The Wavelet Trie: Maintaining an Indexed Sequence of Strings in Compressed Space

CSI 5335 Paper presentation

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Use Case

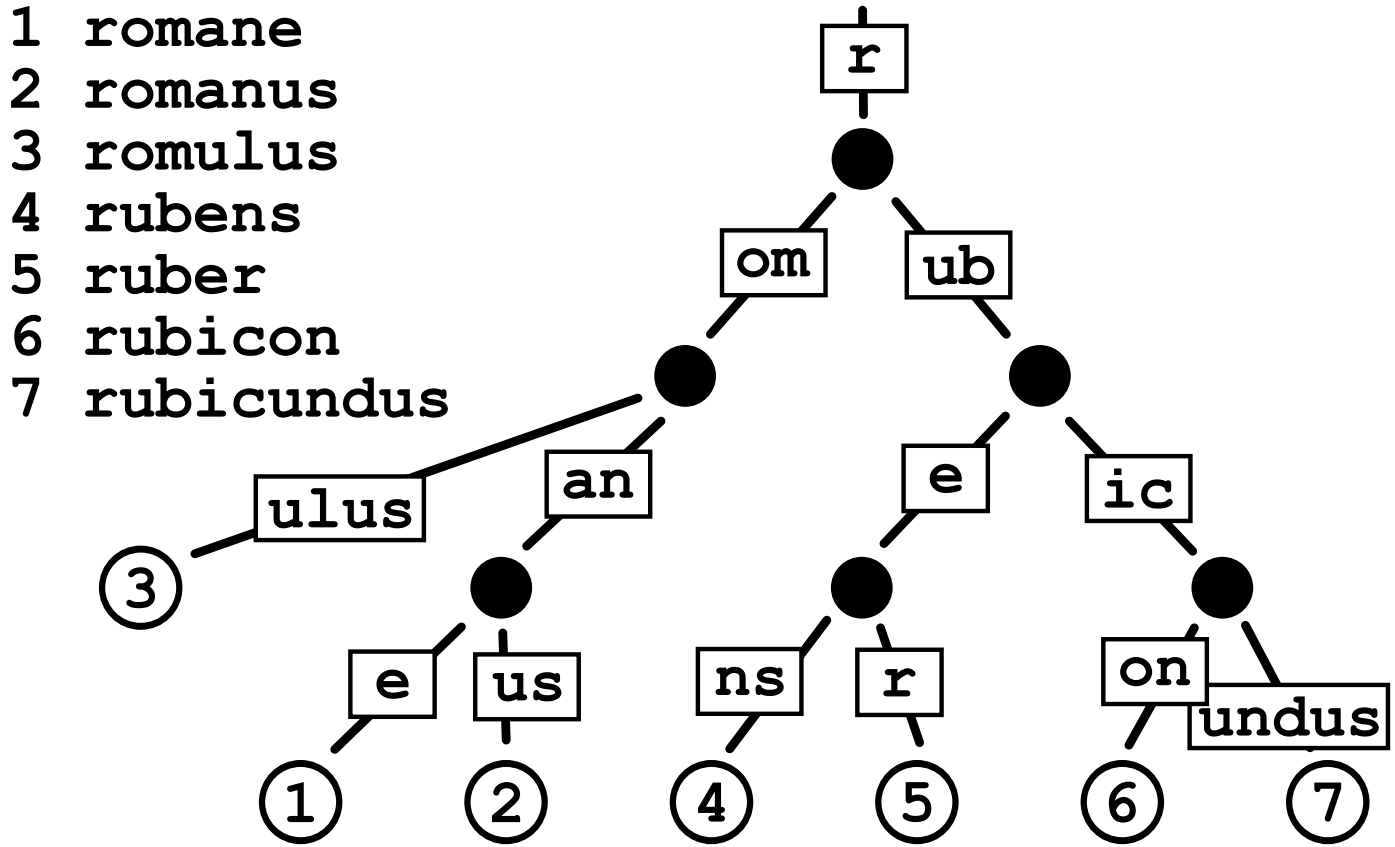
- A lot of things are string sequences.
- Column databases store and index string sequences.
- Great example: access logs

```
188.26.52.117 - - [24/Apr/2013:03:35:48 -0500] "GET /img/welcome/corner.png
188.26.52.117 - - [24/Apr/2013:03:35:49 -0500] "GET /img/welcome/arrowDown.gif
188.26.52.117 - - [24/Apr/2013:03:35:49 -0500] "GET /img/welcome/regionals.jpg
```

- Pretty similar, huh?
- I heard indexes make stuff faster $\rightarrow$ **indexed sequence of strings**
- Rank query: Number of requests for `/img/welcome/corner.png`?
- Select query: Position of i-th occurrence of `/img/welcome/corner.png`
- We can do prefix operations too

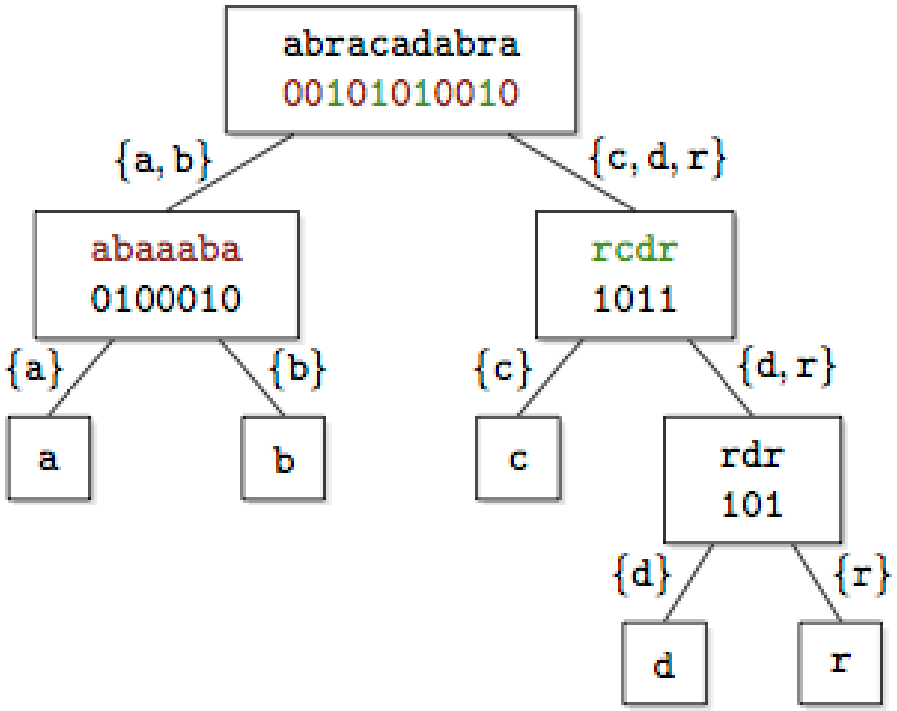
Patricia Trie

- Space-efficient trie.
- Node has always has at least two children.



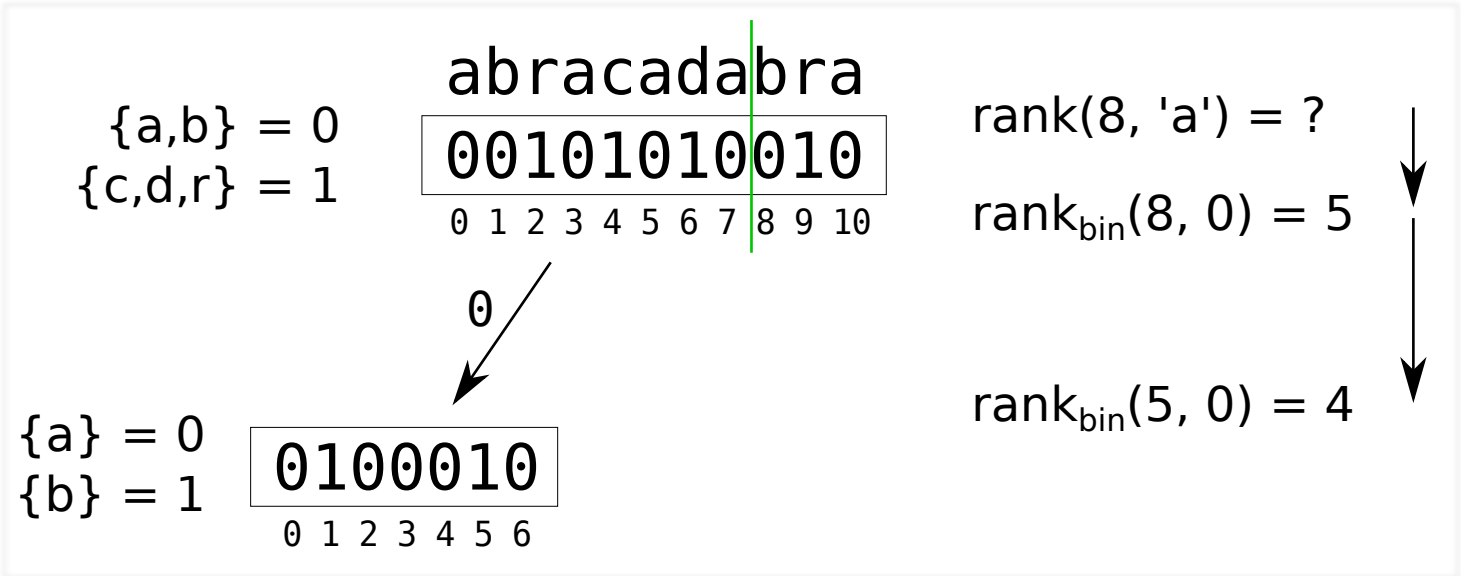
Wavelet Tree

- Organizes a string into a balanced binary tree of bit vectors.
- Alphabet $\sum = \{a, b, c, d, r\} \rightarrow \{0, 0, 1, 1, 1\}$
- At root, we have *ambiguity*, reducing ambiguity towards leaves



Efficient Computation of Rank in Wavelet Tree

- $rank(8, a)$ = how many a's before position 8
- $rank_{bin}(pos, s)$ binary rank, # of occurences of s before pos



E.g.: # of requests to `/img/welcome/corner.png` before April 10th.

Mutable & Compressed Indexed Sequences

- Sequences can change over time:
  $Insert(s, pos)$, $Append(s)$, $Delete(pos)$
- Alphabet not always known in advance

- Traditional approach, store explicitly (e.g. array), make an index
- Space-inefficient, we want to query the compressed representation
- **Succint data structure** – uses space close to lower information-theoretic bound

Wavelet Trie to rescue

Wavelet Trie

- Wavelet Tree + Patricia Trie
- Compressed data structure
- Can support *Insert*, *Append*, *Delete*, and dynamic alphabet
- Static, Append-only, Fully-dynamic
- Note: $O(|s| + h_s)$

	Query	Append	Insert	Delete	Space
Static	$O(\|s\| + h_s)$	–	–	–	$LB + o(\tilde{h}n)$
Append-only	$O(\|s\| + h_s)$	$O(\|s\| + h_s)$	–	–	$LB + PT + o(\tilde{h}n)$
Fully-dynamic	$O(\|s\| + h_s \log n)$	$O(\|s\| + h_s \log n)$	$O(\|s\| + h_s \log n)$	$O(\|s\| + h_s \log n)$	$LB + PT + O(nH_0)$

h_s – number of nodes traversed while searching for s is Patricia Tree
$\tilde{h}$ – average height of the Wavelet Trie
$|s|$ – length of query string

Summary

Thank you.

Resources

- http://alexbowe.com/wavelet-trees/