Variance maximization

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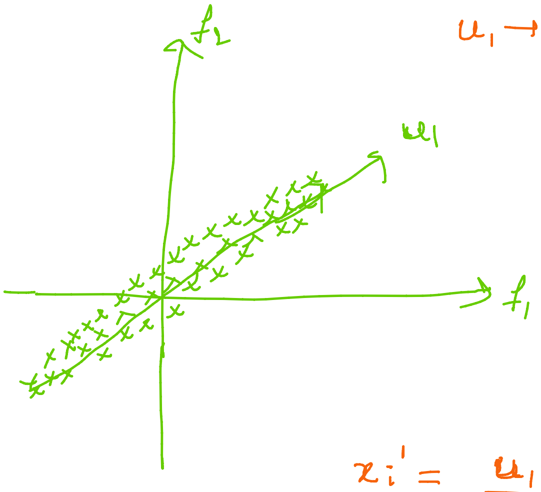
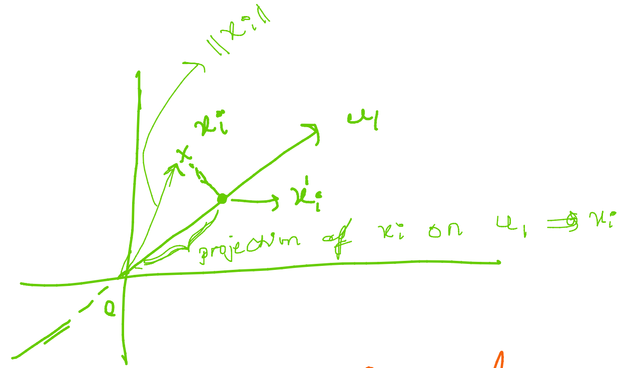
# Variance Maximization Interpretation.




$u_1 \rightarrow$ unit vector.




$x_i' \rightarrow$ projection of $x_i$ on $u_1$

$$D = \{x_i\}_{i=1}^{n}$$

$$D' = \{x_i'\}_{i=1}^{n}$$

$$x_i' = \frac{u_1 \cdot x_i}{\|u_1\|}$$

Here, $\|u_1\| = 1$, Hence, $\boxed{x_i' = u_1 \cdot x_i}$

$$\Rightarrow \boxed{x_i' = u_1^T x_i}$$

$\|^{ly}$, $\boxed{\bar{x}_i' = u_1^T \bar{x}_i}$ → mean of $x_i$'s

mean of $x_i'$

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find $u_1$, such that, variance of projection is maximum.

$$\Rightarrow \underbrace{\operatorname{Var}\{u_1^T x_i\}_{i=1}^{n}}_{\substack{1\times n \;\; n\times 1 \\ =1}} = \frac{1}{n}\sum_{i=1}^{n}\left(u_1^T x_i - u_1^T \bar{x}_i\right)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(u_1^T x_i\right)^2$$

$\bar{x}_i = [0, 0, 0, \ldots, 0]$

$X \rightarrow$ col. std.

Hence mean $= 0$
var $= 1$

So our optimization problem becomes,

$$\max_{u_1} \sum_{i=1}^{n}\left(u_1^T x_i\right)^2 \quad \text{s.t.} \quad \|u_1\|^2 = 1$$

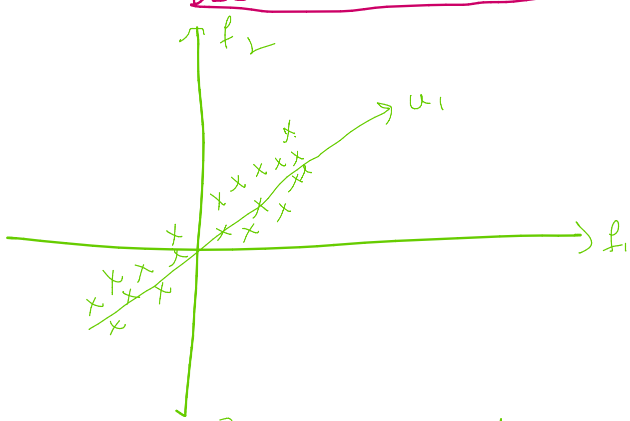
So we have to find $u_1$ s.t. $\sum (u_1^T x_i)^2$ is max. with contraint $\boxed{u_1^T u_1 = 1}$

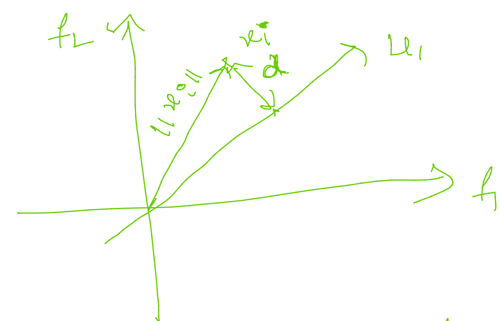
$$\Rightarrow \boxed{\max_{u_1} \sum_{i=1}^{n} (u_1^T x_i)^2 \quad s.t. \quad u_1^T u_1 = 1 = \|u_1\|^2}$$

There is constraint that $\|u_1\|^2 = 1$, i.e. $u_1$ should be unit vector.

Because if we don't have this constraint, since we are maximizing the sum, $u_1$ could go to $\infty$ to maximize the sum. Hence we are restricting $u_1$ to be a unit vector.

## * DISTANCE MINIMIZATION INTERPRETATION




we want to calculate $d$,

By simple pythogoras thm,

$$d^2 = \|x_i\|^2 - (\|x_i\| \cos\theta)^2$$

$$\boxed{d^2 = \|x_i^2\| - (u_1^T x_i)^2}$$

We know, $\|x_i\|^2 = x_i^T x_i$,

We want to minimize distances,

Hence,

$$\boxed{\min_{u_1} \sum_{i=1}^{n} d_i^2}$$



$$\Rightarrow \min_{u_1} \sum_{i=1}^{n} \left( (x_i^T x_i) - (u_1^T x_i)^2 \right)$$

$$\underbrace{\qquad\qquad}_{d^2}$$

So our obj. function becomes,

$$\boxed{\min_{u_1} \sum_{i=1}^{n} (x_i^T x_i) - (u_1^T x_i)^2 \quad \text{such that } u_1^T u = 1}$$

We took squared distance, instead of mod, because it is differentiable for optimization.
mod is not differentiable.
So math is easy.