9.5

a) $\mu = 12$ $\quad H_0: \mu = 12$

$\sigma = 0.5$ $\quad H_1: \mu < 12$

$n = 4$

$$P(\bar{x} < 11.5) = P\left(z < \frac{11.5-12}{\frac{0.5}{\sqrt{4}}}\right)$$

$$= P(z < -2) = 0.0228$$

$$\alpha = (P(z < -2)) = 0.0228$$

b) $\cancel{P(\bar{x} < 11.25)}$

$$P(\bar{x} < 11.5 \mid \mu = 11.25)$$

$$= P\left(z < \frac{11.5 - 11.25}{\frac{0.5}{\sqrt{4}}}\right) = P(z < 1)$$

$$= 0.8413$$

$$\beta = 1 - 0.8413 = 0.1587$$

c) $$P(\bar{x} < 11.5 \mid \mu = 11.5) = P\left(z < \frac{0}{\frac{0.5}{\sqrt{4}}}\right)$$

$$= P(z < 0) = 0.5000$$

$$\beta = 1 - 0.5000 = 0.5$$

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9.6 $\mu = 12$ $H_0: \mu = 12$

$\sigma = 0.5$ $H_1: \mu < 12$

$n = 16$

$$\frac{\sigma}{\sqrt{n}} = \frac{0.5}{4} = 0.125$$

a) $P(\bar{x} < 11.5 \mid \mu = 12)$

$$= P\left(z < \frac{11.5-12}{0.125}\right) = P(z < -4) \simeq 0$$

$\alpha = 0$

b) $P(\bar{x} < 11.5 \mid \mu = 11.25)$

$$= P\left(z < \frac{11.5-11.25}{0.125}\right) = P(z < 2)$$

$= 0.9772$

$\beta = 1 - 0.9772 = 0.0228$

c) $P(\bar{x} < \cancel{11.25}\ 11.5 \mid \mu = 11.5)$

$$= P\left(z < \frac{11.5-11.5}{0.125}\right) = P(z < 0)$$

$= 0.5$

$\beta = 1 - 0.5 = 0.5$

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9.9 $\mu = 12$ $n = 4$ $\sigma = 0.5$

$H_0 : \mu = 12$

$H_1 : \mu < 12$

a) $\bar{x} = 11.25$

$$P(\bar{x} < 11.25) = P\left(z < \frac{11.25 - 12}{\frac{0.5}{4}}\right)$$

$$= \frac{0.5}{\sqrt{4}} = 0.25$$

$$P(z < -3) = 0.0013$$

b) $$P(\bar{x} < 11.0) = P\left(z < \frac{11 - 12}{0.25}\right)$$

$$= P(z < 4) \simeq 0$$

c) $$P(\bar{x} < 11.75) = P\left(z < \frac{11.75 - 12}{0.25}\right)$$

$$P(z < -1) = 0.1587$$

9.20 $\sigma = 0.25$ $H_0 : \mu = 5$

$n = 8$ $H_1 : \mu \neq 5$

$$\frac{\sigma}{\sqrt{n}} = \frac{0.25}{\sqrt{8}} = 0.0883$$

a) $4.85 \leq \bar{x} \leq 5.15$

$$1 - \left(\frac{4.85 - 5}{0.0883} \leq z \leq \frac{\cancel{5.55}\ 5.15 - 5}{0.0883}\right).$$

$= 1 - (-1.6987 \le z \le 1.6987)$

$= 1 - (\Phi(1.6987) - \Phi(-1.6987))$

$= 1 - (0.9545 - 0.0455) = 1 - 0.909$

$\alpha = 0.091$ (Type-I error)

b) Power of the test $\rightarrow 1-\beta$

$\mu = 5.1$

$$P\left(\frac{4.85 - 5.1}{0.0883} < z < \frac{5.15 - 5}{0.0883}\right)$$

$(-2.8312 < z < 0.56625)$

$\Phi(0.56625) - \Phi(-2.8312)$

$= 0.7123 - 0.0023 = 0.71$

Power of the test $= 1-\beta = 0.29$

Prob. of accepting null hypothesis when it is false is 0.71.

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9.21 $n=16$ $\sigma = \frac{1}{16}$ $H_0: \mu=5$ $H_1: \mu \neq 5$

a) $P(4.85 \le \bar{x} \le 5.15)$

$\alpha = 2 \times P(x < 4.85)$

$= 2 \times P\left(z < \frac{4.85 - 5}{1/16}\right)$

$= 2 \times P(z < -2.4) = 2(0.0082)$

$= 0.0164$ for $n=16$

for $n=16$, $\cancel{0.001}$ $\alpha = 0.0164$

for $n=8$, $\alpha = 0.089$

As n increases, α decreases implies as the sample size increases, the probability of Type I error decreases.

b) $\beta = P(4.85 \le \bar{x} \le 5.15)$

$= P\left(\frac{4.85-5.1}{1/16} \le z \le \frac{5.15-5.1}{1/16}\right)$

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$= P(-4 \leq Z \leq 0.8)$

$= 0.7881$ for $n=16$

Power of test $= 1-\beta$

$= 1 - 0.7881$

$= 0.2119$

For $n=8$, power of the test $= 0.288$ ~~16~~

$\therefore$ As n increases, power of the test ~~is~~ increases.

9.42 a) $H_0: \mu \leq 100$

$H_1: \mu > 100$

$\alpha = 0.5 \qquad Z_\alpha = 1.665$

$$Z_0 = \frac{\bar{x} - \mu}{\frac{6}{\sqrt{n}}} = \frac{98 - 100}{\frac{2}{\sqrt{9}}} = -3$$

$|z| > Z_\alpha$

$\therefore$ Reject H_0.

$\therefore$ Mean water temp. is greater than 100.

b) p-value

$$P\left(Z > \frac{98-100}{\frac{2}{\sqrt{9}}}\right)$$

$P(Z > -3) \Rightarrow 1 - P(Z < -3)$

$= 1 - 0.0014$

$= 0.998$

c) true mean $= 104$

$$\beta = P\left(Z < -\frac{(104-100)\sqrt{9}}{2}\right) = P(Z < -4.355)$$

≈ 0