

Robust Self-Supervised Anomaly Detection with Diffusion Models

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Abstract

A robust anomaly detection mechanism should possess the capability to effectively remediate anomalies, restoring them to a healthy state, while preserving essential healthy information. Despite the efficacy of existing generative models in learning the underlying distribution of healthy reference data, they face primary challenges when it comes to efficiently repair larger anomalies or anomalies situated near high pixel-density regions. In this paper, we introduce a self-supervised anomaly detection method based on a diffusion model that samples from multi-frequency, four-dimensional simplex noise and makes predictions using our proposed dynamical hybrid UNet vision transformer (UDHVT). This simplex-based noise function helps address primary problems to some extent and is scalable for three-dimensional and colored images. In the evolution of vision transformers, our developed architecture serving as the backbone for the diffusion model, is tailored to treat time and noise image patches as tokens. We incorporate long skip connections bridging the shallow and deep layers, along with smaller skip connections within these layers. Furthermore, we integrate a partial diffusion Markov process, which reduces sampling time, thus enhancing scalability. Our method surpasses existing generative-based anomaly detection methods across three diverse datasets, which include BrainMRI, Brats2021, and the MVtec Leather dataset. It achieves an average improvement of +10.1% in Dice coefficient, +10.4% in IOU, and +9.6% in AUC.

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supervised learning models poses notable challenges due to the substantial amount of annotated data they necessitate, making the acquisition process expensive and time-consuming. Self-supervised anomaly detection is a pow-

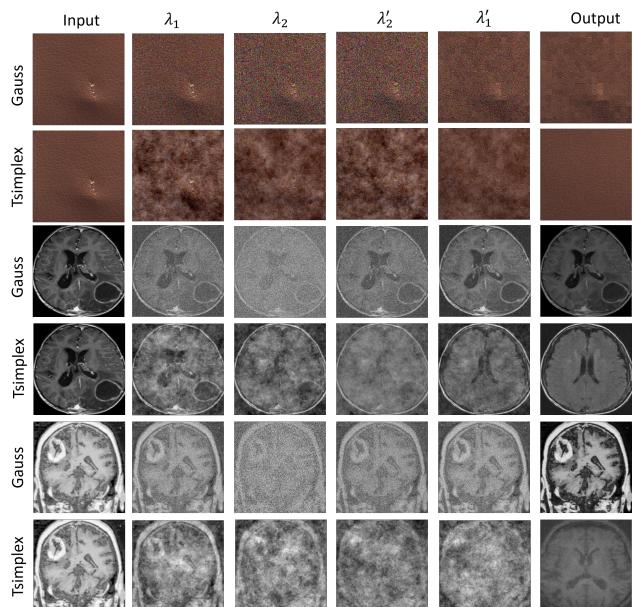


Figure 1. Forward and backward diffusion processes for anomaly detection in Leather (rows 1-2) and BrainMRI (rows 3-6) test data with a partial diffusion step (300), where $\lambda_1 (= 100)$ and $\lambda_2 (= 200)$ represent intermediate steps in the forward process, while $\lambda'_1 (= 200)$ and $\lambda'_2 (= 100)$ indicate diffusion steps in the backward process.

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1. Introduction

The scarcity of experts with the ability to diagnose and treat specific medical conditions is a pressing concern in developing countries [19]. To illustrate, consider the ratio of dermatologists to the general population, which can plummet to as low as 1 per 216,000 people [14]. This motivation fuels the development of a deep learning system with the ability to localize diseases and thereby prevent misdiagnosis or underdiagnosis [10, 27]. Nonetheless, employing

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erfull deep learning algorithms that train on healthy or normal infrance data which is used as a threshold for anomalies. The primary aspect of these algorithms is to address unhealthy or abnormal regions, followed by the calculation of the target anomaly using the difference of squares. When it comes to diverse anatomy, anomaly patterns, and distribution shifts, image data, especially medical image data, can be quite complex. Unsupervised models struggle with sampling from complex data distributions due to their limited assumptions, inadequate representation capac-

052 ity, risk of mode collapse, issues with high-dimensional
 053 data, sample efficiency challenges, difficulties in evalua-
 054 tion, and sensitivity to initialization. For tasks requiring
 055 precise sampling from highly complex data distributions,
 056 alternative, more advanced generative models needed. Genera-
 057 tive models have demonstrated their potency in self-
 058 supervised representation learning of the underlying distri-
 059 bution, particularly in the context of healthy inference data
 [19, 35, 50, 52]. Denoising diffusion probabilistic mod-

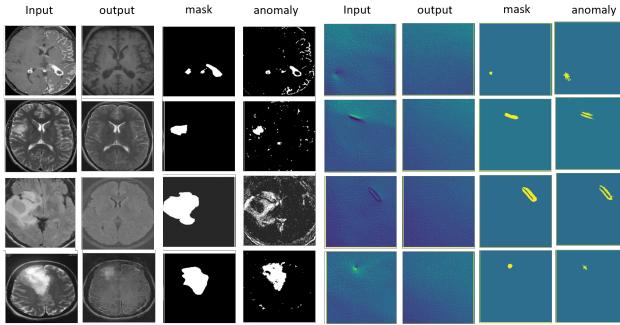


Figure 2. Abnormality repairing under the condition of larger anomaly, anomaly near to the high intensity region and coloured images by our proposed methodology.

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 061 els (DDPMs) [22] have demonstrated remarkable effective-
 062 ness in self-supervised representation learning and are cap-
 063 able of generating samples even from complex data distribu-
 064 tions with superior convergence, as compared to genera-
 065 tive adversarial networks (GANs) and variational autoen-
 066 coders (VAEs) [8, 50]. DDPM consists of two steps: a for-
 067 ward noise injection step and a backward denoising step.
 068 In the forward step, noise is injected from a $N(0, I)$ dis-
 069 tribution, while the denoising backward step stochastically
 070 transforms the samples from a gaussian distribution onto a
 071 learned data distribution. We employ this approach to train
 072 DDPM on healthy reference data, which maps anomaly data
 073 onto the healthy distribution through a diffusion process but
 074 gaussian noise at each diffusion step does not able to re-
 075 cover the anomaly and that results into unrepairs anomaly
 076 [52]. Gaussian noise has a constant power spectral density,
 077 meaning it has equal power across all frequencies which
 078 makes it "white" noise. The author of the paper [52] ex-
 079 plored the use of simplex noise [34], a common choice
 080 for tasks such as procedural terrain generation, texture syn-
 081 thesis, and creating natural-looking patterns like clouds or
 082 marble textures. However, it was discovered that models
 083 trained based on simplex noise have a few disadvantages,
 084 including a decrease in sample quality, particularly when
 085 subjected to higher noise levels (a further t value). These
 086 models also struggle to repair anomalies situated near other
 087 high-frequency information, and they exhibit limited explo-
 088 ration capabilities, especially in the context of complex and
 089 high-dimensional simplex noise, which affects their focus

090 on tasks like processing 3D and colored images. More-
 091 over, the model was trained using a batch size of one due
 092 to the time-complexity, which scales as $\mathcal{O}(\text{batch_size} \times t)$,
 093 where t represents the time required to sample an image of
 094 size (H, W) . This motivation led us to address these chal-
 095 lenges, resulting in the development of a four-dimensional
 096 simplex noise function capable of generating noise for three
 097 dimension and colored images while maintaining the same
 098 processing time for batched images.

099 Additionally, we introduce a simple and versatile Dy-
 100 namic Hybrid UNet architecture inspired by vision trans-
 101 formers called UDHVT (Figure 3a) designed as the back-
 102 bone of diffusion models. Our algorithms offer several ad-
 103 vantages over adversarial training, including improved sam-
 104 ple quality and stable training, particularly beneficial for
 105 smaller datasets.

106 The paper makes several significant contributions, which is
 107 be summarized as follows:

- 108 • We present Tsimplex, a novel sampling strategy tailored
 109 for 3D and colored images. Tsimplex enhances the sam-
 110 pling process, improving the quality of the samples gen-
 111 erated.
- 112 • We have developed a Vision Transformer-based U-net
 113 model as the backbone for diffusion models and improved
 114 the multi-head attention mechanism by integrating inter-
 115 activity between the attention heads.

2. Related Work

116 **Self-supervised anomaly detection:** In the realm of
 117 anomaly detection, Self-Supervised Learning (SSL) plays
 118 a pivotal role in training systems to capture intricate rela-
 119 tionships within data, with a primary focus on detecting ir-
 120 regular patterns. SSL encompasses two distinct approaches
 121 [2]: Invariance-based methods [1, 2, 5, 8, 9] and generative
 122 methods [3, 12, 13, 21–23, 33, 52]. Generative models, in
 123 particular, have made substantial contributions to anomaly
 124 detection and have paved the way for addressing more intri-
 125 cate tasks, particularly in self-supervised understanding and
 126 the generation of natural images.

127 Variational Autoencoders (VAEs) [25], for instance,
 128 have garnered acclaim for their ability to generate data
 129 while also providing interpretable latent representations.
 130 However, VAEs do encounter challenges when tasked with
 131 generating high-quality, sharp images. Conversely, Genera-
 132 tive Adversarial Networks (GANs) [20] have excelled in
 133 producing visually appealing samples, yet they can be chal-
 134 lenging to train and may suffer from mode collapse. More
 135 recently, Flow-based models like Glow [24] have emerged,
 136 offering exact likelihood estimation and high-quality sam-
 137 ples but tending to be computationally intensive. When
 138 selecting a generative model, it's crucial to consider the
 139 specific task requirements and the trade-offs between inter-
 140 pretability, image quality, and computational resources.

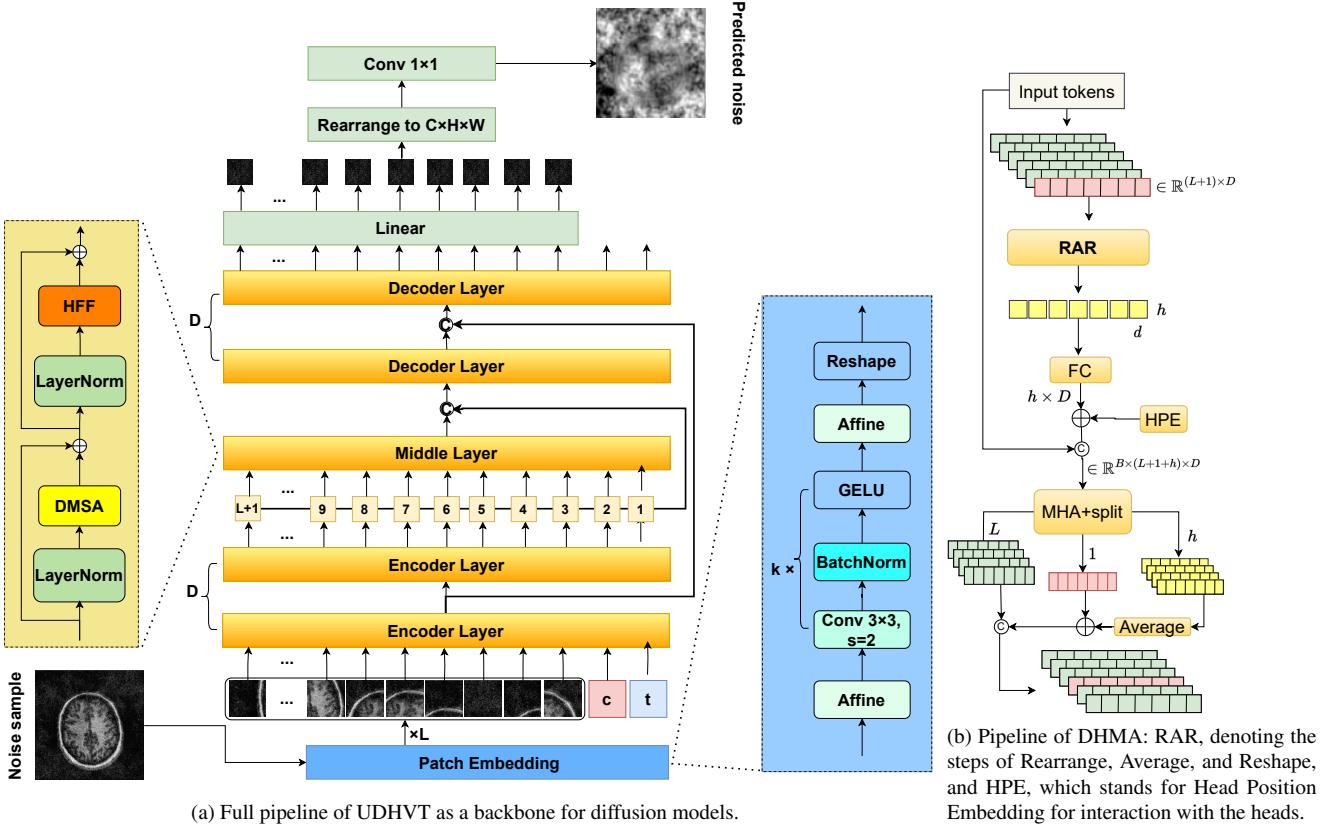


Figure 3. The complete UDHVT architecture designed for partial diffusion models, where it processes noisy image inputs, including diffusion steps as tokens, and predicts the noise.

GANs have proven to be instrumental in learning highly detailed and realistic images [11, 16, 20]. Authors in [37] introduced Deep Convolutional Generative Adversarial Networks (DCGANs), showcasing GANs' ability to capture semantic image content, which has led to intriguing applications like vector arithmetic for manipulating visual concepts. Additionally, [53] trained GANs on natural images and employed the trained models for semantic image inpainting, demonstrating the versatility and potential of GANs in various image-related tasks.

While VAE models are sometimes criticized for their poor sample quality, GANs [44] come with their set of challenges, including their inability to repair anomalies, training instability, model collapse, and a reliance on large datasets.

Recent strides in the field of Diffusion Probabilistic Models (DDPM) [13, 22] have showcased their ability to generate higher-quality samples from complex distributions with superior coverage compared to GANs [44] and VAEs. However, these improvements come at the expense of reduced scalability and increased sampling times, primarily due to the necessity of employing long Markov chain sequences [26]. Furthermore, DDPMs also have limitations in capturing larger anomalies caused by Gaussian noise.

In their paper [52], the authors introduced a noising scheme for diffusion models based on simplex noise. However, this scheme comes with several significant drawbacks. As the noise level increases, there is a noticeable decrease in sample quality, especially when applying noise to higher values of " t ". Moreover, the scheme struggles to effectively repair anomalies located near other high-frequency information, limiting its ability to handle complex data patterns. Additionally, it exhibits limited exploration capabilities, especially concerning 3D and colored images, where its focus is less well-defined. Most crucially, this noising scheme leads to increased sampling times, which can be a significant practical limitation, particularly when dealing with batched images or real-time applications.

Backbone of diffusion models: Indeed, along with the development of diffusion model algorithms [5, 6, 15, 23, 29, 30, 45, 47–49], the revolution in backbone models plays a crucial and integral role. An illustrative instance is U-Net, constructed upon a convolutional neural network (CNN) and previously utilized in research [22, 46]. The CNN-driven U-Net design features a sequence of down-sampling blocks, a series of up-sampling blocks, and extensive skip connections between these two sets of blocks

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[13, 39, 40, 43]. This architectural framework has held a prominent position within diffusion models utilized for image generation assignments. Conversely, vision transformers (ViT) [17] have demonstrated promising, and in some cases, superior performance compared to CNNs in a range of tasks. In their paper [6], the authors introduce a straightforward and versatile architecture for image generation using Vision Transformers within diffusion models. Experimental results illustrate that U-ViT performs on par with, if not better than, a CNN-based U-Net of a similar size. However, recent studies [18, 32, 38] that investigate the reasons behind the difference in data efficiency between ViTs and CNNs have led to the conclusion that it attributed to a lack of inductive bias. The paper [31] attempts to bridge the gap between Vision Transformers and Convolutional Neural Networks as a potential solution to enhance their respective biases.

3. Methodology

3.1. Overview of Diffusion Models

Diffusion models, specifically Diffusion Probabilistic Models (DDPMs) [22], are a class of generative models that employ a diffusion process resembling a Markov chain. This process comprises sequential steps, where each step involves sampling from a Gaussian distribution. Importantly, the mean of this distribution depends on the current state of the chain. As the number of steps increases, the distribution over the chain converges to a Gaussian distribution. Let's begin with data represented as $x_0 \sim q(x_0)$ and a Markov chain process q progressing from x_1 to x_χ , injecting noise at each step from normal distribution with a variance schedule β_t :

$$q(x_t | x_{t-1}) = N(x_t; \sqrt{1 - \beta_t}x_{t-1}, \beta_t I) \quad (1)$$

In DDPM, instead of repeatedly applying q to sample $x_t \sim q(x_t | x_0)$, it expresses $q(x_t | x_0)$ as a Gaussian distribution using an auxiliary noise variable $\eta \sim N(0, I)$:

$$q(x_t | x_0) = N(x_t; \sqrt{\bar{\alpha}_t}x_0, (1 - \bar{\alpha}_t)I) \quad (2)$$

$$= \sqrt{\bar{\alpha}_t}x_0 + \eta\sqrt{1 - \bar{\alpha}_t} \quad (3)$$

Here, $1 - \alpha_t = \beta_t$, and $\bar{\alpha}_t = \prod_{s=0:\chi} \alpha_s$. $1 - \bar{\alpha}_t$ serves as a noise scheduler in place of β_t . To sample from the posterior distribution, which is also Gaussian, Bayes' theorem is applied by sampling from each reverse step of the distribution $q(x_{t-1} | x_t)$ for t ranging from χ to 1, eventually reaching $q(x_0)$. The parameters (mean vector and covariance matrix) of $q(x_{t-1} | x_t)$ can be estimated using neural networks, which approximate this distribution. The objective of these neural networks is to minimize the dissimilarity between probability distributions from step t to $t - 1$ using Kullback-Leibler divergence (D_{KL}). The

loss function for training the parameterized distribution $p_\theta(x_{t-1} | x_t)$ is expressed as the variational lower bound L_{vlb} on the marginal likelihood $p_\theta(x_0)$. It is defined as the sum of terms L_0 through L_T , where $L_0 = -\log p_\theta(x_0 | x_1)$, $L_{t-1} = D_{KL}(q(x_{t-1} | x_t, x_0) || p_\theta(x_{t-1} | x_t))$, and $L_T = D_{KL}(q(x_T | x_0) || p(x_T))$. These terms quantify the reconstruction loss, conditional divergence, and final divergence, respectively, in the context of this variational lower bound formulation.

Conceptually, a neural network is also seen as a mapping from a simpler Gaussian distribution to a more complex distribution of images. This mapping thought of as a non-parametric method for defining the mean function of a Gaussian process. A network denoted as $\eta_\theta(x_t, t)$ with parameters θ for predicting η trained to simplify the objective by DDPM [22] and enhance the quality of sampling. For a given $x_0 \sim q(x_0)$ and $\eta \sim N(0, I)$ at each step t within the range $[0, \chi]$, the following loss function is defined:

$$L(\theta) = \frac{1}{\chi} \sum_{t=0:\chi} \|\eta - \eta_\theta(x_t, t)\|^2 \quad (4)$$

3.2. Noise function

The visual world is endlessly captivating due to its consistency across different scales, a phenomenon known as scale invariance, which can be observed in various visual contexts [42]. In natural images, the distribution of frequencies adheres to a power-law distribution, with lower-frequency components playing a more substantial role in defining the image's characteristics [51, 52]. However, there's a notable discrepancy in how DDPMs treat lower-frequency and high-frequency components when Gaussian noise is employed, mainly due to the uniform spectral density of this noise source as shown in the Figure 1. Conversely, diffusion models employing simplex noise tend to assume that lower-frequency components are relatively less corrupted, leading to the recovery of larger anomalous regions in the reverse process (1 for Tsimplex).

Tsimplex: Simplex noise represents an enhancement of Perlin noise [34], characterized by increased computational efficiency and the generation of smooth and structured randomness. It relies on gradient noise and is generated through the amalgamation of numerous noise octaves. In this process, each octave represents a higher-frequency variation of the noise from the previous octave. These octaves are weighted with decreasing amplitude and increasing frequency, yielding a more intricate and detailed noise pattern. The ultimate outcome is a textured noise that exhibits gradual and continuous variations across spatial dimensions shown in the Figure 5a. The generation of Tsimplex noise is outlined in Algorithm 1, while the pseudocode for the simplified simplex function can be found in Supplementary Material Algorithm 1. In the case of Tsimplex noise, we

286 follow similar steps as detailed in [52], but with additional
 287 exploration for three-dimensional and colored images. Notably,
 288 the total time required to sample noise from Tsimplex
 289 for a batch of 1000 steps is approximately 5.97 seconds,
 290 which is less than simplex noise and close to Gaussian
 291 noise, as demonstrated in Figure 4. This improvement
 292 is attributed to the Honeycomb pattern introduced by the
 293 additional time step parameter, $t \in \mathbb{R}^{batch_size \times num_step}$. Fur-
 294 thermore, we have observed that the sample quality of sim-
 295 plex noise decreases as t increases. One of the reasons for
 296 the lower sample quality is the asymmetry of simplex noise,
 297 as shown in Figure 5c, in contrast to Gaussian noise. In con-
 298 trast, Tsimplex provides similar sample quality to Gaussian
 299 noise, as illustrated in Figure 5b. For a more in-depth ex-
 300 amination of the symmetry of simplex noise, please refer to
 301 SUPPLEMENTARY MATERIAL.

Algorithm 1 GenNoise($S, t, O = 8, p = 0.9, \mu = 128$)

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1: Initialize,  $A \leftarrow 1, N \leftarrow 0$ 
2:  $x, y \leftarrow$  spatial grids,  $C \leftarrow$  channel grids
3: for  $i$  from 1 to  $O$  do
4:    $N \leftarrow$  simplex( $\frac{x}{\mu}, \frac{y}{\mu}, \frac{z}{\mu}, \frac{t}{\mu}$ )
5:    $N \leftarrow N + A \cdot N$ 
6:    $\mu \leftarrow \frac{\mu}{2}, A \leftarrow A \cdot p$ 
7: end for
8: return  $N$ 
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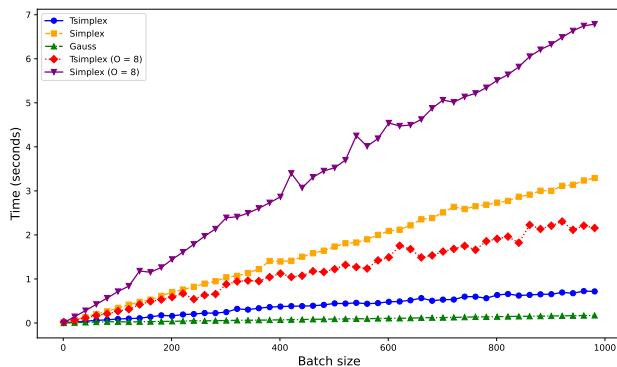


Figure 4. Time Required for sampling upto 1000 steps for grid of 64×64 and O represents number of octave

3.3. Dynamic Hybrid UNet

UDHVT serves as a fundamental component in diffusion models (see Figure 3a) for anomaly detection and can be applied across a variety of tasks within diffusion modeling. The primary goal of UDHVT is to minimize the loss as defined in equation 4 and generate predictions by removing the noise to reconstruct the image. It takes the noisy input x_t , the time step t , and predicts the noise added to x_t .

Sliding window patch embedding : Following the architectural principles of ViT [17], UDHVT divides the input images into patches and treats all patches, along with time, as tokens. The whole process of sliding window patch embedding (SPE) is shown in the Figure 3a and some function can formulated as follows.

$$\text{Affine}(x) = \text{Diag}(\nu)x + \phi \quad (5)$$

Where ν , and ϕ are learnable parameters initialized with 1 and 0 respectively. The output from the affine function undergoes a series of operations, including a Conv(3,3) operation, followed by batch normalization and activation functions, and this sequence is repeated up to k times. Finally, the result is post-processed once again through an affine function to get the sliding window patch embedding ($L = \frac{H \times W}{P^2}$).

UDHVT Layer: Following the architecture of UNet, UDHVT also comprises three types of layers: Encoder, Middle, and Decoder. These layers consist of the same types of blocks, as indicated by the colors in Figure 3a. These blocks primarily include dynamical multi-head attention (DMHA) (refer to Figure 3b) and a hybrid feed forward (HFF) block (see Figure 6). Inspired by [31], we incorporate the Head token into our DMHA. This addresses the issue of inductive bias in ViTs by allowing interactions between multiple heads, in contrast to the hierarchical structure of the vision transformer with window attention [28].

DMHA: The mechanism is formulated as follows: For $x \in \mathbb{R}^{b \times L \times D}$, we first apply Rearrange Average and Reshape (RAR), i.e., we rearrange D into $h \times d$, average with respect to $L + 1$ tokens, and reshape into $h \times d$. The output is then projected into $h \times D$ through a fully connected layer (FC), followed by layer normalization and activation. The head tokens are added with the head position embedding (HPE) so the position embedding of head will not be forgotten. We concatenate the generated head token with shortcut input and feed to the multi-head attention. Finally output is transformed into original shape by splitting into $L \times D$, $1 \times D$, and $h \times D$ as shown in the Figure 3b.

HFF: We employ a channel attention mechanism to consolidate the features of patch tokens into the class token as visualized in Figure 6. Before reaching the projection layers, we split the class token. Subsequently, the patch tokens undergo processing within a depth-wise convolutional (DW-Conv) integrated feed forward network, which includes a shortcut. The resulting output patch tokens are then subjected to averaging, producing a weight vector referred to as W . Following the squeeze-excitation operation, the output weight vector is channel-wise multiplied with the class token. This recalibrated class token is then joined with the output patch tokens to reconstruct the token. In UDHVT, we integrate skip connections, much like those employed in the Unet architecture, into the diffusion

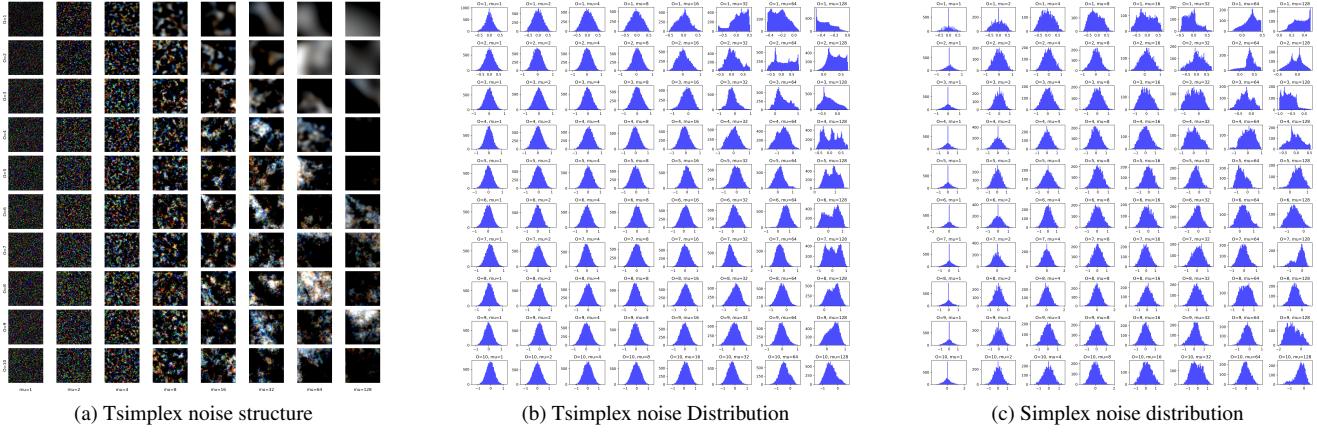


Figure 5. Comparing Tsimplex and Simplex noise, we analyze the impact of two variables: octave (N) on the y-axis and frequency (μ) on the x-axis, examining their influence on both distribution and structure.

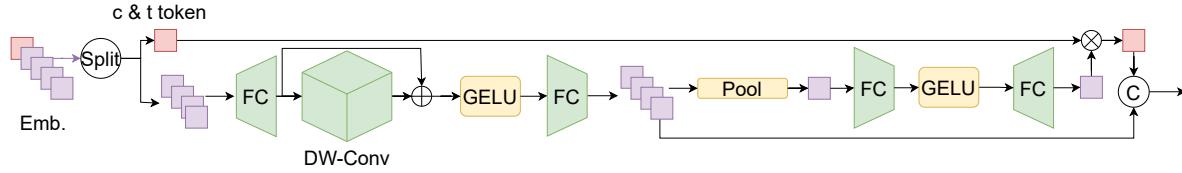


Figure 6. Pipeline of Hybrid Feed Forward network.

models, establishing connections between shallow and deep layers. The primary goal is to furnish pixel-level information, which is particularly sensitive to fine-grained features. Consequently, the incorporation of extensive skip connection shortcuts amplifies feature communication and preserves the fidelity of pixel-level details. Additionally, UDHVT employs a Conv(3, 3) block before predicting the noise. This step is intended to mitigate artifacts that may arise in images due to the attention mechanism.

4. Experiments

4.1. Implementation details

All experiments in this study are conducted using the DDPM algorithm as the foundation. The hyperparameters employed for DHUVT, used for approximating η_θ , closely resemble those of the ViT model as outlined in [32]. In all of our experiments, we employ the hyperparameters detailed in Table 3. The model is implemented using PyTorch and trained on a single GPU, specifically the NVIDIA RTX A4000, which boasts 16GB of GDDR6 VRAM.

4.2. Datasets

Brain MRI: We utilize the healthy brain dataset sourced from the NFBS repository [36]. This dataset comprises T1-weighted MRI scans with dimensions of $256 \times 256 \times 192$.

For our experiments, we focus on 2D slices of size 256×192 in the axial plane. Specifically, we allocate 100 of these slices for training purposes and reserve 25 for testing the algorithms. For anomaly detection, we curate a set of 154 tumor images from Kaggle, deliberately choosing a diverse range to pose a challenging task in tumor detection. In Figures 1 and 2, we exclusively showcase images from this tumor dataset, highlighting the substantial variations compared to the healthy brain dataset.

Lether: To train our model on typical inference data, we employ a dataset comprising 245 normal images. For the testing phase, we evaluate the model’s performance on abnormal inferences, which involve various anomalies in leather such as color variations, cuts, folds, glue marks, and punctures. These anomalies are sourced from the MVTec dataset [7].

AnnoBrats: From the BRATS 2021 (Brain Tumor Segmentation) dataset [4, 26], we initially preprocess it into healthy and anomaly datasets using segmentation masks for model training and testing, respectively. We select the top 1306 (40%) 2D slices with dimensions of $4 \times 240 \times 155$, utilizing all four modalities, as anomalies are more discernible in this perspective. For testing, we employ the top 1935 (60%) 2D slices with dimensions of $4 \times 240 \times 155$, along with segmentation masks.

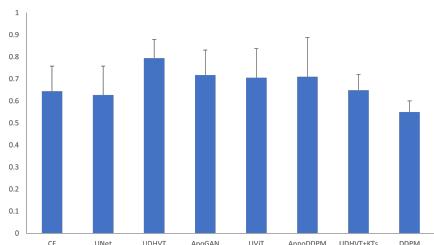
411 4.3. Results

412 To evaluate our method, we segment unhealthy/anomalous
 413 regions in the test dataset and employ segmentation met-
 414 rics, including the Sørensen–Dice coefficient (Dice), Inter-
 415 section over Union (IOU), Precision, and Recall. The re-
 416 sults for comparison are presented in Table 1. Additionally,
 417 we conduct an area under the curve (AUC) comparison, as
 418 depicted in Figure 7.

419 We explore three different backbone models for the
 420 backward diffusion process: Unet, UViT, and our proposed
 421 DHUVT, in conjunction with the Tsimplex noise function.
 422 In autoencoders and Variational Autoencoders (VAE),
 423 we utilize architectures similar to those in [41]. For the
 424 diffusion-based models, we first identify the optimal dif-
 425 fusion step range for anomaly detection and subsequently
 426 compute the results.

427 As illustrated in Figure 7, UDHVT outperforms other
 428 methods and exhibits lower deviation compared to AnnoD-
 429 DPM, though it does exhibit slightly higher deviation com-
 430 pared to DDPM. UDHVT leverages sampling from Tsim-
 431 plex, which exhibits fewer stochastic patterns than sim-
 432 plex, owing to the inclusion of batch sampling. In con-
 433 trast, DDPM samples from Gaussian noise, which has fewer
 434 stochastic patterns.

435 We also showcase the results of sampling from Tsimplex
 436 and Gaussian noise on the BrainMRI and leather (channels
 437 = 3) subset of the MVTec AD dataset, demonstrating excel-
 438 lent healthy reconstructions in Figure 1 and 2.



439 Figure 7. Comparision with the AUC matric on BrainMRI dataset
 440 for CE [33], AnnoGAN [44], AnnoDDPM [52], DDPM [22] with
 441 gaussian noise, and some backbones, including UNet [13], UViT
 442 [6], and proposed UDHVT with Tsimplex.

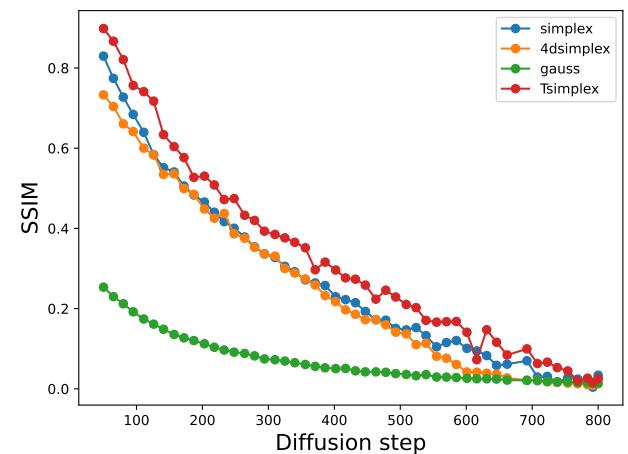
443 4.4. Ablation studies

444 **Noise functions:** Based on our experiments, we have ob-
 445 served a slight decrease in sample quality as the diffusion
 446 step increases. This phenomenon is likely attributable to the
 447 asymmetry in Tsimplex noise. We use structural similarity
 448 index measure (SSIM) to compare the compare the quality
 449 of reconctrction which is shown in the Figure 8. Tsimplex is
 450 giving the better SSIM than other noise function.

451 **Efficet of diffusion steps:** Selecting the diffusion step (t)

452 Table 1. Performance Comparison of the model’s ability to seg-
 453 ment abnormal regions. Square error is employed as a predictor of
 454 the mask. We use same architecture of VAE, AE based on ResNet.

(a) Brain MRI				
Model	Dice (\uparrow)	IOU (\uparrow)	Recall (\downarrow)	Precision (\uparrow)
Autoencoder	0.098±0.015	0.0632 ± 0.043	0.109±0.093	0.130±0.041
VAE	0.111±0.051	0.060±0.033	0.113±0.023	0.132±0.041
Context Encoder	0.242±0.207	0.152±0.14	0.252±0.218	0.276±0.227
F-AnnoGAN	0.140±0.00	0.108±0.001	0.38±0.011	0.018±0.010
DDPM	0.017±0.001	0.006±0.016	0.013±0.021	0.042±0.025
AnoDDPM	0.334 ± 0.299	0.243 ± 0.237	0.607 ± 0.452	0.263 ± 0.257
UNet (Backbone)	0.346 ± 0.224	0.234 ± 0.183	0.313 ± 0.241	0.491 ± 0.253
UViT (Backbone)	0.418±0.315	0.205±.174	0.297±0.253	0.421 ± 0.234
UDHVT (Ours Backbone)	0.466 ± 0.129	0.364 ± 0.186	0.705 ± 0.131	0.383 ± 0.197
(b) AnnoBrats2021				
Model	Dice (\uparrow)	IOU (\uparrow)	Recall (\downarrow)	Precision (\uparrow)
Autoencoder	0.015±0.047	0.063±0.028	0.125±0.028	0.112±0.042
VAE	0.016±0.048	0.05±0.028	0.115±0.029	0.115±0.043
Context Encoder	0.239±0.199	0.154±0.142	0.245±0.212	0.265±0.222
F-AnnoGAN	0.135±0.001	0.098±0.003	0.384±0.009	0.085±0.003
DDPM	0.010±0.013	0.004±0.008	0.007±0.010	0.036±0.049
AnoDDPM	0.334±0.253	0.146±0.130	0.083±0.185	0.042±0.148
UNet (Backbone)	0.371±0.281	0.162±0.144	0.092±0.206	0.047±0.163
UViT (Backbone)	0.409±0.309	0.179±0.158	0.101±0.227	0.051±0.179
UDHVT (Ours Backbone)	0.428±0.131	0.280±0.178	0.210±0.131	0.299±0.124
(c) Leather				
Model	Dice (\uparrow)	IOU (\uparrow)	Recall (\uparrow)	Precision (\downarrow)
Autoencoder	0.003±0.062	0.002±0.010	0.005±0.082	0.004±0.182
VAE	0.003±0.063	0.002±0.010	0.005±0.084	0.004±0.186
Context Encoder	0.141 ± 0.164	0.085 ± 0.111	0.531 ± 0.226	0.207 ± 0.323
DDPM	0.016 ± 0.020	0.008 ± 0.010	0.466 ± 0.250	0.008 ± 0.011
AnoDDPM	0.228±0.213	0.147±0.157	0.509±0.283	0.427±0.410
UNet (Backbone)	0.237±0.023	0.153±0.217	0.536±0.130	0.444±0.101
UViT (Backbone)	0.291±0.244	0.1987±0.201	0.295 ± 0.262	0.548±0.403
UDHVT (Ours Backbone)	0.306±0.134	0.205± 0.138	0.585±0.165	0.298±0.126



453 Figure 8. Effect of diffusion steps on SSIM with the backbone
 454 UDHVT and varity of noise functions.

455 is a critical parameter in the simplex noise diffusion model
 456 for anomalous detection. In our approach, termed the partial
 457 diffusion model, we avoid unnecessary diffusion steps af-
 458 ter the anomaly has been repaired. Consequently, we con-
 459 ducted a series of experiments on various diffusion mod-

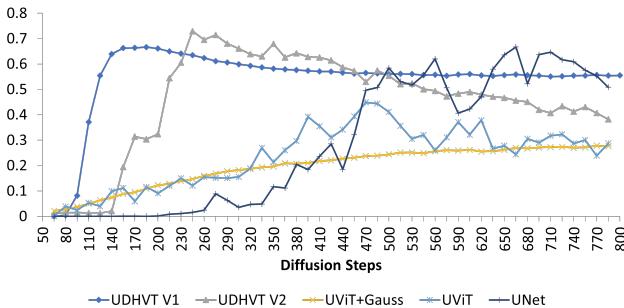


Figure 9. Diffusion steps range selection based on best Dice score for all simplex noise based model.

els, each with different time steps ranging from 0 to 800, all based on simplex noise as illustrated in Figure 9. All PDM involves selecting the optimal range of diffusion steps where the Dice score is maximized across all experiments. It is evident from the figure that our proposed model (UDHVT V1 and UDHVT V2) reaches the highest Dice score with fewer steps.

Backbone Configuration: To assess the impact of all the modifications made to UDHVT, in addition to the ViT model, we conducted a series of experiments involving various backbone variations. We evaluated these variations based on Dice and AUC scores, which are summarized in Table 2. The Leather dataset was chosen for this comparison due to its inherent difficulty in segmenting anomalies, as highlighted in Table 1c. As depicted in Figure 3a, we ex-

Table 2. Comparison of variations with DICE and AUC scores.

UDHVT Variations	Dice	AUC
PE+MHA+MLP+Conv	0.293±0.230	0.640±0.092
SPE+MHA+MLP+Conv	0.140±0.208	0.621±0.099
PE+DMHA+MLP+Conv	0.268±0.232	0.757±0.131
SPE+DMHA+MLP+Conv	0.147±0.204	0.711±0.070
PE+DMHA+HFF+Conv	0.163±0.177	0.706±0.110
SPE+DMHA+HFF+Conv	0.181±0.128	0.667±0.092
SPE+DMHA+HFF+RConv	0.306±0.234	0.636±0.137

amined four types of variations, including changes in Patch Embedding (PE and SPE), Multi-Head Attention (MHA and DMHA), Multi-Layer Perceptron (MLP and HFF), and Refinement Layer (Conv(3,3) and an additional Conv(3,3) layer for output refinement). Among these configurations, UDHVT with the setup of PE+DMHA+MLP+Conv outperformed others in terms of AUC scores. However, the PE+DMHA+MLP+Conv configuration achieved superior results in Dice scores.

5. Discussion

In the realm of anomaly detection, a full Markov chain is not necessarily required. However, when it comes to gener-

ating high-quality images, the full Markov chain becomes an essential component. Our observations reveal that as the number of diffusion steps increases, Tsimplex exhibits a decrease in sample quality, as depicted in Figure 8. To address this issue, a multi-section noise can specifically designed to incorporate multi-frequency ($F_{i,j}$ at i, j grid point) noise functions, aiming to improve sampling quality and subsequently enhance the Structural Similarity Index (SSIM). For example, $F_{i,j} = \delta \times e^{-\left(\frac{(i-\frac{n+1}{2})^2}{2\sigma^2} + \frac{(j-\frac{m+1}{2})^2}{2\sigma^2}\right)}$ with its parameters δ , n , m , and σ , serves as a concise yet powerful tool. It not only provides a means to tailor the average frequency value (δ) but also allows for nuanced control over the spread and smoothness of the noise distribution through the strategic manipulation of σ . This kind of multi-frequency on pixels can be a versatile solution, offering potential avenues to enhance sample quality and tackle the asymmetry inherent in noise patterns.

The presence of zig-zag patterns in the results can be attributed to the stochastic nature of Tsimplex noise. To mitigate this, we adopted a strategy of multiple sampling and the averaging of reconstructions, resulting in a smoother Dice score graph, as illustrated in Figure 9 (UDHVT V2). It is worth considering the integration of such strategies into the training of diffusion models in the future. Furthermore, UDHVT, as the backbone for diffusion models, has been designed to acquire a generalized self-supervised representation of healthy images. Looking ahead, this foundational structure could potentially be further explored for the generation of high-quality images, akin to the capabilities of a UNet architecture. The methodology presented here exhibits robustness when dealing with unannotated data. In the future, we envision its potential to harness additional information in three-dimensional (3D) images, particularly when coupled with the developed Tsimplex noise. This expansion into 3D data could offer exciting prospects for enhancing anomaly detection and image generation.

6. Conclusion

In this study, we develop a self-supervised anomaly detection model using partially observed diffusion steps. We incorporate simplex noise diffusion to control anomaly size and introduce Tsimplex sampling techniques for 3D and color images, reducing noise function stochasticity through output averaging. Additionally, we present UDHVT, a versatile backbone for diffusion models by incorporating the dynamical head intreted multi-head attention in vision transformers. Our method is applied to three diverse image datasets, delivering high-quality results through self-supervised learning without the need for annotated data. We aim to extend this methodology to various medical anomaly detection challenges in the future, as we believe that UDHVT holds promise for advancing backbone research in dif-

531 fusion models and enhancing generative modeling.

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Robust Self-Supervised Anomaly Detection with Diffusion Models

Supplementary Material

Algorithm 2 Simplex Noise

Input: Input point $\mathbf{P} = (x, y, z)$

Output: Noise value $N(\mathbf{P})$

Initialize $N(\mathbf{P}) \leftarrow 0$

Step 1: Grid Setup

Define a regular grid of points by dividing the space into a grid of cells: $\mathbf{G} = \{\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_n\}$.

Step 2: Grid Positioning

Determine the position of \mathbf{P} within the grid by finding the closest grid points to \mathbf{P} .

$\mathbf{V} = \{\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_n\}$, where \mathbf{V}_i is the position of the i -th closest grid point to \mathbf{P} .

Step 3: Gradient Vectors

Calculate the pseudo-random gradient vector \mathbf{G}_i for each grid point: $\mathbf{G}_i = (g_{ix}, g_{iy}, g_{iz})$.

Step 4: Dot Products

Calculate the dot products between \mathbf{G}_i and vectors from the closest grid points to \mathbf{P} : $D_i = \mathbf{G}_i \cdot (\mathbf{P} - \mathbf{V}_i)$.

Step 5: Interpolation

Interpolate the dot products using a smooth function to obtain $N(\mathbf{P})$:

$N(\mathbf{P}) = \sum_{i=1}^n F(D_i)$, where $F(D_i)$ is the interpolation function.

Step 6: Octaves

Repeat Steps 1-5 for multiple octaves with different frequencies and amplitudes, accumulating the results in $N(\mathbf{P})$.

return $N(\mathbf{P})$

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7. Further Noise function

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We have endeavored to enhance the sample quality by introducing two new noise sources: Tsimplex Gauss (Tsg) and Komal-Tsimplex (KTs) noise. Gaussian noise, known for its high-quality samples, is integrated into Tsg, which is formulated as follows:

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$$\text{Tsg}(x) = \alpha \cdot \text{GenNoise}(S, t) + (1 - \alpha) \cdot \text{gauss}(x), \quad (6)$$

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Here, $\alpha = \left(1 - \frac{t}{\chi}\right)$, with t representing the current diffusion step and χ denoting the total diffusion steps. For KTs noise, we introduce the multi-frequency ($F_{i,j}$) for simplex noise, where the position of a pixel (i, j) , and described by the following equation:

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$$F_{i,j} = \delta \times e^{-\left(\frac{(i-\frac{n+1}{2})^2}{2\sigma^2} + \frac{(j-\frac{m+1}{2})^2}{2\sigma^2}\right)} \quad (7)$$

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In this equation, δ represents the desired average frequency value, n and m are the dimensions of the grid, and σ con-

Table 3. Hyperparameters

Parameter	Value
Image Settings	
Img_size	(224, 224)
Batch_Size	32
Epochs	3000
Time_step	1000
Model Configuration	
channels	1 or 3
beta_schedule	cos
loss-type	$l2 - norm$
learning rate	1e-4
patch_size	16
embed_dim	384
depth	6
num_heads	6
mlp_ratio	4
num_class	null or 2
EMA rate	0.9999
Other Parameters	
octave	6
frequency	64
persistence	0.9

trols the spread of the Gaussian distribution. A larger value of σ results in a Komal (smoother) distribution. 780
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