

2nd January 2022

Today's Task

1. Longest subarray with sum divisible by k
2. count of subarrays with sum divisibly by k
3. Longest subarray with equal number of 0s, 1s and 2s.
4. count of subarrays with equal number of 0s, 1s and 2s.
5. Task completion

Longest Subarray with sum divisible by k: \rightarrow

$k = 2$

length = ?

array \rightarrow 2 7 6 1 4 5

if
Remainder is
same
at s_1
and s_2 .

Sum $S_1 = k \cdot n + x$

Sum $S_2 = k \cdot m + x$

$S_1 = k \cdot n + \underline{x}$ \rightarrow Remainder

$S_2 = k \cdot m + \underline{x}$ \rightarrow Remainder $k = 5$

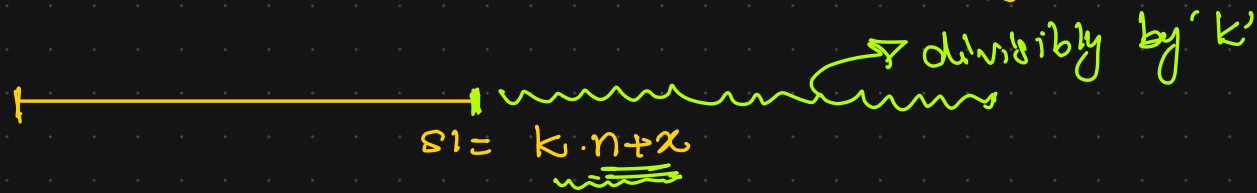
83

$5 \times 16 + 3$
 \uparrow
 $k \cdot n + x$

$$\begin{aligned} \text{Sum of middle part} &= S_2 - S_1 \\ &= k \cdot m + x - (k \cdot n + x) \\ &= k \cdot m + \cancel{x} - k \cdot n - \cancel{x} \\ &= k(m - n) \end{aligned}$$

NOTE!

\Rightarrow Sum of middle part is divisible by k



To prove: sum between i_1 to i_2 is divisible by k } → subarray from i_1 to i_2 .

$$S_1 = k \cdot n + x$$

$$S_2 = k \cdot m + x$$

$$S_3 (\text{sum from } i_1 \text{ to } i_2) = S_2 - S_1 = (k \cdot m + x) - (k \cdot n + x) \\ = k \cdot m + x - k \cdot n - x$$

$$S_3 = k(m - n)$$

⇒ S_3 is divisible by k ,

Sum →
 ↪ Hash map
 ↪ Remainder

Remainder is same for both cyclic sum, S_1 & S_2 process

Key

$$i_1 < i_2$$

& S_1 is sum of element from 0 to i_1 ,

& S_2 is sum of element from 0 to i_2 .

array \rightarrow $\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 2 & 7 & 6 & 1 & 4 & 5 \end{matrix}$

$k=3$

sum \rightarrow 0 2 9 15 16 20 25

Rem = sum % k

Remainder \rightarrow 0 2 0 0 1 2 1

$1 - (-1) = 2$

$0 \times 3 + 2$
 $n \times k + r$

$6 \times 3 + 2$
 $m \times k + r$

length = 0 ~~2~~ ~~2~~ (4)

0 \rightarrow -1

2 \rightarrow 0

1 \rightarrow 2

7 6 1 4

$2 - 1 = 1$

$2 - (-1) = 3$

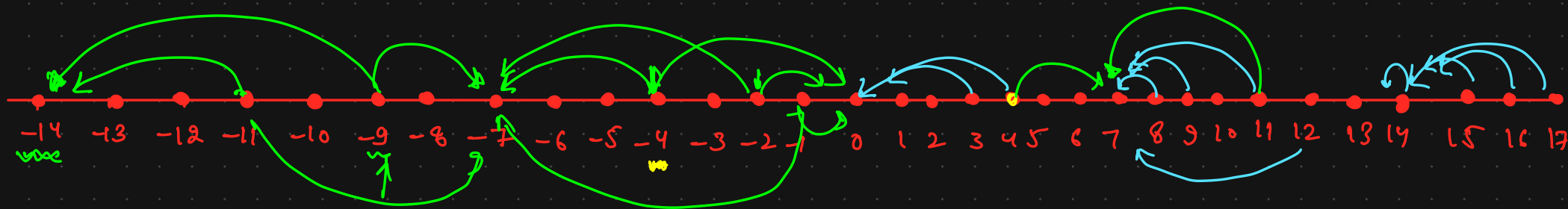
$18 \% 3 = 0$

HashMap < Integer, Integer > map

$4 - 0 = 4$

Remainder \uparrow
First encounter index for that remainder.

if (rem < 0) rem = rem + k



remainder

generic behavior

$$-9 \times 7 = -2$$

gen. beh.

$$k=7$$

$$11 \times 7 = 4$$

$$4 \times 7 = 4 \quad -3$$

$$\begin{bmatrix} -4 \times 7 = -4 \text{ OR } +3 \\ -2 \times 7 = -2 \text{ OR } 5 \end{bmatrix}$$

$$6 \times 7 = 1$$

$$9 \times 7 = 2$$

$$3 \times 7 = 3$$

$$4 \times 7 = 4$$

$$-9 \times 7 = -2 \text{ OR } +5$$

$$-11 \times 7 = -4 \text{ OR } 3$$

$$-1 \times 7 = -1 \text{ OR } 6$$

$$11 \times 7 = 4$$

$$12 \times 7 = 5$$

$$14 \times 7 = 0$$

$$15 \times 7 = 1$$

$$16 \times 7 = 2$$

$$17 \times 7 = 3$$

$$-4 \times 7$$

$$4 - 7 = -3$$

$$|val| \times 7 = \text{rem} \quad (\text{rem})$$

↑
sign of a

$$-4 \times 7 = 4 \times 7 = \underline{\underline{4}}$$

$$-9 \times 7 = \underline{\underline{-2}}$$

generalize-

⇒ if remainder < 0
 +ve → add k

array →

⁰
¹
²
³
⁴
⁵
⁶
⁷
k=7
 -3 -4 -4 8 5 4 2 6 4

Sum = 0

Cracking remainder = 0

(+ve) remainder = 0

-3	-7	-11	-2	2	6	8	14	18
<u>-3</u>	0	-4	<u>-3</u>	2	6	1	0	<u>4</u>
<u>4</u>	0	2	<u>4</u>	2	6	1	0	<u>4</u>

length = 1 2 3 4

0 → -1

4 → 0

3 → 2

2 → 4

6 → 5

1 → 6

-ve remainder -

$$-k < \text{rem} \leq 0$$

add k both

$$\text{true} \rightarrow -k + k < \text{rem} + k \leq k$$

$$0 < \text{rem} + k < k$$

Count of subarrays with sum divisible by K! →

K=5.

array → 2 4 8 1 7 3 6 1 9 2 7 3

sum → 0 2 6 14 15 22 25 31 32 41 43 50 53

Remainder → 0 2 1 4 0 2 0 1 2 1 3 0 3

Rem vs Freq.

0 → ~~1~~ 2 2 4

2 → ~~1~~ 2 3

1 → ~~1~~ 2 3

4 → 1

3 → ~~1~~ 2

→ 2 4 8 1

→ 4 8 1 7

→ 7 3

→ 2 4 8 1 7 3

→ 8 1 7 3 6

→ 3 6 1

→ 4 8 1 7 3 6 1

→ 1 9

Count = ~~1~~ ~~2~~ ~~4~~ ~~5~~ ~~7~~ ~~9~~ ~~12~~ 13

→ 8 1 7 3 6 1 9

→ 6 1 9 2 7

→ 7 3 6 1 9 2 7

→ 2 4 8 1 7 3 6 1 9 2 7

→ 7 3

array \rightarrow ⁰-3 ¹-4 ²-4 ³8 ⁴5 ⁵4 ⁶2 ⁷6 k=7

sum \rightarrow 0 -3 -7 -11 -3 2 6 8 14

rem \rightarrow 0 -3 0 -4 -3 2 6 1 0
(-ve)

rem \rightarrow 0 4 0 2 4 2 6 1 0
(+ve)

count = ~~0~~ ~~1~~ ~~2~~ 4

0 \rightarrow ~~1~~ 2

4 \rightarrow ~~1~~ 2

3 \rightarrow 1

2 \rightarrow 1

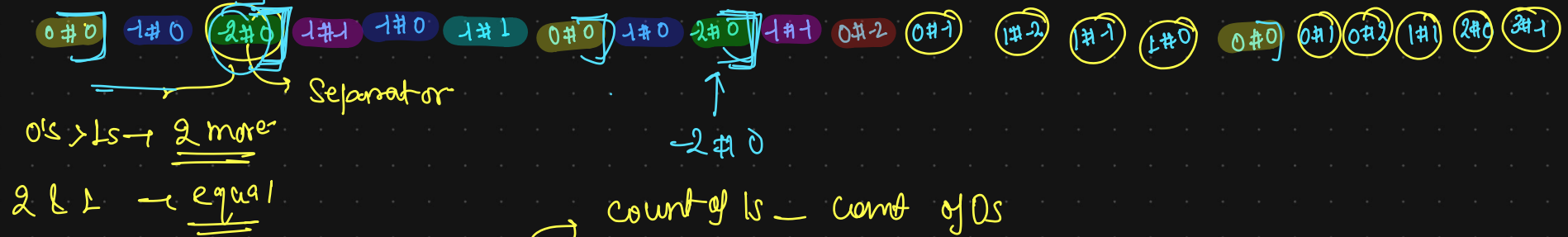
6 \rightarrow 1

1 \rightarrow 1

\rightarrow -3 -4
 \rightarrow -4 -4 8
 \rightarrow -4 8 5 4 2 6
 \rightarrow -3 -4 -4 8 5 4 2 6

longest subarray with equal number of 0s, 1s and 2s →

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
array →	0	0	1	2	2	1	0	0	1	1	2	1	2	2	0	2	2	1	1	
0's →	0	1	2	2	2	2	3	4	4	4	4	4	4	4	5	5	5	5	5	
1's →	0	0	0	1	1	1	2	2	3	4	4	5	5	5	5	5	5	6	7	8
2's →	0	0	0	0	1	2	2	2	2	2	3	3	4	5	5	6	7	7	7	7



Key = a#b

C1s - C0s → string

C2s - C1s → value

longest

a#b → count of 1s - count of 0s

a#b → count of 2s - count of 1s

separator

What?

Why?

How?

Count of Subarrays with equal no. of 0s, 1s and 2s →

Count is same as previous all problems,

we manage count 0, count 1 and count 2.

Make key in format of $a \# b$.

$a \rightarrow \text{count } 1 - \text{count } 0$

$b \rightarrow \text{count } 2 - \text{count } 1$

$\# \rightarrow \text{separator.}$

We have to manage freq in value of hashmap.

Initial push $[0 \# 0 \rightarrow 1]$

Task Completion : →

156
2 5 6 7 9 4

no. of work

Already completed task

①
②

A 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
B

A → 1, 8, 11, 13, 15] → Point

B → 3, 10, 12, 14] → True

① Hashset → already solved work

② Make a loop from 1 to n
↑
no. of work
If already solved skip
↳ Alternate assign