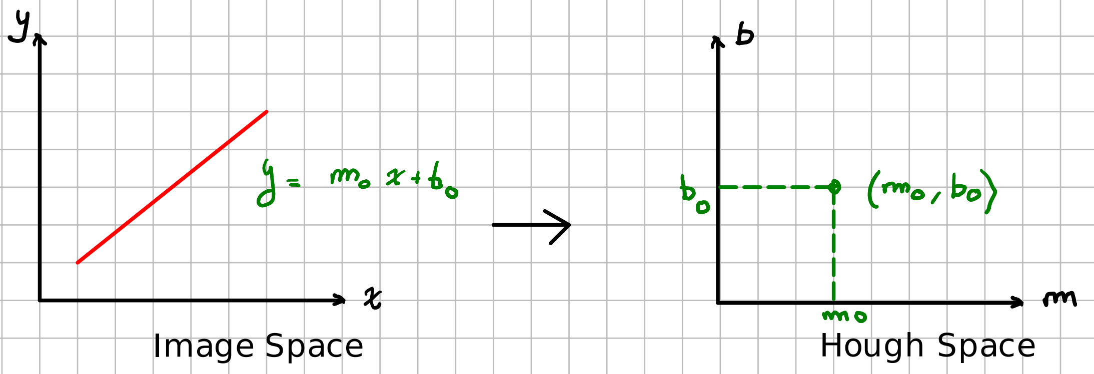
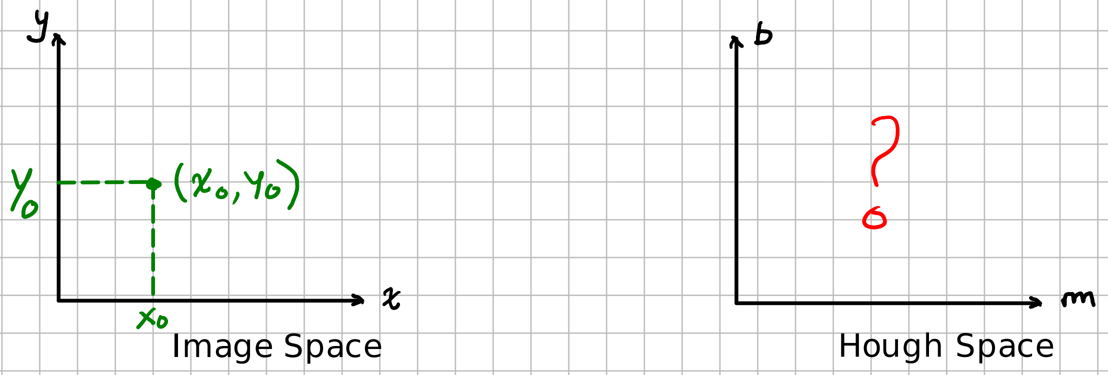
Hough Transform

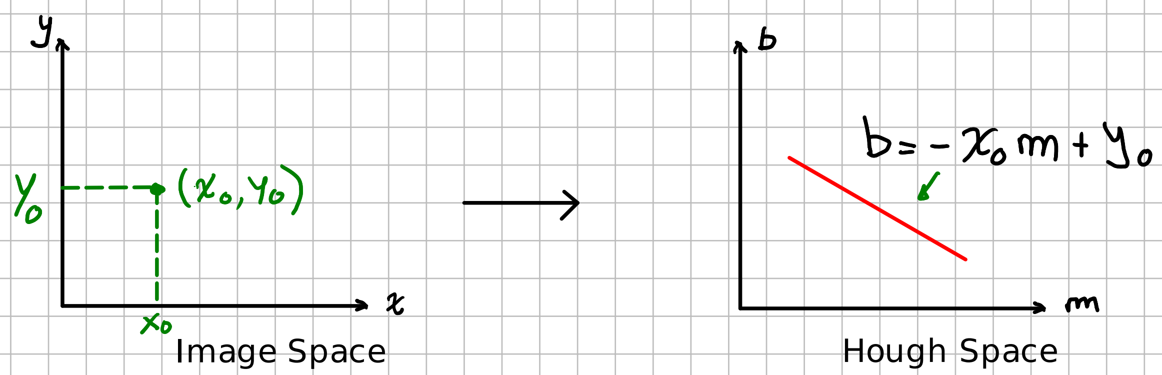


Una linea en la imagen corresponde a un punto en el espacio de Hough

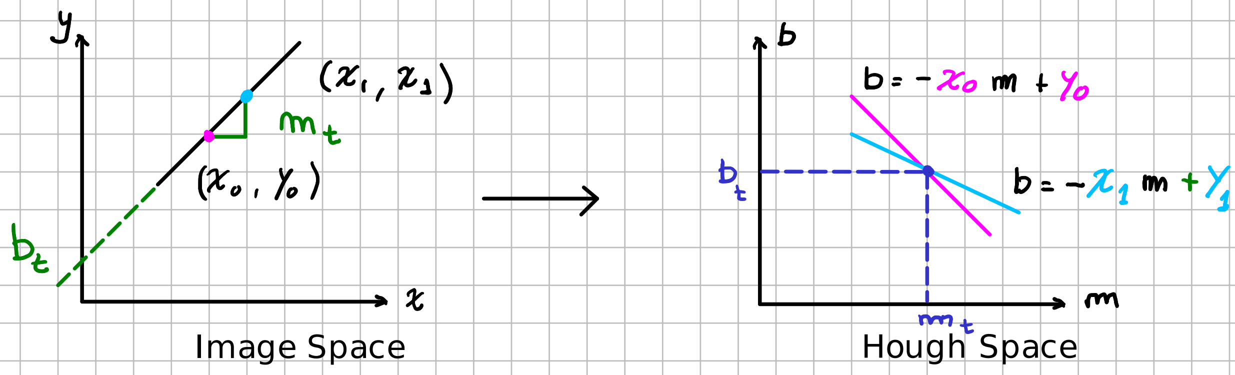
Como se transforma en el espacio de Hough un punto en la imagen?



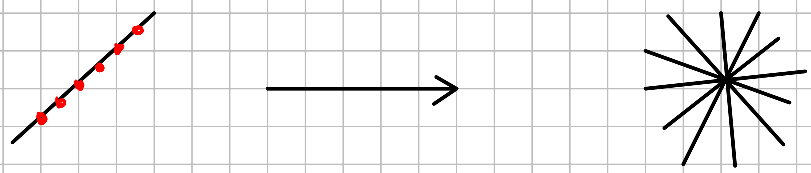
$$y = mx + b \Big|_{x_0, y_0} \rightarrow y_0 = m x_0 + b \rightarrow \boxed{b = -x_0 m + y_0}$$



Que pasa con la linea que contiene a (xo,yo) y a (x1,y1) ?



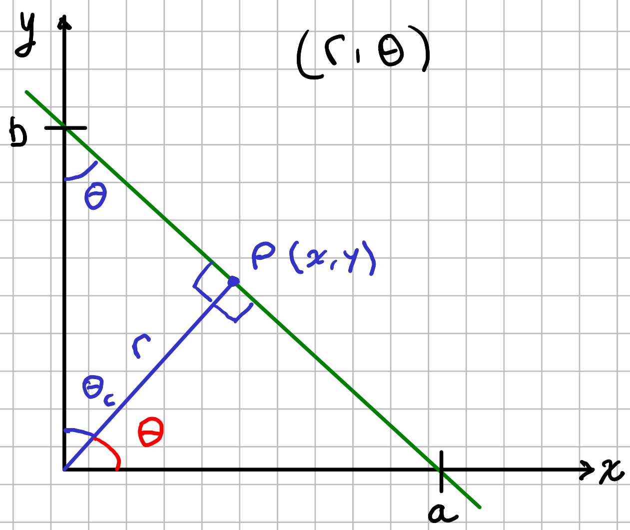
Es la interseccion de las lineas b=-xo m+yo con b=-x1 m+y1



Probema en el espacio (m,b) : Lineas verticales $\Rightarrow m=\infty$

Representacion ALTERNATIVA : Coordenadas polares

Ecuacion de La recta en Coordenadas Polares



(r, θ)

$$\frac{x}{a} + \frac{y}{b} = 1$$ Ecuacion de una recta

$$\cos\theta = \frac{r}{a} \qquad \operatorname{Sen}\theta = \frac{r}{b}$$

$$a = \frac{r}{\cos\theta} \qquad b = \frac{r}{\operatorname{sen}\theta}$$

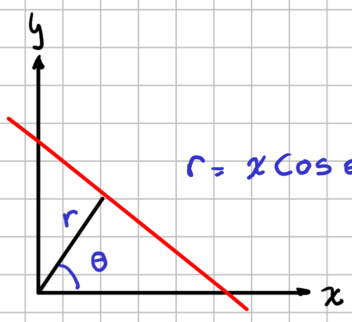
Entonces $$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{\left(\frac{r}{\cos\theta}\right)} + \frac{y}{\left(\frac{r}{\operatorname{Sen}\theta}\right)} = 1$$

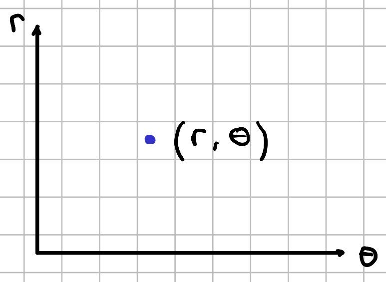
$$\frac{x\cos\theta}{r} + \frac{y\operatorname{Sen}\theta}{r} = 1$$

$$\boxed{x\cos\theta + y\operatorname{Sen}\theta = r}$$

Entonces θ y r definen una recta (asi como lo hacen m y b en la forma $y = mx + b$)

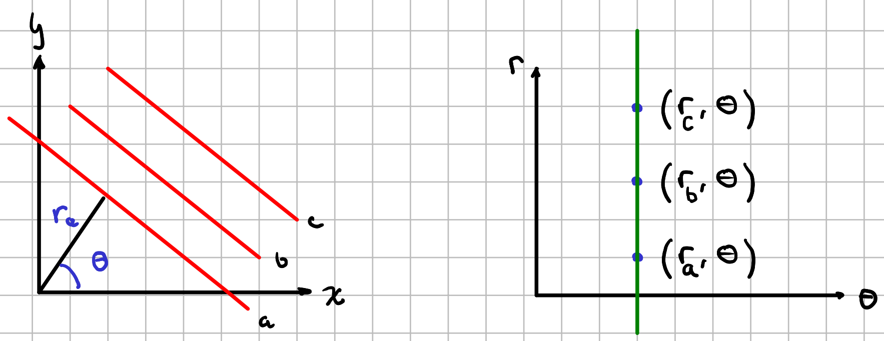


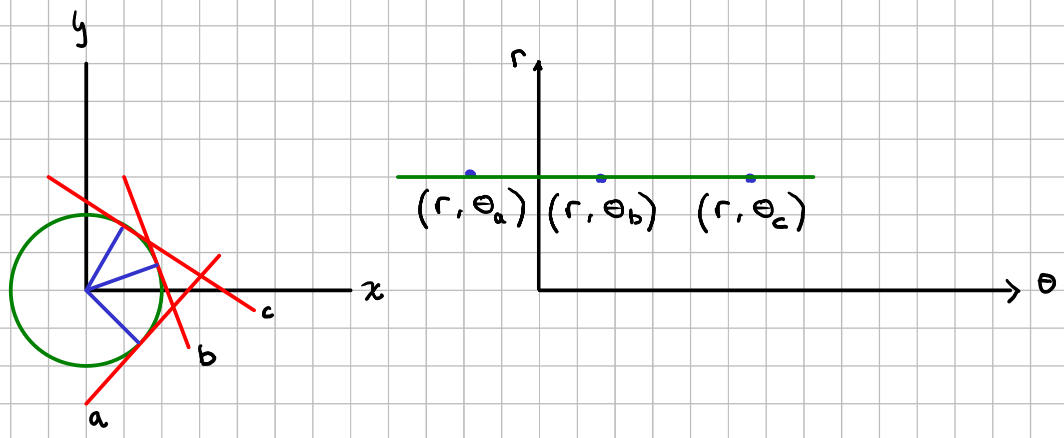
$r = x\cos\theta + y\,\mathrm{sen}\,\theta$



Toda recta en el plano (x,y) se transforma en un punto en plano (r,θ).

Casos particulares





Forma alternativa

$$r = x\cos\theta + y\,\text{sen}\,\theta = A\cos(\theta-\varphi)$$

$$A\cos(\theta-\varphi) = A\left[\cos\theta\cos\varphi + \text{sen}\,\theta\,\text{sen}\,\varphi\right]$$

$$A\cos(\theta-\varphi) = \underbrace{A\cos\varphi}_{x}\,\cos\theta + \underbrace{A\,\text{sen}\,\varphi}_{y}\,\text{sen}\,\theta$$

$$\begin{cases} x = A\cos\varphi \\ y = A\,\text{sen}\,\varphi \end{cases}$$

$$x^2+y^2 = A^2\cos^2\varphi + A^2\text{sen}^2\varphi$$
$$x^2+y^2 = A^2(\cos^2\varphi + \text{sen}^2\varphi)$$
$$x^2+y^2 = A^2$$
$$A = \sqrt{x^2+y^2}$$

$$\frac{y}{x} = \frac{\text{sen}\,\varphi}{\cos\varphi} = \tan\varphi$$

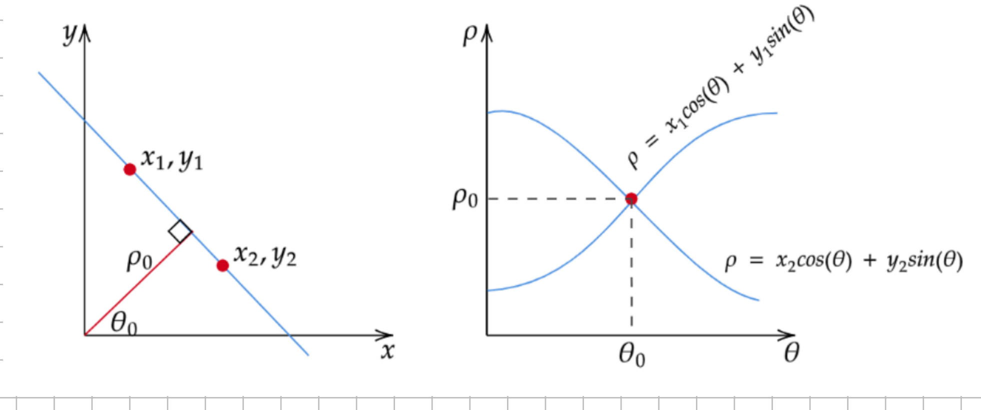
$$\varphi = \text{arc tg}\left(\frac{y}{x}\right)$$

Entonces:

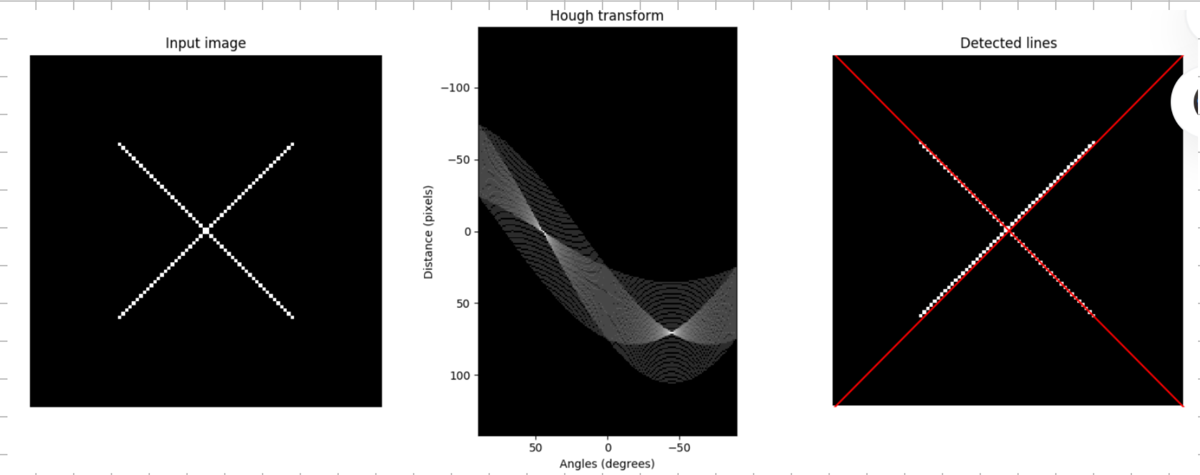
$$\boxed{r = A\cos(\theta-\varphi)}$$

$$\boxed{A = \sqrt{x^2+y^2}} \qquad \boxed{\varphi = \text{arc tg}\left(\frac{y}{x}\right)}$$

$$r = x\cos\theta + y\,\text{sen}\,\theta = A\cos(\theta-\varphi)$$



Ejemplo:


Input image
Hough transform
Detected lines
Distance (pixels)
Angles (degrees)
−100
−50
0
50
100
50
0
−50

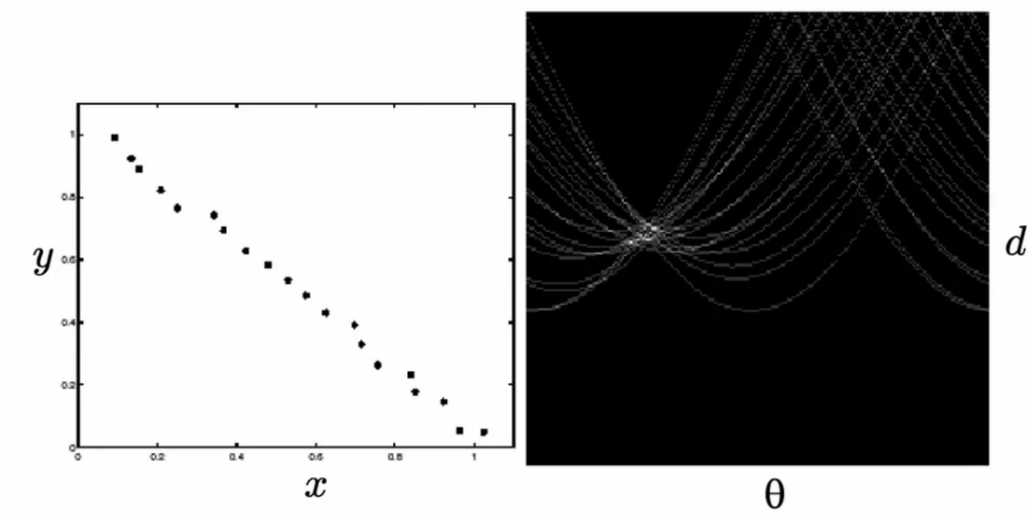

y
x
d
θ
1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

imagen original → Low pass (gaussian) → Deteccion de bordes (Canny)

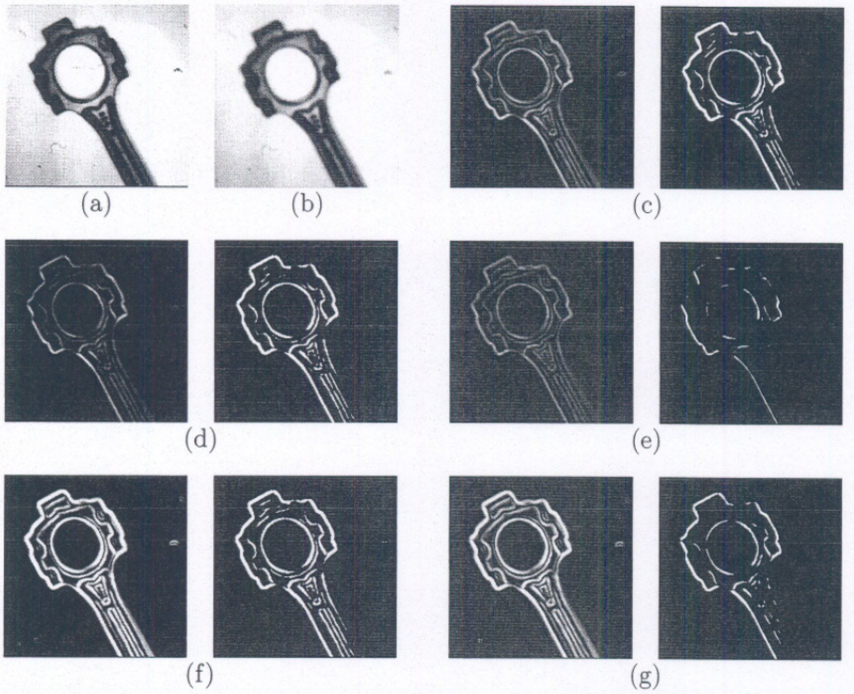
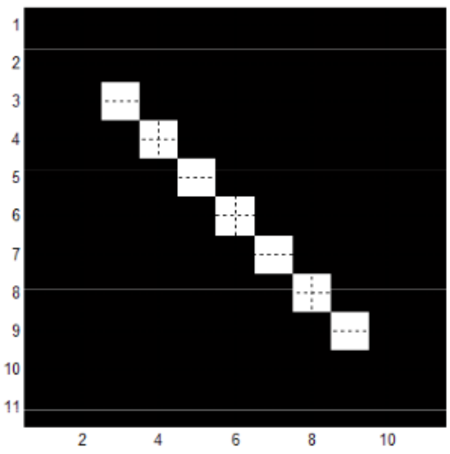
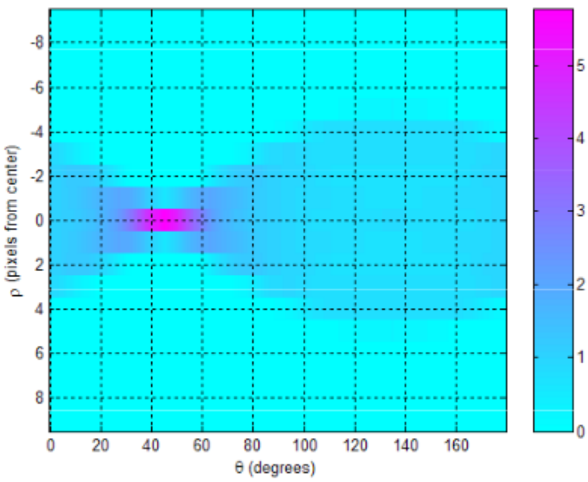


Figure 5.4: A comparison of various edge detectors. (a) Original image. (b) Filtered image. (c) Simple gradient using 1×2 and 2×1 masks, $T = 32$. (d) Gradient using 2×2 masks, $T = 64$. (e) Roberts cross operator, $T = 64$. (f) Sobel operator, $T = 225$. (g) Prewitt operator, $T = 225$.

- Example 1: 11x11 image and its Hough transform:





Particionamos el plano r, θ en celdas $A(r, \theta)$

$A \rightarrow$ Acumulador

Rango de θ $[-\pi/2, \pi/2]$

Rango de r $[-D, D]$ D diagonal de la imagem

$\mathcal{E}_j$ Imagen $N \times N$ $D = \sqrt{2}N$

Para cada punto (x_k, y_k) de la imagen

Para cada θ_j $[-\pi/2, \pi/2]$

Evaluo $r_k = x_k \cos\theta_j + y_k \operatorname{sen}\theta_j$

Redomdeo $r_k \rightarrow r_{kq}$

Incremento $A(r_k, \theta_j)$ si r_{kq} cae en esa celda.

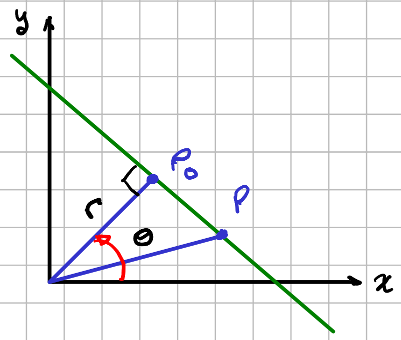
- Una vez finalizado $A(i,j) = P \Rightarrow P$ puntos en el espacio x-y se encuentran sobre la linea $r_j = x \cos\theta_j + y \operatorname{sen}\theta_j$

- Buscamos todos los candidatos donde $A(i,j)$ se encuentran por encima de un umbral

- Representamos las rectas sobre la imagen original

Apendice

Ec de la recta en coord Polares (otra forma)



P_o: Punto mas cercano al origen.

$$P_o = (r\cos\theta, r\,\text{sen}\,\theta)$$

Si $P=(x,y)$ pertenece a la recta

$$\Rightarrow (\vec{P}-\vec{P_o})\cdot\vec{P_o} = 0 \quad \text{·: Producto escalar}$$

$$\vec{P}\cdot\vec{P_o} = \vec{P_o}\cdot\vec{P_o}$$

$$\underbrace{(x,y)}_{\vec{P}}\cdot\underbrace{(r\cos\theta, r\,\text{sen}\,\theta)}_{\vec{P_o}} = \underbrace{(r\cos\theta, r\,\text{sen}\,\theta)}_{\vec{P_o}}\cdot\underbrace{(r\cos\theta, r\,\text{sen}\,\theta)}_{\vec{P_o}}$$

$$r x\cos\theta + r y\,\text{sen}\,\theta = r^2\cos^2\theta + r^2\,\text{sen}^2\theta$$

$$\cancel{r}(x\cos\theta + y\,\text{sen}\,\theta) = r^{\cancel{2}}\underbrace{(\cos^2\theta + \text{sen}^2\theta)}_{1}$$

$$\boxed{x\cos\theta + y\,\text{sen}\,\theta = r}$$