7/4/22
9:30pm

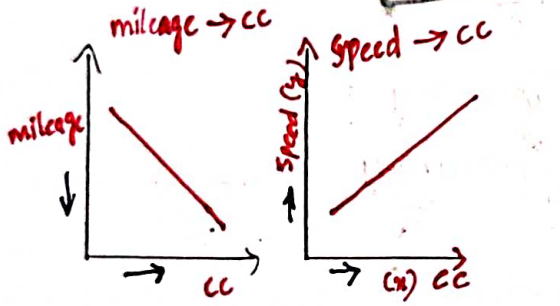
— Regression

Predicting Output which is Continous

- Regression
  - Linear Regression
  - Non-Linear Regression

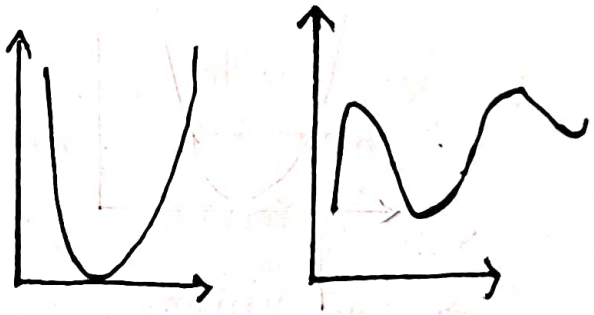
("Assuming the relation between x & y is Linear")

Ex:- In The Form of $(y = mx + c)$

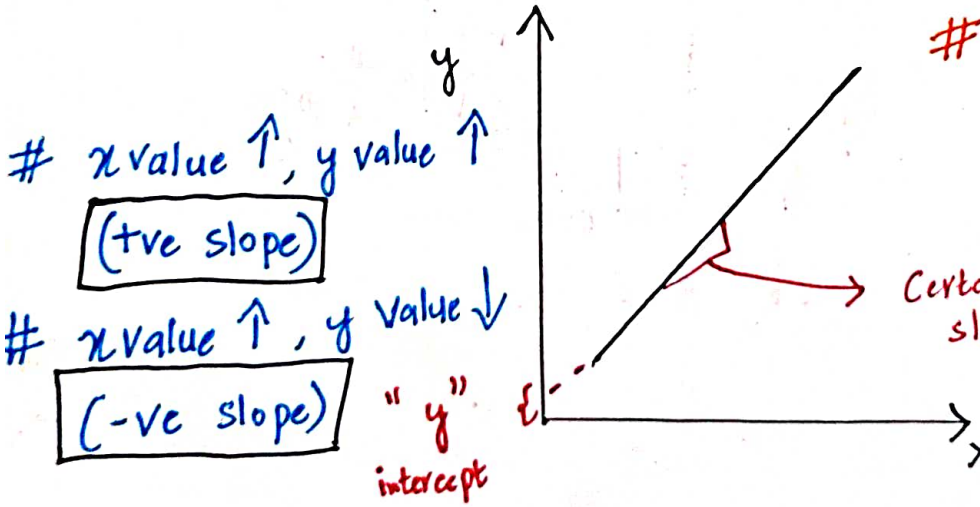


* mileage decreases ↓ cc increases ↑
* cc increases ↑ Speed increases ↑

Any line other than straight line is Non-Linear



Quadratic equation



x value ↑, y value ↑ (+ve slope)

x value ↑, y value ↓ (-ve slope)

Any relation in terms of Straight Line, it is called as "Linear Regression"

Ex:-

X	Y
1	15
2	25
3	35
4	45
5	55
6	? 35
7	35
8	35.

In Early Ages, They took "Avg" values.

Like

$$\frac{15+25+35+45+55}{5} = 35$$

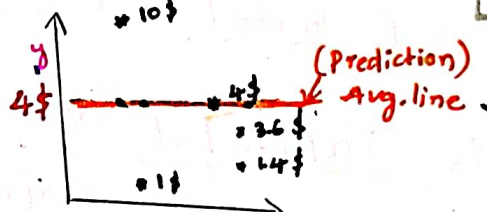
→ They say, Avg Line is Best Fit Line.

* Best rants

tips (waiter)

	Bill amount
1. 10 $	– (not given)
2. 1 $	–
3. 4 $	–
4. 3.6 $	–
5. 1.4 $	–
6 → Avg	4$
7 → 4$ Every time Answer is 4$ (because of Avg line)	

So, For only One Variable / column – (Avg)



→ For X value [Predic Va

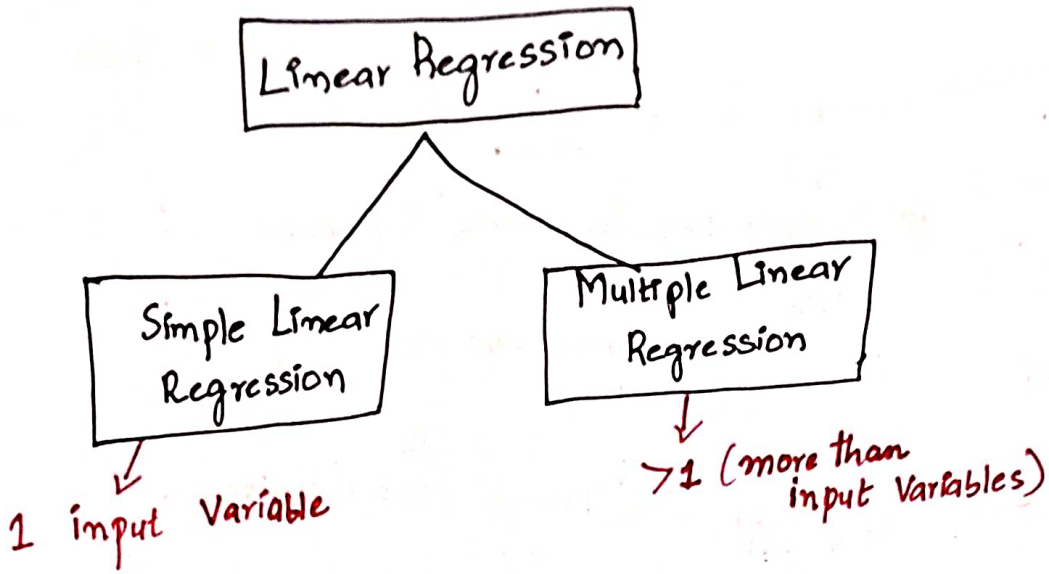
For Single Variable

Best Fit line is Average line

H₀ : Average line is best fit line

H₁ : Regression line > Avg. line.

Y	avg. line Y – Prediction	Error	error²
10	4	6	36
1	4	-3	9
4	4	0	0
3.6	4	-0.4	0.16
1.4	4	-2.6	6.76
		0	51.92



Simple Linear Regression :

→ In Simple Linear regression, We predict the value of One Variable "Y" based on other Variable "X"

→ "X" is independent Variable (or) Input variable (or) Exploratory variable

"y" is dependent Variable (or) output variable (or) response variable

Ex:-

Weight	CC	Speed	Mileage → O/P
indepen dent	in. var	in. var	(it depends on, weight, cc, speed)

Que :- Why it is Simple ?

Ans :- Because, it Examines Relation ship between two Variables Only.

X	Y

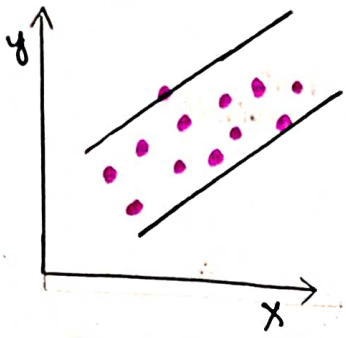
$y = x$

Que :- Why linear ?

Ans :- When the independent Variable increases (or Decreases) The dependent Variable increases (or Decreases) in a linear Fashion.

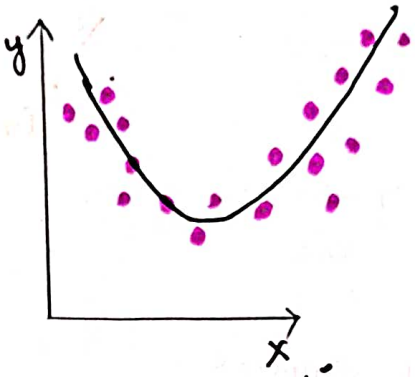
Que: How to check Linear Regression ?
(or) not

Ans:- By scatter plot ← Linearity, direction

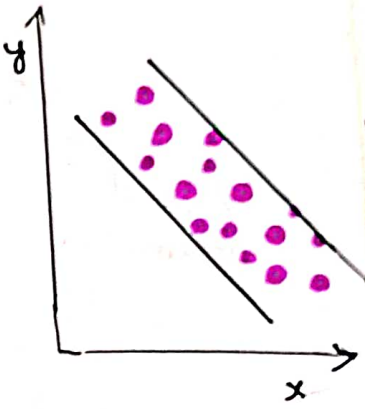


Linear

As "x" value increasing
"y" value also increasing
"Positive Direction"



Quadratic line

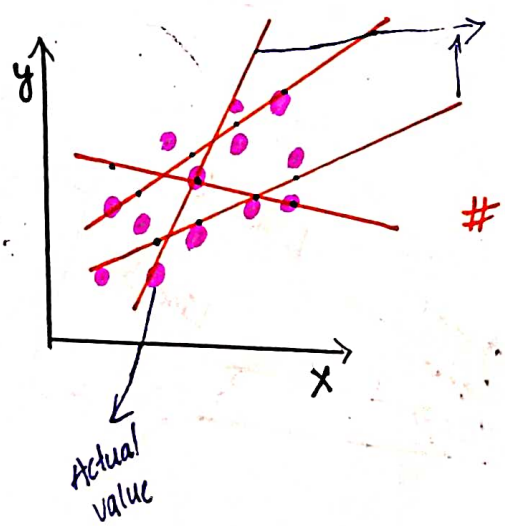


Linear But downward direction

As "x" value increasing
"y" value also decreases
"Negative direction"

Que:- Which Line to be Considered ?

Ans:-



Taking Relation → linear

All lines are straight line

Que:- Different notations ? But Same Concept .?

Ans:-

$y = mx + c$ → slope, "y intercept

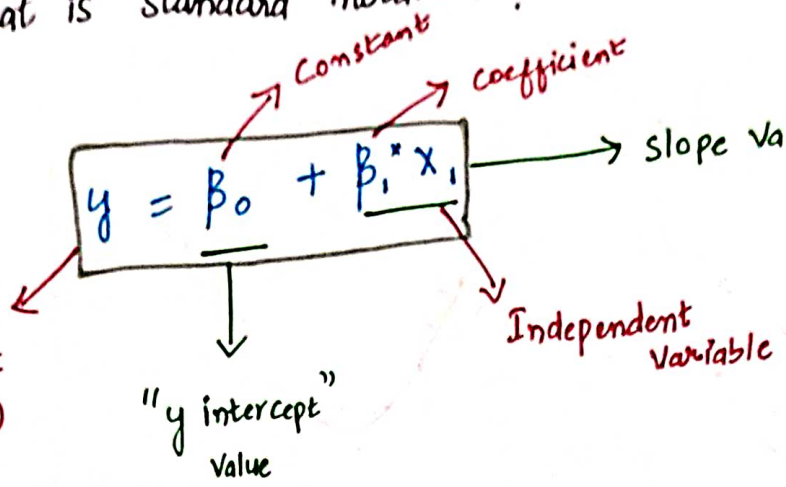
$y = ax + b$ → slope, "y" intercept

$y = b_0 + b_1 x$ → y intercept, slope

$y = \beta_0 + \beta_1 x$ → "y" intercept, slope

Que:- What is Standard notation ?

Ans :-



Ordinary Least squares :

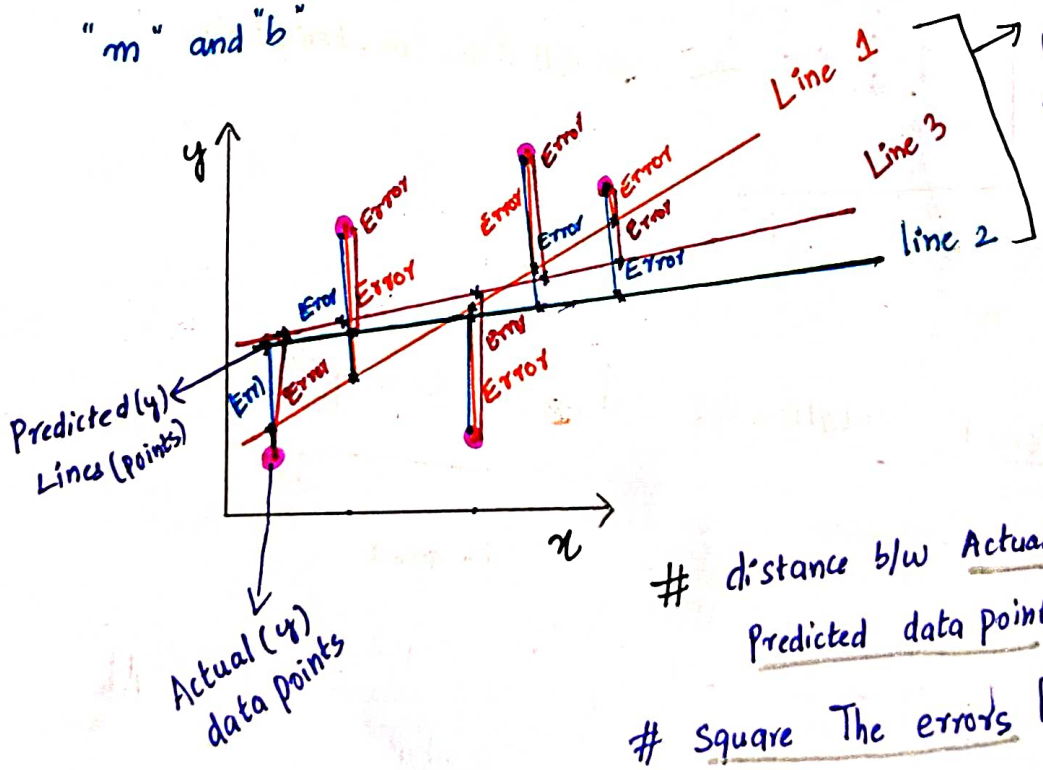
→ Least squares fitting is a way to find The best fit Curve (or) Line for a set of points

→ Sum of Squares of Residuals (Errors) are used to estimate the best fit curve (or) Line

→ Least squares method is used to obtain The coefficient of "m" and "b"

#best fit line ?

Where The Sum of squares of Error is Minimum



distance b/w Actual data point - Predicted data point [Error]

Square The errors [each error do square]

after Squaring Error, Than Sum Error

Que:- What is SSE ?

Ans:- SSE ⇒ Some of squares of Error

Denoted by $\boxed{\Sigma (y-\hat{y})^2}$ → squaring them = SSEmin

Sum, Actual Value, Predicted Value

$\Sigma(y-\hat{y})^2_{min}$ → This value Should be Minimum $\boxed{\Sigma(y-\hat{y})^2_{min}}$

* you Have identify the line Such a way

Que :- How to identify The Exact line ?

Ans :- Explanation :- of Calcus

Minimum point : $y = x^2$ (what is Least "y" value) = 0

if 0.00001 also it get squared

so, "0" is Least y value

how it is identified $y = 0$

$\frac{d(y)}{dx}$ at $x=0$ ⇒ $\frac{dy}{dx}$ at $x=0$ ⇒ $\frac{d(x^2)}{dx}$ at $x=0$ ⇒ $2x \Rightarrow 2(0) = 0$

Ex :- $y = 4x^2+5$ → $4 \times 2 = 8$

$\frac{dy}{dx}$ at $x=0$ ⇒ $8x + 0 \Rightarrow 8(0)+0 \Rightarrow 0$

* $\frac{d}{dx}(4x^2) = 2 \times 4 = [8x]$

* $\frac{d}{dx}(5) = 0$

Mimimum Value $= \frac{dy}{dx}$ at $x = 0$

$SSE_{min} = \varepsilon[y - \hat{y}]^2$

So, in order to get minimum Value For S.S.E

Calculate $\frac{d[\varepsilon[y - \hat{y}]^2]}{dx}$ at $x = 0$ → $\hat{y} = \beta_0 + \beta_1 x$

$$\therefore \frac{d}{dx} \varepsilon (y - \beta_0 + \beta_1 x)$$

Expand This Equation

$$\beta_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\beta_0 = \bar{y} - \beta_1 \bar{x}$$

"y" Mean

"x" Mean

H_0 : Avg. line (Errors) $\leq$ Regression Line

H_1 : Regression line $<$ Avg. line

Example :

X	Y
4	11
6	4
7	6
8	5
12	8

identifying best fit line for These values

By using $y = \beta_0 + \beta_1 x$ [Linear line] ✓

not in $y = \log x$, e^x, ax^2, $\beta x^{-2} x$

For Average Fit Line

X	Y	y-Pred avg	Error	Error^2
4	11	6.8	4.2	17.64
6	4	6.8	-2.8	7.84
7	6	6.8	-0.8	0.64
8	5	6.8	-1.8	3.24
12	8	6.8	1.2	1.44
				30.8

Avg. line

For Random value $2+5x$ (Eqn)

SSE

For $2+5x$ Equation

X	Y	y-Pred	Error	Error^2
4	11	22	-11	121
6	4	32	-28	784
7	6	37	-31	961
8	5	42	-37	1369
12	8	62	-54	2916
				6151

For $5+4x$ Equation

X	Y	y-Pred	Error	Error^2
4	11	21	-10	100
6	4	29	-25	625
7	6	33	-27	729
8	5	37	-32	1024
12	8	53	-45	2025
				4503

SSE

* Avg $= \frac{11+4+6+5+8}{5} = 6.8$

* Error $\Rightarrow$ (Actual - Predicted Answer) = y - $\hat{y}$

$11 - 6.8 = 4.2$

$4 - 6.8 = -2.8$

$6 - 6.8 = -0.8$

$5 - 6.8 = -1.8$

$8 - 6.8 = 1.2$

* Error^2 $= (4.2)^2 = 17.64$

$(2.8)^2 = 7.84$

$(0.8)^2 = 0.64$

$(1.8)^2 = 3.24$

$(1.2)^2 = 1.44$

* Some of square of Error $= 17 + 7.84 + 0.64 + 3.24 + 1.44 = 30.8$

For, $2+5x$ SSE

$\Rightarrow$ y pred $\Rightarrow$ $2+5(4) = 22$

$2+5(6) = 32$

$2+5(7) = 37$

$2+5(8) = 42$

$2+12(12) = 62$

$\Rightarrow$ Error $(y - \hat{y}) = 11 - 22 = -11$

$4 - 32 = -28$

$6 - 37 = -31$

$5 - 42 = -37$

$8 - 62 = -54$

$\Rightarrow$ Error^2 $= (-11)^2 = 121$

$(-28)^2 = 784$

$(-31)^2 = 961$

$(-37)^2 = 1369$

$(-54)^2 = 2916$

$\Rightarrow$ SSE $= 121 + 784 + 961 + 1369 + 2916$

$= 6151$

For $5+4x$ SSE

* y pred $\Rightarrow$ $5+4(4) = 2$

$5+4(6) = 29$

$5+4(7) = 33$

$5+4(8) = 37$

$5+4(12) = 53$

* Error $y - \hat{y}$ pred $= 11 - 21$

$4 - 29$

$6 - 33$

$5 - 37$

$8 - 53$

* Error^2 $= (-10)^2 = 100$

$(25)^2 = 62$

$(27)^2 = 72$

$(32)^2 = 10$

$(45)^2 = 20$

SSE $= 100 + 625 + 729 + 2025 =$

For Regression line

$y = \beta_0 + \beta_1 x$

$\therefore \beta_0 = \bar{y} - \beta_1(\bar{x})$

$\beta_1 = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$

X	Y	$x-\bar{x}_{mean}$	$y-\bar{y}$	$\bar{x}*\bar{y}$	$(x-\bar{x})^2$
4	11	-3.4	4.2	-14.28	11.56
6	4	-1.4	-2.8	3.92	1.96
7	6	-0.4	-0.8	0.32	0.16
8	5	0.6	-1.8	-1.08	0.36
12	8	4.6	1.2	5.52	21.16
				Σ -5.6	Σ 35.2

X Avg: 7.4

Y Avg. 6.8

* Avg. of $X = \frac{4+6+7+8+12}{5} = 7.4$

* Avg. of $Y = \frac{11+4+6+5+8}{5} = 6.8$

* $X-\bar{X}_{(mean)}$ = $4-7.4 = -3.4$, $6-7.4 = -1.4$, $7-7.4 = -0.4$, $8-7.4 = 0.6$, $12-7.4 = 4.6$

* $Y-\bar{Y}_{(mean)}$ = $11-6.8 = 4.2$, $4-6.8 = -2.8$, $6-6.8 = -0.8$, $5-6.8 = -1.8$, $8-6.8 = 1.2$

* $\bar{x}_{mean} * \bar{y}_{mean}$ =

$-3.4 \times 4.2 = -14.28$

$-1.4 \times -2.8 = 3.92$

$-0.4 \times -0.8 = 0.32$

$+0.6 \times -1.8 = -1.08$

$4.6 \times 1.2 = 5.52$

* $\Sigma \bar{x}_{mean} * \bar{y}_{mean}$

$= -14.28 + 3.92 + 0.32 - 1.08 + 5.52 = -5.6$

* $(x-\bar{x})^2$

$= (3.4)^2 = 11.56$

$(-1.4)^2 = 1.96$

$(-0.4)^2 = 0.16$

$(0.6)^2 = 0.36$

$(4.6)^2 = 21.16$

* $\Sigma[x-\bar{x}]^2$

$= 11.56 + 1.96 + 0.16 + 0.36 + 21.16 = 35.2$

$\beta_1 = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} \Rightarrow \frac{-5.6}{35.2} = 0.15909$ → Slope

$\beta_0 = \bar{y} - \beta_1(\bar{x}) = 6.8 - (0.159)(7.4)$

$6.8 - (-0.159) = 7.9771$ → y intercept

Now, The equation is

$y = \beta_0 + \beta_1 x$

$y = 7.977 - 0.15909\,[x]$

Here $y = 7.977273 - 0.159[x]$ Equation.

X	Y	ypred	Err	Error^2
4	11	7.34	3.66	13.39
6	4	7.02	-3.02	9.12
7	6	6.86	-0.86	0.73
8	5	6.70	-1.7	2.89
12	8	6.06	1.94	3.76
				29.89 → SSE

Equation: $7.977 - 0.159(x)$

$7.977 - 0.159(4) = 7.341$
$7.977 - 0.159(6) = 7.023$
$7.977 - 0.159(7) = 6.864$
$7.977 - 0.159(8) = 6.705$
$7.977 - 0.159(12) = 6.069$

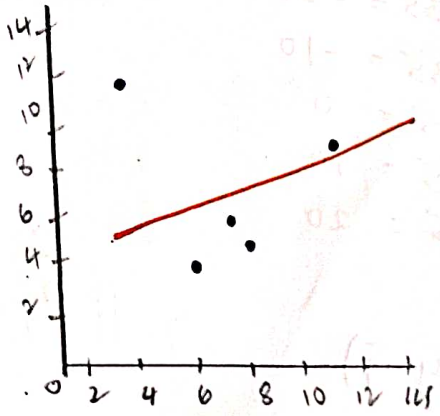
Error
$11 - 7.34 = 3.66$
$4 - 7.02 = -3.02$
$6 - 6.86 = -0.86$
$5 - 6.70 = -1.7$
$8 - 6.06 = 1.94$

Error^2
$= (3.66)^2 = 13.39$
$(-3.02)^2 = 9.12$
$(-0.86)^2 = 0.73$
$(-1.7)^2 = 2.89$
$(1.94) = 3.76$

$SSE = 13.39 + 9.12 + 0.73 + 2.89 + 3.76$
$= 29.89$

Comparing with Avg. line and Other Equations

$y = 7.977 - 0.159[x]$
Best fit line.
$[SSE]_{min}$



For, Average →

For, 10x+5

X	Y	$\bar{y}$	Error $y-\bar{y}$	error^2
1	15	35	+20	400
2	25	35	+10	100
3	35	35	0	0
4	45	35	-10	100
5	55	35	-20	400

SST = 1000

X	Y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x}*y-\bar{y})$	$(x-\bar{x})^2$	$\hat{y}$	Error	Error^2
1	15	-2	-20	40	-4	15	0	0
2	25	-1	-10	10	-1	25	0	0
3	35	0	0	0	0	35	0	0
4	45	1	10	10	1	45	0	0
5	55	2	20	40	4	55	0	0
				100	0			SSE=0

$\bar{x}=3$ $\bar{y}=35$

X Avg $= \frac{1+2+3+4+5}{5} = 3$

y Avg $= \frac{15+25+35+45+55}{5} = 35$

* $x-\bar{x} = 1-3 = -2$

$2-3 = -1$
$3-3 = 0$
$4-3 = 1$
$5-3 = 2$

* $y-\bar{y} = 15-35 = -20$

$25-35 = -10$
$35-35 = 0$
$45-35 = 10$
$55-35 = 20$

* $(x-\bar{x}) \times (y-\bar{y})$

$= -2 \times (-20) = 40$
$-1 \times (-10) = 10$
$1 \times 10 = 10$
$2 \times 20 = 40$

* $\Sigma(x-\bar{x} \times y-\bar{y})$

$= 40+10+0+10+40 = 100$

* $(x-\bar{x})^2$ # For squaring don't consider (+,-)

$= (-2)^2 = +4$
$(-1)^2 = +1$
$(1)^2 = 1$
$(2)^2 = 4$

* $\Sigma[x-\bar{x}]^2 = -4-1+1+4 = 10$

$\beta_0 = \bar{y} - \beta_1 \bar{x}$

$\beta_1 = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$

$\beta_1 = \frac{100}{10} = 10$ → slope

$\beta_0 = 35 - 10(3) = 5$ → y intercept

Equation is

$y = \beta_0 + \beta_1 x$

$y = 5+10x$ ✓

*** Ex [For Interv

$\therefore R^2 = 1 - \frac{0_{avg}}{1000_{avg}}$

$R^2 = 1$

Maximum value

8/4/22
3:30am

8/4/22
1:00 pm

* BIVARIATE STATISTICS *

X	Y

⇒ Simple linear regression :-

it is **2 Sample "T" test**

Ho : Avg. line ≥ Regression line

H1 : Regression line > Avg. line.

Q :- How can This can be proved by statistically

A :- Where $(SSE)_{Regression\ line} < (SSE)_{Average\ line}$

Ex :- 120 < 30

⇒ Multiple linear regression :-

X_1	X_2	X_3	X_4	Y

Where we have multiple "x" values and only single "y". We use **"ANOVA"** ↓ applied when **Statistical Test**. Output variable is **"continous"**.

Q :- What is the hypothesis Test apply For Regression ?

A :- **Anova Test** (> 2 samples) Apply For More Than 2 Samples

if we apply Only For 2 samples it is **"2 sample T Te**

* **"correlation"** is high gives "good" results → Linea Regression

"Relation between two variables"

Modulus of 'r' ← $|r| = \pm$ any value

→ $|r| \geq 0.8$ (strong) correlation

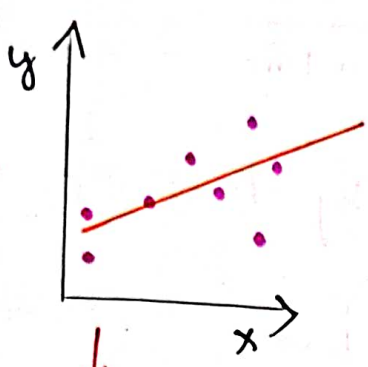
$0.5 < |r| < 0.8$ (moderate)

$|r| < 0.5$ (weak)

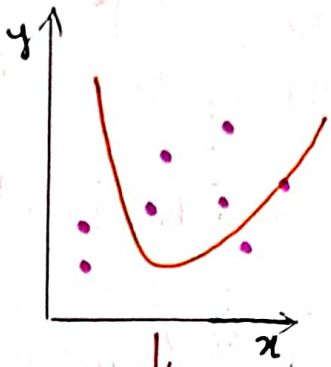
The Value of "One Variable", is a function of "Other Variable"

The Value of y, is a Function of x;

$$y = f(x)$$



$\therefore y = mx + c$



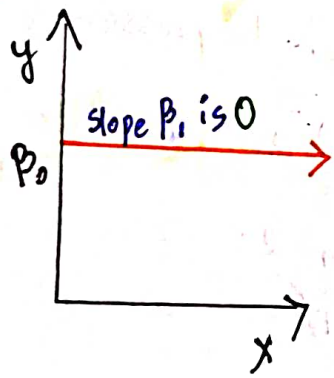
$y = ax^2 + bx + c$

The Value of dependent Variable, is function of independent Variable

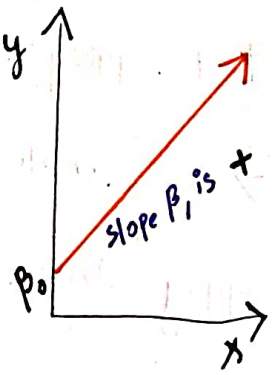
$\Rightarrow E(y) = \beta_0 + \beta_1 x$ → "slope"

→ "y" intercept

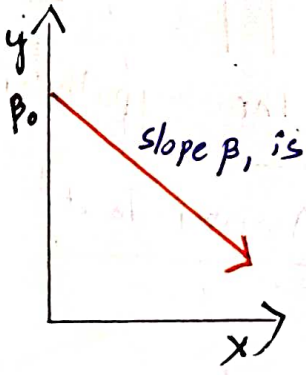
Estimated value



$E(y) = \beta_0 + 0(x)$



$E(y) = \beta_0 + \beta_1 x$



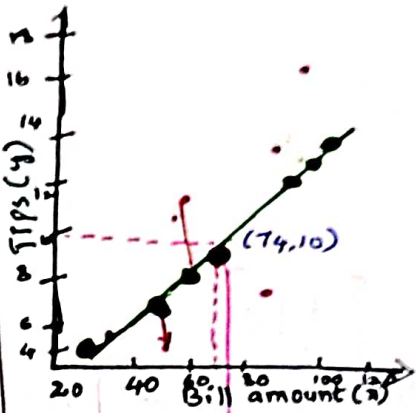
$E(y) = \beta_0 - \beta_1 x$

* Calculations

$y = 0.1462[x] - 0.8188$ Equation

X	Y	$X-\bar{X}$ (Deviations)	$Y-\bar{Y}$ (Deviations)	$(x-\bar{x})[y-\bar{y}]$	$(x-\bar{x})^2$	$y = 0.1462x - 0.8188$	$\hat{y}$
34	5	-40	-5	200	1600	0.1462[34] - 0.8188	4.1505
108	17	34	7	238	1156	0.1462[108] - 0.8188	14.9693
64	11	-10	1	-10	100	0.1462[64] - 0.8188	8.5365
88	8	14	-2	-28	196	0.1462[88] - 0.8188	12.0453
99	14	25	4	100	625	0.1462[99] - 0.8188	13.6535
51	5	-23	-5	115	529	0.1462[51] - 0.8188	6.6359
				$\Sigma = 615$	$\Sigma = 4206$		

$\bar{x} = 74$, $\bar{y} = 10$



Centroid

The line should pass through centroid.

Avg. of x :- $\frac{34+108+64+88+99+51}{6} = \bar{x} = 74$

Avg of y :- $\frac{5+17+11+8+14+5}{6} = \bar{y} = 10$

* Descriptive statistics / Centroid

$\bar{x}, \bar{y} = 74, 10$

* line $[\hat{y} = b_0 + b_1 x]$ min

$b_0 [\bar{y} - b_1 \bar{x}]$

$$b_1 = \frac{\sum [x_i - \bar{x}][y_i - \bar{y}]}{\sum [x_i - \bar{x}]^2}$$

Slope $= \frac{\sum [x-\bar{x}][y-\bar{y}]}{\sum [x-\bar{x}]^2}$

How This Equation is Formed ?

** Variance $(x) = \frac{\Sigma (x-\bar{x})^2}{(n-1)}$ → s

Variance(x) can be written as.

$$\frac{\Sigma (x-\bar{x})(x-\bar{x})}{(n-1)}$$

*** Covariance (x,y) written as

$$\frac{\Sigma (x-\bar{x})(y-\bar{y})}{(n-1)}$$

Substuting & Equating

$$\frac{Cov(x,y)}{V(x)} = \frac{\frac{\sum [x-\bar{x}][y-\bar{y}]}{n-1}}{\frac{\sum [x-\bar{x}]^2}{n-1}}$$

(S.S.E)

* Sum of Squares of Error $= \varepsilon [y - \hat{y}]^2$

$= \varepsilon (y - [\beta_0 + \beta_1 x])^2$

$= \varepsilon [y - \beta_0 - \beta_1 x]^2$

↓ should be minimum

Slope

$$b_1 = \frac{\sum [x - \bar{x}][Y - \bar{Y}]}{\sum [x - \bar{x}]^2}$$

$b_1 = \frac{615}{4206}$

$b_1 = 0.1462$

y intercept $\Rightarrow b_0 = \bar{y} - b_1 \bar{x}$

$b_0 = 10 - 0.1462 [74]$

$b_0 = -0.8188$

Final Equation $\Rightarrow$ $y = b_0 + b_1 x$

$y = -0.8188 + 0.1462 x$

$y = \beta_0 + \beta_1 x$

(or)

$y = \underset{mx}{0.1462 x} \overset{+}{} \underset{c}{0.8188}$

*** "Accuracy" called as "Coeffient of Determination"

$R^2 = 1 - \frac{SSE_{(reg)}}{SST_{(Avg)}}$

"Determing The variation of "y" with respect To variation of "x"

(or)

→ As "x" value changes, how "y" value changes.

(or)

"y" variable Explained by "x" variable.

Que:- What is Max. value of R^2

Ans: $R^2 = 1$ [Max. value]

$(SSE)_{reg} = 0$

$R^2 = -$Infinite

$\therefore R^2 < 0$ (Worst model)

***** Range of R^2.

$[\alpha, 1]$

Infinite

"Regression Squared Error"

$\hat{y}$	Error $y - \hat{y}$	Square Error $(y-\hat{y})^2$
4.1505	(5 - 4.1505) 0.8495	(0.8495)² 0.7217
14.9693	2.0307	4.1237
8.5365	2.4635	6.0688
12.0453	-4.0453	16.3645
13.6535	0.3465	0.1201
6.6359	-1.6359	2.6762

$SSE \Rightarrow \sum = 30.075$

* Avg. line = 120
* Regression line = 30.075

→ difference between these are $\frac{30}{120} \Rightarrow 75\%$.

Co-efficient of Determination.

*** (or)

$$R^2 = 1 - \frac{SSE_{(Reg)}}{SST_{(Avg)}} \rightarrow \frac{SSE\ [\text{Sum of squares of Error [Regress line}}{SST\ [\text{sum of squares of Error [Average line}}$$

$$\sum [y - \hat{y}]^2$$

$$\sum [y - \bar{y}]^2$$

***

$$R^2 = 1 - \frac{\sum [y - \hat{y}]^2_{reg}}{\sum [y - \bar{y}]^2_{Avg}}$$

$$R^2 = 1 - \frac{30}{120} = 1 - \frac{1}{4}$$

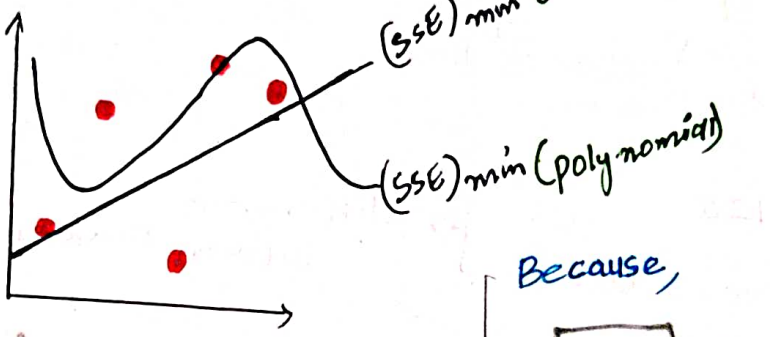
$$R^2 = \frac{4-1}{4} = \frac{3}{4} \Rightarrow 0.75 \Rightarrow \boxed{75\%\ \text{accuracy}}$$

8/4/22
4:30 PM

Dt:- 9/4/22
3:00 pm

Evaluation Metrics in Regression

After Model Fitting, we would like to assess the performance of model by **Comparing model predictions** to actual **[True] data.**



1. **MAE** [Mean Absolute Error] → $\frac{\sum |y - \hat{y}|}{n}$

Bad → differentiation

squaring of units

2. **MSE** [Mean Squared Error] → $\frac{\sum [y - \hat{y}]^2}{n}$

Because,

MAE → $\frac{d[|x|]}{dx}$

$\frac{d[|x|]}{dx}$ at $x=0$ → $\frac{x}{|x|} = \frac{0}{|0|} \Rightarrow \alpha$ (Infinite)

MSE → $\frac{dx^2}{dx}$

$\frac{dx^2}{dx} = 2x$ at $x=0$

**** Only way to make Error Minimum * *

$(error)_{min}$ is

$$\frac{d[\,]}{dx} \text{ at } x = 0$$

x is only notation it's 2 after d( )/dy

Example — milage of a car

Y	$\hat{Y}$	$y-\hat{y}$
182	189	0.7

Here 18.2 Kmph → 18.9 Kmph

Error = $(0.7)^2 = 0.49 \frac{km^2}{h^2}$

If we square it

Problem is Here it's also squaring

Que:- What are Evaluation Metrics used for Regression?

Ans:- We have 1. Mean Absolute Error (MAE)
and 2. Mean Square error [MSE].

Que ⇒ **Mean absolute Error (MAE)** Where, we are going To use?

Ans:- MAE is "Absolute Values of Mean" is called mean absolute Error.

But, Problem with $\frac{d}{dx}$ of (Mod) $|x|$ is $\frac{x}{|x|}$ and at $x=0$, The answer is infinite

So, we are not going to Consider "MAE"

Que ⇒ **Mean Square Error (MSE)**

Ans:- Problem with MSE is with calculating of $\frac{d}{dx}$ of x^2 we will get Some Value of $2x$. But, by Squaring the Error. Units are also squaring up it gives wrong prediction. So, we not going to Consider "MSE".

And Solution For This is :-

$$RMSE = \sqrt{\frac{\Sigma(y-\hat{y})^2}{N}}$$

Same as in Variance

we Take Standard deviation

"Root Mean Square Error"

Que :- What is "Root Mean Square Error" RMSE ?

A :- "RMSE" represents Standard deviation of residuals [Errors] difference between model predictions and True Values (Training data)

* RMSE can be easily intrepreted Compared To MSE because RMSE Units match units of Output.

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[y-\hat{y}]^2}$$

(or)

$$\sqrt{\frac{\sum[y-\hat{y}]^2}{n}}$$

Example

squaring of Errors

Converting negetaive Values To positive Values by adding | |

X	$\hat{y}$	Error	MAE	MSE
1	1.4	0.4	0.4	0.16
2	2.2	0.2	0.2	0.04
3	3.8	0.8	0.8	0.64
4	4.1	0.1	0.1	0.01
5	5.6	0.6	0.6	0.36

$\frac{\Sigma}{n} = 0.42$ $\frac{\Sigma}{n} = 0.242$

Root Mean Squar Error $= \sqrt{0.242}$

⇒ **Mean percentage Error [MPE]**

$$MPE = \frac{100\%}{n} \sum_{i=1}^{n} (y - \hat{y}] / y_i$$

In this we calculate Percentage of Error. But We don't consider "positive" or "Negative" value, without absolute operation.

⇒ **Mean absolute Percentage Error [MAPE]**

In This "MAPE" we convert Every value To positive by applying "Absolute" to the Equation.

$$MAPE = \frac{100\%}{n} \sum_{i=1}^{n} |y - \hat{y}| / |y_i|$$

9/4/22
5:10pm

9/9/22
9:30 pm

"Simple Linear Regression"

⇒ Data Science Project life cycle (crisp-dm) → Data mining.

Cross-Industry Standard Process For

crisp-dm → Methodology

1. Bussiness problem Understanding
2. Data Understanding
   - Data collection [D...]
   - Data variables (research - domain Expert)
   - Dataset understandring
3. Data Preprocessing
   - EDA
   - Data cleaning
   - Data Wrangling
   - Train / test split
4. Modelling → Apply Different Algorithms
5. Evaluation → Accuray of The Model checking (or) If not go back and start again
6. Presentation → Visulazation (Tableau, Power BI)

⇒ Basic Liberaries For all Machine Learning

```
# Import numpy as np
# Import Pandas as pd
# Import matplotlib.pyplot as plt
  % matplotlib inline
# Import Seaborn as sns
```

Step 2 : Business Problem Understanding

- Is there a relationship between total advertising Spend and Sales ?
- Our next ad Campaign will have a total spend of $200, how many units do we expect to sell as a result of this ?

Step 2.1 Data collection

, Comma Seperated values

```
# df = pd.read_csv("Advertisimg.csv")
# df.head()
```

Out

Tv	Radio	newspaper	Sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	9.3
151.5	41.3	59.5	18.5
180.8	10.8	58.4	12.9

Step 2.2 Data Understanding

The sample data displays Sales (in thousands of units) for a particular product as a function of advertising budgets (in thousand of dollars) for Tv, radio and Newspaper media

Independent Variables

→ Tv : Advertising dollars spent on Tv for a Single product in a Given market (in Thousands of dollars) → Ex: 230.1 x 1000 already done scaling

→ Radio :- Advertising dollars Spent on Radio

→ Newspaper : Advertising dollars Spent on news paper

Target Variable

→ Sales : Sales of a Single product in a given market (in thousands of widgets) → Ex:- 22.1 x 1000 = 22100 products sold

Step - 2.3 Dataset Understanding

```
# df.info()
```

Out

	column	Non Null Count	Dtype
0	Tv	200 non null	float 64
1	Radio	200 non null	float 64
2	Newspaper	200 non null	float 64
3	Sales	200 non null	float 64

(200, 4)

Que :- If Some one was to spend a total of $200, what would The expected Sales be ?

A :- We have Simplified This quite a bit by Combining all Features into "total spend"

We add new column, total of 3 column Creating new one

```
# df["total_spend"] = df["TV"] + df["radio"] + df["newspaper"]
```

```
# df.head()
```

Out

	TV	Radio	newspaper	Sales	total_spend
0	230.1	37.8	69.2	22.1	337.1
1	44.5	39.3	45.1	10.4	128.9
2	17.2	45.9	69.3	9.3	132.4
3	151.5	41.3	58.5	18.5	251.3
4	180.8	10.8	58.4	12.9	250.0

We drop 3 columns, Consider only two columns For Calculating

```
#df.drop(columns = ["TV", "radio", "newspaper"], inplace=True)
```

```
# df.head()
```

Out

Sales	Total-Spend
22.1	337.1
10.4	128.9
9.3	132.4
18.5	251.3
12.9	250.3

Step-3.1 Exploratory Data Analysis [EDA]

On The basis of This data. How should you spend advertising money in future? These general questions might lead you to more specific questions:

1. Is there a relation ship between ads and Sales?
2. How strong is that correlation?
3. Given ad spending, can Sales be predicted?

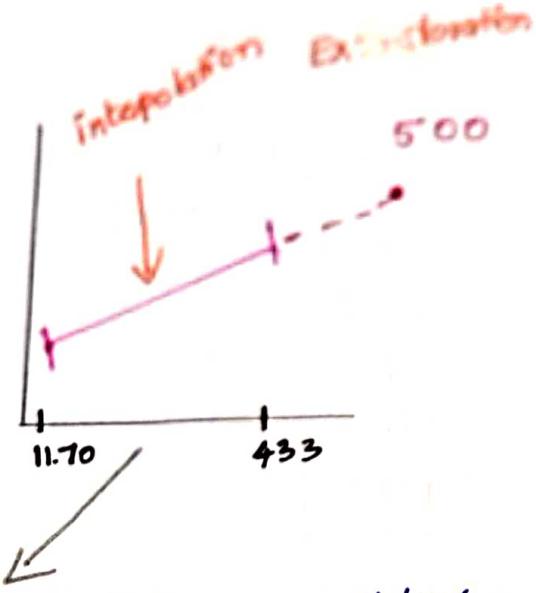
df.describe()

Out :-

	Sales	total_spend
count	200.000000	200.000000
Mean	14.022500	200.860500
std	5.217457	92.985181
min	1.600000	11.700000
25%	10.375000	123.550000
50%	12.900000	207.350000
75%	17.400000	281.125000
max	27.000000	433.600000

How close The values in dataset ← To The mean

Mean and median are close. They normal distribution

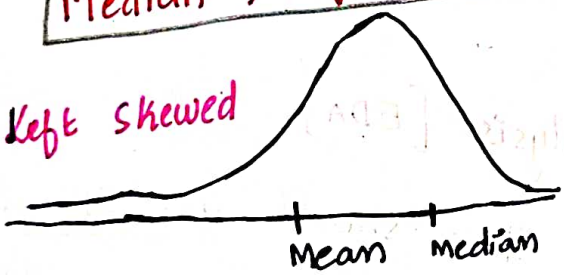


* **Interpolation** :- Estimation of Value with in two known Values in sequence of Values.

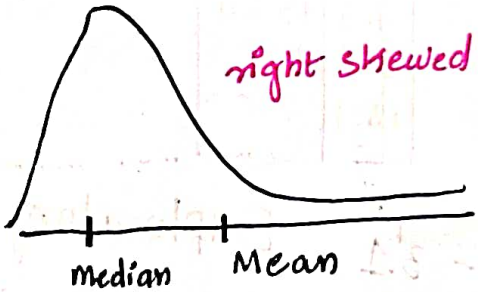
* **Extrapolation** :- Prediction The Value, Outside of known Values it is called Extrapolation.

***

⇒ Median > (greater Than) Mean



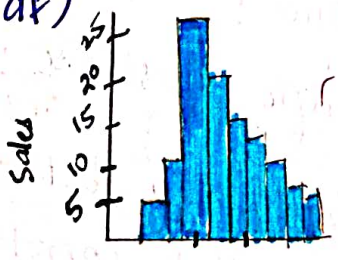
⇒ Median < [Less Than] Mean

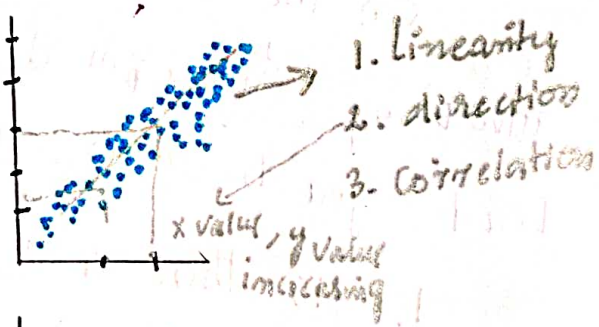


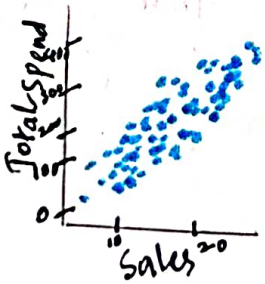
sns.pairplot(df)

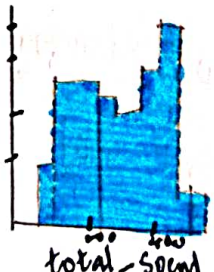
plt.show()

Out :









df.corr()

Out

	Sales	total_spend
Sales	1.000000	0.867712
total_spend	0.867712	1.000000

Step 3.2 Data cleaning

df.isnull().sum()

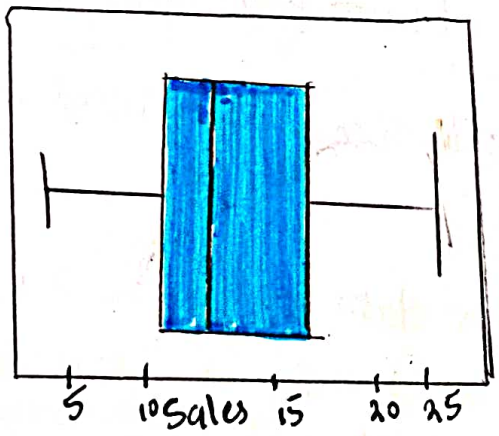
Out

sales 0
total_spend 0

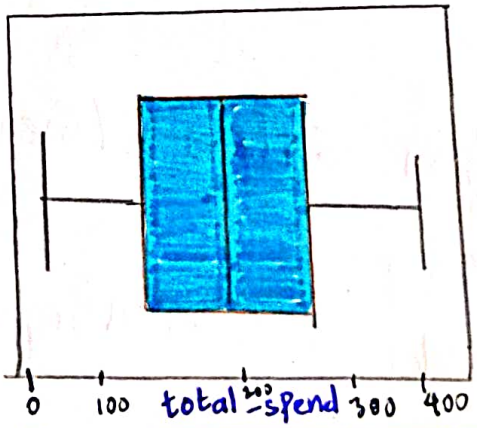
step 3.3 Data Wrangling

* Outliers

sns.boxplot(df.sales)
plt.show()



sns.boxplot(df.total_spend)
plt.show()



* Feature Transformation

```
# df.sales.skew()
```

Out :- 0.407

```
# df.total_spend.skew()
```

Out : 0.049

STep 3.. Train Test split

```
# X = df.drop(columns = "sale")
# Y = df["sales"]

from sklearn.model_selection import train_test_split
# X_train, X_test, y_train, y_test = train_test_split(x, y,
                                     test_size = 0.3, random_state= )
```

STEP-4 Modelling :- If doing with data preprocessing. directly working on modelling is "Base line model (or) raw model.

```
# from sklearn.linear_model import LinearRegression
  storing
# model = LinearRegression()
```

fitting the data

```
# model.fit(x_train, y_train)
```

Out :- LinearRegression()

```
# Print("Model Intercept : ", model.intercept_)
  Print("Model Coefficient : ", model.coef_)
```

Attributes (multiple)

→ y intercept

$\beta_0 + \beta x$

↓ coefficient value

Out :- Model Intercept : 4.33176

Model Coefficient : 0.0480

$\hat{y} = 4.33176 + 0.0480(x)$

⇒ **Predictions**

```
# spend = 200
# Predicted_Sales = 4.33176 + 0.0480 * spend
# Predicted_Sales
```

Out :- 13.93860

If we have multiple equations, we can write this

```
# model.predict([[200]])
```

→ input should given in 2 dimensional

Out array([13.93860])

Predicting on X_train, X_test

	x	y	ŷ	
X Train	1	10	14	→ Train accuracy
	2	15	12	
	3	20	13	
	4	30	14	
X Test	5	50	33	→ Test accuracy

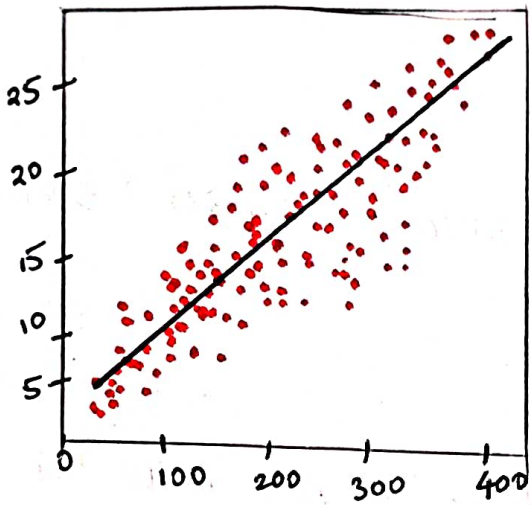
```
# train_predictions = model.predict(x_train)
# test_predictions = model.predict(x_test)
```

Only stores, No output

Regression line

⇒ plotting the Least squares line

```
# plt.scatter (x_train , y_train , color = "red")
# train_predictions = model.predict (x_train)
# plt.plot (x_train, train_predictions , color = "Black")
# plt.show()
```



Steps

⇒ Evaluation metrics

```
from sklearn.metrics Import mean_absolute_error

# print ("MAE For Test data :", mean_absolute_error (y_test,
                                                     test_predictions))

# print ("MAE For Train data :", mean_absolute_error (
                                  y_train , train_predictions))
```

Out: . MAE For Train data : 1.97768
MAE For Test data : 1.90653

```
from sklearn.metrics import mean_squared_error

# print ("MSE for test data : "mean_squared_error (y_test,
                                                    test_predictions))

# print ("MSE for train data : mean_squared_error (y_train,
                                                    train_predictions))
```

Out : MSE for Test data : 6.82220
MSE for Train data : 6.40720

RMSE (Just write np.sqrt).

```
# print ("RMSE for test data", np.sqrt(mean_squared_error
                                        (y_test, test_predictions)))

# print ("RMSE for train data", np.sqrt (mean_squared_error
                                          (y_train, train_predictions)))
```

np.sqrt

Out : RMSE for test data : 2.611935
RMSE for train data : 2.53124

```
from sklearn.metrics import r2_score

# print ("R2 for test data", r2_score (y_test, test_
                                        Predictions))

# print ("R2 for train data", r2_score (y_train, train_
                                         Predictions))
```

Out R2 for test data : 0.74001
R2 for train data : 0.76534

(24)

Another way for R²

```
# model.score(x_train, y_train)
```

Out : 0.76534

```
# model.score(x_test, y_test)
```

Out : 0.740017

Cross_validation ⇒ K-Fold Cross Validation

```
from sklearn.model_selection import cross_val_score
# scores = cross_val_score(model, x, y, cv=5)
```

Here K = 5

```
# print(scores)
# scores.mean()
```

Out : [0.74964192, 0.79455226, 0.76417134, 0.74872042, 0.65980565]

⇒ 0.74337 → should be Equal to Test square
Than it is good model.

Linear Regression assumptions

Errors

L → Linearity
I → Independent
N → Normality
E → Equal variance

10/4/22
3:30 AM

⇒ CHECK LIST ⇐

Assumptions of linear regression

1. check whether model has overfitting (or) underfitting problem
2. Is test Accuracy = Cross Validation Score
3. check assumptions (if it is linear Regression)
4. check model meets bussiness problem requirements
5. Finally, save the model and share to deployment Team.

Purely Assumptions of The Researcher

1 Que :- Is model has overfitting (or) underfitting problem ?

Ans :- it's good model.

2 Que Is test accuracy = Cross Validation Score ?

Ans :- Applied K-Fold cross validation and it is Equal in test accuracy and CV score.

3 Que :- check Assumptions (if it is linear regression)

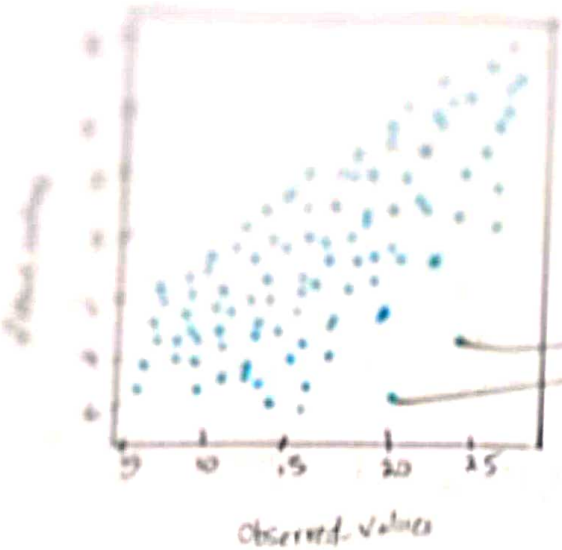
Ans :- Line Assumptions.

1 Linearity of Errors

```
# test_res = y_test - test_predictions.
```

↑ Saved in TRE.

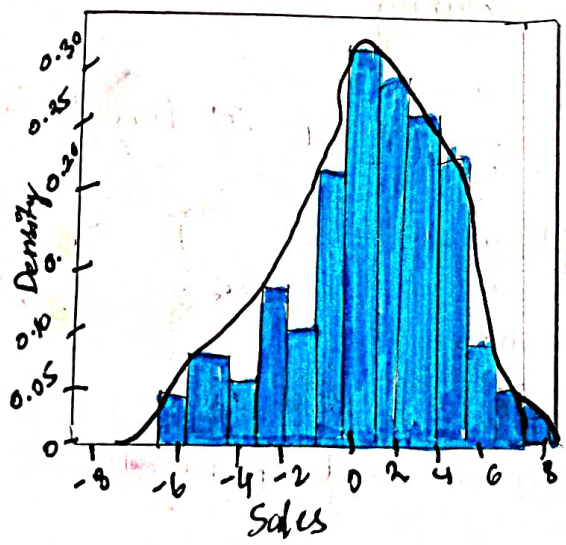
```
# plt.scatter (y_test, test_res)
  plt.xLabel ("Observed_values")
  plt.YLabel ("fitted_values")
  plt.show()
```



2. Normality of Errors *

```
# sns.distplot(test_res, bins = 15, kde=True)
  plt.show()
```

Out



Normal skewed → "Assumption meet"

3. Equal variance of Error (Homoscadesicity)

```
# plt.scatter(test_predictions, test_res, c="r")
# plt.axhline(y=0, color="blue")
```

ax = axis → h = horizontal line

% of Positive Errors
% of negative Errors

color = red

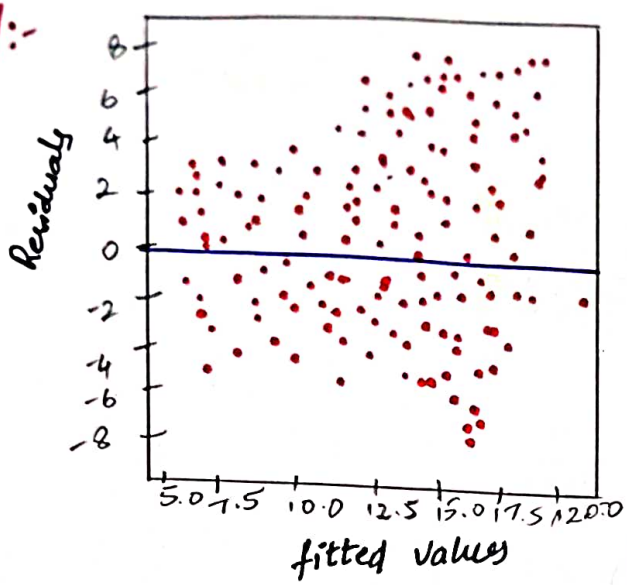
```
# plt.xlabel ("fitted values")
# plt.ylabel ("Residuals")
# plt.show()
```

For model

Ho :- Avg line best fit line
H1 : regression line best line

Anova Test
P: $\leq \alpha$
P $\geq \alpha$

Out :-



1 sample two Tail test

Variable → Significant

Ho :- $\beta_1 = 0$
H :- $\beta_1 \neq 0$

Variable insignificant → don't ⇒ Consider

4. Variable → significance

```
# import statsmodels.formula.api as smf
# m = smf.ols("y~x", data=df).fit()
# m.summary()
```

"y" upon "x"
y multiple column
ordinary least square (SSE) min

Out

OLS Regression Results.

Train

Dep. variable : Y
Model : OLS
method : Least squares

R-squared : 0.753
Adj. R-Squared : 0.752
F. static : 603.4
prob.(F.static) : 5.06×10^{-62}

P = 5.06^{-62}
P < 0.05
Reject Ho

	coeff	std.err	t	P>|t|	[0.025	0.975]
intercept	4.2730	0.499	9.676	0.000	3.378	5.508
x	0.0487	0.002	24.564	0.000	0.045	0.053

Confidence Interval
Std.Error = $\sigma/\sqrt{n}$
$\bar{X} \pm Z \frac{\sigma}{\sqrt{n}}$
Central limit theorem

One sample
Test applied
1 Sample Test
Two Tail
95%
2.5
2.5
Error

Ans: When Average line is going to fail in Regression?

When Sum of Square (SSE) of Average line is Less than the Sum of Squares of regression line $(SSE)_{reg}$ it going To fail in this Situation.

$$(SST)_{avg} < (SSE)_{reg}$$

(Errors)

Step 6 Final Inferences

```
# model.predict([[200]])
```

Out array([13.9898])

⇒ Save a Model

```
from joblib import dump
# dump(model, "Sales_model.joblib")
```

Out: ["Sales_model.joblib"] → Saves in working directory.

⇒ Load a Model → From client side

```
# from joblib import load
```

```
# loaded_model = load ("sales_model.joblib")

# loaded_model.predict ([[200]])
```

Out array ([13.989])

```
# loaded_model.predict ([[500]]) # Check with multiple values.
```

Out :: array ([[28.3488]])

with 74% Accuracy.

19/4/22
5:00pm

Date :- 11/04/22
10:30 pm

Multiple Linear Regression

#Multiple Linear Regression :-

Examines Relationship between [two] (or) [more Than two] Variables

* Each [independent Variable] has its own corresponding coefficient.

$$y = b_0 + b_1 * x_1 + b_2 * x_2 + \dots + b_n x_n$$

dependent Variable

Independent Variables.

⇒ Example :

TV	Radio	np	Sales

TV, Radio, news paper, Sales

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta x_3$$

TV, News paper, newspaper

y- intercept :- $\hat{y}$

Coefficient :- $\beta_1, \beta_2, \beta_3$

Same Example : But different method

Step-1 Bussiness Problem Understanding.

* What is The relation b/w each adversting channel [TV, Radio, Newspaper] and Sales ? $y = f[x_1, x_2, x_3]$
* Previously, we Explored is There a relationship between Total adversting Spend and Sales ? as well as predicting the total Sales For Some Value of Total spend.

Step: 2.1 Data collection

```
# df = pd.read_csv("Advertising.csv")
# df.head()
```

Out

TV	Radio	Newspaper	Sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	9.3
151.5	41.3	58.5	18.5
180.8	10.8	58.4	12.9

Step 2.2 Data Understanding

Same as Simple linear Regression Dataset.

Step 2.3 Data Set Understanding

```
# df.info()
```

Step-3.1

Exploratory Data Analysis [EDA]

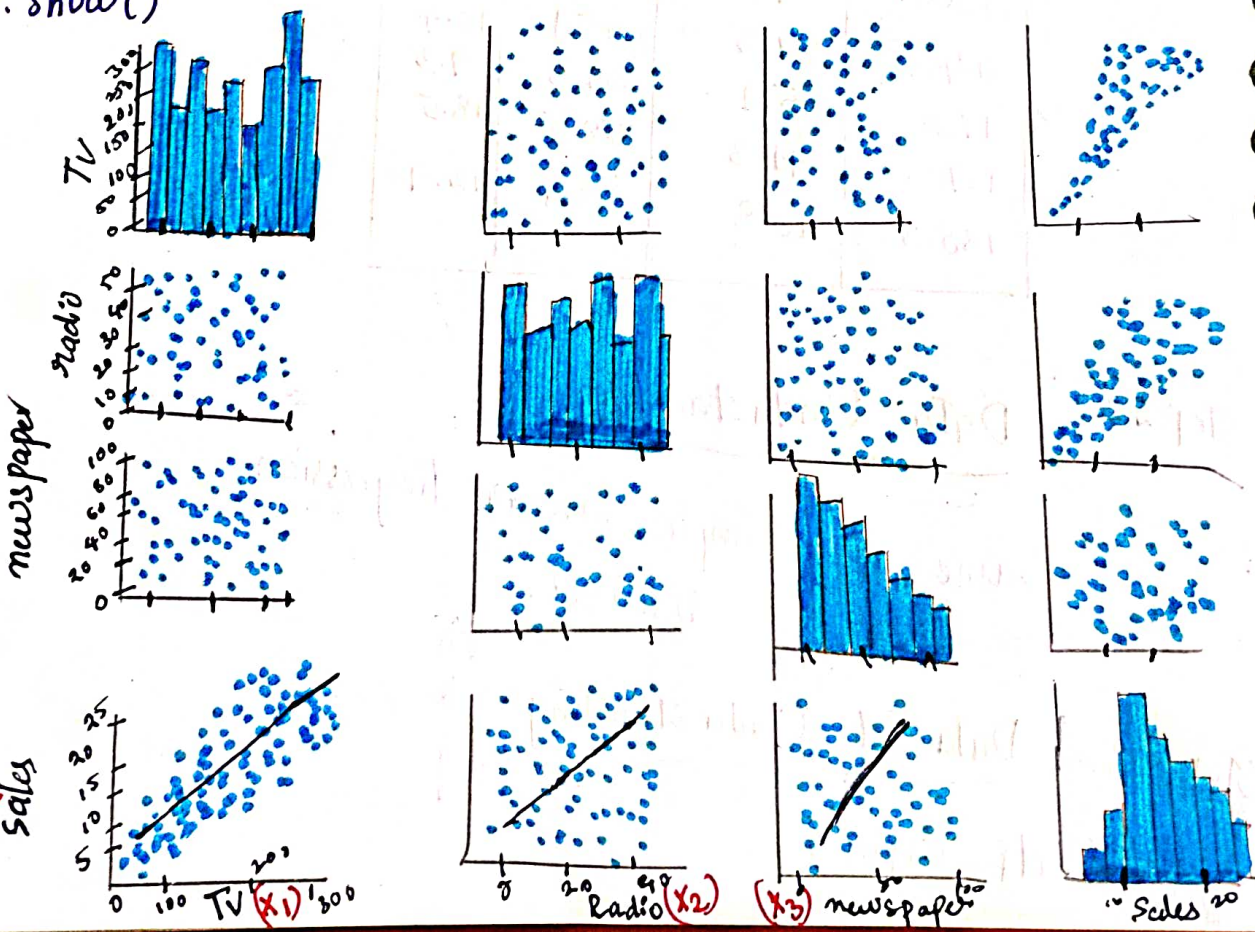
df.describe()

Out:

	TV	Radio	newspaper	Sales
count	200.000000	200.000000	200.000000	200.000000
Mean	147.042500	23.264000	30.554000	14.022500
Std	85.854236	14.846809	21.778621	5.217457
min	0.700000	0.000000	0.300000	1.600000
25%	74.375000	9.975000	12.750000	10.375000
50%	149.750000	22.900000	25.750000	12.900000
75%	218.825000	36.525000	45.100000	17.400000
max%	296.40000	49.600000	114.000000	27.000000

sns.pairplot(df)

plt.show()



⇒ By observing the scatter plot, we made an assumption of relation between [y] and [$x_1 + x_2 + x_3$] is Linear

(symmertical Matrix)

df.corr() →

	TV	Radio	News paper	Sales
TV	1.000000	0.054809	0.056648	0.782224
Radio	0.054809	1.000000	0.354104	0.576223
News paper	0.056648	0.354104	1.000000	0.228299
Sales	0.782224	0.576223	0.228299	1.000000

* Where upper Triangle, lower Triangle is same
The diagonal elements = 1

i) y (vs) x_1
ii) y [vs] x_2
iii) y [vs] x_3
→ relation strong → High Accuracy

x_1 [vs] x_2
x_1 (vs) x_3
x_2 [vs] x_3
→ Low ↓
If strong
collinearity problem

⇒ The relation between "y" and "x" be high ...
dependent
Independent variable.

⇒ The higher the value - stronger the correlation

⇒ The relation between any two independent Variables should be "Low"

**** ⇒ if the correlation between any two (2) independent Variables is strong, then it is called "collinearity problem".

Should be "weak"

Should be weak

* idenpendent variable [student 2]

Should be strong

* idenpendent Variable [students]
* dependent Variable [Teacher]

Example

12/4/22
3:30 pm

Dt: 12/4/2
1:00pm

STEP: 3.2 Data cleaning

Same as Simple Linear

STEP 3.3 Data Wrangling

not Same •

STEP 3.4 Train_Test split

```
# X = df.drop(columns = "sales")
# y = df["sales"]

from sklearn.model_selection import train_test_split

# X_train, Xtest, y_train, y_test = train_test_split(x, y,
                                    test_size = 0.3, Random_
                                    state = 29)
```

STEP-4 Modelling

Here, $y = \beta_0 + \beta_x \times [TV] + \beta_2 \times [radio] + \beta_3 \times [Newspaper]$

```
from sklearn.linear_model import LinearRegression
                                  ↑ capital
# model = LinearRegression()
# model.fit(x_train, y_train)
```

Out: LinearRegression()

model.coef_

Out : array([0.04422917, 0.18781641, 0.00758744])
β_1 β_2 β_2

model.intercept_

Out : 2.974097357
β_0

⇒ Predictions → Predicting on "X"

train_predictions = model.predict(x_train)

test_predictions = model.predict(x_test)

STEP: 5 Evaluations → on "y"

from sklearn.metrics import mean_squared_error

test_RMSE = np.sqrt(mean_squared_error(y_test, test_predictions))

train_RMSE = np.sqrt(mean_squared_error(y_train, train_predictions))

print(train_RMSE, test_RMSE)

Out : 1.64082 , 1.7805

model.score (Xtrain, y_train) (train R^2)

Out: 0.888817

model.score (X_test, y_test) (test R^2)

Out: 0.905258

⇒ checklist

Que: Is model has Underfitting (or) Overfitting problem?

Ans:- Good model

Que: Is Test Accuracy = Cross Validation Score

Ans:

```
from sklearn.model_selection import Cross_Val_score
# scores = cross_val_score (model, X, y, cv=5)
# print (scores)
# scores.mean()
```

Out [0.87865198, 0.9176312, 0.92933, 0.81443, 0.895...

→ 0.887106349, compare with Xtest, ytest Accuracy.

3. Check Assumptions

* Linearity of errors

test_res = y_test - test_predictions (Xtest)

↓ Residuals

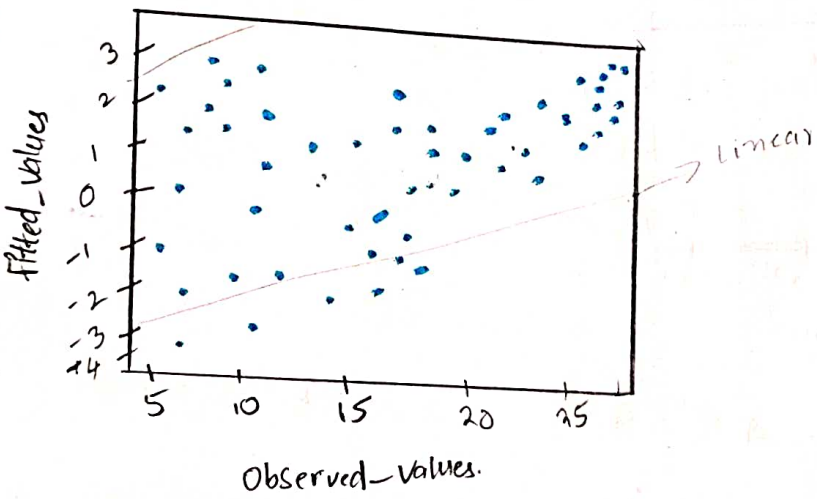
plt.scatter (y_test, test_res)

plt.xlabel ("Observed_values")

plt.ylabel ("fitted_values")

plt.show()

Out:



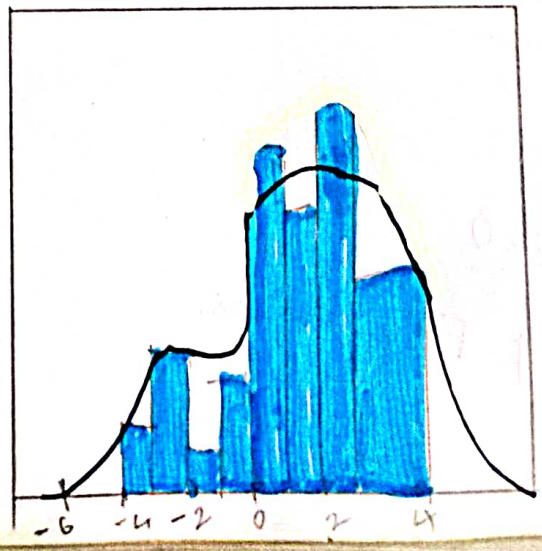
* Normality of errors

sns.distplot (test_res, bins = 15, kde = True)

plt.show()

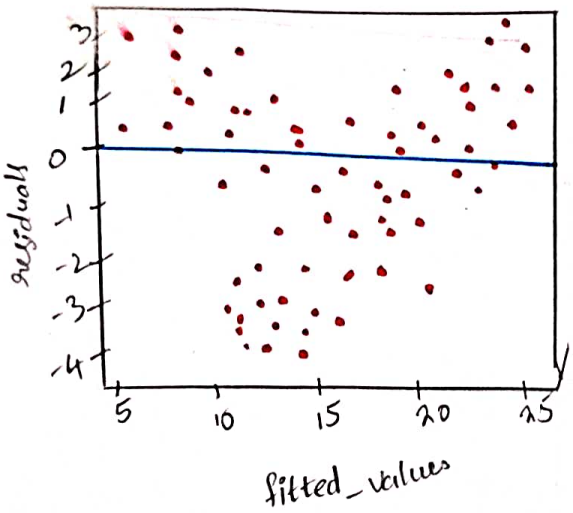
Out:

Left skewed →



3. Equal Variance of Errors (Homoscadesicity) ✗

```
# plt.scatter(test_predictions, test_res, c="r")
# plt.axhline(y=0, color="blue")
# plt.xlabel("fitted_values")
# plt.ylabel("residuals")
# plt.show()
```



* Assumptions Failed, $R^2 = 86\%$ [We reject the model]

⇒ Every condition in checklist should satisfy Than, Only we consider the model

4. Variable Significance *

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

⇒ Hypothesis Testing For Variables.

import Statsmodels.formula.api smf

```
# model 1 = smf.ols("y ~ x", data = df).fit()
# model 1.summary()
```

Out: OLS Regression Results.

Dep variable : Y
Model : OLS
method : Least Squars

R^2-squared : 0.897
Adj. R-squared : 0.896
F statistic : 570.3
Prob (F-statistic) : $1.58e^{-96}$

For Model: Accept H_0 :- 1.58×10^{-96} - $P < 0.05$
↳ Reject H_0

	Coeff	Std err	t	$P > \|t\|$	[0.025	0.975]
Intercept	2.9389	0.312	9.422	0.000	2.324	3.554
β_1 X(0)	$\neq 0$ 0.2458	0.001	32.809	0.000	0.043	0.049
β_2 X(1)	$\neq 0$ 0.1885	0.009	21.893	0.000	0.472	0.206
β_3 X(2)	$= 0$ -0.0010	0.006	-0.177	0.860	-0.013	0.011

Variable wise → $P < 0.05$ → P. low Null. go → Reject H_0

Variable 1
H_0 : $\beta_1 = 0$ ✗
H_1 : $\beta_1 \neq 0$ ✓

Variable 2
$\beta_2 = 0$ ✗
$\beta_2 \neq 0$ ✓

Variable 3
H_0 : $\beta_3 = 0$ ✓
H_1 $\beta_3 \neq 0$ ✗

$P > 0.05$

⇒ Checking whether data has any influence value using influence index plots

1) * Influence Index plots, For Making "p" value equal < 0.05)

```
import statsmodels.api as sm
# sm.graphics.influence_plot(model1)
```

Why?

$\beta_3 = 0$,

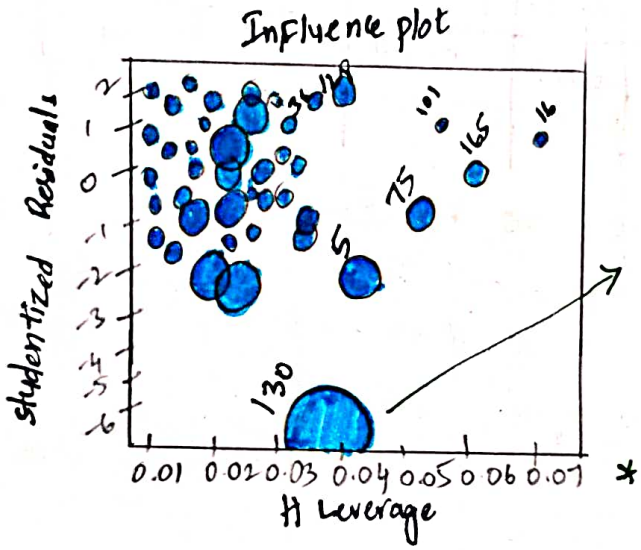
$\hat{y} = \beta_0 + \beta_1 x + \beta_2 x_2 + \beta_3 x_3$

By dropping, we are losing 1/3 of data = 33% data

In That Case, we drop some records, That are influencing More.

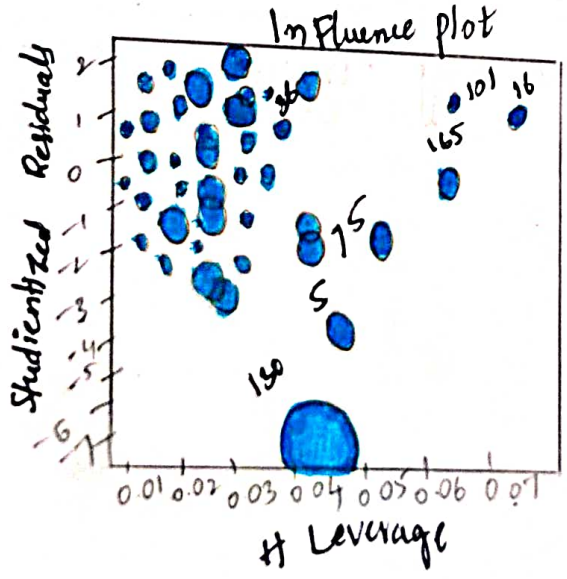
Ex:- 1/200 = 0.05%.

Out:



Index 130 is showing high influence, So, we exclude that entire row.

* Studentized Residuals = Residuals / standard deviation of residuals.



row

```
# df_new = df.drop(df.index[[130]], axis=0)
# df_new.
```

Out

	Tv	Radio	News Paper	Sales
0	230.1	37.8	69.2	22.1
1	44.5	39.3	45.1	10.4
⋮	⋮	⋮	⋮	⋮
199	232.1	8.6	8.7	13.4

199 x 4 columns

Once again Rebuild model

```
# model 2 = smf.ols(formula = "Sales ~ Tv + radio + newspaper",
                    data = df_new).fit()

# model2.summary()
```

Out : OLS Regression MODEL

→ This is one method of reducing the P value.

	Coeff	Std.err	t	P>|t|
Intercept	3.0931	0.290	10.654	0.000
Tv X(0)	0.0448	0.001	34.425	0.000
radio X(1)	0.1939	0.009	24.130	0.000
news paper X(2)	-0.043	0.005	-0.777	0.438

→ Reduced From 0.86 to 0.43 By Reducing.

2)

⇒ Variance inflation Factor [VIF]

Variance inflation Factor [VIF] measures ratio between the variance for a given regression coefficient with only that variable in the model versus the variance for given regression coefficient with all variables in the model.

1 independent variable influence on other independent variable is called as ("VIF")

affecting

X_1 X_2 X_3

TV → radio → news

TV ← Radio → news

TV ← Radio ← news

$$V.I.F = \frac{1}{1-R^2}$$

```
# rsq_Tv = smf.ols ("Tv ~ radio + newspaper", data = df)
                                              .fit()

# rsq_Tv.summary()
```

Out : OLS Regression Results

R-squared = 0.005

$$V.IF = \frac{1}{1-(0.05)^2}$$

V.IF = 1.0000

Calculating VIF's values of independent variables

```
# rsq_Tv = smf.ols ("Tv ~ radio + newspaper", data = df).
                                    .fit().rsquared

# Vif_Tv = 1/(1 - rsq_Tv)
```

```
# rsq_radio = smf.ols("radio ~ TV + newspaper", data=df)
                .fit().rsquared

# vif_radio = 1/(1-rsq_radio)

.....

# rsq_newspaper = smf.ols("newspaper ~ radio + TV",
                  data = df).fit().rsquared

# vif_newspaper = 1/(1-rsq_newspaper)

.....

Storing VIF values in a dataFrame

# d1 = {"variables": ["TV", "radio", "newspaper"],
        "VIF": [vif_TV, vif_radio, vif_newspaper]}

# vif_frame = pd.DataFrame(d1)

# vif_frame
```

2nd way : VIF

Out

	variable	VIF
0	TV	1.004611
1	Radio	1.144952
2	newspaper	1.145187

If the VIF model > (greater) 4. For any independent variable, drop that particular variable

Rishu
12/04/22
4:40 pm

3) Dt: 16/9/22

$[SSE)_{min}$

$[E(y-\hat{y})^2]_{min}$

$\Rightarrow$ AV plot [Added Variable plot)

$\frac{d(o)}{dx}$ at $x=0$

Partial differention instead of normal differentition. → Apply $\frac{dou}{dau(x}$ $\Rightarrow$ $\frac{\partial}{\partial x}$

#

What is partial differention ?

$\sum [y-\hat{y}]^2$ → it should be Minimum

$\{[y-[\beta_0+\beta_1 x_1+\beta_2 x_2+\beta_3 x_3]]^2$ minimum

$\Sigma[y-\beta_0-\beta_1 x-\beta_2 x_2+\beta_3 x_3]]^2$ minimum

In Simple linear Regression ?

$\frac{d}{dx}[E[y-\beta_0-\beta_1 x_1)^2]$ at $x=0$ → He we have only one variable.

In Multiple linear Regression ?

Constant Calculate Constant

$\frac{\partial}{\partial x}\Sigma[y-\beta_0-\underline{\beta_1 x}-\underline{\beta_2 x_2}-\underline{\beta_3 x_3}]]^2$

calculate Constant constant

↓

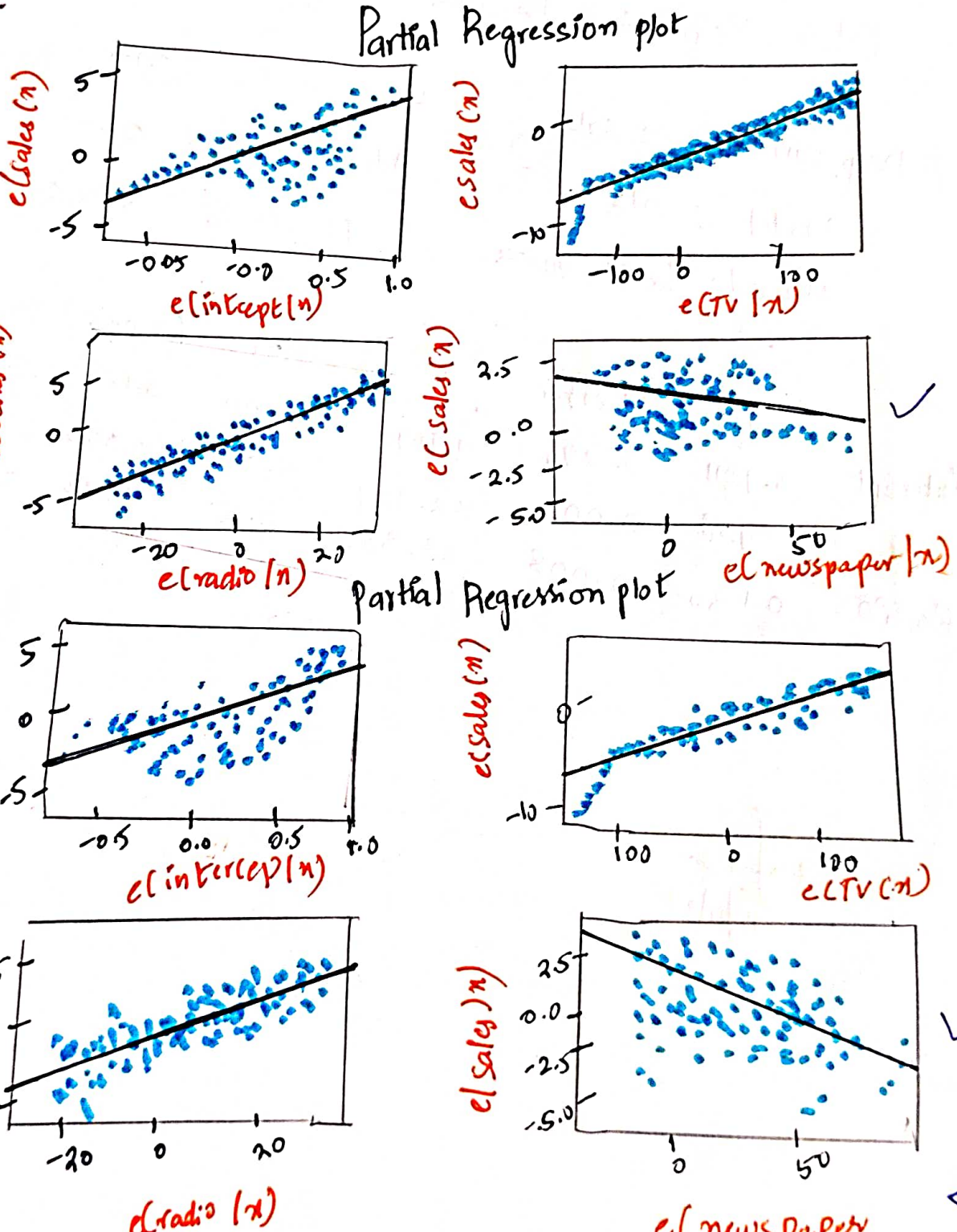
when we calculate with $\frac{dou}{dau(x)}$ $\frac{\partial}{\partial x}$.

When we calculate with one variable other variable will be Constant.

⇒ it is apply Simple Linear Regression on x_1 and x_2 Simple Linear Reg on x_3. it is Applying individually S.L.R on Each and Every individual. variable.

sm.graphics.plot_(partregress)- grid (lm)
Partregress

Out:



Added variable plot is not showing any significance For

⇒ Final model including "TV" and "Radio" Only

```
# final_model = smf.ols(formula = "Sales ~TV + radio",
                        data = df).fit()

# final_model.Summary()
```

Out

OLS Regression Results

Dep. Variable : sales
Model : OLS
Method : Least Squares

R-squared : 0.897 = 90%
Adj. R-squared : 0.896
F-statics = 859.6

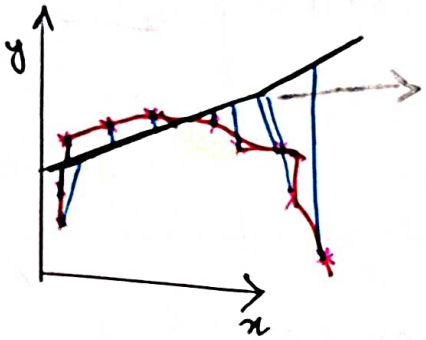
	Coeff	Std. error	t	P(>|t|)	(0.025)	(0.975)
Intercept	2.9211	0.294	9.919	0.000	2.340	3.502
TV	0.0458	0.001	32.909	0.000	0.043	0.048
Radio	0.1880	0.008	23.382	0.00	0.172	0.204

16/4/22

Dt: 16/04/23
1:00pm

Polynomial Regression

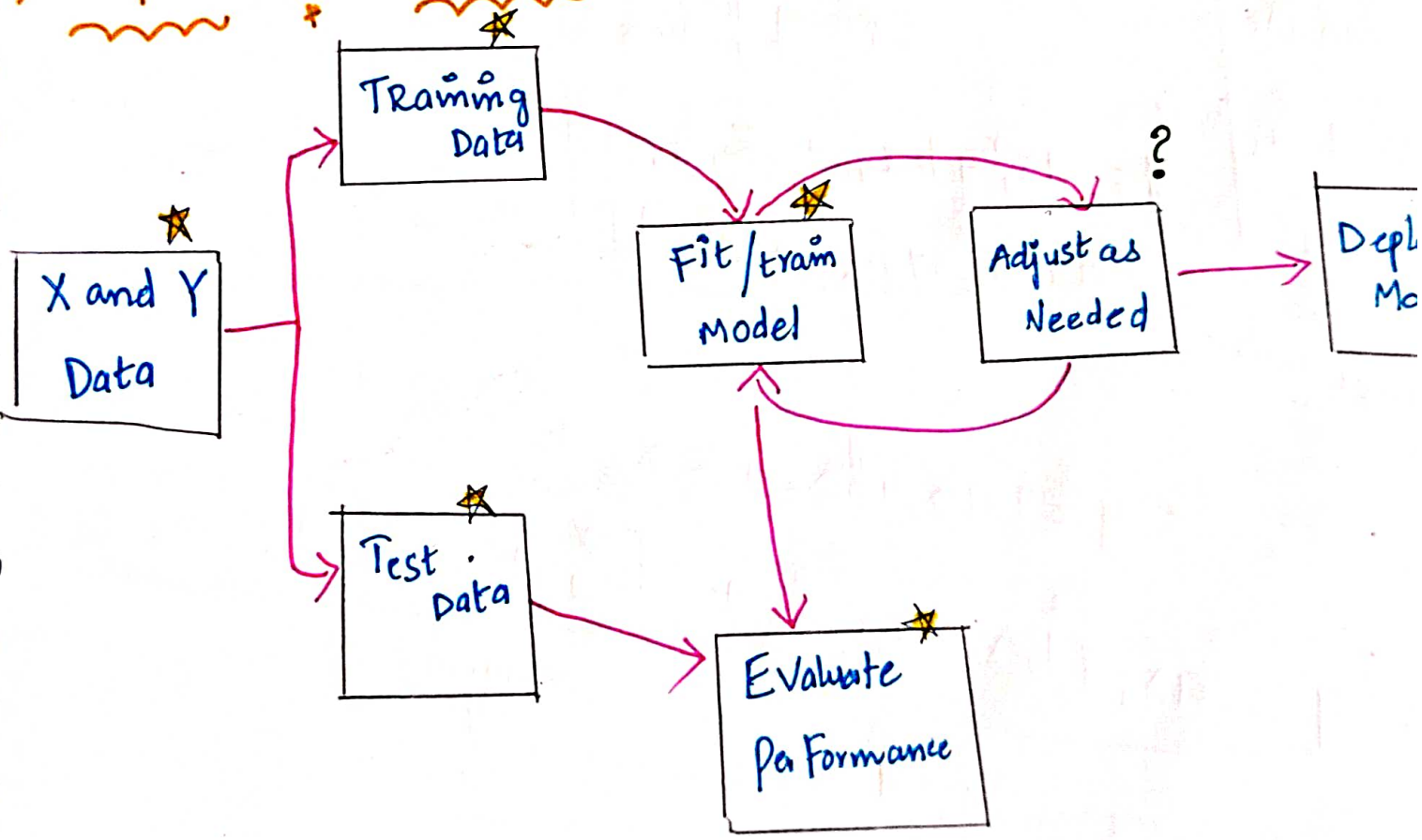
⇒ Any Regression problem. $[SSE]_{min}$ ⇒ $[\varepsilon[y-\hat{y}]^2]_{min}$



straight line (Linear line)

$[\varepsilon(y-\hat{y})^2]$

⇒ Supervised Machine learning process



S.L.R → 1 Variable

$y = ax + b$

Linear

M.L.R → 2 Variables

$y = ax + b + cx + d$

$y = ax_1 + bx_2 + c$

it is came from joining two equations

$= ax_1 + b + cx_2 + d$ (b, d: intercepts)

$y = ax_1 + cx_2 + e$ (always intercept have one value only)

3 Variable

$y = ax_1 + bx_2 + cx_3 + d$

it is came from joining three equations

$y = ax_1 + b$

$y = cx_2 + d$

$y = ex_3 + f$

Quadratic

1 Variable

$y = ax^2 + bx + c$

Here we write intercept value as "Z"

$y = ax_1^2 + bx_1 + dx_2^2 + ex_2 + z$

it is came from joining two equations

$y = ax_1^2 + bx_1 + z_1 c_1 + dx_2^2 + ex_2 + z_2 f_2$

* $y = ax_1 + bx_2 + cx_3 + z$

* $y = ax_1 + bx_2 + cx_3 + dx_1^2 + ex_2^2 + fx_3^3 + z$

(squaring of)

$+ gx_1x_2 + hx_2x_3 + ix_3x_1$

Combinations of variable [iteration terms]

original data | Squaring original Data | Combination (or) Iteration terms

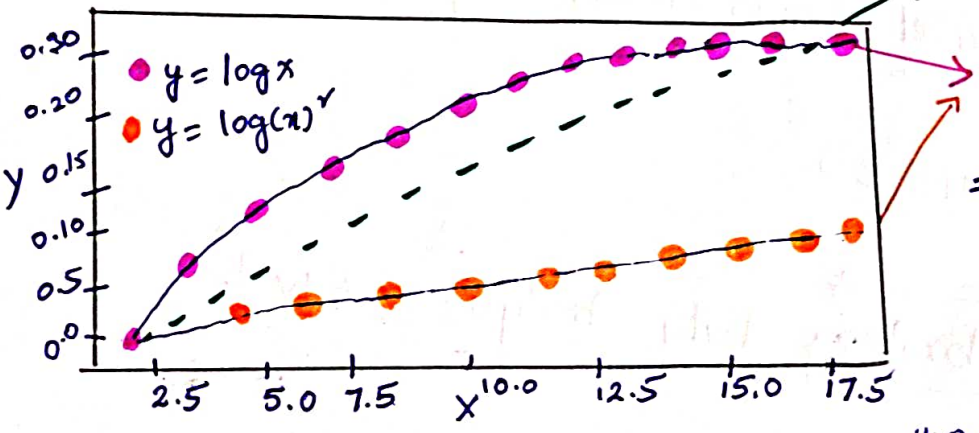
TV	Radio	newspaper	Sales	TV^2	$Radio^2$	$Newspaper^2$	TV*Radio	Radio*Newspaper	Newspaper*TV
—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—

* Quadratic equation : $x_1' * x_2' = x_1' + x_2'$?

multiply → degree 2 ; if we add two variables → degree 1

$a^m \times a^n = a^{m+n}$

⇒ Polynomial Regression



#quadratic – ax^2+bx+c

⇒ If we want to calculate Combinations with Squaring.

* Degree 2 = TV^2 (01) $x_1^2+x_2^2+x_3^2$ + $x_1x_2+x_2x_3+x_3x_1$ + $x_1+x_2+x_3$ (interaction terms)

* Degree 3 = $x_1^3+x_2^3+x_3^3$ + $x_1x_2^2+x_2x_3^2+x_3x_1^2$ + $x_1^2x_2+x_2^2x_3+x_3^2x_2$ + $x_1^2+x_2^2+x_3^2$ + $x_1+x_2+x_3$

cubic Equation : ax^3+bx^2+cx+d

Step 1 — Problem Understanding

Step 2 — Data Collection

```
# df = pd.read_csv("Advertising.csv")
# df.head()
```

Step 3 — Data Understanding

```
# df.shape
```

Step 4 — Dataset Understanding

```
# df.info()
```

Step 5 — Exploratory Data Analysis [EDA]

```
# df.describe()
# sns.pairplot(df)
```

Step 6 — Data cleaning

```
# df.isnull().sum()
```

Step 7 — Data wrangling

```
# X = df.drop("sales", axis=1)
# y = df["sales"]
```

Adding new Features to original Data

Polynomial Regression with Sci-kit Learn

```
from sklearn.preprocessing import PolynomialFeatures
# polynomial_converter = PolynomialFeatures(degree=2, include_bias=False)
```

include_bias = False → don't include "z" value

$ax_1^2 + bx_2^2 + cx_3^2$
$dx_1x_2 + ex_2x_3 + fx_3x_1$
$gx_1 + hx_2 + ix_3$

```
# X_poly = polynomial_converter.fit_transform(X)
```

fit_transform: Calculate, Convert

```
# X_poly.shape
```

Out (200, 9) — changed 3 to 9

rows = 200, columns

STEP

```
from sklearn.model_selection import train_test_split
# X_train, X_test, y_train, y_test = train_test_split(X_poly, y, test_size=0.3, Random_state=29)
```

200 → 140 × 9 = train, 60 × 9 = test

MODEL Fitting On Polynomial Data.

```
from sklearn.linear_model import LinearRegression

# model = LinearRegression()
# model.fit(x_train, y_train)
```

Predictions

```
# train_pred = model.Predict(x_train)
# test_pred = model.predict(X_test)
```

Evaluation

```
# model.score(x_train, y_train)   [# Train R^2]
```

Out 0.986

```
# model.score(x_test, y_test)   [# Test R^2]
```

Out 0.980

Cross - Validation

```
from sklearn.model_selection import cross_val_score

# scores = cross_val_score (model, X_poly, y, cv=5)

# print (scores)

# scores.mean()
```

Out [0.987 , 0.989, 0.991, 0.958, 0.993]

mean : 0.984

RMSE

```
from sklearn.metrics import mean_squared_error

# test_RMSE = np.sqrt (mean_squared_error (y_test, test_pred))

# train_RMSE = np.sqrt (mean_squared_error (y_train, train_pred))

# print (train_RMSE, test RMSE)
```

Out 0.5950 , 0.7233

Earlier,

Multiple Linear Regression
* RMSE - 1.94

Polynomial Regression
* RMSE - 0.72

Applying loop for knowing better Accuracy From First to last.

- from sklearn.preprocessing import PolynomialFeatures
- from sklearn.model_selection import train_test_split
- from sklearn.linear_model import LinearRegression.

```
# train_rmse_errors = [ ]
# test_rmse_errors = [ ]

for d in range [1,10):
# polynomial_converter = PolynomialFeatures (degree = d, include_bias = False)
# X_poly = polynomial_converter.fit_transform(x)
# X_train, X_test, y_train, y_test = train_test_split (X_poly, y, test_size = 0.3, Random_state = 29)
# model = LinearRegression()
# model.fit (x_train, y_train)
# train_pred = model.predict (x_train)
# test_pred = model.predict (X_test)
# train_RMSE = np.sqrt (mean_squared_error (y_train, train_pred)
# train_rmse_errors.append (train_RMSE)
# test_RMSE = np.sqrt (mean_squared_error (y_test, test_Pred)
# test_rmse_errors.append (test_RMSE)
```

train_rmse_errors

Out : 0 [1.7345 d=1 ← degree
1 0.5879 d=2
2 0.4339 d=3
3 0.3517 d=4
4 0.2509 d=5
5 0.19704 d=6
6 5.4214 d=7
7 0.14180 d=8
8 0.16654 d=9

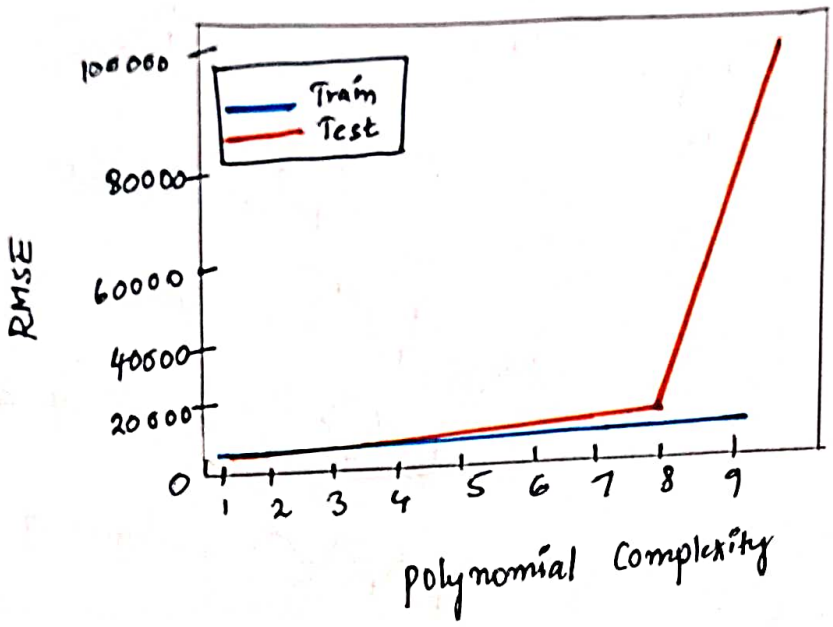
No. of bends
↓
∪ = Quadratic
∩∪ = Cubic
∪∩∪ = degree of 4
∪∩∪∩∪∩∪ = degree of 7

test_rmse_errors

Out : 0 1.5161 d=1 ↓
1 0.6646 d=2
2 0.5803 d=3
3 0.5077 d=4
4 2.5758 d=5 ↑
5 4.4926 d=6
6 1381.404 d=7
7 4449.599 d=8
8 9591.24 d=9

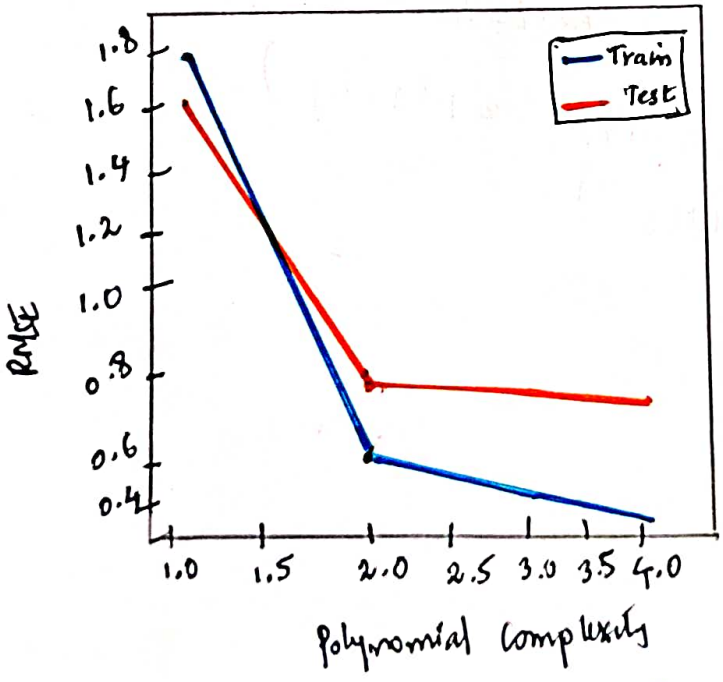
plt.plot (range (1,10), train_rmse_errors, Label = "TRAIN")
plt.plot (range (1,10), test_rmse_errors, Label = "TEST")
plt.xlabel ("Polynomial complexity")
plt.ylabel ("RMSE")
plt.legend()
plt.show()

Out :




```
# plt.plot(range(1,5), train_rmse_errors[:4], label = "TRAIN")
# plt.plot(range(1,5), test_rmse_errors[:4], label = "TEST")
# plt.xlabel("Polynomial Complexity")
# plt.ylabel("RMSE")
# plt.legend()
# plt.show()
```

Out :



S.No.5 Finalizing Model choice

```
# final_poly_converter = polynomial Features (degree = 2,
                                              include_bias = False)

# final_model = Linear Regression()
# final_model.fit (final_poly_converter.fit_transform(x), y)
```

Out: Linear Regression()

* Saving MODEL and CONVERTER *

```
from joblib import dump
# dump (final_model, "Sales_poly_model.joblib")
```

Out ["Sales_poly_model.joblib"]

```
# dump (final_poly_converter, "poly_converter.joblib")
```

Out: ["poly_converter.joblib"]

Deployment & predictions :

Client wants to spend 149k on "TV", 22k on "radio" 12k on "Newspaper" Ads. How many units could we expect to sell as a result ?

```
from joblib import load
```

```
# Loaded_poly = load ("poly_converter.joblib")
# loaded_model = load ("Sales_poly_model.joblib")

-
# Campaign_poly = loaded_poly.transform ([[149, 22, 12]])

-
# final_model.predict (campaign_poly)
```

Out: array ([14.51114])

16/04/22
9:25 pm