4.69. $\mu = 5$ $\sigma = 4$

a) $P(X < 11) = P\left(\frac{X-5}{4} < \frac{11-5}{4}\right)$

$P\left(Z < \frac{6}{4}\right) = P(Z < 1.5)$

$= 0.933193$

b) $P(X > 0) = 1 - P(X \leq 0)$

$= 1 - P\left(\frac{x-5}{4} \leq \frac{0-5}{4}\right)$

$= 1 - P\left(Z \leq -\frac{5}{4}\right) = 1 - P(Z \leq -1.25)$

$= 1 - 0.105650 = 0.894350$

c) $P(3 < X < 7) = P\left(\frac{3-5}{4} < \frac{X-5}{4} < \frac{7-5}{4}\right)$

$= P\left(-\frac{2}{4} < \frac{X-5}{4} < \frac{2}{4}\right) = P(-0.5 < Z < 0.5)$

$= \Phi(0.5) - \Phi(-0.5) = 0.691462 - 0.308538$

$= 0.382924$

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d) $P(-2 < x < 9)$

$$P\left(\frac{-2-5}{4} < \frac{x-5}{4} < \frac{9-5}{4}\right)$$

$$= P\left(\frac{-7}{4} < \frac{x-5}{4} < 1\right) = P(-1.75 < z < 1)$$

$$= \Phi(1) - \Phi(-1.75) = 0.841345 - 0.040059$$

$$= 0.801286$$

e) $P(2 < x < 8) = P\left(\frac{2-5}{4} < \frac{x-5}{4} < \frac{8-5}{4}\right)$

$$= P\left(-\frac{3}{4} < z < \frac{3}{4}\right) = P(-0.75 < z < 0.75)$$

$$= \Phi(0.75) - \Phi(-0.75) = 0.773373 - 0.226627$$

$$= 0.546746$$

4.76 $\mu = 12.4$ $\sigma = 0.1$

a) $P(x < 12) = P\left(\frac{x-12.4}{0.1} < \frac{12-12.4}{0.1}\right)$

$$= P\left(z < \frac{-0.4}{0.1}\right) = P(z < -4) = 0.00003$$

b) $1-P(12.1 < X < 12.6) = 1-P\left(\frac{12.1-12.4}{0.1} < Z < \frac{12.6-12.4}{0.1}\right)$

$= 1-\left(\frac{-0.3}{0.1} < Z < \frac{0.2}{0.1}\right) = 1-(-3 < Z < 2)$

$= 1-(\Phi(2) - \Phi(-3)) = 1-(0.977250 - 0.001350)$

$= 1-(0.9759) = 0.0241 = 2.41\%$ are scrapped

c)

Margin of error, E = CV × standard error

$\alpha = 100\% - 99\% = 1\%$

$\frac{\alpha}{2} = \frac{1}{2}\% = 0.005$

$1 - 0.005 = 0.995$

Z-score for $0.995 = 2.58$

$E = 2.58 \times 0.1 = 0.258$

(Mean $- 0.258$, Mean $+ 0.258$)

$= (12.142, 12.658)$

4.78

a) $P(X > 0.5) = 1 - P(X \leq 0.5)$

$$= 1 - P\left(\frac{X - 0.05}{0.4} < \frac{0.5 - 0.4}{0.05}\right)$$

$$= 1 - P(Z < 2) = 1 - 0.977250 = 0.02275$$

b) $P(0.4 < X < 0.5) = \left(\frac{0.4 - 0.4}{0.05} < Z < \frac{0.5 - 0.4}{0.05}\right)$

$$= (0 < Z < 2) = \Phi(2) - \Phi(0) = 0.977250 - 0.5$$

$$= 0.4772$$

c) $P(X > x) = 0.9$

$$\cancel{P\left(Z > \frac{x - 0.4}{0.05}\right) = 0.9 \Rightarrow P\left(Z < \frac{x - 0.4}{0.05}\right) = 0.10}$$

$$P(X > x) = 1 - P(X \leq x)$$

$$= 1 - P\left(Z \leq \frac{x - 0.4}{0.05}\right)$$

Hence, $\phi\left(\frac{x - 0.4}{0.05}\right) = 0.10$

$$\frac{x - 0.4}{0.05} = -1.28$$

$x = 0.336$ is the reaction time that is exceeded 90% of the time.

4.83 a) $\mu = 7000$ $\sigma = 600$

$$P(X < 5000) = P\left(\frac{X-7000}{600} < \frac{5000-7000}{600}\right)$$

$$= P\left(Z < -\frac{2000}{600}\right)$$

$$= P(Z < -3.33) = 0.000434$$

b) $P(X > x) = 0.95 = 1 - P(X \leq x)$

$$1 - P\left(\frac{X-7000}{600} \leq \frac{x-7000}{600}\right) = 0.95$$

$$1 - P\left(Z \leq \frac{x-7000}{600}\right) = 0.95$$

$$\phi\left(Z \leq \frac{x-7000}{600}\right) = 0.05$$

$$\frac{x-7000}{600} = -1.64 \Rightarrow x = 6016$$

c) $P(x > 7000) = \frac{1}{2}$

$\frac{1}{2} \Rightarrow$ fails $\qquad \frac{1}{2} \Rightarrow$ works

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

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4.98 $n = 1000$ $p = 2\% = 0.02$

$1-p = 0.98$

a) $P(X > 25)$

$\mu = np = 20$

$\sigma = \sqrt{np(1-p)} = \sqrt{20(0.98)} = \sqrt{19.6} = 4.42$

$np > 5$ and $n(1-p) > 5$, hence,

we can use normal approximation

$\therefore P(X > 25.5) = P\left(\frac{X-20}{4.42} > \frac{25.5-20}{4.42}\right)$

$= P(Z > 1.244) = 1 - P(Z \leq 1.244)$

$= \cancel{1-0.08}\ 1 - 0.8925 = 0.1075$

b) $P(20 < X < 30) = P(19.5 < X < 30.5)$

$= P(-0.11 < Z < 2.37) = P(Z < 2.37) - P(Z < 0.11)$

$= 0.991106 - \cancel{0.452205}\ 0.456205$

$= 0.534901$

53

a) $f(-1,-2) + (-0.5,-1) = \frac{1}{8} + \frac{1}{4}$

$= \frac{1+2}{8} = \frac{3}{8}$

b) $P(X<0.5) = f(-1,-2) + \cancel{f(-1,-0.5)}$
$f(-0.5,-1)$

$= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

c) $P(Y<1.5) = f(-1,-2) + f(-0.5,-1)$
$+ f(0.5,1)$

$= \frac{1}{8} + \frac{1}{4} + \frac{1}{2} = \frac{1+2+4}{8} = \frac{7}{8}$

d) $P(X>0.25, Y<4.5)$

$= f(0.5,1) + f(1.0,2) = \frac{1}{2} + \frac{1}{8}$

$= \frac{4+1}{8} = \frac{5}{8}$

e) $E(X) = \sum_{x} x f_x(x,y)$

$= \frac{-1}{8} - \frac{0.5}{4} + \frac{0.5}{2} + \frac{1}{8}$

$= \frac{-2+2+1}{8} = \frac{1}{8}$

$$E(Y) = \sum_y y f_y(x,y)$$

$$= \frac{-2}{8} - \frac{1}{4} + \frac{1}{2} + \frac{2}{8} = \frac{-2-2+4+2}{8} = \frac{2}{8}$$

$$= \frac{1}{4}$$

$$E(X^2) = \sum_x x^2 f_x(x,y)$$

$$= 1\left(\frac{1}{8}\right) + \frac{0.25}{4} + \frac{0.25}{2} + \frac{1}{8}$$

$$= \frac{1+0.5+1+1}{8} = \frac{\frac{7}{2}}{8} = \frac{7}{16}$$

$$E(Y^2) = \sum_y y^2 f_y(x,y)$$

$$= 4\left(\frac{1}{8}\right) + 1\left(\frac{1}{4}\right) + 1\left(\frac{1}{2}\right) + 4\left(\frac{1}{8}\right)$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{2} + \frac{1}{2} = \frac{2+1+2+2}{4} = \frac{7}{4}$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= 0.4375 - 0.0156 = 0.4219$$

$$V(Y) = E(Y^2) - (E(Y))^2 = \frac{7}{4} - \left(\frac{1}{4}\right)^2$$

$$= 1.75 - 0.0625 = 1.6875$$

f) $g(x)$ = Marginal probability distribution of X.

X	-1	-0.5		1
$g(x)$	$1/8$	$1/4$	$1/2$	$1/8$

g) $P(Y/X=1)$

$P(Y=-2 \mid X=1) = 0$
$P(Y=-1 \mid X=1) = 0$
$P(Y=1 \mid X=1) = 0$
$P(Y=2 \mid X=1) = \dfrac{P(Y=2 \ \& \ X=1)}{P(X=1)}$

$= \dfrac{\frac{1}{8}}{\frac{1}{8}} = 1$

$\therefore P(Y|X=1) = 1$

h) $P(X|Y=1)$

$P(X=-1 \mid Y=1) = 0$
$P(X=-0.5 \mid Y=1) = 0$
$P(X=0.5 \mid Y=1) = \dfrac{\frac{1}{2}}{\frac{1}{2}} = 1$

$P(X|Y=1) = 1$

i) $E(X|Y=1) = \sum_{x} (X) P(x|Y=1)$

$= 0.5$

j) ~~If~~ X & Y are not independent because $P(X=-1) = \frac{1}{8}$ and $P(X=-1|Y=-2) = \frac{\frac{1}{8}}{\frac{1}{8}} = 1$.

∴ X & Y are independent if $P(X|Y) = P(X)$ for all values of X & Y.

But $P(X|Y) \neq P(X)$, hence, X & Y are <u>not</u> independent.

<u>5.8</u>

a) $P(X=2) = f(2,1,1) + f(2,1,2) + f(2,2,1) + f(2,2,2)$

$= 0.20 + 0.15 + 0.10 + 0.05 = 0.5$

b) $P(X=1, Y=2) = f(1,2,1) + f(1,2,2)$
$= 0.15 + 0.20 = 0.35$

c) $P(\cancel{x}\ Z<1.5) = f(1,1,1) + f(1,2,1) + f(2,1,1) + f(2,2,1)$

$= 0.05 + 0.15 + 0.20 + 0.10 = 0.5$

d) $P(X=1$ or $Z=2)$

$= f(1,1,1) + f(1,1,2) + f(1,2,1) + f(1,2,2) + f(2,1,2) + f(2,2,2)$

$= 0.05 + 0.10 + 0.15 + 0.20 + 0.15 + 0.05$
$= 0.7$

e) $E(X) = \sum x f(x) = 1(0.05) + 1(0.10) + 1(0.15) + 1(0.20) + 2(0.20) + 2(0.15) + 2(0.10) + 2(0.05)$

$= 1.5$

f) $P(X=1|Y=1) = \dfrac{P(X=1)\ \&\ P(Y=1)}{P(Y=1)}$

$= \dfrac{0.05+0.10}{0.05+0.10+0.15+0.20}$

$= \dfrac{0.15}{0.50} = \dfrac{3}{10} = 0.3$

g) $P(X=1, Y=1 \mid Z=2) = \dfrac{P(X=1, Y=1, Z=2)}{P(Z=2)}$

$= \dfrac{0.10}{0.10+0.20+0.15+0.05}$

$= \dfrac{0.10}{0.50} = \dfrac{1}{5} = 0.2$

h) $P(X=1 \mid Y=1, Z=2) = \dfrac{P(X=1, Y=1, Z=1)}{P(Y=1, Z=2)}$

$= \dfrac{0.10}{0.10+0.15} = \dfrac{0.10}{0.25} = \dfrac{2}{5}$

$= 0.4$

i) X has two possible values: 1 and 2.

$P(X=1 \mid Y=1, Z=2) = \dfrac{P(X=1, Y=1, Z=1)}{P(Y=1, Z=2)}$

$= \dfrac{0.10}{0.25} = 0.4$

$P(X=2 \mid Y=1, Z=2) = \dfrac{P(X=2, Y=1, Z=2)}{P(Y=1, Z=2)}$

$= \dfrac{0.15}{0.10+0.15} = \dfrac{0.15}{0.25} = \dfrac{3}{5} = 0.6$