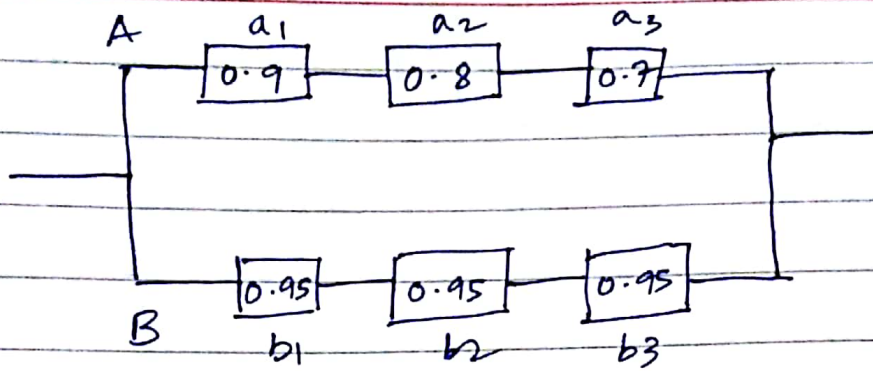


2.156



Let A = Top path,
 B = bottom path

A & B are independent

Top path is functional if $a_1 \cap a_2 \cap a_3$ occurs.

Bottom path is functional if $b_1 \cap b_2 \cap b_3$ occurs.

~~$P(A \cup B)$~~

The circuit is functional iff $A \cup B$ occurs

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

By independence,

$$= P(A) + P(B) - P(A)(B)$$

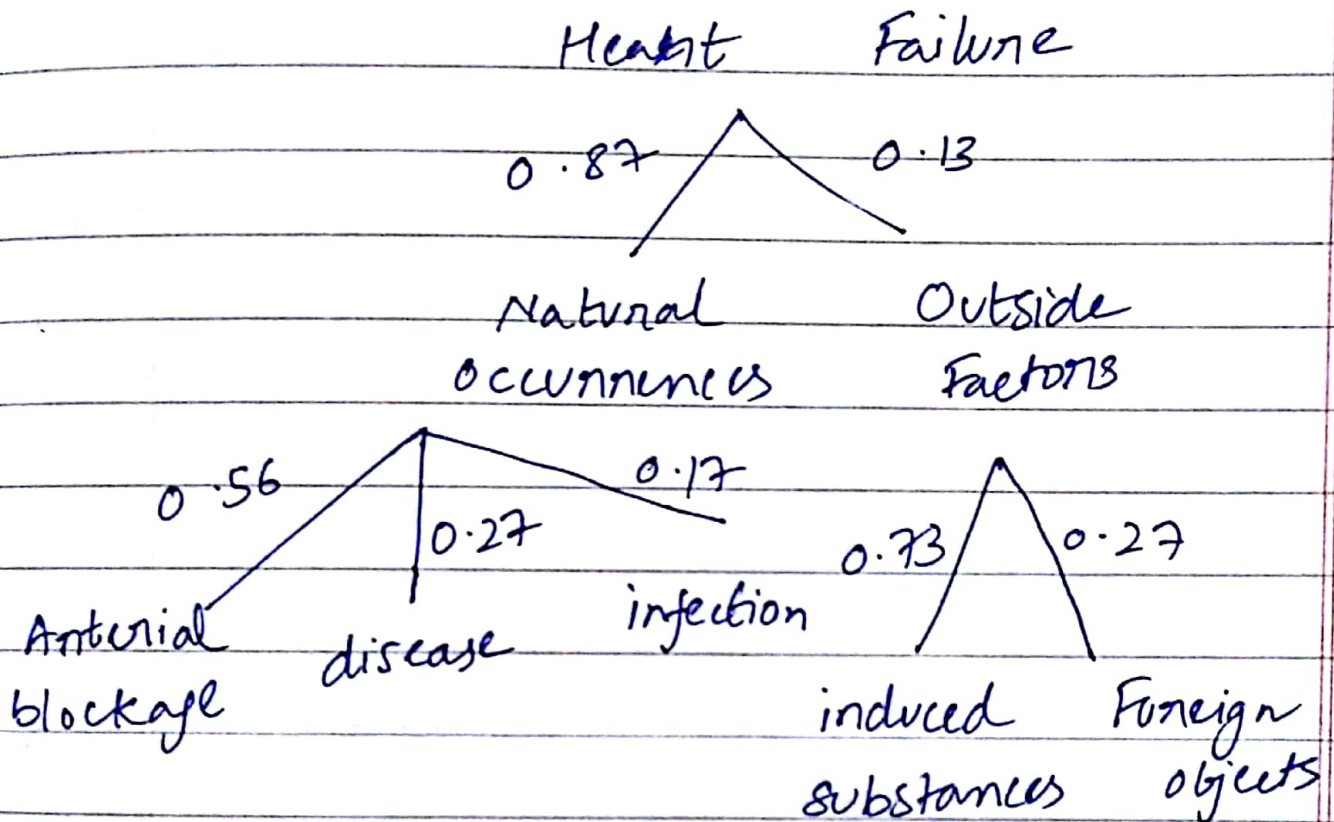
$$= (0.9)(0.8)(0.7) + (0.95)(0.95)(0.95) - (0.9)(0.8)(0.7)(0.95)^3$$

$$= 0.504 + 0.857 - 0.432 = 0.929$$

The probability that the circuit operates
is 0.929.

2.128

Using tree diagram



a)

$$P(\text{failure due to induced substance}) = (0.13)(0.73) = 0.0949$$

b)

$$P(\text{failure due to disease on infection}) = 0.87(0.27) + 0.87(0.17) = 0.382$$

2.102

Total = 100 parts

		Length	
		Excellent	Good
Surface	Excellent	80	2
finish	Good	10	8

A = sample has excellent finish

B = sample has excellent length

$$a) \quad P(A) = \frac{80+2}{100} = \frac{82}{100} = 0.82$$

$$b) \quad P(B) = \frac{80+10}{100} = \frac{90}{100} = 0.90$$

$$c) \quad P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{80}{80+10} = 0.88$$

$$d) \quad P(B|A) = \frac{P(B \text{ and } A)}{P(A)} = \frac{80}{80+2} = 0.975$$

$$e) \quad P(B|A) = 0.975$$

$$f) \quad P(A|B') = \frac{P(A \text{ and } B')}{P(B')} = \frac{2}{2+8} = \frac{2}{10} = 0.20$$

2.172

I = identifying a defective item
 D = defective item is chosen

$$P(D) = 0.009$$

$$P(I|D) = 0.99$$

$$P(D') = 0.991$$

$$P(I|D') = 0.005$$

$$P(I'|D) = 0.995$$

a)

$$P(I) = P(I|D)P(D) + P(I|D')P(D')$$

$$= 0.99(0.009) + (0.005)(0.991)$$

$$= 0.0139$$

The probability that an item selected for inspection is classified as defective is 0.0139.

$$b) P(D|I') = \frac{P(D \cap I')}{P(I')} = \frac{P(I'|D)P(D)}{P(I')} \\ = \frac{(0.995)(0.991)}{(1 - 0.0139)} = 0.9999$$

3.103.

X = No. of mornings with green lights as you approach it

Using binomial distribution,
 $p = 0.2$ $q = 0.8$

a) PMF of X is $f(x) = P(X=x)$

$$P(X=1) = {}^5C_1 (0.2)^1 (0.8)^9 \\ = 0.4096$$

$$b) P(X=4) = {}^{20}C_4 (0.2)^4 (0.8)^{16} \\ = 4845 (0.0016) (0.02814) \\ = 0.2181$$

$$c) P(X \geq 4) = 1 - P(X \leq 4) = 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)] \\ = 1 - [0.011 + 0.057 + 0.1369 + 0.2053 + 0.2181] \\ = 0.3717$$

3.66

$x = \text{Nickel charge}$

x	$f(x)$	$x - 2.29$	$(x - 2.29)^2$	$(x - 2.29)^2 f(x)$
0	0.17	-2.29	5.2441	0.891
2	0.35	-0.29	0.0841	0.029
3	0.33	0.71	0.5041	0.166
4	0.15	1.7	2.89	0.4335
				<u>1.524</u>

$$\text{CDF}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.17 & 0 \leq x < 2 \\ 0.52 & 2 \leq x < 3 \\ 0.85 & 3 \leq x < 4 \\ 1 & x \geq 4 \end{cases}$$

$$\text{Mean} = \sum x f(x) = 2.29$$

$$\text{Variance} = \sum x^2 f(x) - \mu^2$$

$$\sum_{i=1}^4 f(x_i) (x_i - \mu)^2$$

$$= 1.524$$

3.101

$x = \text{No. of times the line is occupied}$

binomial distribution

$$n=10 \quad p=0.4 \quad q=0.6$$

$$\begin{aligned} \text{a) } P(X=3) &= {}^{10}C_3 (0.4)^3 (0.6)^7 = 120 (0.064) (0.027) \\ &= 0.2149 \end{aligned}$$

$$\begin{aligned} \text{b) } P(X \geq 1) &= 1 - P(X < 1) = 1 - P(X=0) \\ &= 1 - [{}^{10}C_0 (0.4)^0 (0.6)^{10}] = 1 - [(0.6)^{10}] \\ &= 0.994 \end{aligned}$$

$$\text{c) } E(X) = np = 10(0.4) = 4$$