10-6 $\sigma_1 = \sigma_2 = 3$

$n_1 = 20$ $\bar{x}_1 = 18$

$n_2 = 20$ $\bar{x}_2 = 24$

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$\alpha = 0.05$

a) $$z_0 = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{18 - 24}{\sqrt{\frac{3^2}{20} + \frac{3^2}{20}}} = \frac{-6}{\sqrt{\frac{18}{20}}} = -6.32$$

P-value: $p = P(z < -6.32 \text{ or } z > 6.32)$
$= 2(z < -6.32) = 2(0) = 0$

reject H_0 because p-value < 0.05

There is enough evidence to support that the population means are not equal

b) 95% CI
$\alpha = 0.05$ $\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

$z_{\alpha/2} = z_{0.025} = 1.96$

$$(\bar{x}_1 - \bar{x}_2) - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < x$$

$$\bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} > x$$

$$(18-24) - 1.96\sqrt{\frac{3^2+3^2}{20}} < x < (18-24) + \frac{1.96}{\sqrt{\frac{3^2+3^2}{20}}}$$

$$-7.859 \le x \le -4.140$$

10-18 $H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$n_1 = 10 \quad n_2 = 10 \quad \bar{x}_1 = 7.8 \quad \bar{x}_2 = 5.6 \quad \sigma_1^2 = 4 \quad \sigma_2^2 = 9$

a) since $\sigma_1^2 = \sigma_2^2$

$n_1 + n_2 - 2 = 18$

$t_{0.025, 18} = 2.101$

$$S_p^2 = \frac{(n-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{9\times4 + 9\times9}{10}$$

$S_p^2 = 6.5$

$S_p = \sqrt{6.5} = 2.549$

$$T_0 = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{7.8-5.6}{2.55\sqrt{\frac{2}{10}}}$$

$= 1.93$

Now $T = 1.93$ is between $t_{0.025,18}$ & $t_{0.05,18}$ i.e 2.101 & 1.734, we can say that $0.025 < p < 0.05$

Since p-value is not less than 0.025 we can reject the alternative H_1 & accept H_0.

b) $\bar{x}_1 - \bar{x}_2 - t_{0.025,18}\ S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_2 \le t_{0.025,18}\ S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} + (\bar{x}_1 - \bar{x}_2)$

$\Rightarrow (7.8-5.6) - 2.101\left(\sqrt{\frac{2}{10}}\right)(2.55) \le \mu_1 - \mu_2 \le (7.8-5.6) + (2.101)\left(\frac{\sqrt{2}}{\sqrt{10}}\right)(2.55)$

$= -0.195 \le \mu_1 - \mu_2 \le 4.595$

Since 0 is between -0.195 & 4.595, we cannot reject H_0.

10-51 Calculate the differences between the two samples:

$36925 - 34318 = 2607$
$45300 - 42280 = 3020$
$36240 - 35500 = 740$
$3210 - 38015 = -805$
$48360 - 47800 = 500$
$38200 - 37810 = 390$
$33500 - 33215 = 285$

$H_0: \mu_1 = \mu_2$
$H_1: \mu_1 \neq \mu_2$ $\quad n = 8$

99% $\quad \alpha = 0.01$

$$\text{mean} = \frac{2607 + 3020 + 740 + 150 - 805 + 500 + 390 + 285}{8}$$

$= 868.37$

$$S_d = \sqrt{\frac{(2607 - 868.37)^2 + \ldots + (285 - 868.37)^2}{8-1}}$$

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$sd = 1290.03$

$t_{\alpha/2, n-1} \Rightarrow t_{0.005,7} = 3.499$

$\bar{d} - t_{\alpha/2} \cdot \frac{sd}{\sqrt{n}} = 868.37 - 3.499 \times \frac{1290.03}{\sqrt{8}}$

$= -727.503$

$\bar{d} + t_{\alpha/2} \cdot \frac{sd}{\sqrt{n}} = 868.37 + 3.499 \times \frac{1290.03}{\sqrt{8}}$

$= 2464.253$

Since the intervals contain 0, neither of the brands is better.

Reject H_1 & accept H_0.

10-68 $n_1 = 10$ $n_2 = 16$

$s_1 = 4.7$ $s_2 = 5.8$

$\alpha = 0.05$

Given that mean conc are same for both the samples:

$H_0: \sigma_1^2 = \sigma_2^2$

$H_1: \sigma_1^2 \neq \sigma_2^2$

$F = \frac{s_1^2}{s_2^2} = \frac{4.7^2}{5.8^2} = 0.657$

$dof = 16 - 1 = 15$

$15, 9$

$f_{0.025, 9, 15} = 3.77$

$\frac{1}{3.77} = 0.265$

$0.265 \leq F \leq 3.12$

$0.265 < 0.657 \leq 3.12$

Failed to reject Ho.

∴ There is not enough evidence to conclude that the two population variance differ.