Big-O

- How code slows as data grows
- Not the same as running time
- Big trend over time
- 10× data → ??× time
- Mathy, but doesn’t have to be

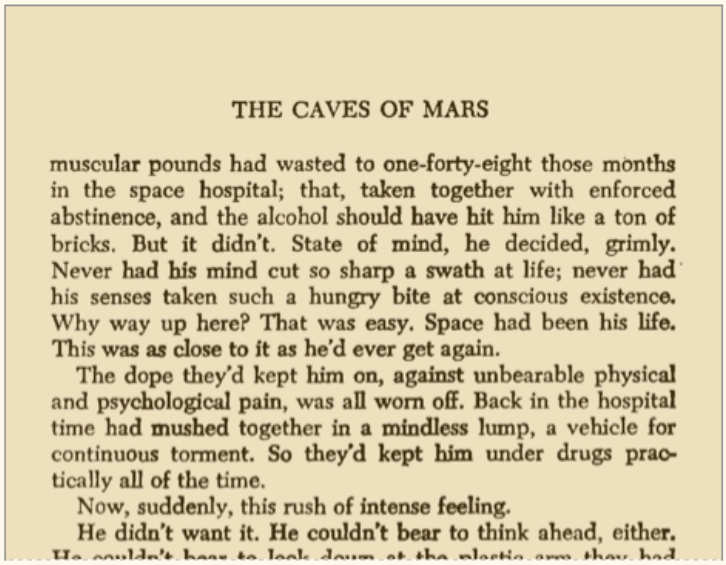
Terminology

- O(*blah blah* N *blah*)
- N: how much data
- “Order of”
- Not a function!

Counting beans

Finding words

Novel



THE CAVES OF MARS

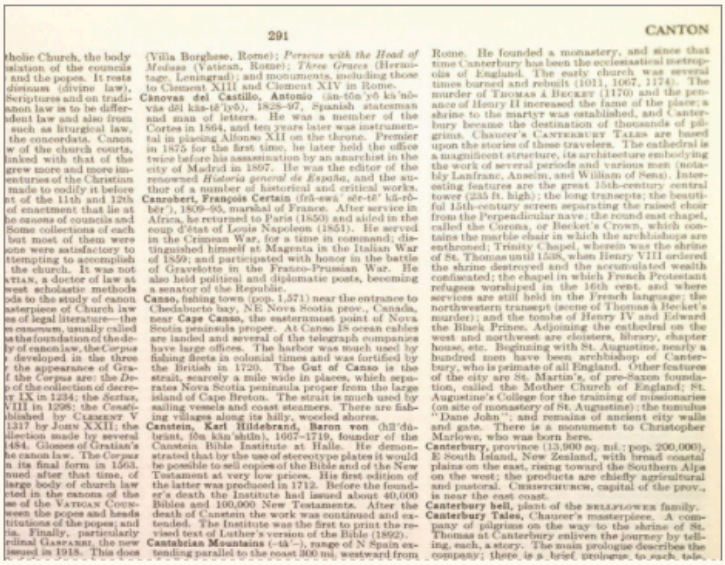
muscular pounds had wasted to one-forty-eight those months in the space hospital; that, taken together with enforced abstinence, and the alcohol should have hit him like a ton of bricks. But it didn't. State of mind, he decided, grimly. Never had his mind cut so sharp a swath at life; never had his senses taken such a hungry bite at conscious existence. Why way up here? That was easy. Space had been his life. This was as close to it as he'd ever get again.

The dope they'd kept him on, against unbearable physical and psychological pain, was all worn off. Back in the hospital time had mushed together in a mindless lump, a vehicle for continuous torment. So they'd kept him under drugs practically all of the time.

Now, suddenly, this rush of intense feeling.

He didn't want it. He couldn't bear to think ahead, either. He couldn't bear to look down at the plastic arm they had

Encyclopedia



291 CANTON

tholic Church, the body (Villa Borghese, Rome); *Perseus with the Head of* Rome. He founded a monastery, and since that
slation of the councils *Medusa* (Vatican, Rome); *Three Graces* (Hermi- time Canterbury has been the ecclesiastical metrop-
and the popes. It rests tage, Leningrad); and monuments, including those olis of England. The early church was several
divinum (divine law), to Clement XIII and Clement XIV in Rome. times burned and rebuilt (1011, 1067, 1174). The
Scriptures and on tradi- **Cánovas del Castillo, Antonio** (än-tō'n yō̇ kä'nō- murder of THOMAS À BECKET (1170) and the pen-
anon law is to be differ- vās dĕl käs-tē'lyō), 1828–97, Spanish statesman ance of Henry II increased the fame of the place; a
dent law and also from and man of letters. He was a member of the shrine to the martyr was established, and Canter-
such as liturgical law, Cortes in 1864, and ten years later was instrumen- bury became the destination of thousands of pil-
the concordats. Canon tal in placing Alfonso XII on the throne. Premier grims. Chaucer's CANTERBURY TALES are based
w of the church courts, in 1875 for the first time, he later held the office upon the stories of these travelers. The cathedral is
linked with that of the twice before his assassination by an anarchist in the a magnificent structure, its architecture embodying
grew more and more in- city of Madrid in 1897. He was the editor of the the work of several periods and various men (nota-
ventures of the Christian renowned *Historia general de España*, and the au- bly Lanfranc, Anselm, and William of Sens). Inter-
made to codify it before thor of a number of historical and critical works. esting features are the great 15th-century central
nt of the 11th and 12th **Canrobert, François Certain** (frăn-swä' sĕr-tăⁿ' kă-rō- tower (235 ft. high); the long transepts; the beauti-
of enactment that lie at bĕr'), 1809–95, marshal of France. After service in ful 15th-century screen separating the raised choir
he *canons* of councils and Africa, he returned to Paris (1850) and aided in the from the Perpendicular nave; the round east chapel,
Some collections of each coup d'état of Louis Napoleon (1851). He served called the Corona, or Becket's Crown, which con-
but most of them were in the Crimean War, for a time in command; dis- tains the marble chair in which the archbishops are
one were satisfactory to tinguished himself at Magenta in the Italian War enthroned; Trinity Chapel, wherein was the shrine
ttempting to accomplish of 1859; and participated with honor in the battle of St. Thomas until 1538, when Henry VIII ordered
the church. It was not of Gravelotte in the Franco-Prussian War. He the shrine destroyed and the accumulated wealth
ATIAN, a doctor of law at also held political and diplomatic posts, becoming confiscated; the chapel in which French Protestant
west scholastic methods a senator of the Republic. refugees worshiped in the 16th cent. and where
ods to the study of canon **Canso,** fishing town (pop. 1,571) near the entrance to services are still held in the French language; the
asterpiece of Church law Chedabucto bay, NE Nova Scotia prov., Canada, northwestern transept (scene of Thomas à Becket's
es of legal literature—the near **Cape Canso,** the easternmost point of Nova murder); and the tombs of Henry IV and Edward
canonum, usually called Scotia peninsula proper. At Canso 18 ocean cables the Black Prince. Adjoining the cathedral on the
as the foundation of the de- are landed and several of the telegraph companies west and northwest are cloisters, library, chapter
y of canon law, the *Corpus* have large offices. The harbor was much used by house, etc. Beginning with St. Augustine, nearly a
developed in the three fishing fleets in colonial times and was fortified by hundred men have been archbishop of Canter-
the appearance of Gra- the British in 1720. The Gut of Canso is the bury, who is primate of all England. Other features
f the *Corpus* are: the *De-* strait, scarcely a mile wide in places, which sepa- of the city are St. Martin's, of pre-Saxon foundation, called the Mother Church of England; St. Augustine's College for the training of missionaries (on site of monastery of St. Augustine); the tumulus "Dane John"; and remains of ancient city walls and gate. There is a monument to Christopher Marlowe, who was born here.
Canterbury, province (13,900 sq. mi.; pop. 200,000), E South Island, New Zealand, with broad coastal plains on the east, rising toward the Southern Alps on the west; the products are chiefly agricultural and pastoral. CHRISTCHURCH, capital of the prov., is near the east coast.
Canterbury bell, plant of the BELLFLOWER family.
Canterbury Tales, Chaucer's masterpiece. A company of pilgrims on the way to the shrine of St. Thomas at Canterbury enliven the journey by telling, each, a story. The main prologue describes the company; there is a brief prologue to each tale,

rates Nova Scotia peninsula proper from the large island of Cape Breton. The strait is much used by sailing vessels and coast steamers. There are fishing villages along its hilly, wooded shores.
Canstein, Karl Hildebrand, Baron von (hĭl'dē-brănt, fŏn kän'shtīn), 1667–1719, founder of the Canstein Bible Institute at Halle. He demonstrated that by the use of stereotype plates it would be possible to sell copies of the Bible and of the New Testament at very low prices. His first edition of the latter was produced in 1712. Before the founder's death the Institute had issued about 40,000 Bibles and 100,000 New Testaments. After the death of Canstein the work was continued and extended. The Institute was the first to print the revised text of Luther's version of the Bible (1892).
Cantabrian Mountains (-tā'-), range of N Spain extending parallel to the coast 300 mi. westward from

O(N)

O(log N)

Other terms

- O(1): constant time
- O(N): linear
- $O(N^2)$: quadratic
- Big-O:
  - complexity
  - time complexity
  - algorithmic complexity
  - asymptotic complexity

Determining Big-O

- Identify your code
- Identify N
- Count the steps in a *typical* run
- Keep the most significant part

An Example

```
moms = [
    ("Ned", "Eleanor"),
    ("Max", "Susan"),
    ("Susan", "Shelly"), ...
]
def find_mom(moms, child):
    """Find the mom of `child`."""
    for child_name, mom_name in moms:    # 3 * N/2
        if child == child_name:          # 1 * N/2
            return mom_name              # 1
    return None
```

3N/2 + N/2 + 1 → 2N + 1 → O(N)

find_mom is O(N)

Another example

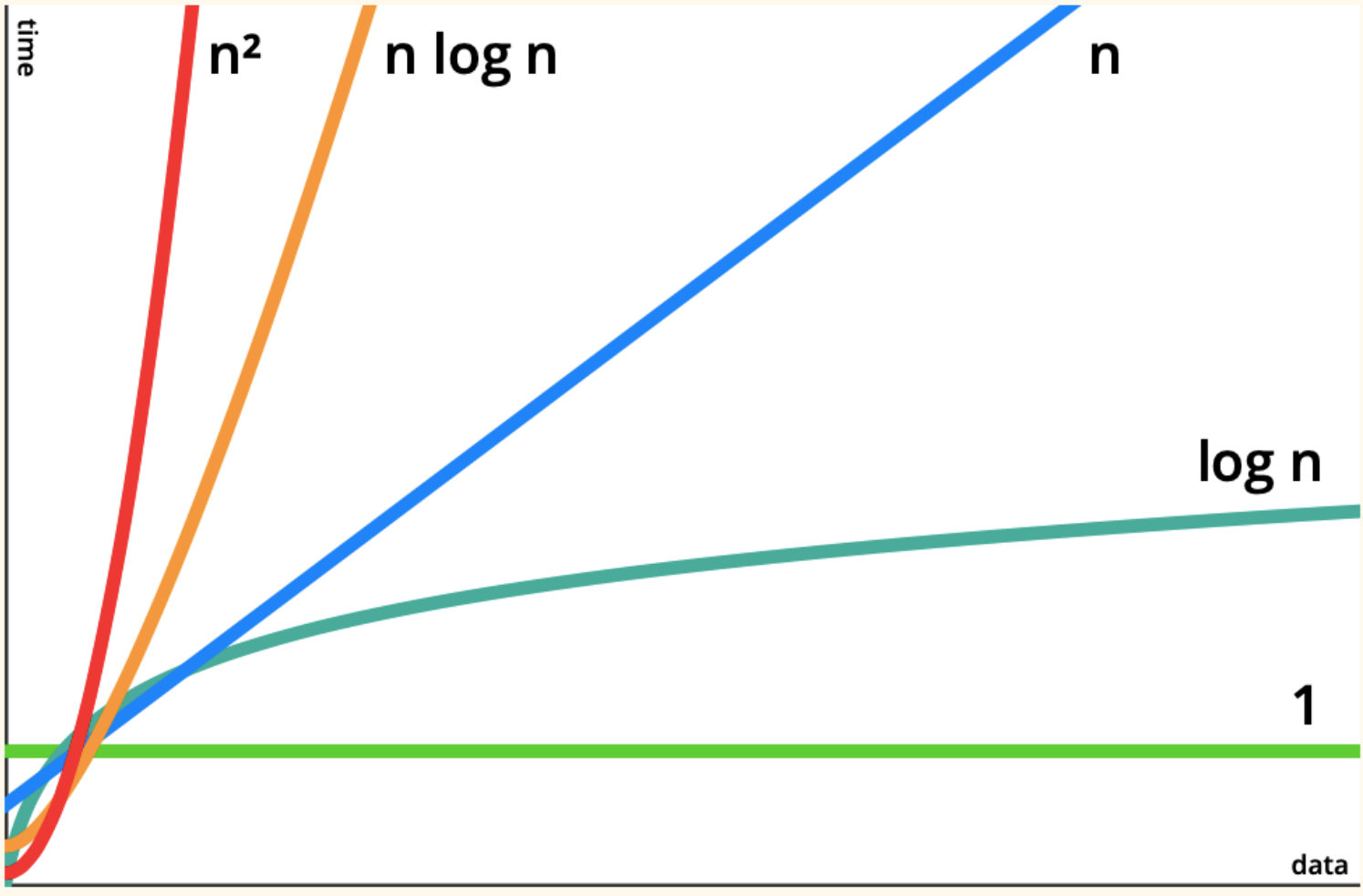
```
def how_many_grandmothers(moms):
    """How many moms are grand-moms?"""
    grandmothers = 0
    for child, mom in moms:                  # N
        grandma = find_mom(moms, mom)        # N * N
        if grandma:                          # N
            grandmothers += 1                # kN
    return grandmothers                      # 1
```

$N^2 + kN + 1 \quad \rightarrow \quad O(N^2)$

Ideal: O(1)

- Seems impossible!?
- `len(mylist)`
- `mydict[some_key]`

The Graph



Python complexities

Lists `[a, b, c, ...]`	
`mylist.append(val)`	O(1)
`mylist[i]`	O(1)
`val in mylist`	O(N)
`for val in mylist:`	O(N)
`mylist.sort()`	O(N log N)

Dicts `{k:v, ...}`	
`mydict[key] = val`	O(1)
`mydict[key]`	O(1)
`key in mydict`	O(1)
`for key in mydict:`	O(N)

Sets `{a, b, c, ...}`	
`myset.add(val)`	O(1)
`val in myset`	O(1)
`for val in myset:`	O(N)

Pro-tip: replace lists with sets

Trade-offs

"Replace list lookup with set lookup"

```
#.. make a list ..
if thing in my_list:     # O(N)
```

Good

```
#.. make a set ..
if thing in my_set:      # O(1)
```

Bad

```
#.. make a list ..
my_set = set(my_list)    # O(N)
if thing in my_set:      # O(1)
```

Good

```
#.. make a list ..
my_set = set(my_list)    # O(N)
for many_times:
    if thing in my_set: # O(1)
```

Slow

```
def __init__(self):
    self.items = []

def __getitem__(self, pt):
    for key, value in self.items:
        if key == pt:
            return value

    value = []
    self.items.append((pt, value))
    return value
```

Fast

```
def __init__(self):
    self.items = {}      # pt -> value
    self.rounds = {}     # pt -> pt

def __getitem__(self, pt):
    val = self.items.get(pt)
    if val is not None:
        return val

    for jitter in [0, 0.5]:
        pt_rnd = rounded(pt, jitter)
        pt0 = self.rounds.get(pt_rnd)
        if pt0 is not None:
            return self.items[pt0]

    self.items[pt] = val = []
    for jitter in [0, 0.5]:
        pt_rnd = rounded(pt, jitter)
        self.rounds[pt_rnd] = pt

    return val
```

Slow

```
def __init__(self):
    self.items = []

def __getitem__(self, pt):
    for key, value in self.items:
        if key == pt:
            return value

    value = []
    self.items.append((pt, value))
    return value
```

Fast

O(N) O(1)

```
def __init__(self):
    self.items = {}     # pt -> value
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def __getitem__(self, pt):
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        if pt0 is not None:
            return self.items[pt0]

    self.items[pt] = val = []
    for jitter in [0, 0.5]:
        pt_rnd = rounded(pt, jitter)
        self.rounds[pt_rnd] = pt

    return val
```
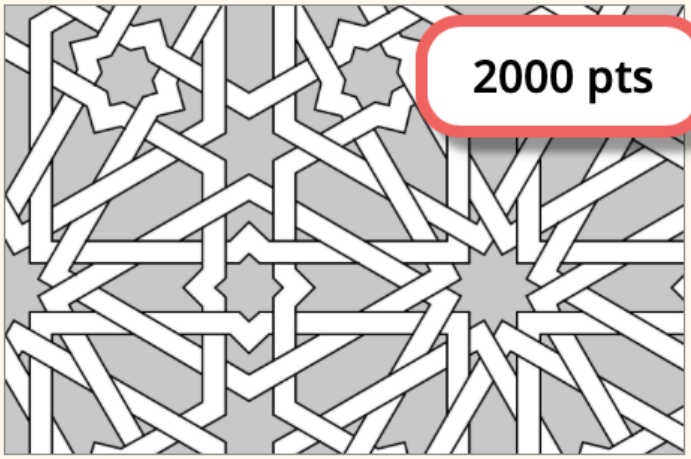

Slow

20s ☹

```
def __init__(self):
    self.items = []

def __getitem__(self, pt):
    for key, value in self.items:
        if key == pt:
            return value

    value = []
    self.items.append((pt, value))
    return value
```



Fast

O(N) O(1)

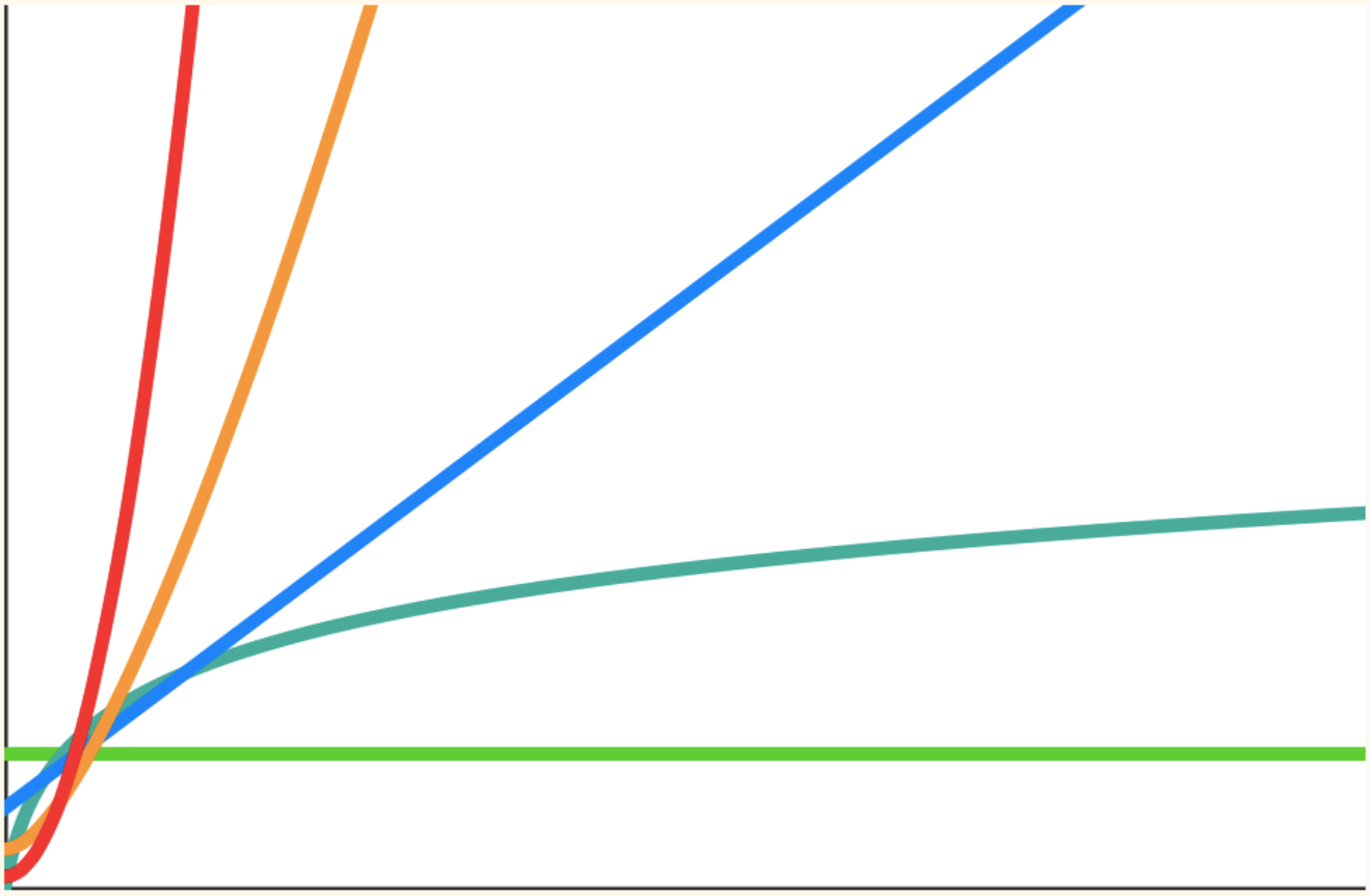
0.4s! ☺

```
def __init__(self):
    self.items = {}
    self.rounds = {}     # pt -> pt

def __getitem__(self, pt):
    val = self.items.get(pt)
    if val is not None:
        return val

    for jitter in [0, 0.5]:
        pt_rnd = rounded(pt, jitter)
        pt0 = self.rounds.get(pt_rnd)
        if pt0 is not None:
            return self.items[pt0]

    self.items[pt] = val = []
    for jitter in [0, 0.5]:
        pt_rnd = rounded(pt, jitter)
        self.rounds[pt_rnd] = pt

    return val
```

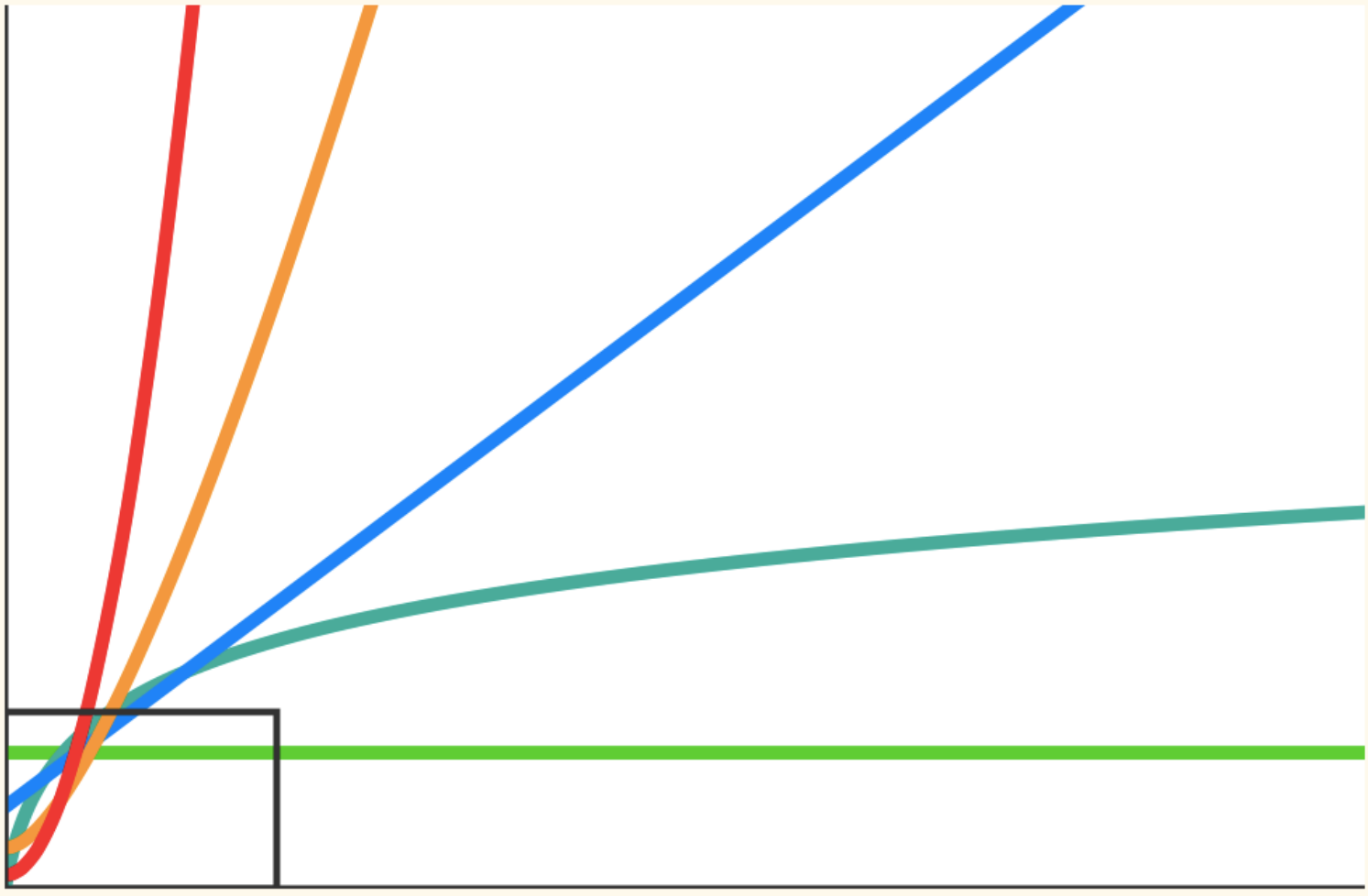
More possibilities

- Complexities
  - $O(N^3)$, $O(N^4)$, ...
  - $O(2^N)$, $O(N^N)$
  - O(N!)
- Dimensions
  - N, M, k, etc
  - O((n+k) log n)

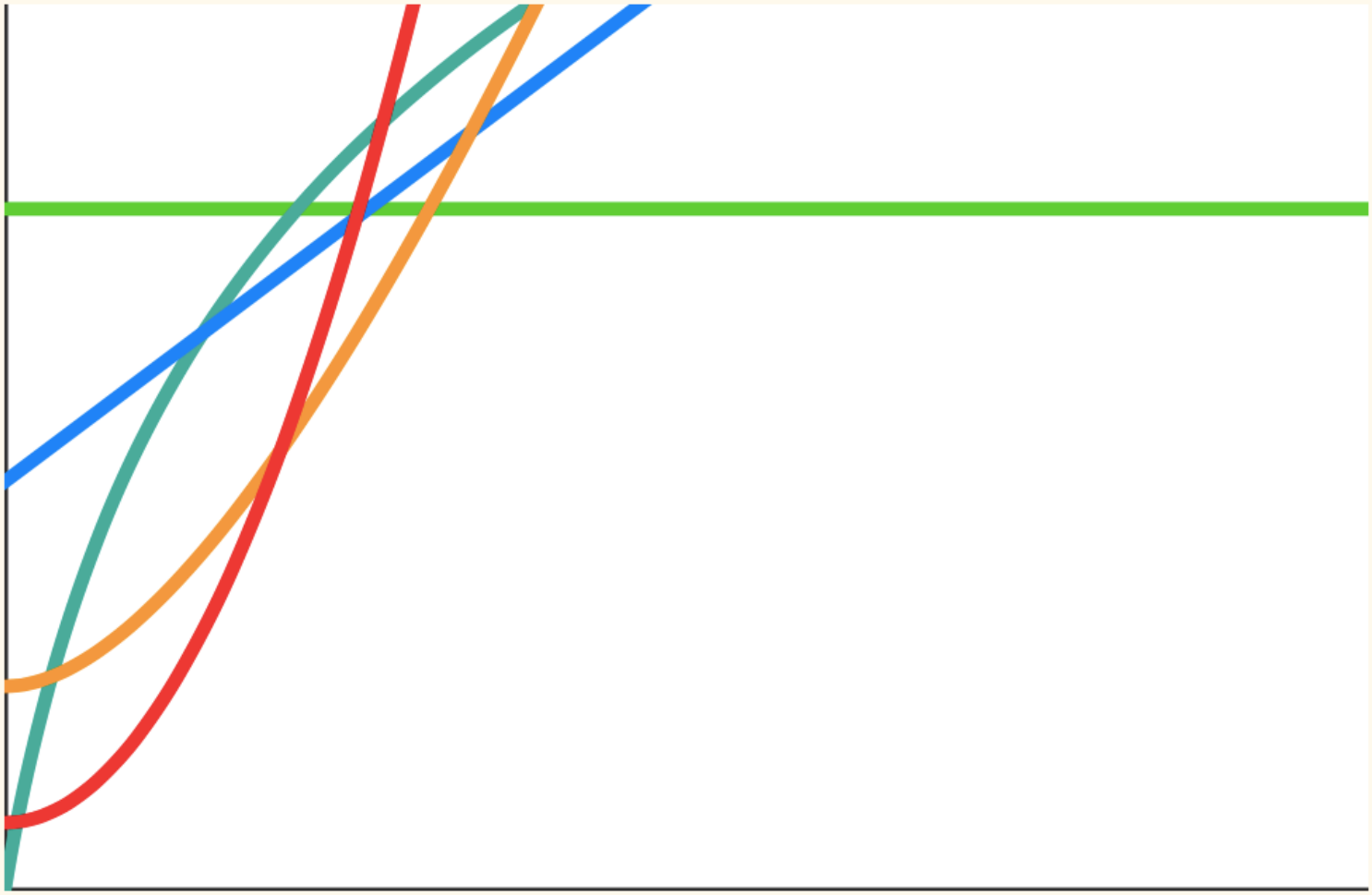
Small numbers



Small numbers



Small numbers



Small numbers

http://www.globalnerdy.com/2014/12/02/programming-poster-of-the-day-and-rob-pikes-5-rules-of-programming/

Advanced: Worst case

- Typical case vs worst case

```
# Typical
s = set()
for i in range(50000):
    s.add(i * 47)                       # O(1)
# O(N), wall time: 10.3 ms
```

```
# Worst
s = set()
for i in range(50000):
    s.add(i * (2**61-1))                # O(N)
# O(N²), wall time: 34.2 s   (3300x slower!)
```

- Dicts also
- Hash randomization prevents DDOS

Advanced: more math

- Lots more: O, o, Ω, ω, Θ
- You don't need it