HomeWork 12

Ques 1. A variation of the satisfiability problem is the MIN 2-SAT problem. The goal is the MIN 2-SAT problem is to find a truth assignment that minimizes the number of satisfied clauses. Give a 1/2 approximation algorithm that you can find for the problem.

Ans. We will construct a graph $G(V, E)$ such that each vertex represents clause c. For any two nodes v_i and v_j in V, the edge (v_i, v_j) is in E if and only if the corresponding clauses C_i and C_j are such that a variable appears in its complemented form in C_i and uncomplemented form in C_j or vice-versa. To construct truth-assignment, we construct a vertex cover V' for graph G such that $|V'|$ is at most twice that of minimum vertex cover for G. Then we construct a truth assignment that causes all clauses in $V - V'$ to be false.

Ques2. Write down the problem of finding a Min-s-t-Cut of a directed network with source s and sink t as an Integer Linear Program and explain your program.

Ans.

- variable - $f(e)$ (flow over edge e for all edges $e \in E$)
- Objective function -

  Minimize $\sum f(e)$ for all e out of s or into t.
- Constraints -

  $f(e) >= l_e$ for all edges $e \in E$

  $\sum f(e)$ for all e into $v = \sum f(e)$ for all e out of v, for all $v \in V$ except for s and t.

Ques 6. 720 students have pre-enrolled for the "Analysis of Algorithms" class of Fall. Each student must attend one of the 16 discussion sections, and each discussion section i has capacity for D_i students. The happiness level of a student assigned to a discussion section i is proportionate to $\alpha_i(D_i - S_i)$, where α_i is a parameter reflecting how well the air-conditioning system works for the room used for section i (the higher the better), and S_i is the actual number of students assigned to that section. We want to find out how many students to assign to each section in order to maximize total student happiness. Express the problem as a linear programming problem.

Ans.
- Variable - S_i
- Objective function -

$$\text{Maximize} \quad \sum_{i=1}^{16} \alpha_i (D_i - S_i)$$

- Constraints -

$$D_i - S_i \geq 0 \quad \text{for } 0 < i \leq 16$$
$$S_i \geq 0 \quad \text{for } 0 < i \leq 16$$

$$\sum_{i=1}^{16} S_i = 720$$

Ques4. A set of n space stations need your help in building a radar system to track spaceships traveling between them. The ith space station is located in 3D space at coordinates (x_i, y_i, z_i). The space stations never move. Each space station i will have a radar with power r_i, where r_i is to be determined. You want to figure how powerful to make each space stations radar transmitter, so that whenever any spaceship travels in a straight line from one station to another, it will always be in radar range of either the first space station (its origin) or the second space station (its destination). A radar with power r is capable of tracking space ships anywhere in the sphere with radius r centered at itself. Thus, a space ship is within radar range through its trip from space station i to space station j if every point along the line from (x_i, y_i, z_i) to (x_j, y_j, z_j) falls within either the sphere of radius r_i centered at (x_i, y_i, z_i) or the sphere of radius r_j centered at (x_j, y_j, z_j). The cost of each radar transmitter is proportional to its power, and you want to minimize the total cost of all the radar transmitters. You are given all of the $(x_1, y_1, z_1), \ldots, (x_n, y_n, z_n)$ values, and your job is to choose values for $r_1, \ldots, r_n$. Express this problem as a linear program.

(a) Describe your variables for the linear program.
(b) Write out the objective function.
(c) Describe the set of constraints for LP. You need to specify the number of constraints needed and describe what each constraint represents.

Ans. (a) variables -

r_i = the power of ith radar r transmitter

$i = 1, 2, \ldots n$

(b) Objective function -

Minimize $r_1 + r_2 + \ldots + r_n$

(c) Constraints -

$r_i + r_j \geq \text{sqrt}((x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2)$