DIP Assignment 5: Question 3

October 24, 2015

1 Question

Consider a 1D image (for example, a single row from a 2D image). You know that given such an image, computing its gradients is trivial. An inquisitive student frames this as a convolution problem to yield an equal $g = h * f$ where g is the gradient image, h is the convolution kernel to represent the gradient operation, and f is the original 1D image. The student tries to develop a method to determine f given g and h. What are the fundamental difficulties he/she will face in this task? Justify your answer. You may assume appropriate boundary conditions.

Now consider that you are given the gradients of a 2D image in the X and Y directions, and you wish to determine the original image. What are the difficulties you will face in this task? Justify your answer. Again, you may assume appropriate boundary conditions.

2 Solution

h is the difference filter here. If we take fourier transform of the whole equation, we have

$$G = HF$$

And then we can simply evaluate $\frac{G}{H}$ to retrieve Fourier transform of original image from the gradient. Fourier inverse of F will five back the original image.
Concerns here will be same as in previous questions.

- The gradient operator for 1-D will be

  $$h = [-1, 1]$$

  Value of the transform at zero is just sum of all the elements. And so,

  $$H(0) = 0$$

  This will lead to a huge blow up in F at zero frequency, and so the DC component will be lost

- Say the gradient image we have is noisy, i.e

$$G = HF + N$$

then no knowledge of N can be a problem and otherwise the retrieval equation is:

$$F = \frac{G}{H} - \frac{N}{H}$$

Now, the frequencies at which H is very low as compared to N, the contribution of even a tiny amount of noise to F increases. And the results will deteriorate a lot.

In case of 2-D image we have

$$\mathbf{G_x} = \mathbf{H_x F}$$

$$\mathbf{G_y} = \mathbf{H_y F}$$

Now, this is an over-determined system, we have two equations and just one unknown. Ideally, solving either of the 2 should work and give back exactly same answer. But we can instead use this extra information to combat the kind of issues that are mentioned above. For example:

- In case the H_x value is zero at certain frequencies and H_y value is non-zero at those same frequencies (or vice versa), we can conveniently pick up appropriate frequency bins from either of the two equations.

- In presence of noise, we can set a threshold t such that

$$\textbf{if } \mathbf{H_x(w)} \geq t \textbf{ use } \mathbf{F} = \frac{\mathbf{G_x}}{\mathbf{H_x}} - \frac{\mathbf{N_x}}{\mathbf{H_x}}$$

$$\textbf{else if } \mathbf{H_y(w)} \geq t \textbf{ use } \mathbf{F} = \frac{\mathbf{G_y}}{\mathbf{H_y}} - \frac{\mathbf{N_y}}{\mathbf{H_y}}$$

- Or the above threshold can also be kept as compared to the noise transform value at that frequency bin instead of a hard-set same threshold for every frequency bin.

- In case the noise values added are same in both settings (which is not so probable as noise is in general random), we can just subtract the two equations and evaluate F more accurately as

$$\mathbf{F} = \frac{\mathbf{G_x} - \mathbf{G_y}}{\mathbf{H_x} - \mathbf{H_y}}$$

- Likewise, a better algorithm can be formulated using the extra information, depending on the setting.

Again, the concerns here will be more-or-less same as before.

- There will be an issue when $\mathbf{H_x}$ and $\mathbf{H_y}$ are both zero for same frequency bin.
- A problem will arise in case of noise when the frequency bins have very low value for both $\mathbf{H_x}$ and $\mathbf{H_y}$
- When using threshold as mentioned above, we might result in a very peaky discontinuous Fourier transform, which can result into deteriorated results. So, we need better algorithm which detects appropriately and uses best of both results