Longest Palindromic Subsequence:

Ex. a b c c b c b → b c c b

String → longest Pal. subseq. LPS

a b c d.

lps(abcd) → lps(string) / OR lps(str)

lps(str) → s(str) → set of seq. of str.

s(abcd) =

–	–	–	–
–	–	–	d
–	–	c	–
–	–	c	d
–	b	–	–
–	b	–	d
–	bc	–	
–	b	c	d
a	–	–	–
a	–	–	d
a	–	c	–
a	–	c	d
a	b	–	–
a	b	–	d
a	bc	–	
a	b	c	d

s(bc) → s(abcd) = a s(bc) d

option, opt

a & d deny

–	–	–	–
–	–	c	–
–	b	–	–
–	b	c	–

a deny.

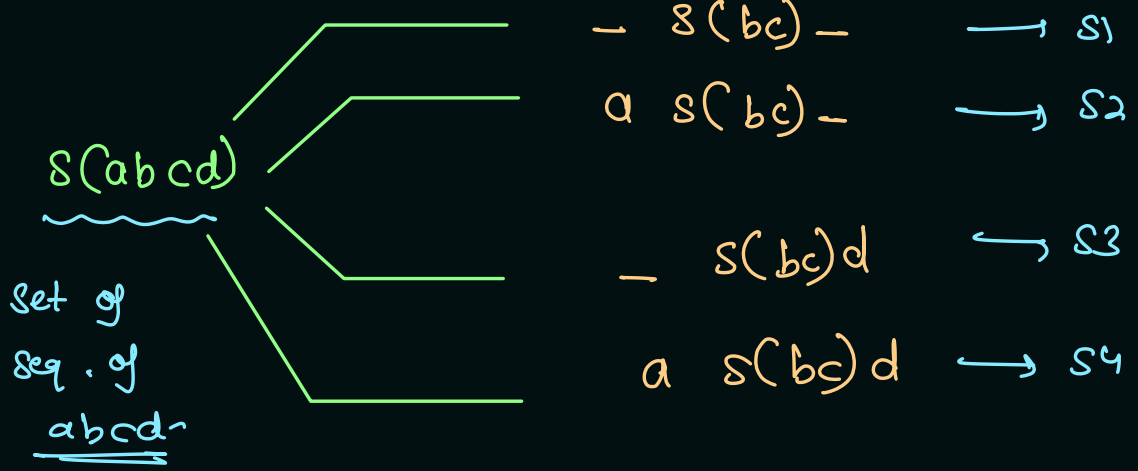
–	–	–	d
–	–	c	d
–	b	–	d
–	b	c	d

'd' deny.

a	–	–	–
a	–	c	–
a	b	–	–
a	b	c	–

a, a

a	–	–	d
a	–	c	d
a	b	–	d
a	b	c	d



$S(abcd)$

set of seq. of abcd

- $S(bc)$ — S_1
- $a\,S(bc)$ — S_2
- $S(bc)d$ — S_3
- $a\,S(bc)d$ — S_4

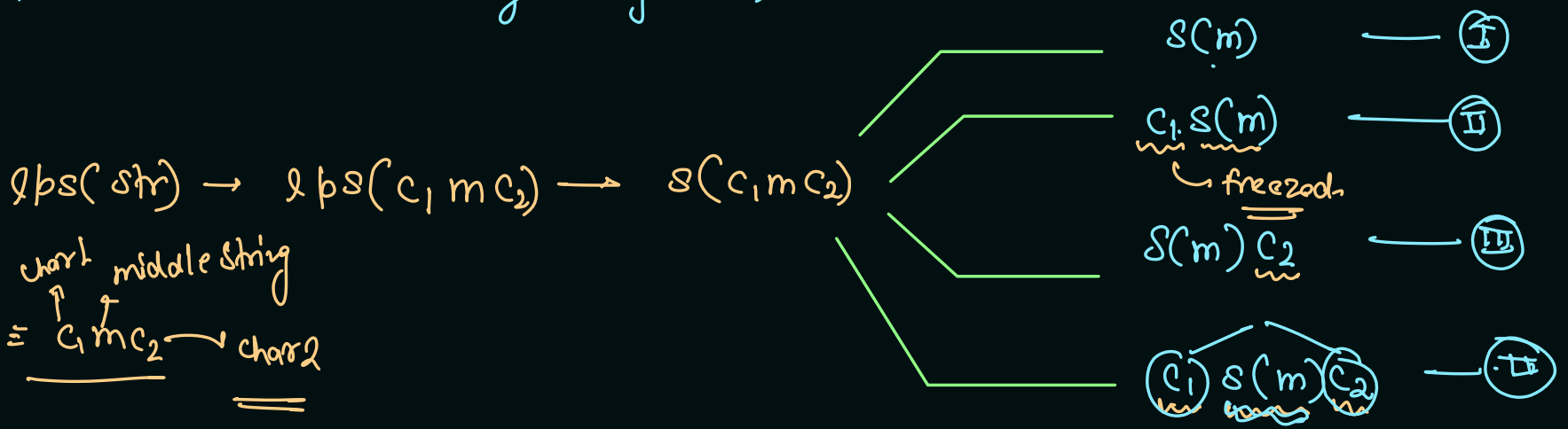
Now move toward genericity →

$lps(str) \rightarrow lps(c_1 m c_2) \rightarrow S(c_1 m c_2)$

- $S(m)$ — I
- $c_1 S(m)$ — II
  - freezed
- $S(m)\,c_2$ — III
- $c_1\,S(m)\,c_2$ — IV

$Str = c_1 m c_2$ → char 2

char 1, middle string



$c_1 == c_2$	$c_1 \neq c_2$
[I, II, III] ✗	IV ✗ Result
IV ✓	[I, II, III] ✓
$l(Str) = 2 + l(m)$	longest from I, II & III

$lps(str) \rightarrow l(c_1 m c_2) \rightarrow s(c_1 m c_2)$

- $\rightarrow s(m)$
- $\rightarrow c_1 s(m)$
- $\rightarrow s(m) c_2$
- $\rightarrow c_1 s(m) c_2$

$l(c_1 m) \rightarrow s(c_1 m)$

- $\rightarrow - s(m)$
- $\rightarrow c_1 s(m)$

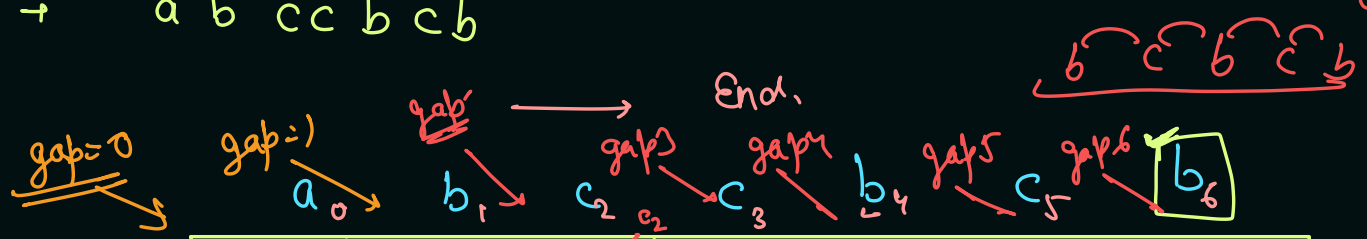
$l(m c_2) \rightarrow s(m c_2)$

- $\rightarrow s(m) -$
- $\rightarrow s(m) c_2$

a b c c b c b

$c_1 == c_2$	$c_1 \neq c_2$
4 ✓ 1, 2, 3] ✗	4 ✗ [1, 2, 3] ✓
$l(str) = 2 + l(m)$	$l(str) = \max(l(c_1 m), l(m c_2))$
$l(c_1 m c_2) = 2 + l(m)$	$l(c_1 m c_2) = \max(l(c_1 m), l(m c_2))$

string → a b cc b cb



	gap=0	gap=1	gap=2	gap=3	gap=4	gap=5	gap=6
a_0	1 a	1 ab	1 abc	2 abcc	4 abccb	4 abccbc	5 abccbcb
b_1	x	1 b	1 bc	2 bcc	4 bccb	4 bccbc	5 bccbcb
c_2	x	x	1 c	2 cc	2 ccb	3 (1,c) cbc	3 cbcb
c_3	x	x	x	1 c	1 (x+1,c) cb	3 cbc	3 cbcb
b_4	x	x	x	x	1 b	1 bc	2 (4,c) bcb
c_5	x	x	x	x	x	1 c	1 cb
b_6	x	x	x	x	x	x	1 b

abc → $c_1 m c_2$

$c_1 m$ → ab

$m c_2$ → bc

a b c c

m

c_1 c_2

$c_1 m$ → abc

$m c_2$ → bcc

Count all palindromic subseq. from given string :

count (S) → C[S(str)] → C[S(ch1 m ch2)]

- C[S(m)] → C_1 (count)
- C[ch1 S(m)] → C_3
- C[S(m) ch2] → C_2
- C[ch1 S(m) ch2] → C_4

bc

m

S(bc) → a - - a

a - c a

a b - a

a bc a

Palindrome = 2

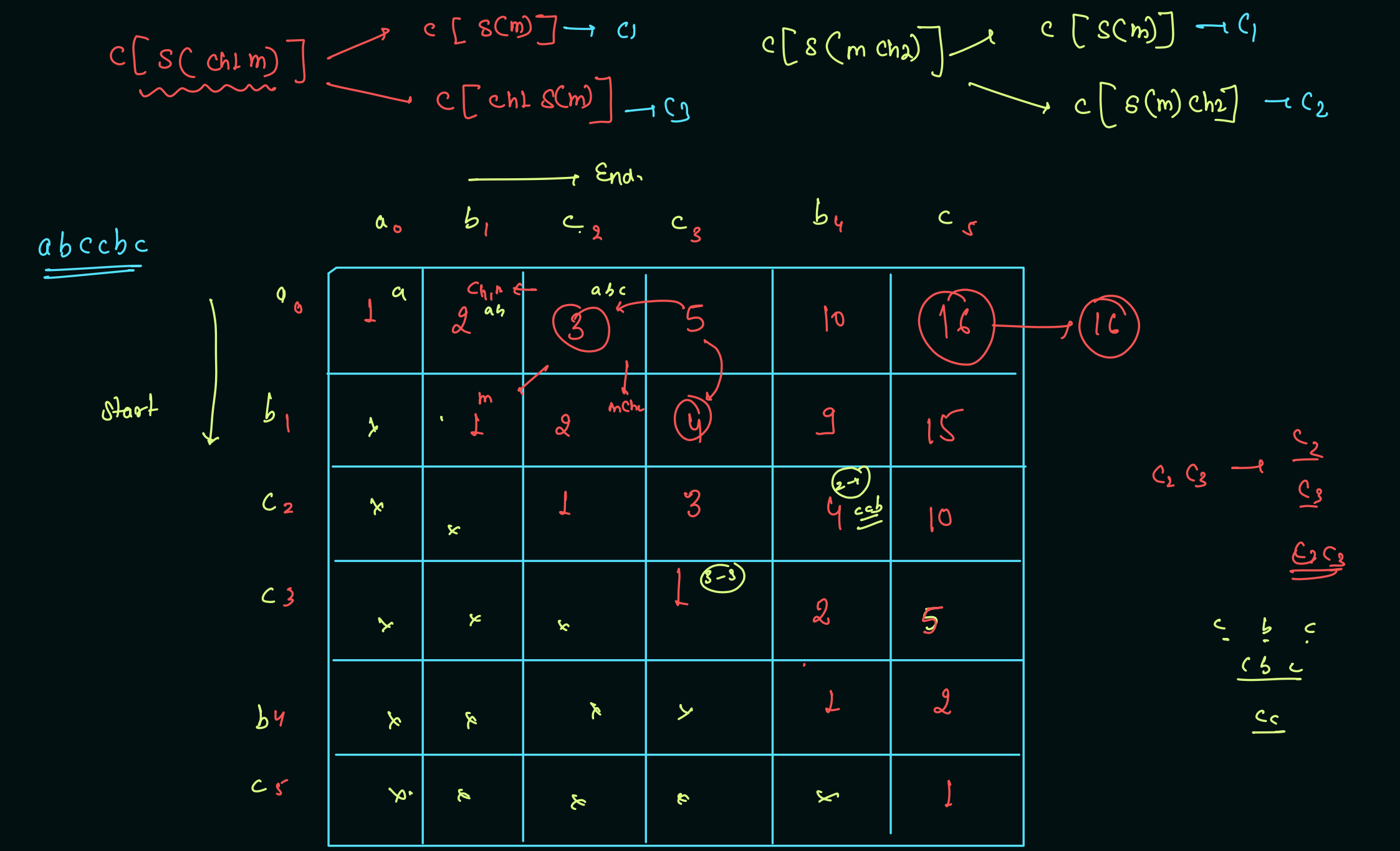
S(m) → 2

a S(m) a = S(m) + 1

Total count [without impact] → $C_1 + C_2 + C_3 + C_4$.

ch1 == ch2	ch1 ≠ ch2
Total count = $C_1 + C_2 + C_3 + C_4$	Total Count = $C_1 + C_2 + C_3 + \cancel{C_4} \rightarrow \boxed{0}$
we know that ch1 == ch2	$= C_1 + C_2 + C_3 + C_1 - C_1$
$\Rightarrow C_4 = C_1 + 1$	$= (C_1 + C_2) + (C_1 + C_3) - C_1$
Total Count = $C_1 + C_2 + C_3 + C_1 + 1$	$\boxed{\text{Total Cout} = C[S(m\,ch_2)] + C[S(ch_1\,m)] - C[S(m)]}$
$= (C_1 + C_2) + (C_1 + C_3) + 1$	

$$\boxed{\text{Total Cout} = C[S(m\,ch_2)] + C[S(ch_1\,m)] + 1}$$



$c[s(ch1\ m)] \rightarrow c[s(m)] \rightarrow c_1$

$c[s(ch1\ m)] \rightarrow c[ch1\ s(m)] \rightarrow c_3$

$c[s(m\ ch2)] \rightarrow c[s(m)] \rightarrow c_1$

$c[s(m\ ch2)] \rightarrow c[s(m)\ ch2] \rightarrow c_2$

End.

abccbc

Start

	a_0	b_1	c_2	c_3	b_4	c_5
a_0	1 a	2 ab ch1 m	3 abc	5	10	16
b_1	x	1 m	2 mch2	4	9	15
c_2	x	x	1	3	4 (2-4) cab	10
c_3	x	x	x	1 (3-3)	2	5
b_4	x	x	x	x	1	2
c_5	x	x	x	x	x	1

16 → 16

$c_2\ c_3 \rightarrow c_2$

c_3

$c_2 c_3$

c b c

cbc

cc

Ugly Number: →

0 based →	0	1	2	3	4	5	6	7	8	9	10
Index	1	2	3	4	5	6	8	9	10	12	14

wrong

2 → ~~2~~ ~~4~~ ~~6~~ ~~8~~ ~~10~~ ~~12~~ 14

3 → ~~3~~ ~~6~~ ~~9~~ ~~12~~ 15

5 → ~~5~~ ~~10~~ 15

7 × 2

val * indx

2, 3, 5

Ungly Number Seq

Index

2p → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ 6

3p → ~~0~~ ~~1~~ ~~2~~ ~~3~~ ~~4~~ 5

5p → ~~0~~ ~~1~~ ~~2~~ 3

2, 3, 5

2×3×5

1	2	2	3		3p	2p						
1	2	3	4	5	6	8	9	10	12	15	16	- - -
0	1	2	3	4	5	6	7	8	9	10	11	12

3p

Index

2×4 = 8
3×3 = 9
5×2 = 10

2×5 = 10
3×3 = 9
5×2 = 10

2×5 = 10
3×4 = 12
5×2 = 10

5×3 = 15
3×4 = 12
2×6 = 12

3×5 = 15
5×3 = 15
2×8 = 16

2×8 = 16
3×6 = 18
5×4 = 20