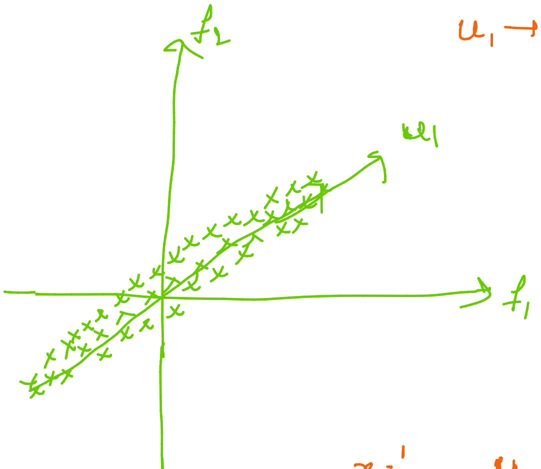
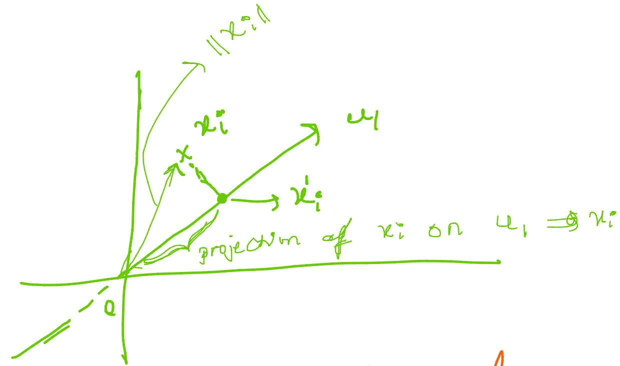
Variance maximization

25 June 2021 19:18

Variance Maximization Interpretation.

$u_1 \rightarrow$ unit vector.





$x_i' \rightarrow$ projection of x_i on u_1

$$x_i' = \frac{u_1 \cdot x_i}{\|u_1\|}$$

$D = \{x_i\}_{i=1}^{n}$

$D' = \{x_i'\}_{i=1}^{n}$

Here, $\|u_1\| = 1$, Hence, $\boxed{x_i' = u_1 \cdot x_i}$

$\Rightarrow \boxed{x_i' = u_1^T x_i}$

$\|^{ly}$, $\boxed{\bar{x}_i' = u_1^T \bar{x}_i}$ $\rightarrow$ mean of x_i's

mean of x_i'

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find u_1, such that, variance of projection is maximum.

$$\Rightarrow \operatorname{Var}\{\underbrace{u_1^T x_i}_{1\times n \; n\times 1}\}_{i=1}^{n} = \frac{1}{n}\sum_{i=1}^{n}(u_1^T x_i - u_1^T \bar{x}_i)^2$$

$= 1$

$$= \frac{1}{n}\sum_{i=1}^{n}(u_1^T x_i)^2$$

$\bar{x}_i = [0, 0, 0. \ldots, 0]$

$X \rightarrow$ col. std.

Hence mean $= 0$

var $= 1$

So our optimization problem becomes,

$$\max_{u_1} \sum_{i=1}^{n}(u_1^T x_i)^2 \quad \text{s.t. } \|u_1\|^2 = 1$$

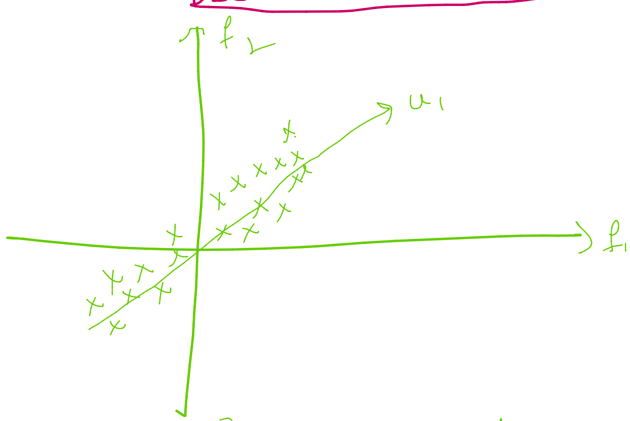
So we have to find u_1 s.t. $\sum(u_1^T x_i)^2$ is max. with contraint $\boxed{u_1^T u_1 = 1}$

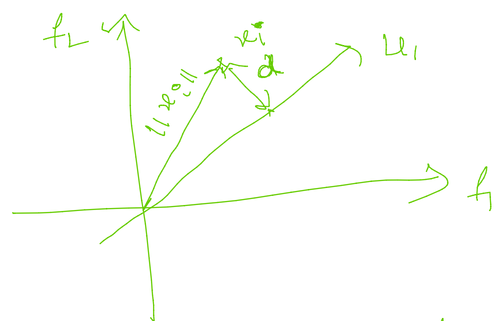
$$\Rightarrow \boxed{\max_{u_1} \sum_{i=1}^{n} (u_1^T x_i)^2 \quad \text{s.t.} \quad u_1^T u_1 = 1 = \|u_1\|^2}$$

There is constraint that $\|u_1\|^2 = 1$, i.e. u_1 should be unit vector.

Because if we don't have this constraint, since we are maximizing the sum, u_1 could go to ∞ to maximize the sum. Hence we are restricting u_1 to be a unit vector.

✱ DISTANCE MINIMIZATION INTERPRETATION





we want to calculate d,

By simple pythogoras thm,

$$d^2 = \|x_i\|^2 - (\|x_i\| \cos\theta)^2$$

$$\boxed{d^2 = \|x_i{}^2\| - (u_1^T x_i)^2}$$

We know, $\|x_i\|^2 = x_i^T x_i$,

We want to minimize distances,

Hence,

$$\boxed{\min_{u_1} \sum_{i=1}^{n} d_i^2}$$

$$\Rightarrow \min_{u_1} \sum_{i=1}^{n} \left(\underbrace{(x_i^T x_i) - (u_1^T x_i)^2}_{} \right)$$

$$\sum_{i=1} \underbrace{\qquad\qquad}_{d^2}$$

So our obj. function becomes,

$$\boxed{\min_{u_1} \sum_{i=1}^{n} (x_i^T x_i) - (u_1^T x_i)^2 \quad \text{such that } u_1^T u = 1}$$

We took squared distance, instead of mod, because it is differentiable for optimization.

mod is not differentiable.

So math is easy.