Distance

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Distances

There are many types of distances.

Most common distance is, $\boxed{\text{Euclidean distance.}}$

Consider any 2 points $x(x_1, x_2)$ & $y(y_1, y_2)$

$$\boxed{d(x,y) = \sqrt{(x_1-y_1)^2 + (x_2-y_2)^2}}$$

Generally,

$$\boxed{d(x,y) = \left(\sum_{i=1}^{n} (x_i - y_i)^2\right)^{1/2}}$$

This is the direct / shortest distance

Manhattan Distance :→

This is the total distance.

$$\boxed{d(x,y) = \sum_{i=1}^{n} |x_i - y_i|}$$

Minkowski distance

General form

$$d(x,y) = \left(\sum_{i=1}^{n} (x_i - y_i)^p\right)^{1/p}$$

$$d(x,y) = \left(\sum_{i=1} \right)$$

If $p = 1$, $d(x,y) \longrightarrow$ manhattan dist.

$p = 2$, $d(x,y) \Rightarrow$ Euclidean dist.

Cosine distance :—

$$\boxed{\text{Cosine dist.} = (1 - \text{cosine similarity})}$$

$$\boxed{\cos\theta = \frac{x_1 \cdot x_2}{\|x_1\| \, \|x_2\|}}$$

Relationship between Euclidean distance & cosine distance.

$$(\text{Euclidean dist})^2 = 2(\text{cosine dist.}) \quad \text{if } \|x_1\| = \|x_2\| = 1$$

$$\|x_1 - x_2\|^2 = (x_1 - x_2)^T (x_1 - x_2)$$

$$= x_1^T x_1 - x_1^T x_2 - x_2^T x_1 + x_2^T x_2$$

$$= \|x_1\|^2 - x_1^T x_2 - x_2^T x_1 + \|x_2\|^2$$

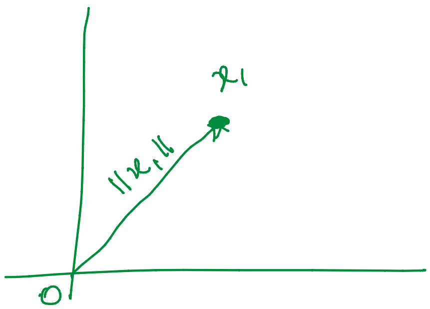
$$= 2 - 2(x_1^T x_2)$$

$$\|x_1 - x_2\|^2 = 2(1 - x_1^T x_2)$$

$$\|x_1 - x_2\|^2 = 2(1 - \cos\theta)$$

$$\boxed{(\text{Euclidean})^2 = 2(\text{cosine dist})}$$

Distance of a point from origin

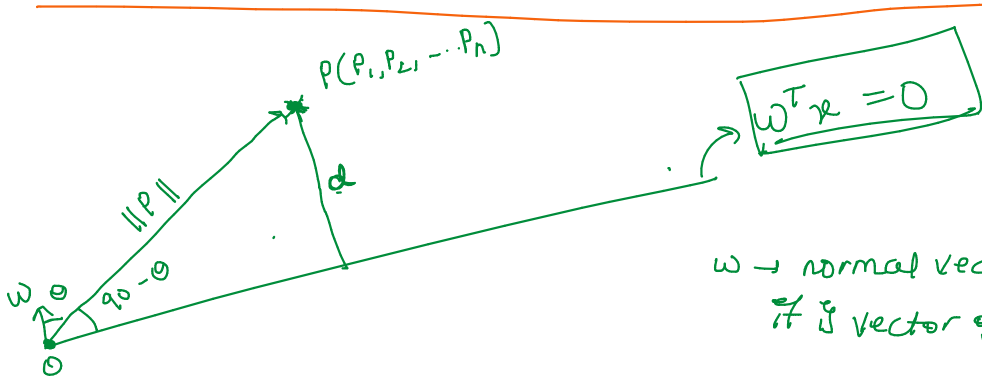


$$||x_1|| = \sqrt{\sum x_i^2}$$

$$\Rightarrow \boxed{||x_1|| = \sqrt{\sum x_i^T x_i}}$$

$$\Rightarrow \boxed{||x_1||^2 = x_1^T x_1}$$

Distance of a point from plane passing through origin



$w \rightarrow$ normal vector to the plane
it is vector of co-efficients

$$\frac{d}{||P||} = \sin(90-\theta)$$

$$\boxed{d = ||P|| \cos\theta}$$

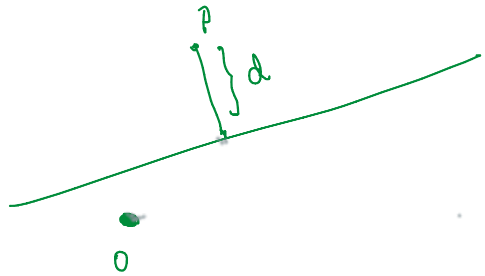
We know that, $w \cdot P = w^T P = ||w|| \, ||P|| \cos\theta$

$$\Rightarrow ||P|| \cos\theta = \frac{w^T P}{||w||}$$

Hence, $\boxed{d = \frac{w^T p}{\|w\|}}$ if $\|w\| = 1$, then $d = w^T p$

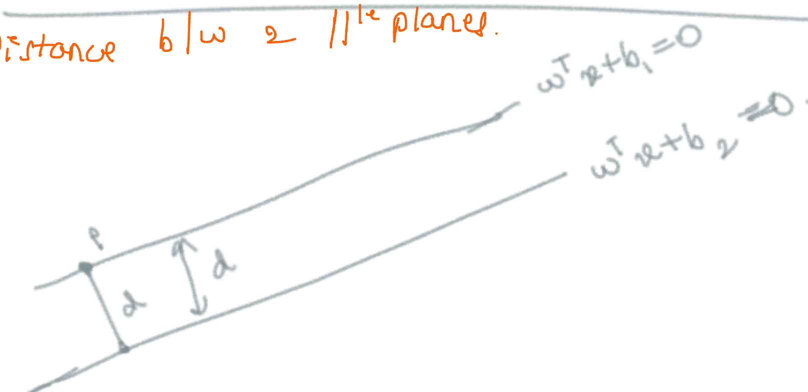
If line is not passing through origin :-

$\leftarrow w^T x + b = 0$



$$\boxed{d = \frac{w^T p + b}{\|w\|}}$$

Distance b/w 2 $\|^{le}$ planes.



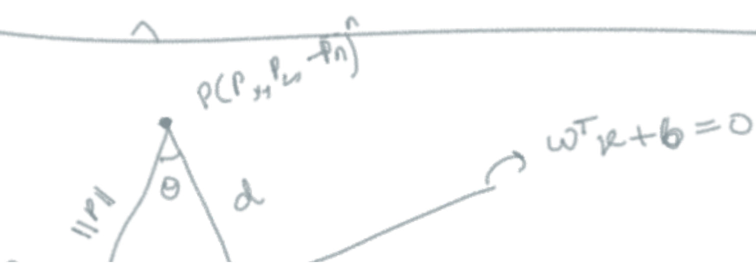
p is on $w^T x + b_1 = 0$

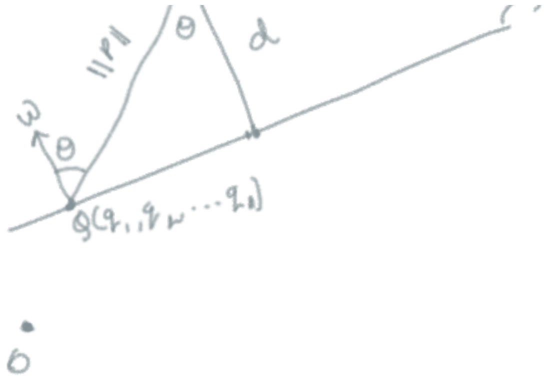
hence $w^T p + b_1 = 0$

$$w^T p = -b_1$$

$$d = \frac{w^T p + b_2}{\|w\|}$$

$$\Rightarrow \boxed{d = \frac{b_2 - b_1}{\|w\|}}$$





$$d = \|PQ\| \cos\theta.$$

$$d = \frac{w^T \cdot (PQ)}{\|w\|} = \frac{w \cdot (PQ)}{\|w\|}$$

$$PQ = \left((p_1 - q_1), (p_2 - q_2), \dots (p_n - q_n)\right)$$

$$PQ \cdot w = \left((p_1 - q_1), (p_2 - q_2), \dots (p_n - q_n)\right) \cdot (w_1, w_2, \dots w_n)$$

$$= (p_1 - q_1) w_1 + (p_2 - q_2) w_2 + \dots + (p_n - q_n) w_n$$

$$= p_1 w_1 - q_1 w_1 + p_2 w_2 - q_2 w_2 + \dots + p_n w_n - q_n w_n$$

$$= (p_1, p_2, p_3, \dots p_n) \cdot (w_1, w_2, \dots w_n) + \underbrace{(-q_1, -q_2, \dots -q_n)(w_1, w_2, \dots w_n)}_{-w^T q}$$

$$\boxed{w^T PQ = w^T p + b}$$

we k.t.,

$$w^T q + b = 0$$

$$b = -w^T q$$

Hence, $\boxed{d = \dfrac{w^T p + b}{\|w\|}}$