Baby Warmer: PI Control and Temperature Disturbances

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1 Executive Summary

After modelling and experimenting with the effects of PI Control and outside temperature disturbances on a baby warmer, there are certain design specifications a system designer should consider when creating a final product based on our research. This report details the important metrics for a baby warmer (within Design Considerations), recommendations and information about the mathematical modelling (within Recommendations and Justification), experimental set-up, results, and analysis of the experimental set-up (within Testing and Performance), and conclusions about key factors that go into designing a baby warmer (within Conclusion).

2 Design Considerations

The main goal of a baby warmer is to bring a baby to a safe temperature without overheating the baby or exposing it to dangerous temperatures. The system should reach a steady state relatively quickly and not fluctuate. This is so that the baby can be warmed quickly and safely, serious fluctuations can be dangerous to a baby too. Some of the design considerations to be made when creating such a system is what materials to use, where the heaters should be placed, the shape of the system and various metrics involving the regulation of the warmer. We decided to focus our research on determining which constants (K_P and K_I) for a proportional integral controlled system would produce the least steady state and transient state error, including decreasing overshoot. Steady state is when a system stops rapidly changing and is within a certain acceptable amount of the desired temperature. Transient state is when a system is moving towards steady state and therefore is significantly changing constantly. We were interested in studying these metrics because having error in a baby warmer can be very dangerous. The choice to use a system controlled by a PI controller was made because in our preliminary research we determined that a PI controller was theoretically better at reducing error compared to just a proportionally controlled system. We also wanted to see how our system would react to a disturbance in outside temperature, such as an air conditioner being turned on or a window being opened. Therefore, we theorized that a PI controller would help the system more quickly adjust to a change in conditions.

3 Recommendations and Justification

In order to test the effect of PI Control on steady-state error, overshoot, and disturbances, we designed two experimental set-ups. One, specifically targeting minimizing steady state error and temperature overshoot, involved using our optimized K_P and K_I values and measuring the effect on the temperature of the heater and the polycarbonate, as well as keeping track of power consumption (both desired and actual power). The second experiment had the same exact set-up as the first one except after it had reached steady state, the set-up was moved so that the outside temperature was significantly colder.

3.1 Thermal Model

The stock-and-flow diagram of the system that we modelled can be seen in Figure 1. Power goes into the heater, which then transferred to the polycarbonate through conduction. Heat then dissipated to the air through convection. The rest of the heat was 100% transferred to the baby.

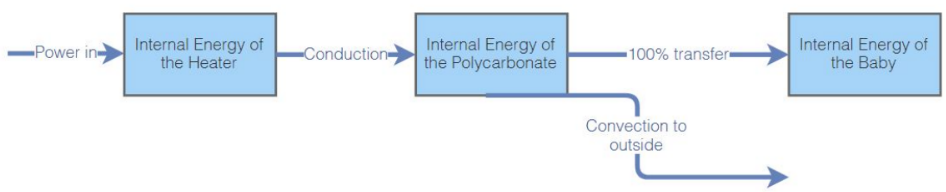

Figure 1: Stock-and-Flow Diagram of our Modelled Thermal System

Because we are assuming 100% transfer of heat from the polycarbonate to the baby (other than the heat that was dissipated through the air) we were able to simplify our model into the stock-and-flow seen in Figure 2, and so we could measure/model the temperature of the polycarbonate instead of adding in an arbitrary baby stock.

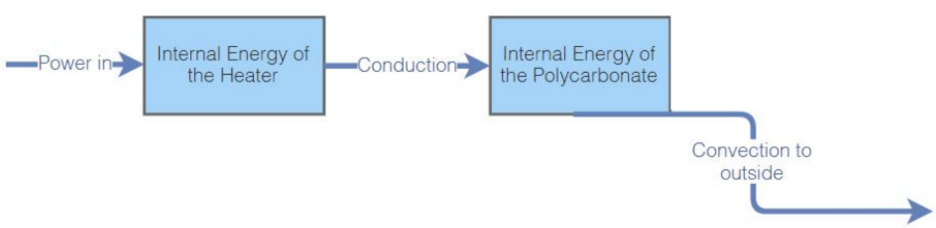

Figure 2: Revised Stock-and-Flow Diagram of our Modelled Thermal System

3.2 Block Diagram

In our control mechanism, we implemented PI (proportional-integral) control. This is similar to proportional control, which computes a proportionality constant (for the error signal) based on the difference between the output and the desired output. PI control uses proportional control, except it has a separate integral constant. The integral is taken and the difference between the desired and the actual output is computed that way. Then, the constant factors in and the output is changed through that. With this constant rechecking of the error, PI control is normally prone to oscillations (and thus a PID controller, or a proportional-integral-derivative controller, would normally be used). However, our findings show that a carefully chosen K_P and K_I value can minimize these oscillations.

Figure 3 shows a block diagram of our control system, with the relevant equations shown within their blocks. Note that the K_P and K_I were the desired values from our mathematical modelling - and that these values would then be used for the experimental set-up.

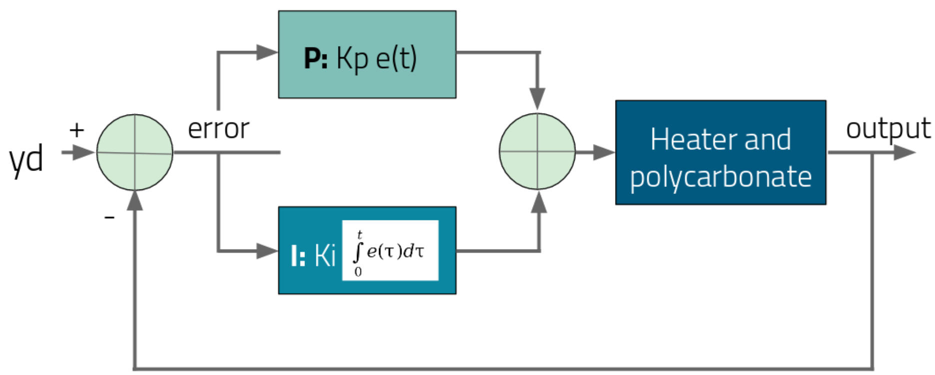

Figure 3: Block diagram of the PI Control system for the baby warmer

3.3 Assumptions

Our model, and our subsequent mathematical analysis, does include a fair amount of assumptions. The largest assumption would have to be the fact that the baby is not a stock. Namely, the fact that the 100% of the heat from the polycarbonate transfers to the baby. In reality, the baby doesn't absorb heat perfectly and in fact has it's own very complex control mechanism (i.e. thermal regulation in the form of homeostasis). However, we felt that this assumption was fair for the model - if the polycarbonate reached 37 C, and that was the material used to create the baby warmer, then eventually the baby would also reach the same temperature as well. The time to steady state would be significantly shorter starting at a higher temperature than room temperature (though the rate of increase to the steady state would be decreased). In addition, we knew that the assumption would not cause danger to the baby - the polycarbonate would never heat up to a high temperature that would endanger the baby.

Another assumption we made was the the system is very efficient. For instance, we assume that no heat is lost through the bottom of the heater, and that all heat transfers occur exactly as we mathematically modelled them. This is one reason why the results of our experiment - because the experiment was done on an 'inefficient system' (i.e. more heat loss than we modelled), the results vary from our model.

Though we would not suggest fully designing a baby warmer based on our model, we do believe it is a reasonable first pass model, and can serve as a base for more iterations that can eventually lead to a final design for a baby warmer.

3.4 Mathematical Analysis

From our model, we were able to create the following ordinary differential equations for the change in temperature of the heater and the change in temperature of the polycarbonate:

$$\frac{dT_H}{dt} = \frac{p(t) - \frac{kA(T_H - T_P)}{d}}{m_H c_H}$$

$$\frac{dT_P}{dt} = \frac{\frac{kA(T_H - T_P)}{d} - hA(T_P - T_O)}{m_P c_P}$$

In this equation, T_H is the temperature of the heater, T_P is the temperature of the polycarbonate, T_o is the temperature of the outside. and k, h, m_P, m_H c_P, c_H are various heat constants.

From this equation, we wanted to generate an equation for T_P(t) that also included the desired temperature values. Therefore, we first took the Laplace transform of both equations and then solved for the temperature of the polycarbonate in the s-domain, or $T_P(s)$. Then, using the block diagram in Figure 3 (and general rules for mathematically solving equations in block diagrams and transfer functions), we found an equation for $T_P(t)$ that related to the desired temperature values (and thus, as a result, included K_P and K_I, values which we could tweak in order to get the model behavior we desire).

At the end of our mathematical analysis, we had an equation for T_P(t) and were able to visualize graphs of that equations with changing K_P and K_I values. See Figure 4 for an example of one of our modelled graphs:

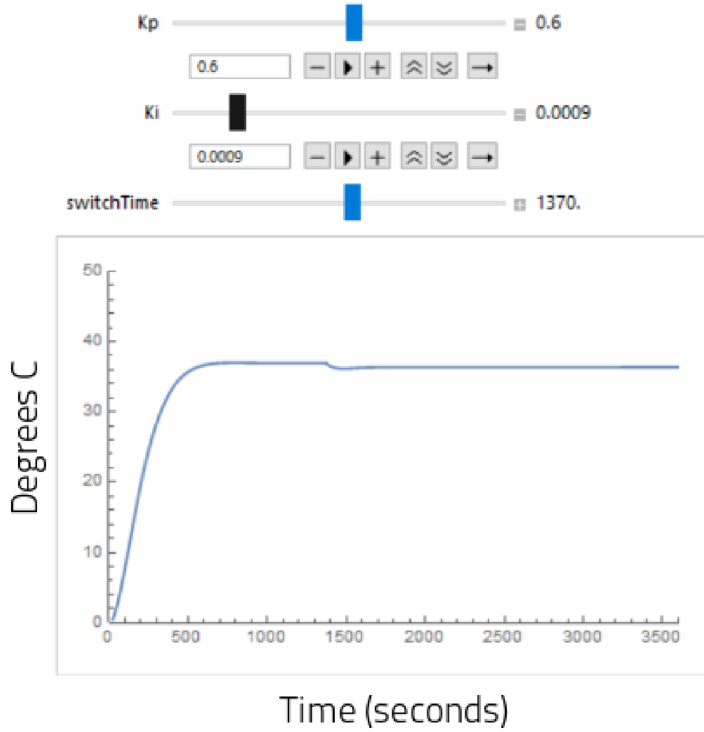

Figure 4: One of the results of our model - specifically, the model that accounted for a variation in outside temperature (i.e. a temperature disturbance). Using the Manipulate capability of Mathematica (the program we used to model the system), we were about to discern the values of K_P and K_I to yield the expected graph behavior - minimal overshoot and steady state error - and reach the desired temperature of 37 C

4 Testing and Performance

In our initial experiment, we altered the provided arduino code so that the the desired power supplied to the heater was controlled by the following equation:

$$P = K_p(T_d - T_h) + K_i \int_0^t (T_d - T_h)dt$$

Where P is the desired power, T_d is the desired temperature, T_h is the temperature of the heater and t is time. K_P and K_i are constants that we determined through mathematical modelling to be .6 and .0009 respectively. We placed the heater on a towel, attached a piece of polycarbonate to the heater, and attached thermistors to the polycarbonate and heater (see setup in Figure 5). Then we ran the experiment for over an hour.

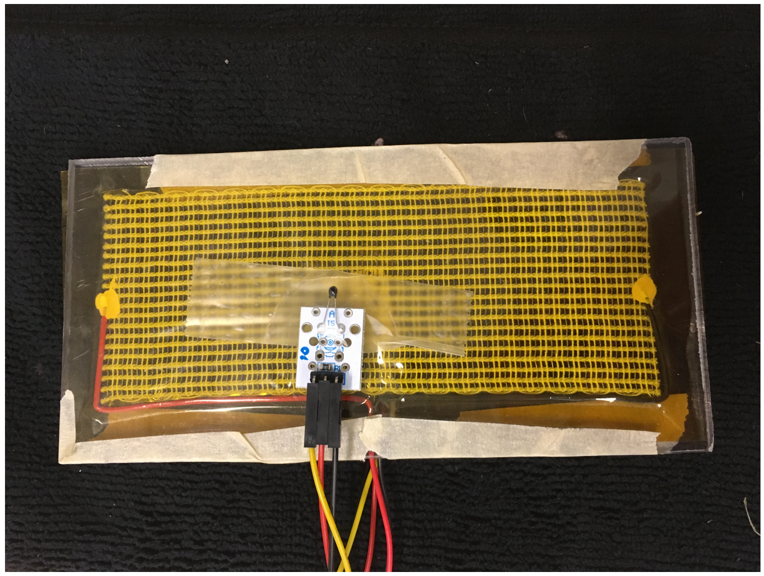

Figure 5: Photo of the experimental setup.

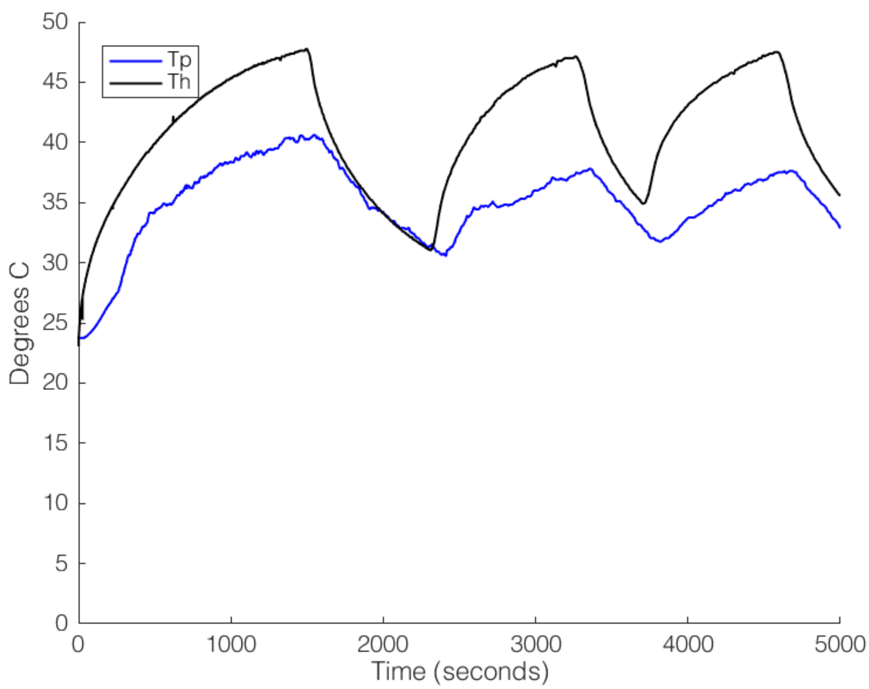

Figure 6: Graph of temperature of polycarbonate (T_P) and temperature of heating pad (T_H) over time

Figure 6 shows the temperature of both the polycarbonate and the heating pad over time. Even though our mathematical model predicted no oscillation with the specific K_P and K_i we determined this was not the case when we ran the experiment. One of the reasons for this is because the power desired was not always the power that could be provided.

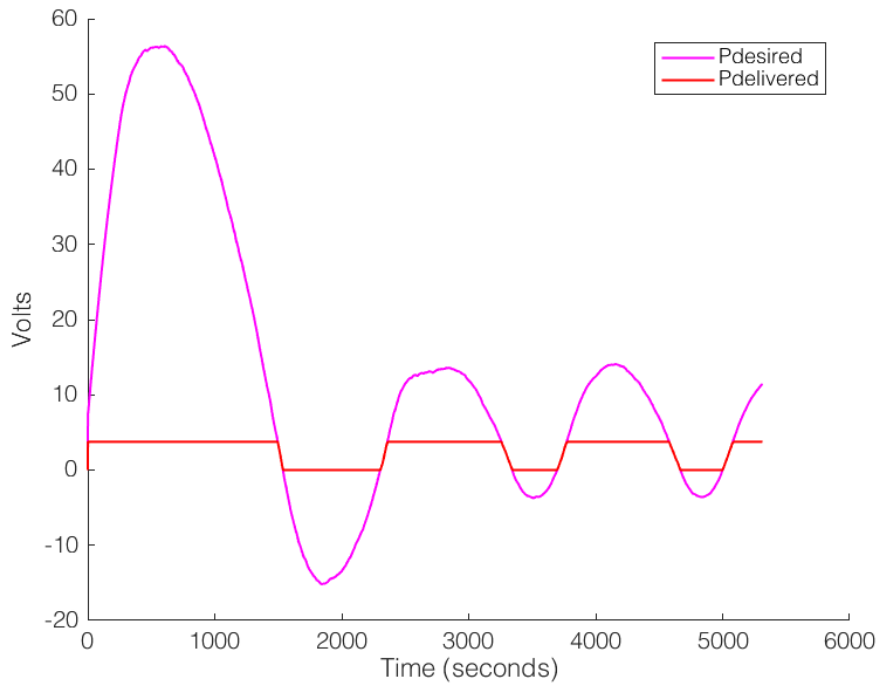

Figure 7: Graph of the desired power $P_{desired}$ and in delivered power $P_{delivered}$ in volts/second over the course of the experiment.

Another reason could be that the specifications for the materials might have been a bit different than the ones that were provided and since the system is very sensitive, a slight difference in constants would alter the mathematical model significantly. However, the polycarbonate was able to stay within an acceptable range to still keep a baby warm so we believe with further experimentation constants could be found that would reduce the oscillation even more.

In the second phase of our experiment, we had the same setup but once the system reached what we accepted as a relatively steady state we introduced an outside temperature change. So at first the system was run in a room at normal room temperature, then at around 5000 seconds the system was put in a room that was significantly cooler due to air conditioning.

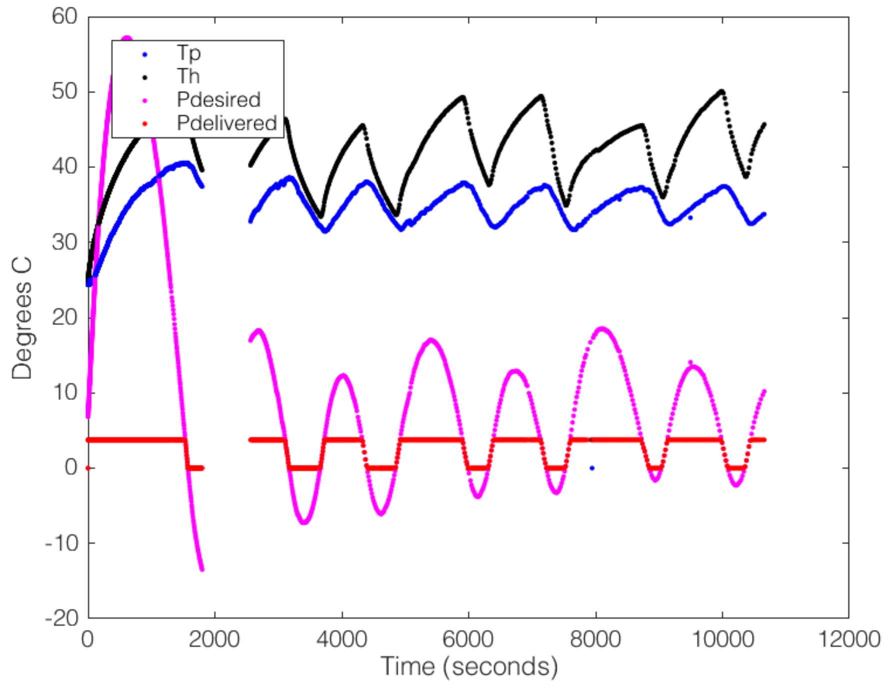

Figure 8: Graph of the desired power $P_{desired}$ and the delivered power $P_{delivered}$ in volts/second and temperature of the polycarbonate (T_P) and temperature of heating pad(T_H over the course of the temperature disturbance experiment. The gap in data is caused when the computer that was recording the data froze for a few minutes.

As seen in Figure 8, at around 5000 seconds when the temperature of the environment changes there is also some changes in how the system runs. The frequency of the oscillation of the temperature of the polycarbonate decreases because it takes longer for it to heat up in the colder enviroment. Also the system takes less time cooling. However we were pleased to see that system continued to stay within a few degrees of 37 degrees celsius, therefore staying safe and effective for the baby. It was unsurprising that the there were oscillations in temperature in this run because the same values of K_P and K_I were used as in the initial run and the same issues that may have caused the oscillations in the initial experiment were still relevant.

5 Conclusion

Our primary recommendations are to further iterate and experiment with this model, to have more accurate modelling (in terms of extraneous heat loss and the addition of a baby), and further research into heat constants and variable values in order to make the model simulate real life more effectively.

However, based on the results of our experimentation itself, our first recommendation is to include a power supply capable of generating at least 60 Watts of power - and not in discrete steps, but can vary continuously over a range. If you notice in the graphs of the experimental results, the power desired did not match the power output. If the baby warmer was powered with a supply that could output the model's desired power output, the results would be much better.

Another conclusion, based strictly on the raw results themselves, is to wait a while before placing the baby within the baby warmer. The oscillations get smaller as the experiment goes on, so optimally it would eventually reach 'steady state' as predicted in our model. We didn't let the experiment run long enough to reach this steady state, but a designer might consider the fact that with a design based strictly on the results of our first pass model, the warmers should be turned on quite a while before babies should be placed within the warmer.

Finally, a K_P value of 0.6 and K_I value of 0.0009 seems to work for many situations - in terms of decreasing steady state, transient, and disturbance-related error. Although our ultimate takeaway is that more work needs to be done on this model in order for it to serve as a true foundation for the creation of a baby warmer, it serves as a great first-pass subject to future iterations. We do feel that the K_P and K_I values that we got, since they worked fairly well for both experiments, are also values that designers working on the software aspect of the control mechanism could use.

In addition, future steps could be taken to limit power consumption, limit the cost of materials needed, and make the entire mechanism as easy as possible to transport and use - especially if it going to be used in poorer and more rural areas.