



COMPUTATIONAL HEAT TRANSFER



Department of Chemical Engineering
Indian Institute of Technology, Kanpur

MENTEE

Name : Avinash Bansal
Roll No. : 190205

MENTOR

Name : Shubh Maheshwari
Roll No. : 180744



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- Convection
- Conduction
- Radiation





CONDUCTION



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Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles. Conduction can take place in solids, liquids, or gases.

Rate of heat conduction through a medium depends on the *geometry* of the medium, its *thickness*, and the *material* of the medium, as well as the *temperature difference* across the medium.

Fourier's Law : The rate of heat transfer through a material is proportional to the negative gradient of the temperature and to the area, at right angles to that gradient, through which the heat flows.

$$Q' = kA(dT/dX)$$

Where, k is the thermal conductivity of the material



Conduction of Heat



Conduction of Electricity

Everyday Examples of Conduction :

1. When ironing a piece of clothing, the iron is hot and the heat is transferred to the clothing.
2. Sand can conduct heat. This is why walking on the beach on a hot summer day will warm your feet.



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CONVECTION



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Convection is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of *conduction* and *fluid motion*. The faster the fluid motion, the greater the convection heat transfer.

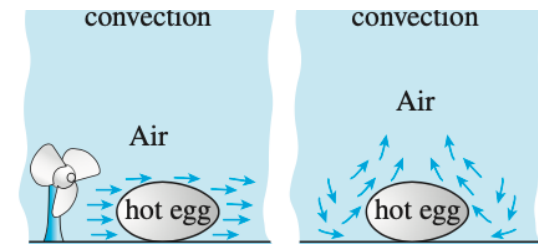
There are two types of Convection:

Forced Convection - When fluid is forced to flow over the surface by external means such as a fan, pump, or the wind.

Natural Convection - When fluid motion is caused by buoyancy forces that are induced by density differences due to the variation of temperature in the fluid.

The rate of convection heat transfer is observed to be proportional to the temperature difference, and is conveniently expressed by **Newton's law of cooling** as :

$$Q' = hA(T_s - T(\infty))$$



Cooling of boiled egg by Forced and Natural Convection





RADIATION



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Radiation is the energy emitted by matter in the form of electromagnetic waves as a result of the changes in the electronic configurations of the atoms or molecules. Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an intervening medium.

- The heat flux emitted by a real surface is less than that of a blackbody at the same temperature is given by

$$E_b = \sigma T_s^4$$

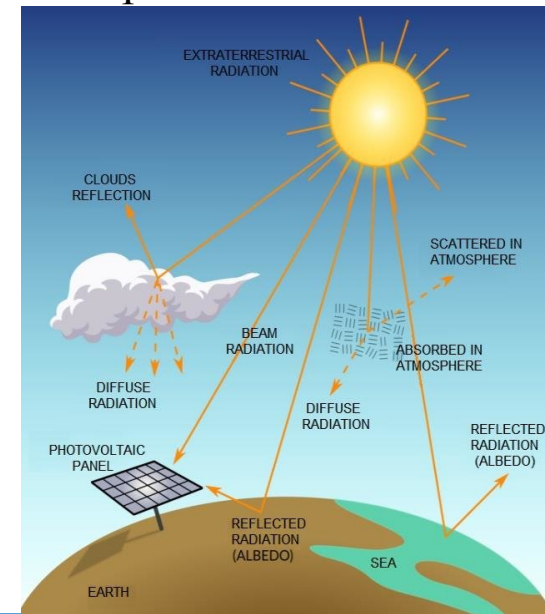
- The rate at which energy is released per unit area is called surface emissive power E.

$$E = \epsilon \sigma T_s^4 \quad 0 \leq \epsilon < 1$$

- $Q' = \epsilon A \sigma (T_s^4 - T_{surr}^4)$

Examples :

1. Ultraviolet light from the sun
2. Electromagnetic radiation from your cell phone.



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HEAT EQUATION



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NON -TRANSIENT HEAT CONDUCTION :

$$(1/\alpha)dT/dx = \partial^2 T/\partial X^2 + \partial^2 T/\partial Y^2 + \partial^2 T/\partial Z^2 + e^\circ/k$$

Where α (thermal diffusivity) = $k/\rho CP$

TRANSIENT HEAT CONDUCTION :

$$\sum \text{all sides } Q^\circ + E^\circ_{\text{gen,element}} = \Delta E_{\text{element}}/\Delta t = \rho v(\text{element}) CP \Delta T/\Delta t$$





FINITE DIFFERENCE METHOD



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For interior nodes (m, n) :

$$(\partial^2 T / \partial x^2)_{m,n} = (T_{m+1,n} - 2T_{m,n} + T_{m-1,n}) / \Delta x^2 \quad (\partial T / \partial x) = (T_{m+1,n} - T_{m,n}) / \Delta x$$

We can generalise heat equation for $\Delta x = \Delta y = L$:

$$T(m-1,n) + T(m+1,n) + T(m,n+1) + T(m,n-1) - 4T(m,n) + e_{\text{gen}} L^2 / k = 0$$



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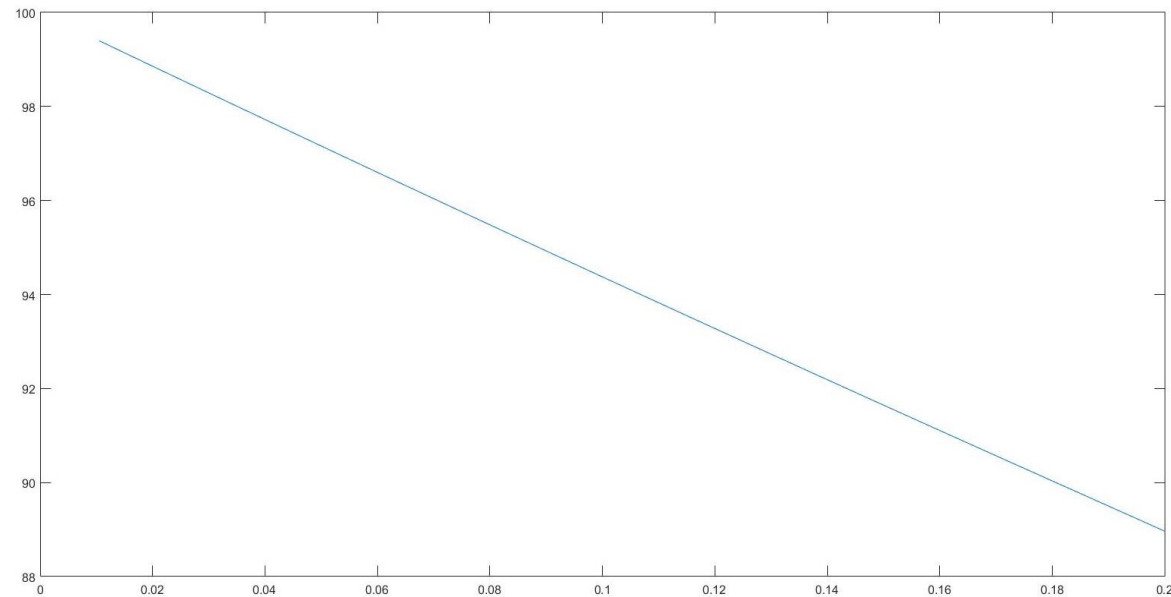
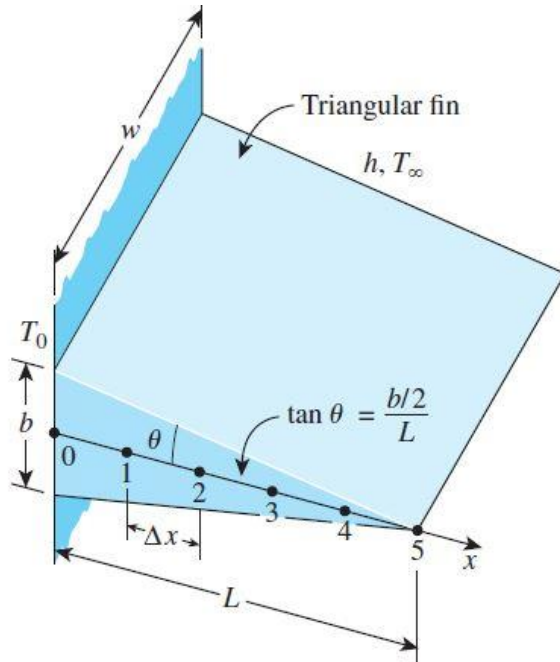
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FIN LOSSING HEAT(NON-TRANSIENT)



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Temp vs Distance (using Finite difference method)

$$\sum_{\text{All sides}} \dot{Q} = 0 \rightarrow kA_{\text{left}} \frac{T_{m-1} - T_m}{\Delta x} + kA_{\text{right}} \frac{T_{m+1} - T_m}{\Delta x} + hA_{\text{conv}}(T_{\infty} - T_m) = 0$$

GOVERNING EQUATION



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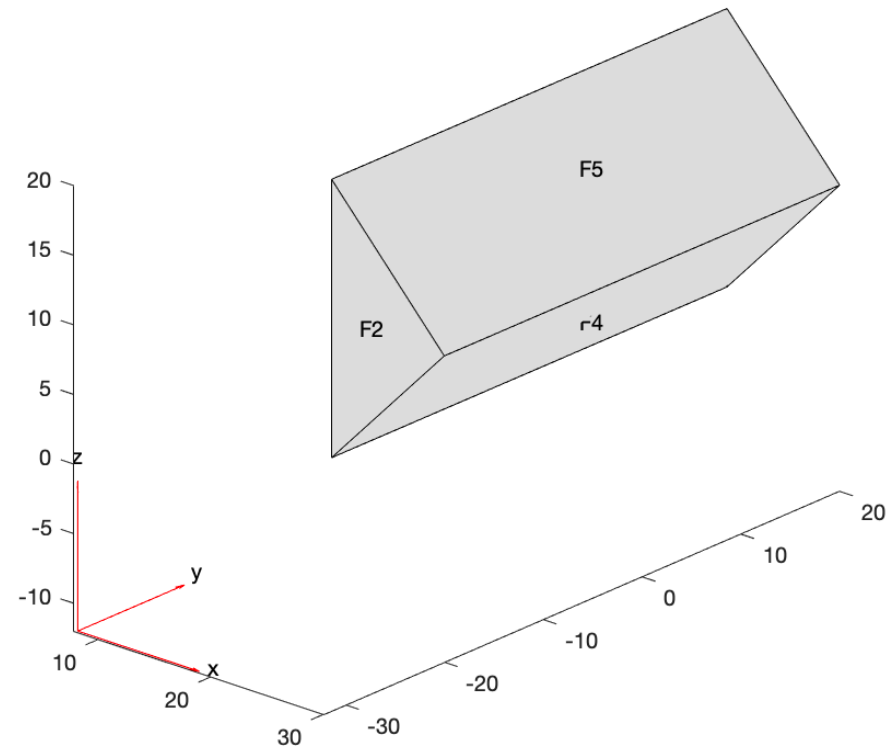
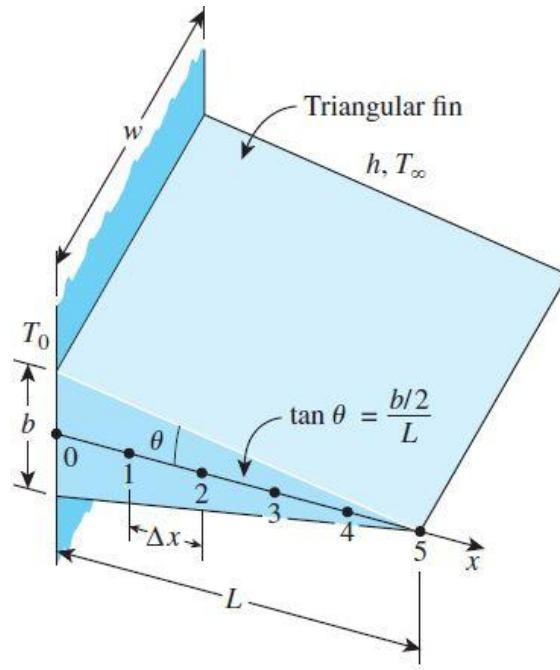
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FIN LOSSING HEAT (PDE TOOLBOX)



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Geometry Model of the fin



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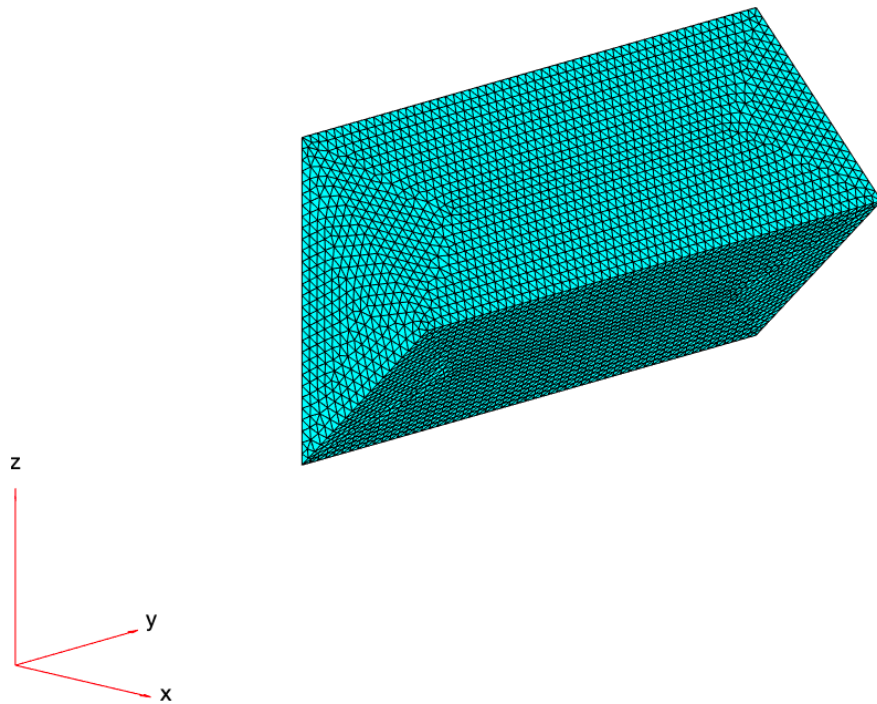


FIN LOSSING HEAT (PDE TOOLBOX)

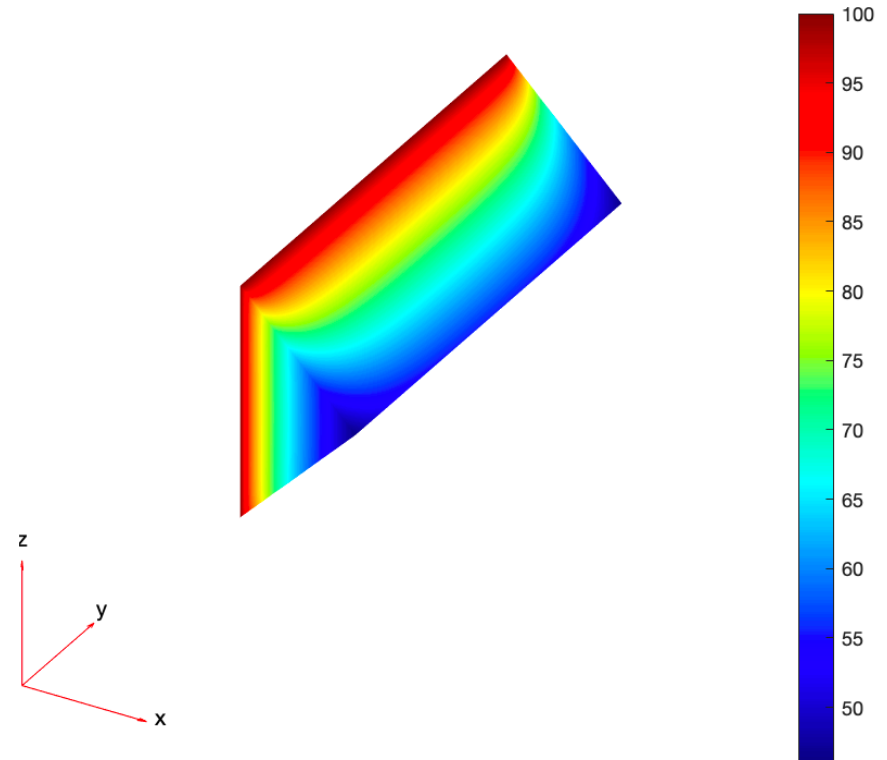


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Temperature profile with respect to distance using colormap model of Partial Differential Equation(PDE) toolbox.



Mesh Model of the fin



Colormap of the fin (using PDE toolbox)

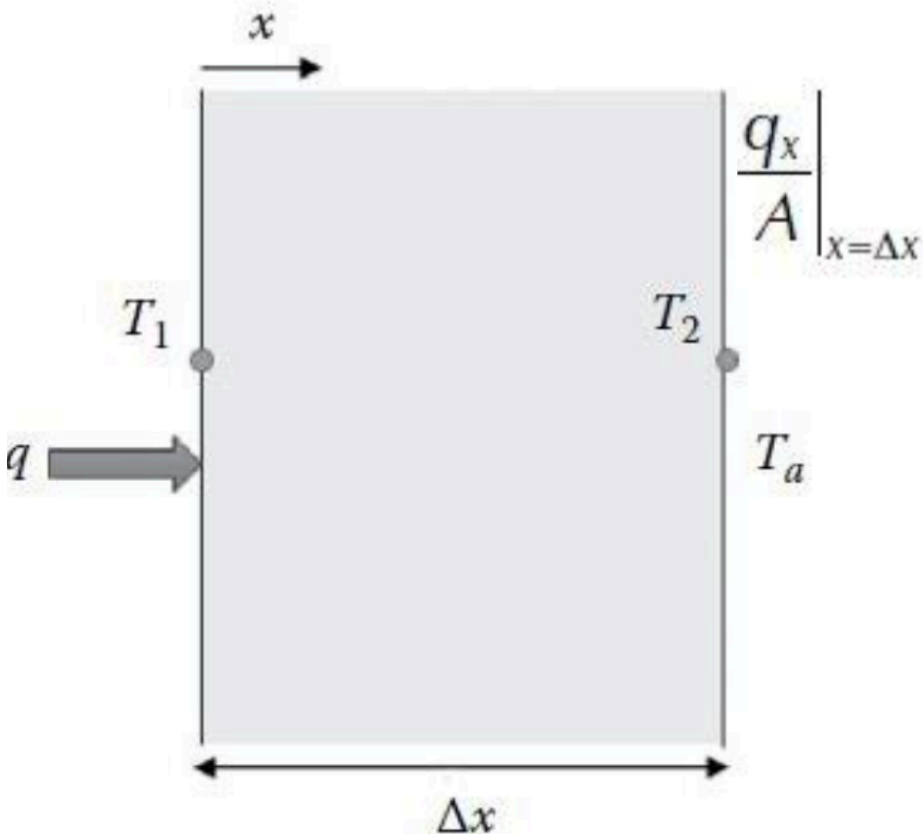


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Here we are solving the problem of radiation and conduction combined using ODE45 toolbox.



$$\left. \frac{q_x}{A} \right|_{x=\Delta x} = \sigma (T_2^4 - T_a^4) \bigg|_{x=\Delta x} \quad \left(\sigma = 5.676 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \right)$$

Governing Equation For Radiation

$$q(x) = -k \frac{dT(x)}{dx}$$

Governing Equation For Conduction

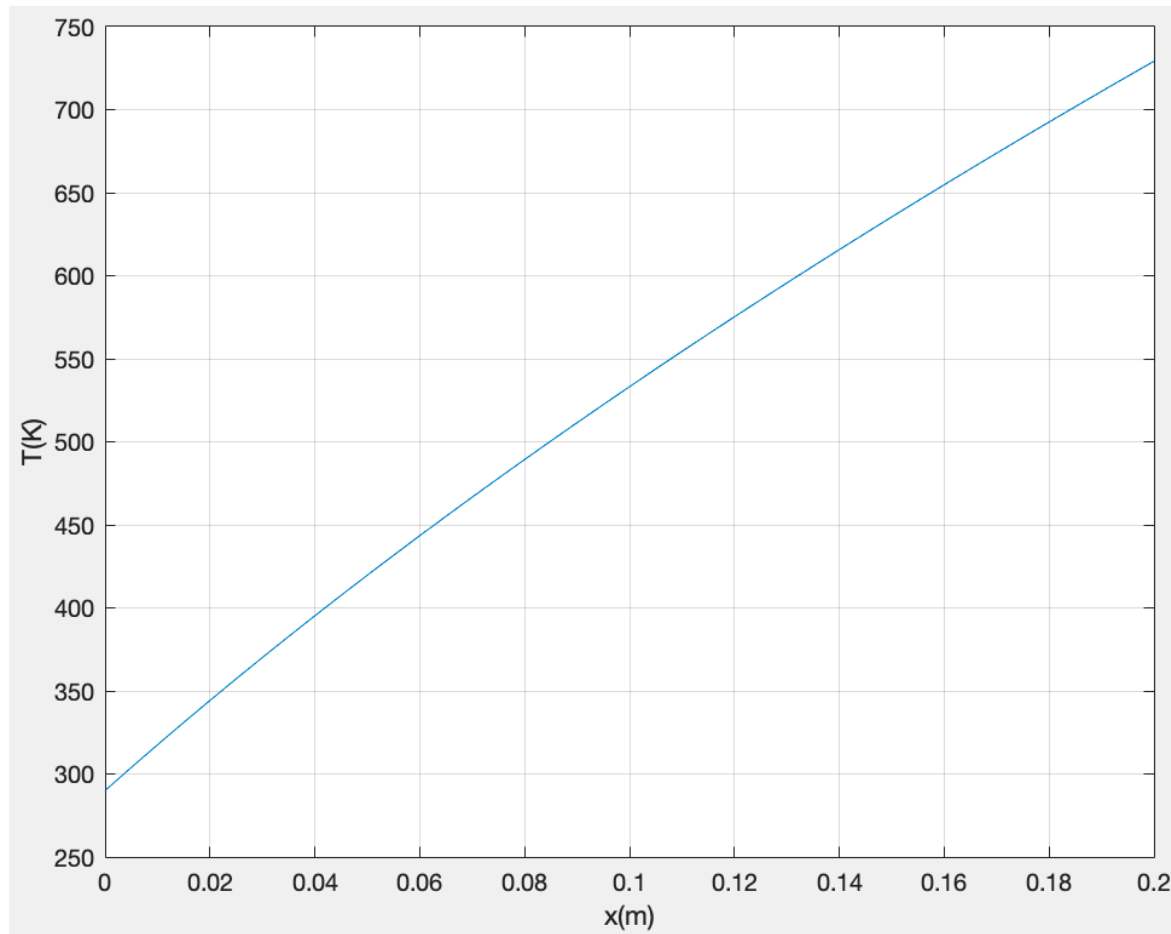




ODE45 TOOLBOX



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Temperature Profile w.r.t. distance using ODE45



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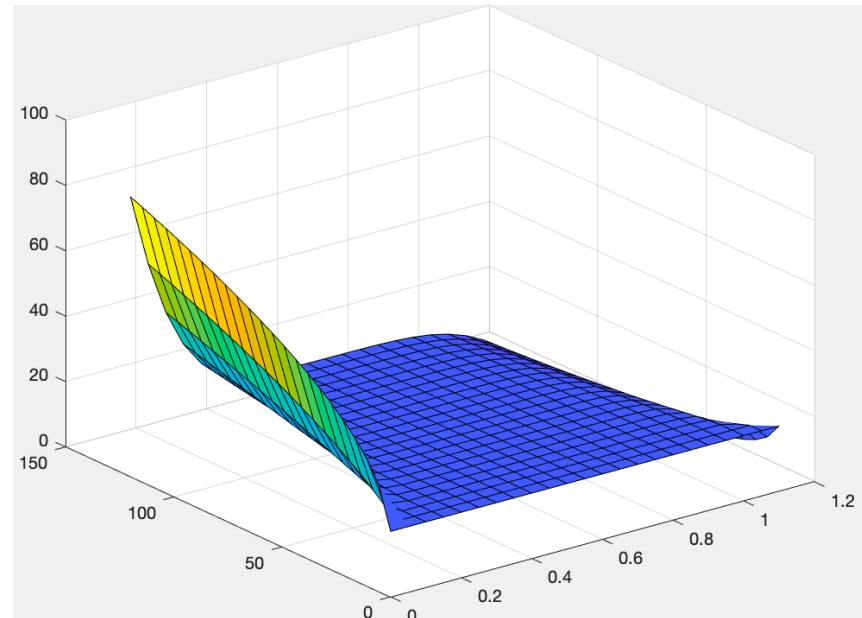
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FINITE DIFFERENCE - EXPLICIT METHOD



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Temperature Distribution Using Explicit Method

$$\sum_{\text{All sides}} \dot{Q}^i + \dot{E}_{\text{gen, element}}^i = \rho V_{\text{element}} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Governing Equation for Explicit method



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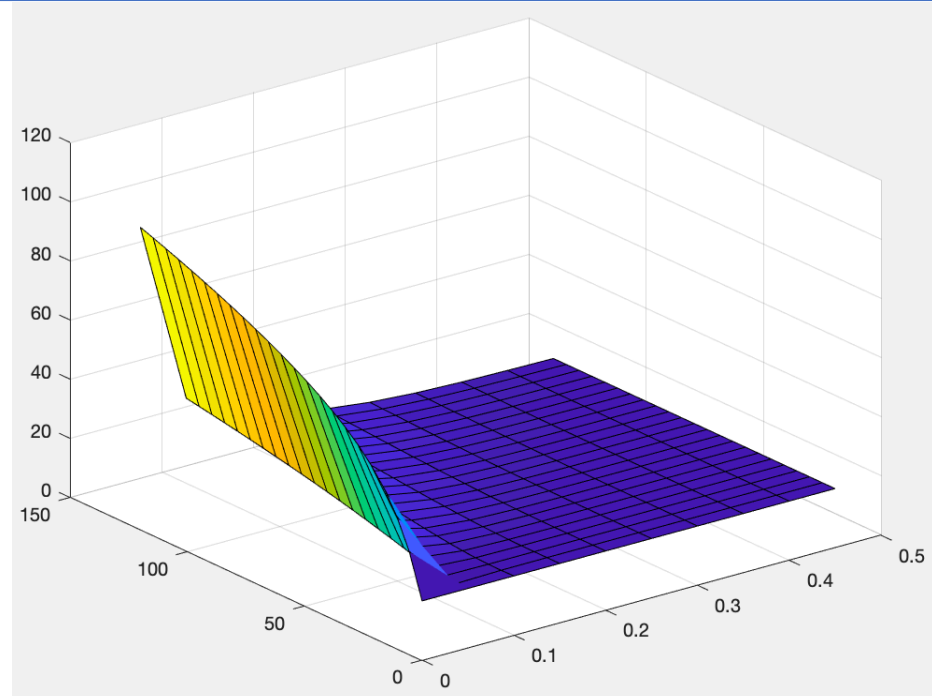
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FINITE DIFFERENCE - IMPLICIT METHOD



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Temperature Distribution Using Implicit Method

$$\sum_{\text{All sides}} \dot{Q}^{i+1} + \dot{E}_{\text{gen, element}}^{i+1} = \rho V_{\text{element}} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Governing Equation for Implicit method



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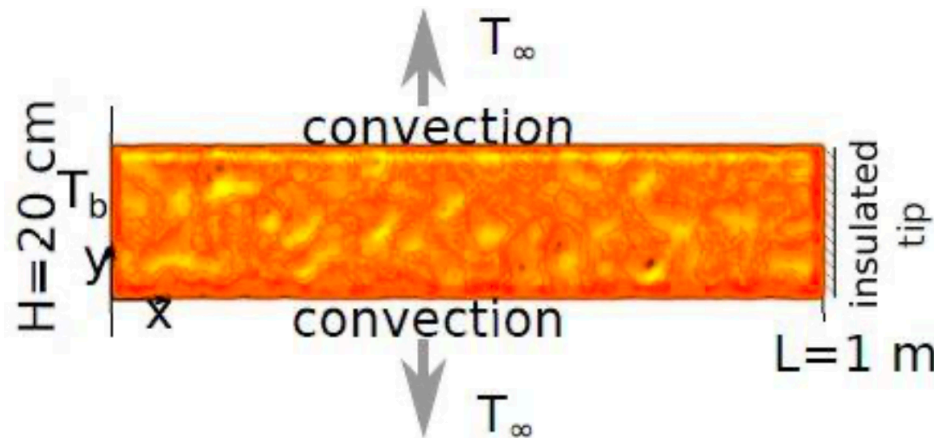
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COPPER FIN IN XY PLANE - STEADY STATE



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Copper Fin in XY Plane

$$\sum_{\text{All sides}} \dot{Q} + \dot{e}V_{\text{element}} = 0$$

Equation for Boundary Nodes

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{e}_{\text{node}} l^2}{k} = 0$$

General Equation for interior Nodes

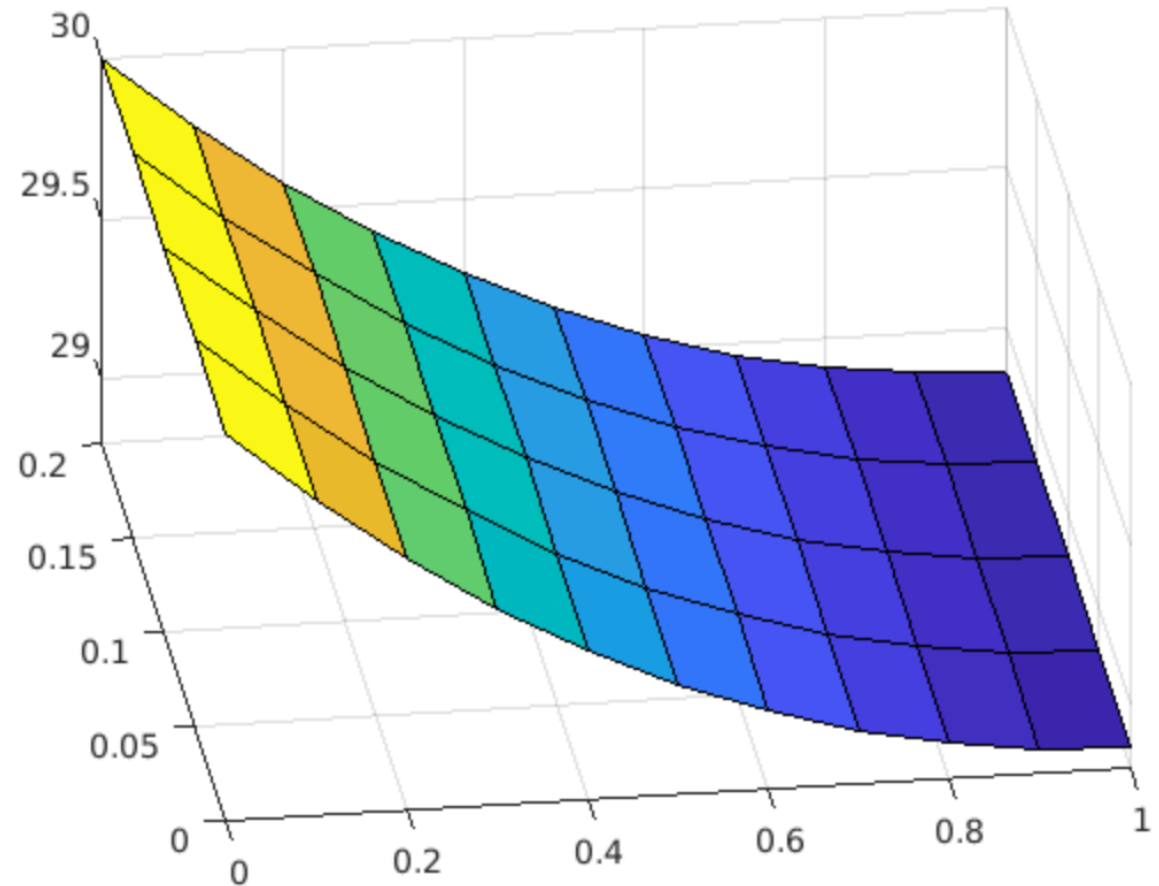
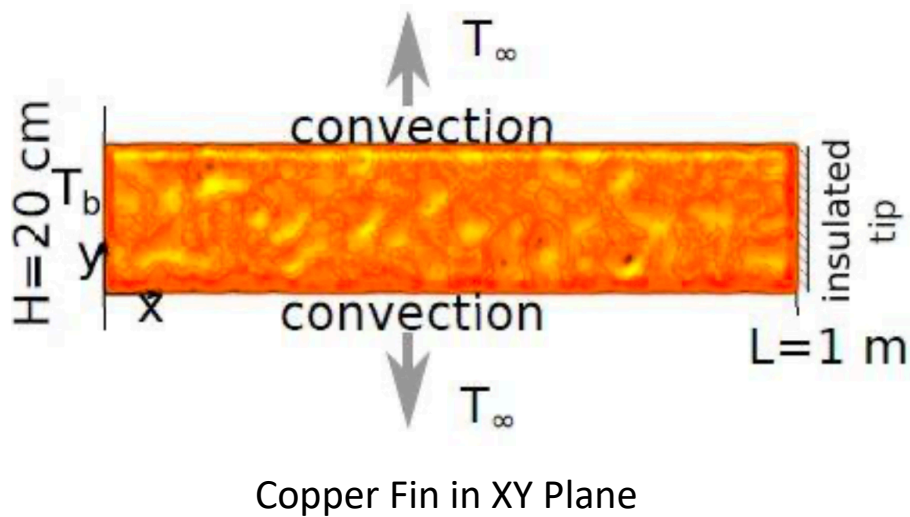




COPPER FIN IN XY PLANE - STEADY STATE



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Temperature profile w.r.t. distance in Steady State



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COPPER FIN IN XY PLANE - TRANSIENT STATE



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$$T_0^{i+1} = \left(1 - 2\tau - 2\tau \frac{h\Delta x}{k}\right) T_0^i + 2\tau T_1^i + 2\tau \frac{h\Delta x}{k} T_\infty + \tau \frac{\dot{e}_0^i \Delta x^2}{k}$$

Equation for Boundary Nodes in Transient State

$$T_{m-1} - 2T_m + T_{m+1} + \frac{\dot{e}_m \Delta x^2}{k} = \frac{\Delta x^2}{\alpha \Delta t} (T_m^{i+1} - T_m^i)$$

Equation for Interior Nodes in Transient State

$$\alpha = k/\rho c_p$$

Thermal Diffusivity



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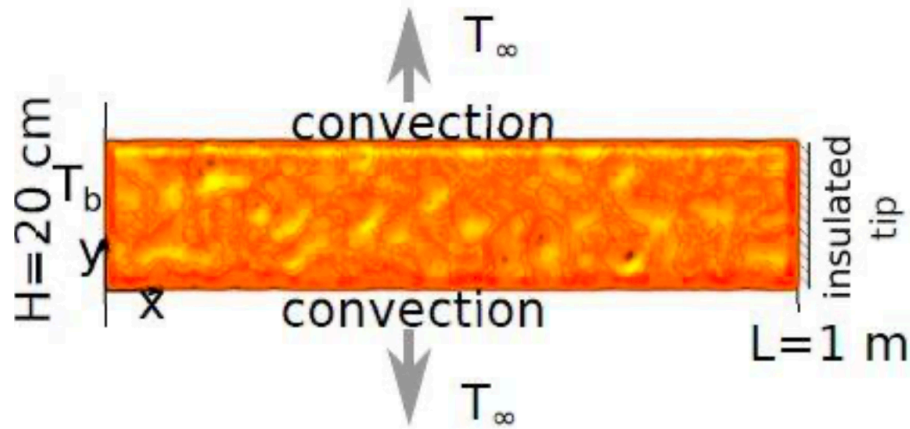
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COPPER FIN IN XY PLANE - TRANSIENT STATE



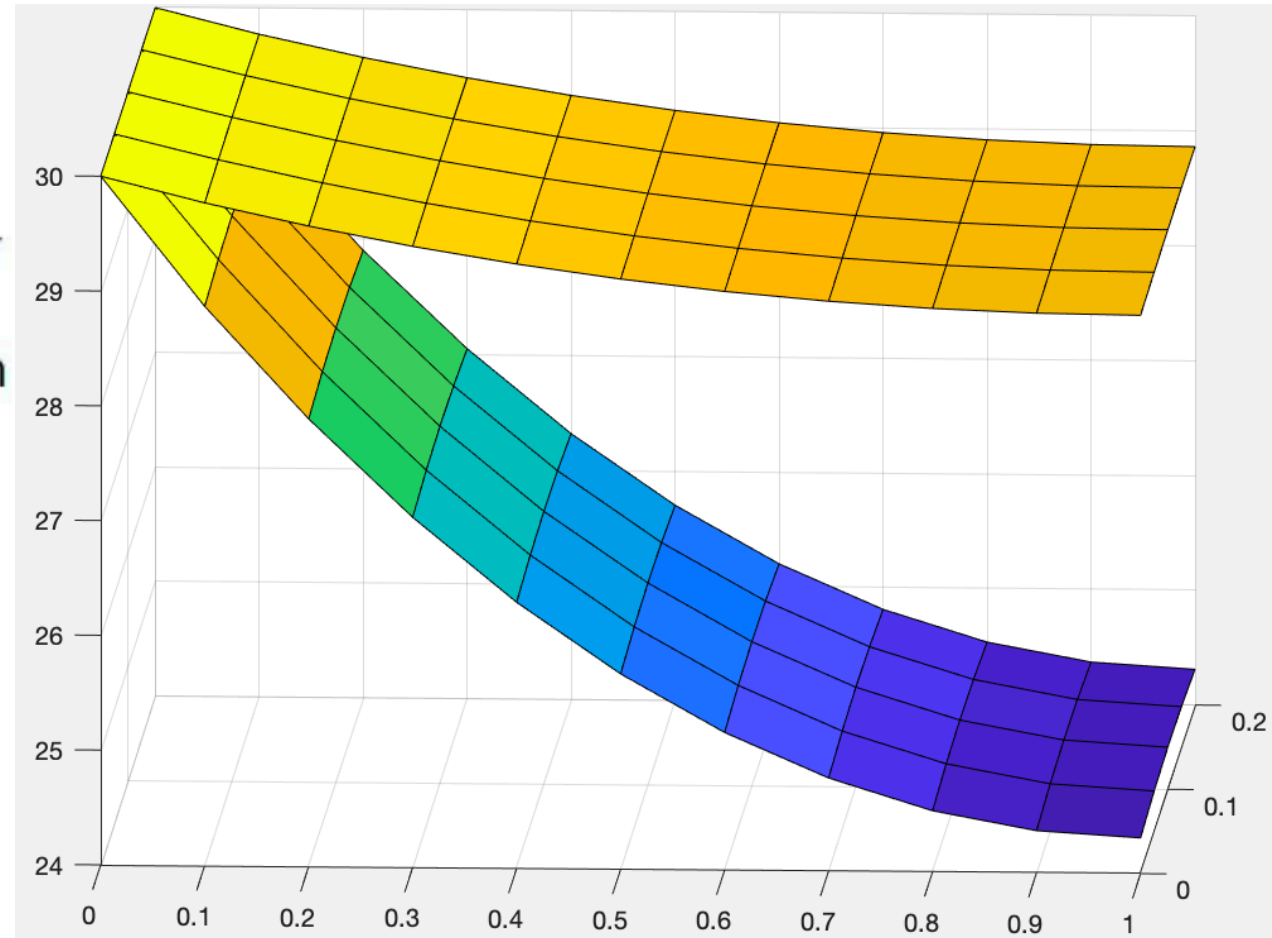
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Copper Fin in XY Plane

$$\tau = \frac{\alpha \Delta t}{\Delta x^2}$$

Mesh Fourier Number



Temperature profile w.r.t. distance in Transient State



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Thank You

