9.43

a) $H_0 : \mu = 3$

$H_1 : \mu \neq 3$ $\quad \alpha = 0.05$

$n = 15 \quad \bar{x} = 2.78 \quad \sigma = 0.9$

$z_{0.05} = 1.645$

$z_0 = \dfrac{2.78 - 3}{\frac{0.9}{\sqrt{15}}} = \dfrac{-0.22}{0.2323} = -0.947$ (lies in -1.645 to 1.645)

We do not reject Ho.

b) $\mu = 3.25 \quad \beta = \phi\left[z_{0.025} + \dfrac{3 - 3.25}{0.9/\sqrt{15}}\right] - \phi\left[-z_{0.025} + \dfrac{3 - 3.25}{\frac{0.9}{\sqrt{15}}}\right]$

~~$p = 2[1 - \phi|z_0|] = 2(1 - (-0.947)) = 2(1.947) = 3.896$~~

$= \phi[1.96 - 1.075) - \phi(-1.96 - 1.075) = 0.81057 - 0.001223$

$= 0.8098 \quad$ Power $= 1 - 0.8098 = 0.1902$

c) $\beta = 1 - 0.9 = 0.1$

true mean $= 3.75 \quad z_{\alpha/2} = 1.96 \quad z_\beta = 1.28$

$n = \dfrac{(z_{\alpha/2} + z_\beta)^2 (0.9)^2}{(3.75 - 3)^2} = \dfrac{(3.24)^2 (0.81)}{(0.75)^2} = 15.11$

$n \simeq 15$

9.45 $\sigma = 1.25$

$n = 10$

$\bar{x} = 40.5$

a) $\alpha = 0.05$

$H_0 : \mu = 40$ $\Rightarrow$ $z_0 = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

$H_1 : \mu > 40$

Reject H_0 if $z_0 > z_\alpha$ where $z_{0.05} = 1.65$

$\bar{x} = 40.5$ $z_0 = \dfrac{40.5 - 40}{\frac{1.25}{\sqrt{10}}} = \dfrac{0.5}{0.39} = 1.282$

Since $1.282 < 1.65$, we do not reject H_0.

b) P-value

$= 1 - \phi(z_0) = 1 - 0.8997$
$= 0.1003$

c) $\beta = \phi\left(z_{0.05} + \dfrac{40 - 42}{\frac{1.25}{\sqrt{10}}}\right) = \phi(1.65 + (-5.06))$

$= \phi(-3.41) \approx 0.0003$

d) $n = \dfrac{(z_\alpha + z_\beta)^2 \sigma^2}{\delta^2} = \dfrac{(z_{0.05} + z_{0.10})^2 \sigma^2}{(40 - 44)^2}$

$$= \frac{(1.65+1.29)^2 (1.25)^2}{4^2} = 0.844$$

e)

95% CI

$$\bar{x} + z_{0.05} \frac{\sigma}{\sqrt{n}} \le \mu$$

$$40.5 + 1.65\left(\frac{1.25}{\sqrt{10}}\right) \le \mu \Rightarrow 41.152 \le \mu$$

The lower bound must be greater than 40 to verify that the true mean exceeds 40hrs.

9.88 $n=15$

$s = 0.008$

$\alpha = 0.01$

a) Parameter of interest : Population standard deviation, σ.

$H_0 : \sigma = 0.01$

$H_1 : \sigma > 0.01$

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(15-1)^2 (0.008)^2}{(0.01)^2} = 8.96$$

Critical region : $\chi_0^2 > \chi^2_{\alpha, n-1} \Rightarrow \chi^2_{0.01, 14} = 29.14$

but $8.96 < 29.14$, hence, H_0 is rejected.

Degrees of freedom = $n-1 = 14$

$$\chi^2_{0.9,14} = 7.79$$

$$\chi^2_{0.5,14} = 13.34$$

$$7.79 < 8.96 < 13.34$$

$$0.5 < \text{p-value} < 0.9$$

9.98

a) $H_0: p = 0.5$

$H_1: p \neq 0.5$

Test is 2-sided with $\alpha = 0.05$

test statistic, $z_0 = \dfrac{x - np}{\sqrt{np_0(1-p_0)}} = \dfrac{117 - 485(0.5)}{\sqrt{(484)(0.5)(0.5)}}$

$= -11.36$

Critical region: $z_0 > z_{\alpha/2}$

$z_0 < -z_{\alpha/2}$ with $z_{0.025} = 1.96$

$z_0 > 1.96$ or $z < -1.96$, $z_0 = -11.36$

Hence H_0 is rejected.

$\text{P-value} = P(z > |z_0|) = 2\phi(z_0) = 2\phi(-11.36)$

$= 2(0) = 0$

b) $\hat{p} = \frac{x}{n} = \frac{117}{484} = 0.2417$

95% CI is $\hat{p} = z_{0.025}\sqrt{\hat{p}(1-\hat{p})/n}$

$= 0.247 \pm 1.96\sqrt{(0.2417)(1-0.2417)/485}$

$= (0.2036, 0.2798)$

$p_0 = 0.5$ is not included in this C.I.

Hence, we have enough evidence that true population is different from 0.5.

9.103

$n = 500$
$x = 466$

$\alpha = 0.05$

$\hat{p} = \frac{x}{n} = \frac{466}{500} = 0.932$

$H_0: p = 0.9$
$H_1: p > 0.9$

$z_0 = \frac{\hat{p} - p}{\sqrt{p_0(1-p_0)/n}} = \frac{0.932 - 0.9}{\sqrt{0.9(0.1)/500}} = 2.385$

P-value $= P(z > 2.385) = 1 - \phi(2.385) = 0.0009$

p value < α, So H_0 is rejected.
There is sufficient ~~ag~~ evidence that rate is atleast 90% correct.

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9.113 Ho: Sample of trees & time periods are independent
H_1: Sample of trees & time periods are <u>not</u> independent.

$\alpha = 0.005$

Test statistic → $\chi_0^2 = \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$

Expected frequency:

$$E_{ij} = \frac{1}{n} \sum_{j=1}^{c} O_{ij} \sum_{i=1}^{r} O_{ij}$$

$df = (r-1)(c-1) = (3-1)(3-1)$
$= 2 \times 2 = 4$

degrees of freedom

$$\chi^2_{0.05,4} = 9.4877$$

If calculated test statistic is greater than 9.4877, we reject Ho.

$E_{11} = \frac{r_1 \times c_1}{n} = \frac{131 \times 133}{387} = 45.02$

$E_{21} = \frac{r_1 \times c_2}{n} = \frac{133 \times 92}{387} = 31.62$

$E_{31} = \frac{r_1 \times c_3}{n} = \frac{133 \times 164}{387} = 56.36$

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$E_{12} = \frac{114 \times 131}{387} = 38.59$

$E_{22} = \frac{114 \times 92}{387} = 27.10$

$E_{32} = \frac{114 \times 164}{387} = 48.310$

$E_{31} = \frac{140 \times 131}{387} = 47.39$

$E_{32} = \frac{140 \times 92}{387} = 33.28$

$E_{33} = \frac{140 \times 164}{387} = 59.32$

Expected values:

Year	Species Penn.	Rub	Secc	Total
1936	45.02	31.62	56.36	133
1972	38.59	27.10 ~~48.31~~	48.31	114
2011	47.39	33.28 ~~59.33~~	59.33	140
Expected Total	131.00	92.00 ~~164~~	164.00	387

$$\chi_0^2 = \sum_{i=1}^{c}\sum_{j=1}^{c} \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = \sum_{i=1}^{3}\sum_{j=1}^{3} \left(\frac{O_{ij} - E_{ij}}{E_{ij}}\right)^2$$

$$= \left(\frac{(12 - 45.02)^2}{45.02} + \frac{(27 - 31.62)^2}{31.62} + \cdots + \frac{(18 - 59.33)^2}{59.33}\right)$$

$$= 26.2192 + 0.6744 + \cdots + 28.78$$

$$\chi_0^2 = 166.3608$$

$\chi_0^2 = 146.3648$

We reject H_0.

P-value : $df = \cancel{9-1=8}\ (3-1)(3-1) = 4$

pchisq (146.36, 4) this gives 1

but we need area to the right,

hence = 1 - pchisq (146.36, 4)

= 1 - 1 = 0

$\therefore$ pvalue = 0.

Since p-value < 0.05,

we reject H_0.