/20
60000/60000 [=====] - 9s 149us/step - loss: 0.0140 - accuracy: 0.9954 - val_loss: 0.0753 - val_accuracy: 0.9783
Epoch 12/20
60000/60000 [=====] - 9s 149us/step - loss: 0.0131 - accuracy: 0.9956 - val_loss: 0.0736 - val_accuracy: 0.9804
Epoch 13/20
60000/60000 [=====] - 9s 151us/step - loss: 0.0097 - accuracy: 0.9969 - val_loss: 0.0787 - val_accuracy: 0.9806
Epoch 14/20
60000/60000 [=====] - 9s 149us/step - loss: 0.0072 - accuracy: 0.9976 - val_loss: 0.0863 - val_accuracy: 0.9797
Epoch 15/20
60000/60000 [=====] - 9s 156us/step - loss: 0.0099 - accuracy: 0.9969 - val_loss: 0.0792 - val_accuracy: 0.9801
Epoch 16/20
60000/60000 [=====] - 9s 150us/step - loss: 0.0099 - accuracy: 0.9966 - val_loss: 0.0754 - val_accuracy: 0.9819
Epoch 17/20
60000/60000 [=====] - 9s 151us/step - loss: 0.0095 - accuracy: 0.9968 - val_loss: 0.0881 - val_accuracy: 0.9809
Epoch 18/20
60000/60000 [=====] - 9s 149us/step - loss: 0.0074 - accuracy: 0.9977 - val_loss: 0.0819 - val_accuracy: 0.9814
Epoch 19/20
60000/60000 [=====] - 9s 151us/step - loss: 0.0067 - accuracy: 0.9980 - val_loss: 0.0770 - val_accuracy: 0.9829
Epoch 20/20
60000/60000 [=====] - 9s 156us/step - loss: 0.0050 - accuracy: 0.9983 - val_loss: 0.0911 - val_accuracy: 0.9800
```

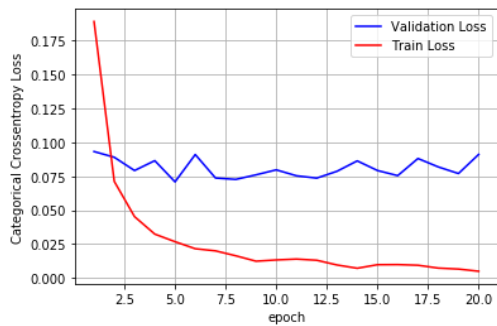
```
In [28]: score = model_batch.evaluate(X_test, y_test, verbose=0)
print('Test score:', score[0])
print('Test accuracy:', score[1])
score3=score[0]
score4=score[1]
train_acc2=history12.history['accuracy']

fig,ax12 = plt.subplots(1,1)
ax12.set_xlabel('epoch') ; ax12.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy12 = history12.history['val_loss']
ty12 = history12.history['loss']
plt_dynamic(x, vy12, ty12, ax12)
```

Test score: 0.09108898642615058
 Test accuracy: 0.9800000190734863



1.3 MLP + Dropout + AdamOptimizer

```
In [24]: from keras.layers import Dropout

model_drop = Sequential()

model_drop.add(Dense(472, activation='relu',
                    input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_drop.add(Dense(168, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_drop.add(Dense(output_dim, activation='softmax'))

model_drop.summary()
```

Model: "sequential_3"

Layer (type)	Output Shape	Param #
=====		
dense_7 (Dense)	(None, 472)	370520
batch_normalization_3 (Batch Normalization)	(None, 472)	1888
dropout_1 (Dropout)	(None, 472)	0
dense_8 (Dense)	(None, 168)	79464
batch_normalization_4 (Batch Normalization)	(None, 168)	672
dropout_2 (Dropout)	(None, 168)	0
dense_9 (Dense)	(None, 10)	1690
=====		
Total params: 454,234		
Trainable params: 452,954		
Non-trainable params: 1,280		

```
In [25]: model_drop.compile(optimizer='adam',
                        loss='categorical_crossentropy',
                        metrics=['accuracy'])

history13 = model_drop.fit(X_train, y_train,
                        batch_size=batch_size,
                        epochs=nb_epoch, verbose=1,
                        validation_data=(X_test, y_test))
```

Train on 60000 samples, validate on 10000 samples

```
Epoch 1/20
60000/60000 [=====] - 10s 175us/step - loss: 0.4305 - accuracy: 0.8688 - val_loss: 0.1433 - val_accu
racy: 0.9556
Epoch 2/20
60000/60000 [=====] - 10s 175us/step - loss: 0.2054 - accuracy: 0.9384 - val_loss: 0.1075 - val_accu
racy: 0.9667
Epoch 3/20
60000/60000 [=====] - 11s 176us/step - loss: 0.1641 - accuracy: 0.9507 - val_loss: 0.0917 - val_accu
racy: 0.9695
Epoch 4/20
60000/60000 [=====] - 11s 176us/step - loss: 0.1362 - accuracy: 0.9586 - val_loss: 0.0856 - val_accu
racy: 0.9727
Epoch 5/20
60000/60000 [=====] - 10s 163us/step - loss: 0.1198 - accuracy: 0.9634 - val_loss: 0.0748 - val_accu
racy: 0.9769
Epoch 6/20
60000/60000 [=====] - 10s 166us/step - loss: 0.1105 - accuracy: 0.9656 - val_loss: 0.0707 - val_accu
racy: 0.9773
Epoch 7/20
60000/60000 [=====] - 10s 162us/step - loss: 0.0991 - accuracy: 0.9693 - val_loss: 0.0716 - val_accu
racy: 0.9771
Epoch 8/20
60000/60000 [=====] - 10s 172us/step - loss: 0.0939 - accuracy: 0.9708 - val_loss: 0.0663 - val_accu
racy: 0.9790
Epoch 9/20
60000/60000 [=====] - 10s 170us/step - loss: 0.0849 - accuracy: 0.9732 - val_loss: 0.0683 - val_accu
racy: 0.9774
Epoch 10/20
60000/60000 [=====] - 11s 177us/step - loss: 0.0823 - accuracy: 0.9741 - val_loss: 0.0616 - val_accu
racy: 0.9816
Epoch 11/20
60000/60000 [=====] - 11s 181us/step - loss: 0.0784 - accuracy: 0.9756 - val_loss: 0.0622 - val_accu
racy: 0.9804
Epoch 12/20
60000/60000 [=====] - 10s 171us/step - loss: 0.0747 - accuracy: 0.9762 - val_loss: 0.0583 - val_accu
racy: 0.9826
Epoch 13/20
60000/60000 [=====] - 11s 182us/step - loss: 0.0719 - accuracy: 0.9772 - val_loss: 0.0593 - val_accu
racy: 0.9814
Epoch 14/20
60000/60000 [=====] - 10s 168us/step - loss: 0.0678 - accuracy: 0.9784 - val_loss: 0.0574 - val_accu
racy: 0.9832
Epoch 15/20
60000/60000 [=====] - 10s 170us/step - loss: 0.0633 - accuracy: 0.9797 - val_loss: 0.0601 - val_accu
racy: 0.9826
Epoch 16/20
60000/60000 [=====] - 11s 181us/step - loss: 0.0601 - accuracy: 0.9798 - val_loss: 0.0558 - val_accu
racy: 0.9828
Epoch 17/20
60000/60000 [=====] - 10s 173us/step - loss: 0.0598 - accuracy: 0.9808 - val_loss: 0.0569 - val_accu
racy: 0.9839
Epoch 18/20
60000/60000 [=====] - 10s 167us/step - loss: 0.0581 - accuracy: 0.9811 - val_loss: 0.0587 - val_accu
racy: 0.9824
Epoch 19/20
60000/60000 [=====] - 10s 174us/step - loss: 0.0550 - accuracy: 0.9818 - val_loss: 0.0560 - val_accu
racy: 0.9847
Epoch 20/20
60000/60000 [=====] - 11s 184us/step - loss: 0.0533 - accuracy: 0.9829 - val_loss: 0.0559 - val_accu
racy: 0.9840
```

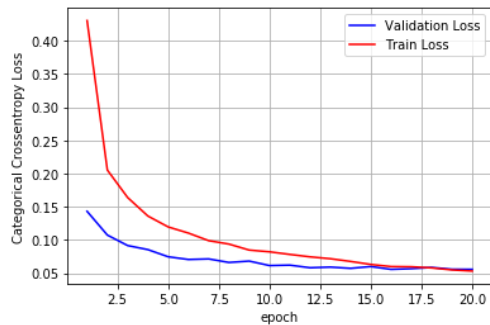
```
In [29]: score = model_drop.evaluate(X_test, y_test, verbose=0)
score5=score[0]
score6=score[1]
train_acc3=history13.history['accuracy']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax13 = plt.subplots(1,1)
ax13.set_xlabel('epoch') ; ax13.set_ylabel('Categorical Crossentropy Loss')

vy13 = history13.history['val_loss']
ty13 = history13.history['loss']
plt_dynamic(x, vy13, ty13, ax13)
```

Test score: 0.055900052050140224

Test accuracy: 0.984000027179718



2) 3-Hidden layer architecture (784-352-164-124 architecture)

2.1 MLP + ReLU + ADAM

```
In [27]: model_relu = Sequential()
model_relu.add(Dense(352, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_relu.add(Dense(164, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )

model_relu.add(Dense(124, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(output_dim, activation='softmax'))

print(model_relu.summary())

model_relu.compile(optimizer='adam',
                  loss='categorical_crossentropy',
                  metrics=['accuracy'])

history21 = model_relu.fit(X_train, y_train,
                          batch_size=batch_size,
                          epochs=nb_epoch, verbose=1,
                          validation_data=(X_test, y_test))
```

Model: "sequential_4"

Layer (type)	Output Shape	Param #
dense_10 (Dense)	(None, 352)	276320
dense_11 (Dense)	(None, 164)	57892
dense_12 (Dense)	(None, 124)	20460
dense_13 (Dense)	(None, 10)	1250
Total params: 355,922		
Trainable params: 355,922		
Non-trainable params: 0		

None

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

60000/60000 [=====] - 8s 127us/step - loss: 0.2361 - accuracy: 0.9291 - val_loss: 0.1118 - val_accuracy: 0.9678

Epoch 2/20

60000/60000 [=====] - 7s 123us/step - loss: 0.0900 - accuracy: 0.9721 - val_loss: 0.0906 - val_accuracy: 0.9712 0.0900 - accuracy

Epoch 3/20

60000/60000 [=====] - 8s 127us/step - loss: 0.0573 - accuracy: 0.9821 - val_loss: 0.0881 - val_accuracy: 0.9738

Epoch 4/20

60000/60000 [=====] - 7s 115us/step - loss: 0.0428 - accuracy: 0.9858 - val_loss: 0.0817 - val_accuracy: 0.9767

Epoch 5/20

60000/60000 [=====] - 7s 110us/step - loss: 0.0315 - accuracy: 0.9896 - val_loss: 0.0765 - val_accuracy: 0.9784

Epoch 6/20

60000/60000 [=====] - 7s 117us/step - loss: 0.0270 - accuracy: 0.9912 - val_loss: 0.0878 - val_accuracy: 0.9756

Epoch 7/20

60000/60000 [=====] - 7s 121us/step - loss: 0.0191 - accuracy: 0.9941 - val_loss: 0.0982 - val_accuracy: 0.9737

Epoch 8/20

60000/60000 [=====] - 7s 112us/step - loss: 0.0228 - accuracy: 0.9923 - val_loss: 0.0801 - val_accuracy: 0.9788

Epoch 9/20

60000/60000 [=====] - 7s 115us/step - loss: 0.0197 - accuracy: 0.9934 - val_loss: 0.0806 - val_accuracy: 0.9800

Epoch 10/20

60000/60000 [=====] - 7s 109us/step - loss: 0.0161 - accuracy: 0.9948 - val_loss: 0.0968 - val_accuracy: 0.9780

Epoch 11/20

60000/60000 [=====] - 7s 111us/step - loss: 0.0119 - accuracy: 0.9964 - val_loss: 0.1039 - val_accuracy: 0.9771

Epoch 12/20

60000/60000 [=====] - 7s 110us/step - loss: 0.0149 - accuracy: 0.9951 - val_loss: 0.0917 - val_accuracy: 0.9785

Epoch 13/20

60000/60000 [=====] - 7s 112us/step - loss: 0.0133 - accuracy: 0.9954 - val_loss: 0.0919 - val_accuracy: 0.9798

Epoch 14/20

60000/60000 [=====] - 7s 110us/step - loss: 0.0114 - accuracy: 0.9964 - val_loss: 0.0995 - val_accuracy: 0.9789

Epoch 15/20

60000/60000 [=====] - 7s 111us/step - loss: 0.0104 - accuracy: 0.9966 - val_loss: 0.0963 - val_accuracy: 0.9805

Epoch 16/20

60000/60000 [=====] - 7s 116us/step - loss: 0.0119 - accuracy: 0.9961 - val_loss: 0.0847 - val_accuracy: 0.9828

Epoch 17/20

60000/60000 [=====] - 7s 116us/step - loss: 0.0088 - accuracy: 0.9974 - val_loss: 0.0848 - val_accuracy: 0.9827

Epoch 18/20

60000/60000 [=====] - 7s 113us/step - loss: 0.0113 - accuracy: 0.9964 - val_loss: 0.1197 - val_accuracy: 0.9771

Epoch 19/20

60000/60000 [=====] - 7s 112us/step - loss: 0.0106 - accuracy: 0.9964 - val_loss: 0.0983 - val_accuracy: 0.9804

Epoch 20/20

60000/60000 [=====] - 7s 113us/step - loss: 0.0098 - accuracy: 0.9969 - val_loss: 0.1023 - val_accuracy: 0.9790

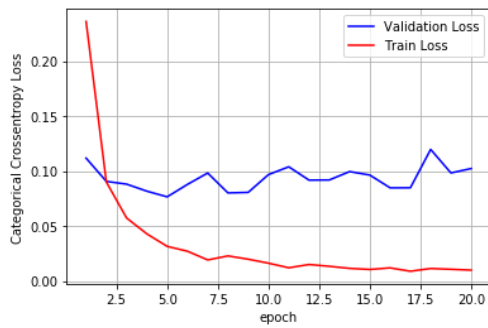
```
In [30]: score = model_relu.evaluate(X_test, y_test, verbose=0)
score7=score[0]
score8=score[1]
train_acc4=history21.history['accuracy']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax21 = plt.subplots(1,1)
ax21.set_xlabel('epoch') ; ax21.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy21 = history21.history['val_loss']
ty21 = history21.history['loss']
plt_dynamic(x, vy21, ty21, ax21)
```

Test score: 0.10229123631868356
Test accuracy: 0.9789999723434448



2.2 MLP + Batch-Norm on hidden Layers + AdamOptimizer

```
In [31]: from keras.layers.normalization import BatchNormalization

model_batch = Sequential()

model_relu.add(Dense(352, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_batch.add(BatchNormalization())
model_relu.add(Dense(164, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_relu.add(Dense(124, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(output_dim, activation='softmax'))
```

```
In [33]: model_batch.compile(optimizer='adam', loss='categorical_crossentropy',
                             metrics=['accuracy'])

history22 = model_batch.fit(X_train, y_train,
                             batch_size=batch_size,
                             epochs=nb_epoch, verbose=1,
                             validation_data=(X_test, y_test))
```

Train on 60000 samples, validate on 10000 samples

Epoch	Train Samples	Validation Samples	Time	Loss	Accuracy	Val Loss	Val Accuracy
Epoch 1/20	60000/60000		6s 105us/step	0.4919	0.8541	0.3529	
Epoch 2/20	60000/60000		5s 82us/step	0.3065	0.9117	5.3663	
Epoch 3/20	60000/60000		5s 83us/step	0.2863	0.9199	2.4895	
Epoch 4/20	60000/60000		5s 82us/step	0.2774	0.9216	0.8210	
Epoch 5/20	60000/60000		5s 91us/step	0.2714	0.9236	1.0564	
Epoch 6/20	60000/60000		5s 86us/step	0.2681	0.9251	0.5624	
Epoch 7/20	60000/60000		5s 91us/step	0.2653	0.9253	4.5390	
Epoch 8/20	60000/60000		5s 78us/step	0.2616	0.9261	1.0641	
Epoch 9/20	60000/60000		5s 79us/step	0.2605	0.9280	4.9788	
Epoch 10/20	60000/60000		5s 80us/step	0.2614	0.9271	0.9586	
Epoch 11/20	60000/60000		5s 79us/step	0.2590	0.9283	2.0622	
Epoch 12/20	60000/60000		5s 77us/step	0.2562	0.9283	1.0619	
Epoch 13/20	60000/60000		5s 81us/step	0.2553	0.9280	1.3594	
Epoch 14/20	60000/60000		5s 79us/step	0.2550	0.9290	2.7772	
Epoch 15/20	60000/60000		5s 78us/step	0.2540	0.9284	7.5427	
Epoch 16/20	60000/60000		5s 83us/step	0.2543	0.9291	1.4970	
Epoch 17/20	60000/60000		5s 82us/step	0.2530	0.9295	0.5769	
Epoch 18/20	60000/60000		5s 84us/step	0.2530	0.9279	3.2516	
Epoch 19/20	60000/60000		5s 78us/step	0.2512	0.9297	0.7772	
Epoch 20/20	60000/60000		5s 81us/step	0.2512	0.9300	0.6023	

```
In [34]: model_batch.summary()
```

Model: "sequential_5"

Layer (type)	Output Shape	Param #
=====		
batch_normalization_5 (Batch Normalization)	(None, 784)	3136

batch_normalization_6 (Batch Normalization)	(None, 784)	3136

batch_normalization_7 (Batch Normalization)	(None, 784)	3136

dense_17 (Dense)	(None, 10)	7850
=====		
Total params: 17,258		
Trainable params: 12,554		
Non-trainable params: 4,704		

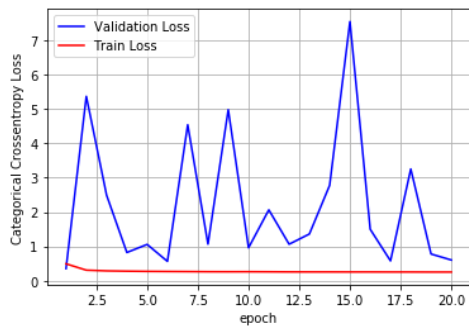

```
In [35]: score = model_batch.evaluate(X_test, y_test, verbose=0)
score9=score[0]
score10=score[1]
train_acc5=history22.history['accuracy']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax22 = plt.subplots(1,1)
ax22.set_xlabel('epoch') ; ax22.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy22 = history22.history['val_loss']
ty22 = history22.history['loss']
plt_dynamic(x, vy22, ty22, ax22)
```

Test score: 0.6022788046717644
Test accuracy: 0.8690000176429749



2.3 MLP + Dropout + AdamOptimizer

```
In [36]: from keras.layers import Dropout

model_drop = Sequential()
model_drop.add(Dense(352, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))

model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))
model_drop.add(Dense(164, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_drop.add(Dense(124, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_drop.add(Dense(output_dim, activation='softmax'))
```

```
In [37]: model_drop.compile(optimizer='adam',
                        loss='categorical_crossentropy',
                        metrics=['accuracy'])

history23 = model_drop.fit(X_train, y_train,
                        batch_size=batch_size,
                        epochs=nb_epoch, verbose=1,
                        validation_data=(X_test, y_test))
```

Train on 60000 samples, validate on 10000 samples

```
Epoch 1/20
60000/60000 [=====] - 8s 135us/step - loss: 1.3556 - accuracy: 0.5682 - val_loss: 0.4809 - val_accuracy:
0.8837
Epoch 2/20
60000/60000 [=====] - 8s 134us/step - loss: 0.8602 - accuracy: 0.7193 - val_loss: 0.4449 - val_accuracy:
0.8909
Epoch 3/20
60000/60000 [=====] - 8s 129us/step - loss: 0.8353 - accuracy: 0.7277 - val_loss: 0.4308 - val_accuracy:
0.8929
Epoch 4/20
60000/60000 [=====] - 8s 135us/step - loss: 0.8351 - accuracy: 0.7289 - val_loss: 0.4315 - val_accuracy:
0.8932
Epoch 5/20
60000/60000 [=====] - 9s 154us/step - loss: 0.8302 - accuracy: 0.7289 - val_loss: 0.4225 - val_accuracy:
0.8955
Epoch 6/20
60000/60000 [=====] - 8s 127us/step - loss: 0.8301 - accuracy: 0.7298 - val_loss: 0.4249 - val_accuracy:
0.8951
Epoch 7/20
60000/60000 [=====] - 8s 129us/step - loss: 0.8223 - accuracy: 0.7341 - val_loss: 0.4165 - val_accuracy:
0.8958
Epoch 8/20
60000/60000 [=====] - 8s 134us/step - loss: 0.8211 - accuracy: 0.7341 - val_loss: 0.4143 - val_accuracy:
0.8962
Epoch 9/20
60000/60000 [=====] - 7s 125us/step - loss: 0.8194 - accuracy: 0.7341 - val_loss: 0.4128 - val_accuracy:
0.8959
Epoch 10/20
60000/60000 [=====] - 8s 126us/step - loss: 0.8258 - accuracy: 0.7321 - val_loss: 0.4142 - val_accuracy:
0.8978
Epoch 11/20
60000/60000 [=====] - 8s 132us/step - loss: 0.8229 - accuracy: 0.7322 - val_loss: 0.4165 - val_accuracy:
0.8936
Epoch 12/20
60000/60000 [=====] - 8s 126us/step - loss: 0.8138 - accuracy: 0.7372 - val_loss: 0.4138 - val_accuracy:
0.8978
Epoch 13/20
60000/60000 [=====] - 8s 138us/step - loss: 0.8128 - accuracy: 0.7363 - val_loss: 0.4108 - val_accuracy:
0.8987
Epoch 14/20
60000/60000 [=====] - 8s 132us/step - loss: 0.8123 - accuracy: 0.7379 - val_loss: 0.4039 - val_accuracy:
0.8985
Epoch 15/20
60000/60000 [=====] - 8s 133us/step - loss: 0.8164 - accuracy: 0.7355 - val_loss: 0.4061 - val_accuracy:
0.8978
Epoch 16/20
60000/60000 [=====] - 8s 133us/step - loss: 0.8174 - accuracy: 0.7362 - val_loss: 0.4056 - val_accuracy:
0.8993
Epoch 17/20
60000/60000 [=====] - 9s 143us/step - loss: 0.8069 - accuracy: 0.7393 - val_loss: 0.4048 - val_accuracy:
0.8953
Epoch 18/20
60000/60000 [=====] - 8s 135us/step - loss: 0.8126 - accuracy: 0.7373 - val_loss: 0.4030 - val_accuracy:
0.8994
Epoch 19/20
60000/60000 [=====] - 8s 132us/step - loss: 0.8087 - accuracy: 0.7380 - val_loss: 0.4040 - val_accuracy:
0.8994
Epoch 20/20
60000/60000 [=====] - 8s 133us/step - loss: 0.7998 - accuracy: 0.7410 - val_loss: 0.4019 - val_accuracy:
0.8968
```

```
In [38]: model_drop.summary()
```

Model: "sequential_6"

Layer (type)	Output Shape	Param #
batch_normalization_8 (Batch Normalization)	(None, 784)	3136
dropout_3 (Dropout)	(None, 784)	0
batch_normalization_9 (Batch Normalization)	(None, 784)	3136
dropout_4 (Dropout)	(None, 784)	0
batch_normalization_10 (Batch Normalization)	(None, 784)	3136
dropout_5 (Dropout)	(None, 784)	0
dense_21 (Dense)	(None, 10)	7850
Total params: 17,258		
Trainable params: 12,554		
Non-trainable params: 4,704		

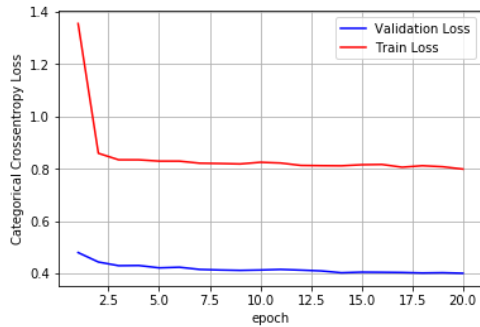
```
In [40]: score = model_drop.evaluate(X_test, y_test, verbose=0)
score11=score[0]
score12=score[1]
train_acc6=history23.history['accuracy']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig, ax23 = plt.subplots(1,1)
ax23.set_xlabel('epoch') ; ax23.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy23 = history23.history['val_loss']
ty23 = history23.history['loss']
plt_dynamic(x, vy23, ty23, ax23)
```

Test score: 0.40186178770065306
 Test accuracy: 0.8967999815940857



3) 5-Hidden layer architecture (784-216-170-136-80-38-10 architecture)

3.1 MLP + ReLU + ADAM

```
In [41]: model_relu = Sequential()
model_relu.add(Dense(216, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_relu.add(Dense(170, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )

model_relu.add(Dense(136, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(80, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )

model_relu.add(Dense(38, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_relu.add(Dense(output_dim, activation='softmax'))

print(model_relu.summary())

model_relu.compile(optimizer='adam',
                  loss='categorical_crossentropy',
                  metrics=['accuracy'])

history31 = model_relu.fit(X_train, y_train,
                          batch_size=batch_size,
                          epochs=nb_epoch, verbose=1,
                          validation_data=(X_test, y_test))
```

Model: "sequential_7"

Layer (type)	Output Shape	Param #
dense_22 (Dense)	(None, 216)	169560
dense_23 (Dense)	(None, 170)	36890
dense_24 (Dense)	(None, 136)	23256
dense_25 (Dense)	(None, 80)	10960
dense_26 (Dense)	(None, 38)	3078
dense_27 (Dense)	(None, 10)	390

=====
 Total params: 244,134
 Trainable params: 244,134
 Non-trainable params: 0

None

Train on 60000 samples, validate on 10000 samples

Epoch 1/20

60000/60000 [=====] - 6s 102us/step - loss: 0.2802 - accuracy: 0.9160 - val_loss: 0.1414 - val_accuracy: 0.9583

Epoch 2/20

60000/60000 [=====] - 5s 80us/step - loss: 0.1050 - accuracy: 0.9682 - val_loss: 0.1064 - val_accuracy: 0.9660

Epoch 3/20

60000/60000 [=====] - 5s 81us/step - loss: 0.0720 - accuracy: 0.9775 - val_loss: 0.0974 - val_accuracy: 0.9697

Epoch 4/20

60000/60000 [=====] - 5s 82us/step - loss: 0.0515 - accuracy: 0.9841 - val_loss: 0.0929 - val_accuracy: 0.9727

Epoch 5/20

60000/60000 [=====] - 5s 78us/step - loss: 0.0429 - accuracy: 0.9864 - val_loss: 0.0790 - val_accuracy: 0.9780

Epoch 6/20

60000/60000 [=====] - 6s 99us/step - loss: 0.0330 - accuracy: 0.9892 - val_loss: 0.0935 - val_accuracy: 0.9762

Epoch 7/20

60000/60000 [=====] - 5s 82us/step - loss: 0.0305 - accuracy: 0.9901 - val_loss: 0.0921 - val_accuracy: 0.9766

Epoch 8/20

60000/60000 [=====] - 5s 79us/step - loss: 0.0263 - accuracy: 0.9918 - val_loss: 0.0936 - val_accuracy: 0.9735

Epoch 9/20

60000/60000 [=====] - 5s 80us/step - loss: 0.0224 - accuracy: 0.9927 - val_loss: 0.0869 - val_accuracy: 0.9791

Epoch 10/20

60000/60000 [=====] - 5s 82us/step - loss: 0.0216 - accuracy: 0.9930 - val_loss: 0.0926 - val_accuracy: 0.9774

Epoch 11/20

60000/60000 [=====] - 5s 77us/step - loss: 0.0194 - accuracy: 0.9937 - val_loss: 0.0950 - val_accuracy: 0.9802

Epoch 12/20

60000/60000 [=====] - 5s 81us/step - loss: 0.0196 - accuracy: 0.9937 - val_loss: 0.0909 - val_accuracy: 0.9767

Epoch 13/20

60000/60000 [=====] - 5s 82us/step - loss: 0.0151 - accuracy: 0.9951 - val_loss: 0.0911 - val_accuracy: 0.9792

Epoch 14/20

60000/60000 [=====] - 5s 83us/step - loss: 0.0160 - accuracy: 0.9948 - val_loss: 0.0981 - val_accuracy: 0.9775

Epoch 15/20

60000/60000 [=====] - 5s 80us/step - loss: 0.0170 - accuracy: 0.9950 - val_loss: 0.0963 - val_accuracy: 0.9794

Epoch 16/20

60000/60000 [=====] - 5s 82us/step - loss: 0.0129 - accuracy: 0.9959 - val_loss: 0.0993 - val_accuracy: 0.9774

Epoch 17/20

60000/60000 [=====] - 5s 77us/step - loss: 0.0149 - accuracy: 0.9954 - val_loss: 0.0963 - val_accuracy: 0.9809

Epoch 18/20

60000/60000 [=====] - 4s 75us/step - loss: 0.0121 - accuracy: 0.9963 - val_loss: 0.1088 - val_accuracy: 0.9779

Epoch 19/20

60000/60000 [=====] - 5s 82us/step - loss: 0.0126 - accuracy: 0.9958 - val_loss: 0.0910 - val_accuracy: 0.9817

Epoch 20/20

60000/60000 [=====] - 5s 80us/step - loss: 0.0088 - accuracy: 0.9972 - val_loss: 0.1104 - val_accuracy: 0.9751

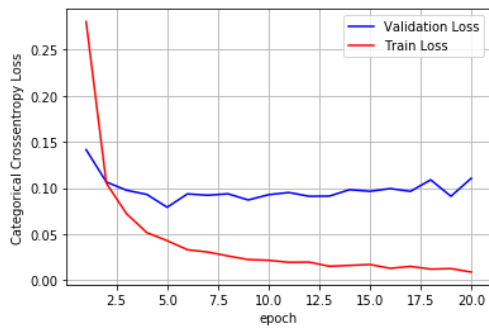
```
In [43]: score = model_relu.evaluate(X_test, y_test, verbose=0)
score13=score[0]
score14=score[1]
train_acc7=history31.history['accuracy']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax31 = plt.subplots(1,1)
ax31.set_xlabel('epoch') ; ax31.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy31 = history31.history['val_loss']
ty31 = history31.history['loss']
plt_dynamic(x, vy31, ty31, ax31)
```

Test score: 0.11039771217970347
 Test accuracy: 0.9750999808311462



3.2 MLP + Batch-Norm on hidden Layers + AdamOptimizer

```
In [44]: from keras.layers.normalization import BatchNormalization

model_batch = Sequential()

model_relu.add(Dense(216, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_batch.add(BatchNormalization())
model_relu.add(Dense(170, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_relu.add(Dense(136, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())
model_relu.add(Dense(80, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_relu.add(Dense(38, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_batch.add(BatchNormalization())

model_batch.add(Dense(output_dim, activation='softmax'))
```

```
In [45]: model_batch.compile(optimizer='adam', loss='categorical_crossentropy',
                             metrics=['accuracy'])

history32 = model_batch.fit(X_train, y_train,
                             batch_size=batch_size,
                             epochs=nb_epoch, verbose=1,
                             validation_data=(X_test, y_test))
```

Train on 60000 samples, validate on 10000 samples

```
Epoch 1/20
60000/60000 [=====] - 11s 180us/step - loss: 0.4971 - accuracy: 0.8524 - val_loss: 14.5068 - val_accurac
y: 0.0992
Epoch 2/20
60000/60000 [=====] - 8s 130us/step - loss: 0.3063 - accuracy: 0.9121 - val_loss: 14.6796 - val_accurac
y: 0.0892
Epoch 3/20
60000/60000 [=====] - 8s 130us/step - loss: 0.2869 - accuracy: 0.9177 - val_loss: 14.4918 - val_accurac
y: 0.1009
Epoch 4/20
60000/60000 [=====] - 8s 128us/step - loss: 0.2753 - accuracy: 0.9219 - val_loss: 14.2871 - val_accurac
y: 0.1136
Epoch 5/20
60000/60000 [=====] - 8s 127us/step - loss: 0.2730 - accuracy: 0.9228 - val_loss: 14.4644 - val_accurac
y: 0.1026
Epoch 6/20
60000/60000 [=====] - 8s 139us/step - loss: 0.2678 - accuracy: 0.9259 - val_loss: 14.2887 - val_accurac
y: 0.1135
Epoch 7/20
60000/60000 [=====] - 8s 133us/step - loss: 0.2663 - accuracy: 0.9249 - val_loss: 14.2855 - val_accurac
y: 0.1137
Epoch 8/20
60000/60000 [=====] - 8s 138us/step - loss: 0.2630 - accuracy: 0.9259 - val_loss: 14.5450 - val_accurac
y: 0.0976
Epoch 9/20
60000/60000 [=====] - 8s 130us/step - loss: 0.2615 - accuracy: 0.9266 - val_loss: 14.5563 - val_accurac
y: 0.0969
Epoch 10/20
60000/60000 [=====] - 9s 143us/step - loss: 0.2605 - accuracy: 0.9276 - val_loss: 14.4762 - val_accurac
y: 0.1018
Epoch 11/20
60000/60000 [=====] - 8s 136us/step - loss: 0.2596 - accuracy: 0.9272 - val_loss: 14.5353 - val_accurac
y: 0.0982
Epoch 12/20
60000/60000 [=====] - 8s 128us/step - loss: 0.2581 - accuracy: 0.9282 - val_loss: 14.4534 - val_accurac
y: 0.1032
Epoch 13/20
60000/60000 [=====] - 8s 129us/step - loss: 0.2559 - accuracy: 0.9286 - val_loss: 14.2855 - val_accurac
y: 0.1137
Epoch 14/20
60000/60000 [=====] - 8s 141us/step - loss: 0.2561 - accuracy: 0.9289 - val_loss: 14.4563 - val_accurac
y: 0.1031
Epoch 15/20
60000/60000 [=====] - 8s 129us/step - loss: 0.2551 - accuracy: 0.9288 - val_loss: 14.5240 - val_accurac
y: 0.0989
Epoch 16/20
60000/60000 [=====] - 8s 133us/step - loss: 0.2529 - accuracy: 0.9294 - val_loss: 14.5482 - val_accurac
y: 0.0974
Epoch 17/20
60000/60000 [=====] - 8s 133us/step - loss: 0.2524 - accuracy: 0.9299 - val_loss: 14.6884 - val_accurac
y: 0.0887
Epoch 18/20
60000/60000 [=====] - 9s 145us/step - loss: 0.2533 - accuracy: 0.9290 - val_loss: 14.5546 - val_accurac
y: 0.0970
Epoch 19/20
60000/60000 [=====] - 7s 117us/step - loss: 0.2512 - accuracy: 0.9293 - val_loss: 14.4747 - val_accurac
y: 0.1019
Epoch 20/20
60000/60000 [=====] - 8s 134us/step - loss: 0.2515 - accuracy: 0.9297 - val_loss: 13.2606 - val_accurac
y: 0.1016
```

```
In [46]: model_batch.summary()
```

Model: "sequential_8"

Layer (type)	Output Shape	Param #
=====		
batch_normalization_11 (Batch Normalization)	(None, 784)	3136
batch_normalization_12 (Batch Normalization)	(None, 784)	3136
batch_normalization_13 (Batch Normalization)	(None, 784)	3136
batch_normalization_14 (Batch Normalization)	(None, 784)	3136
batch_normalization_15 (Batch Normalization)	(None, 784)	3136
dense_33 (Dense)	(None, 10)	7850
=====		
Total params: 23,530		
Trainable params: 15,690		
Non-trainable params: 7,840		

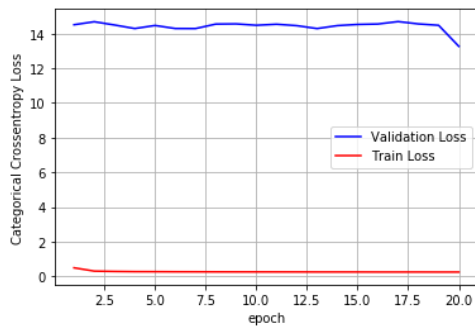
```
In [47]: score = model_batch.evaluate(X_test, y_test, verbose=0)
score15=score[0]
score16=score[1]
train_acc8=history32.history['accuracy']
print('Test score:', score[0])
print('Test accuracy:', score[1])

fig,ax32 = plt.subplots(1,1)
ax32.set_xlabel('epoch') ; ax32.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy32 = history32.history['val_loss']
ty32 = history32.history['loss']
plt_dynamic(x, vy32, ty32, ax32)
```

Test score: 13.260629919433594
 Test accuracy: 0.1015999877214432



3.3 MLP + Dropout + AdamOptimizer

```
In [48]: from keras.layers import Dropout

model_drop = Sequential()
model_relu.add(Dense(216, activation='relu', input_shape=(input_dim,),
                    kernel_initializer=he_normal(seed=None)))
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))
model_relu.add(Dense(170, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_relu.add(Dense(136, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))
model_relu.add(Dense(80, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_relu.add(Dense(38, activation='relu',
                    kernel_initializer=he_normal(seed=None)) )
model_drop.add(BatchNormalization())
model_drop.add(Dropout(0.5))

model_drop.add(Dense(output_dim, activation='softmax'))
```



```
In [49]: model_drop.compile(optimizer='adam',
                        loss='categorical_crossentropy',
                        metrics=['accuracy'])

history33 = model_drop.fit(X_train, y_train,
                        batch_size=batch_size,
                        epochs=nb_epoch, verbose=1,
                        validation_data=(X_test, y_test))
```

Train on 60000 samples, validate on 10000 samples

```
Epoch 1/20
60000/60000 [=====] - 15s 249us/step - loss: 2.1432 - accuracy: 0.3119 - val_loss: 1.0702 - val_accurac
y: 0.8327
Epoch 2/20
60000/60000 [=====] - 13s 220us/step - loss: 1.6152 - accuracy: 0.4400 - val_loss: 0.9769 - val_accurac
y: 0.8457
Epoch 3/20
60000/60000 [=====] - 13s 220us/step - loss: 1.5940 - accuracy: 0.4475 - val_loss: 0.9606 - val_accurac
y: 0.8457
Epoch 4/20
60000/60000 [=====] - 14s 231us/step - loss: 1.5865 - accuracy: 0.4529 - val_loss: 0.9527 - val_accurac
y: 0.8472
Epoch 5/20
60000/60000 [=====] - 12s 207us/step - loss: 1.5784 - accuracy: 0.4535 - val_loss: 0.9404 - val_accurac
y: 0.8495
Epoch 6/20
60000/60000 [=====] - 13s 221us/step - loss: 1.5918 - accuracy: 0.4486 - val_loss: 0.9414 - val_accurac
y: 0.8485
Epoch 7/20
60000/60000 [=====] - 13s 217us/step - loss: 1.5800 - accuracy: 0.4522 - val_loss: 0.9423 - val_accurac
y: 0.8506
Epoch 8/20
60000/60000 [=====] - 13s 210us/step - loss: 1.5859 - accuracy: 0.4500 - val_loss: 0.9480 - val_accurac
y: 0.8464
Epoch 9/20
60000/60000 [=====] - 13s 221us/step - loss: 1.5832 - accuracy: 0.4530 - val_loss: 0.9457 - val_accurac
y: 0.8468
Epoch 10/20
60000/60000 [=====] - 13s 214us/step - loss: 1.5747 - accuracy: 0.4529 - val_loss: 0.9386 - val_accurac
y: 0.8490
Epoch 11/20
60000/60000 [=====] - 13s 210us/step - loss: 1.5605 - accuracy: 0.4601 - val_loss: 0.9280 - val_accurac
y: 0.8552
Epoch 12/20
60000/60000 [=====] - 13s 213us/step - loss: 1.5729 - accuracy: 0.4553 - val_loss: 0.9393 - val_accurac
y: 0.8517
Epoch 13/20
60000/60000 [=====] - 13s 212us/step - loss: 1.5725 - accuracy: 0.4517 - val_loss: 0.9361 - val_accurac
y: 0.8456
Epoch 14/20
60000/60000 [=====] - 14s 226us/step - loss: 1.5665 - accuracy: 0.4575 - val_loss: 0.9368 - val_accurac
y: 0.8520
Epoch 15/20
60000/60000 [=====] - 13s 215us/step - loss: 1.5652 - accuracy: 0.4577 - val_loss: 0.9357 - val_accurac
y: 0.8520
Epoch 16/20
60000/60000 [=====] - 13s 216us/step - loss: 1.5646 - accuracy: 0.4608 - val_loss: 0.9374 - val_accurac
y: 0.8516
Epoch 17/20
60000/60000 [=====] - 13s 221us/step - loss: 1.5632 - accuracy: 0.4571 - val_loss: 0.9393 - val_accurac
y: 0.8546
Epoch 18/20
60000/60000 [=====] - 14s 230us/step - loss: 1.5609 - accuracy: 0.4591 - val_loss: 0.9339 - val_accurac
y: 0.8494
Epoch 19/20
60000/60000 [=====] - 14s 226us/step - loss: 1.5607 - accuracy: 0.4599 - val_loss: 0.9320 - val_accurac
y: 0.8470
Epoch 20/20
60000/60000 [=====] - 13s 219us/step - loss: 1.5509 - accuracy: 0.4657 - val_loss: 0.9236 - val_accurac
y: 0.8496
```

In [50]: `model_drop.summary()`

Model: "sequential_9"

Layer (type)	Output Shape	Param #
batch_normalization_16 (Batch Normalization)	(None, 784)	3136
dropout_6 (Dropout)	(None, 784)	0
batch_normalization_17 (Batch Normalization)	(None, 784)	3136
dropout_7 (Dropout)	(None, 784)	0
batch_normalization_18 (Batch Normalization)	(None, 784)	3136
dropout_8 (Dropout)	(None, 784)	0
batch_normalization_19 (Batch Normalization)	(None, 784)	3136
dropout_9 (Dropout)	(None, 784)	0
batch_normalization_20 (Batch Normalization)	(None, 784)	3136
dropout_10 (Dropout)	(None, 784)	0
dense_39 (Dense)	(None, 10)	7850
Total params: 23,536		
Trainable params: 15,696		
Non-trainable params: 7,840		

In [53]: `score = model_drop.evaluate(X_test, y_test, verbose=0)`

```

score17=score[0]
score18=score[1]
train_acc9=history33.history['accuracy']
print('Test score:', score[0])
print('Test accuracy:', score[1])

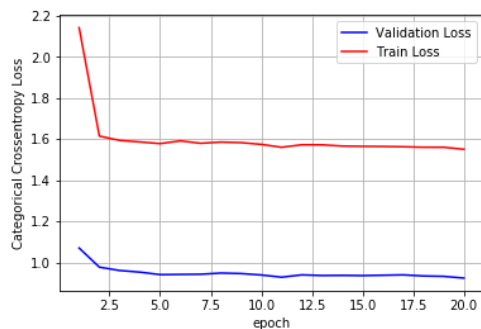
fig,ax33 = plt.subplots(1,1)
ax33.set_xlabel('epoch') ; ax33.set_ylabel('Categorical Crossentropy Loss')

# List of epoch numbers
x = list(range(1,nb_epoch+1))

vy33 = history33.history['val_loss']
ty33 = history33.history['loss']
plt_dynamic(x, vy33, ty33, ax33)

```

Test score: 0.923618637752533
 Test accuracy: 0.8496000170707703



Final Summary:

```
In [62]: from prettytable import PrettyTable

models=[ '2_hidden_layer MLP+ReLU+Adam',
         '2_hidden_layer MLP+ReLU+adam+BN',
         '2_hidden_layer MLP+ReLU+Adam+BN+Drop-out',
         '3_hidden_layer MLP+ReLU+Adam',
         '3_hidden_layer MLP+ReLU+adam+BN',
         '3_hidden_layer MLP+ReLU+Adam+BN+Drop-out',
         '5_hidden_layer MLP+ReLU+Adam',
         '5_hidden_layer MLP+ReLU+adam+BN',
         '5_hidden_layer MLP+ReLU+Adam+BN+Drop-out' ]

training_accuracy=[np.mean(train_acc1),np.mean(train_acc2),np.mean(train_acc3),np.mean(train_acc4),
                   np.mean(train_acc5),np.mean(train_acc6),np.mean(train_acc7),np.mean(train_acc8),
                   np.mean(train_acc9)]

test_score=[score1,score3,score5,score7,score9,score11,score13,score15,
            score17]

test_accuracy=[score2,score4,score6,score8,score10,score12,score14,
               score16,
               score18]
INDEX = [1,2,3,4,5,6,7,8,9]

# Initializing prettytable
Model_Performance = PrettyTable()
# Adding columns
Model_Performance.add_column("INDEX.",INDEX)
Model_Performance.add_column("MODEL_NAME",models)
Model_Performance.add_column("TRAINING ACCURACY",np.around(training_accuracy,2))
Model_Performance.add_column("TESTING ACCURACY",np.around(test_accuracy,2))
Model_Performance.add_column("TEST SCORE",np.around(test_score,2))

# Printing the Model_Performance
print(Model_Performance)
```

INDEX.	MODEL_NAME	TRAINING ACCURACY	TESTING ACCURACY	TEST SCORE
1	2_hidden_layer MLP+ReLU+Adam	0.99	0.98	0.1
2	2_hidden_layer MLP+ReLU+adam+BN	0.99	0.98	0.09
3	2_hidden_layer MLP+ReLU+Adam+BN+Drop-out	0.97	0.98	0.06
4	3_hidden_layer MLP+ReLU+Adam	0.99	0.98	0.1
5	3_hidden_layer MLP+ReLU+adam+BN	0.92	0.87	0.6
6	3_hidden_layer MLP+ReLU+Adam+BN+Drop-out	0.73	0.9	0.4
7	5_hidden_layer MLP+ReLU+Adam	0.99	0.98	0.11
8	5_hidden_layer MLP+ReLU+adam+BN	0.92	0.1	13.26
9	5_hidden_layer MLP+ReLU+Adam+BN+Drop-out	0.45	0.85	0.92