

Distance

There are many types of distances.

Most common distance is, Euclidean distance.

Consider any 2 points $x(x_1, x_2)$ & $y(y_1, y_2)$

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$$

Generally,

$$d(x, y) = \left(\sum_{i=1}^n (x_i - y_i)^2 \right)^{1/2}$$

This is the direct / shortest distance

Manhattan Distance :-

This is the total distance.



Manhattan distance

$$d(x, y) = \sum_{i=1}^n |x_i - y_i|$$

**Praveen Hegde
CONFIDENTIAL**

Minkowski distance

General form

$$d(x, y) = \left(\sum_{i=1}^n (x_i - y_i)^p \right)^{1/p}$$

$$\text{euclidean} = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$$

If $p=1$, $d(x,y) \rightarrow$ manhattan dist.

$p=2$, $d(x,y) \Rightarrow$ Euclidean dist.

Cosine distance:

$$\text{cosine dist.} = (1 - \text{cosine similarity})$$

$$\cos\theta = \frac{\mathbf{x}_1 \cdot \mathbf{x}_2}{\|\mathbf{x}_1\| \|\mathbf{x}_2\|}$$

Relationship between Euclidean distance & cosine distance.

$$(\text{Euclidean dist})^2 = 2(1 - \text{cosine dist.}) \quad \text{if } \|\mathbf{x}_1\| = \|\mathbf{x}_2\| = 1$$

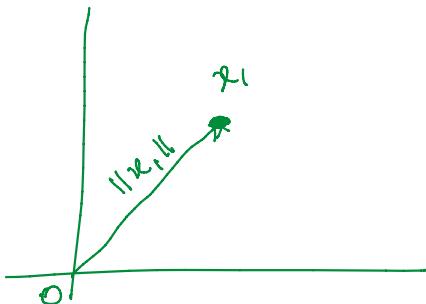
$$\begin{aligned} \|\mathbf{x}_1 - \mathbf{x}_2\|^2 &\equiv (\mathbf{x}_1 - \mathbf{x}_2)^T (\mathbf{x}_1 - \mathbf{x}_2) \\ &= \mathbf{x}_1^T \mathbf{x}_1 - \mathbf{x}_1^T \mathbf{x}_2 - \mathbf{x}_2^T \mathbf{x}_1 + \mathbf{x}_2^T \mathbf{x}_2 \\ &= \|\mathbf{x}_1\|^2 - \mathbf{x}_1^T \mathbf{x}_2 - \mathbf{x}_2^T \mathbf{x}_1 + \|\mathbf{x}_2\|^2 \\ &= 2 - 2(\mathbf{x}_1^T \mathbf{x}_2) \end{aligned}$$

$$\|\mathbf{x}_1 - \mathbf{x}_2\|^2 = 2(1 - \mathbf{x}_1^T \mathbf{x}_2)$$

$$\|\mathbf{x}_1 - \mathbf{x}_2\|^2 = 2(1 - \cos\theta)$$

$$(\text{Euclidean})^2 = 2(\text{cosine dist})$$

Distance of a point from origin

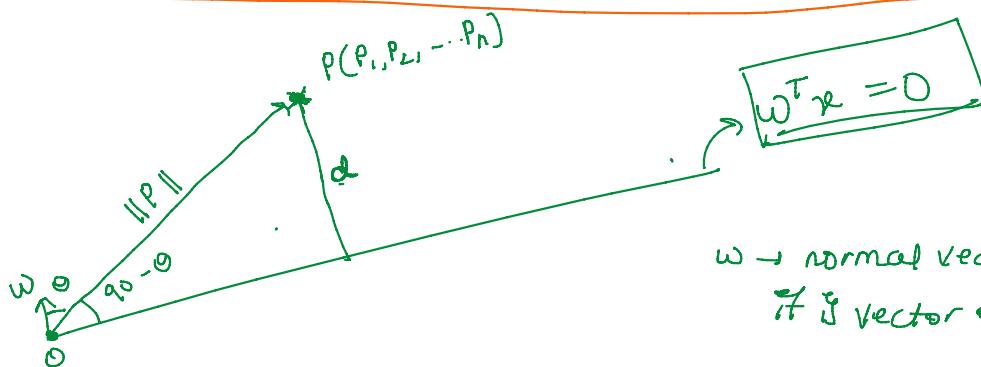


$$\|x_i\| = \sqrt{\sum x_i^2}$$

$$\Rightarrow \|x_i\| = \sqrt{\sum x_i^T x_i}$$

$$\Rightarrow \|x_i\|^2 = x_i^T x_i$$

Distance of a point from plane passing through origin



$w \rightarrow$ normal vector to the plane
it is vector of ω -coefficients

$$\frac{d}{\|p\|} = \sin(90 - \theta)$$

$$d = \|p\| \cos \theta$$

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We know that, $w \cdot p = w^T p = \|w\| \|p\| \cos \theta$

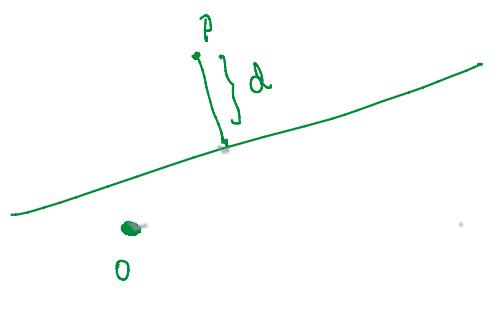
$$\Rightarrow \|p\| \cos \theta = \frac{w^T p}{\|w\|}$$

Hence,

$$d = \frac{\omega^T p}{\|\omega\|}$$

if $\|\omega\| = 1$,
then $d = \omega^T p$

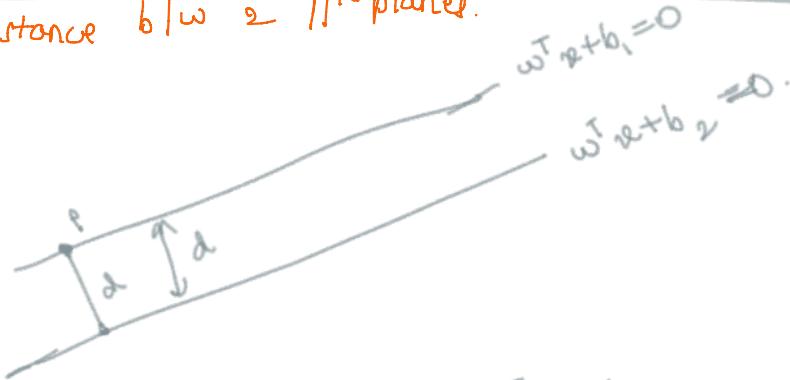
If line is not passing through origin \rightarrow



$\omega^T p + b = 0$

$$d = \frac{\omega^T p + b}{\|\omega\|}$$

Distance $b/\omega \equiv \parallel$ plane.



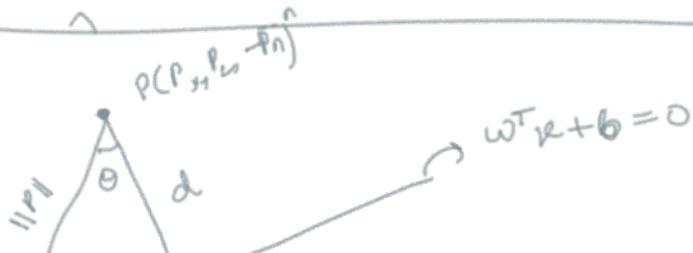
$$d = \frac{\omega^T p + b_2}{\|\omega\|}$$

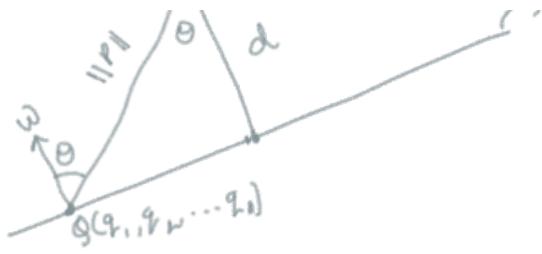
p is on $\omega^T p + b_1 = 0$

Hence $\omega^T p + b_1 = 0$

$$\underline{\omega^T p = -b_1}$$

$$\Rightarrow d = \frac{b_2 - b_1}{\|\omega\|}$$





$$d = \|\mathbf{pq}\| \cos \theta.$$

$$d = \frac{\omega^T (\mathbf{pq})}{\|\omega\|} = \frac{\omega \cdot (\mathbf{pq})}{\|\omega\|}$$

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6

$$\mathbf{pq} = (p_1 - q_1, (p_2 - q_2), \dots, (p_n - q_n))$$

$$\mathbf{pq} \cdot \omega = ((p_1 - q_1), (p_2 - q_1), \dots, (p_n - q_n)) \cdot (\omega_1, \omega_2, \dots, \omega_n)$$

$$= (p_1 - q_1)\omega_1 + (p_2 - q_1)\omega_2 + \dots + (p_n - q_n)\omega_n$$

$$= p_1\omega_1 - q_1\omega_1 + p_2\omega_2 - q_2\omega_2 + \dots + p_n\omega_n - q_n\omega_n$$

$$= (p_1, p_2, p_3, \dots, p_n) - (q_1, q_2, \dots, q_n) \cdot (\omega_1, \omega_2, \dots, \omega_n)$$

$$\boxed{\underline{\omega^T \mathbf{pq}}} = \boxed{\underline{\omega^T p + b}}$$

$$\begin{aligned} & \omega_k t, \quad -\omega^T q \\ & \omega^T q + b = 0 \\ & b = -\omega^T q \end{aligned}$$

Hence,

$$\boxed{d = \frac{\omega^T p + b}{\|\omega\|}}$$

REVIEWED

By Praveen Hegde at 6:59 pm, Aug 29, 2021