count all distinct Palindromic subseq:

string str = $a_0 d_1 a_2 a_3 d_4$

$a_0 d_1 a_2$, $a_0 d_1 a_3$ → ada

$d_1 a_2 d_4$, $d_1 a_3 d_4$ → dad

$d_1 a_2 a_3 d_4$ → daad

$d_1 d_4$ →

distinct from Repitition

Total 6 Answers

Discussion →

ds(adcad) → ds(ch1 m ch2)

brute force → Generate all subseq. Pick unique palindrome using HashSet or HashMap.

ds(adcad) → s(ch1 m ch2)

- _ s(m) _
- ch1 s(m) _
- _ s(m) ch2
- ch1 s(m) ch2

→ I

s("abc")

- → s("ab") _
- → s("ab") c

s(s1) → s(ch1 m)

- _ s(m)
- ch1 s(m)

↓ opposite

s(s2) → s(m ch2)

- s(m) _
- s(m) ch2

→ II

① _ _ _, _ b _, a _ _, a b _

② _ _ c, _ b c, a _ c, a b c

Required set and unions :-

① $S(S1 \cup S2) = S(S1) + S(S2) - S(S1 \cap S2)$

② $S(S1 \cup S2 \cup S3) = S(S1) + S(2) + S(3) - S(S1 \cap S2) - S(S2 \cap S3)$
$\quad - S(S1 \cap S3) + S(S1 \cap S2 \cap S3)$

③ $S(S1 \cup S2 \cup S3 \cup S4) = S(S1) + S(S2) + S(S3) + S(S4) - S(S1 \cap S2) - S(S1 \cap S3)$
$\quad - S(S1 \cap S4) - S(S2 \cap S3) - S(S2 \cap S4) - S(S3 \cap S4) +$
$\quad S(S1 \cap S2 \cap S3) + S(S1 \cap S2 \cap S4) + S(S1 \cap S3 \cap S4) + S(S2 \cap S3 \cap S4)$
$\quad - S(S1 \cap S2 \cap S3 \cap S4)$

dps(s) → dps(ch1 m ch2)

- _ dps(m) _ → case - A
- _ dps(m) ch2 → case - B
- ch1 dps(m) _ → case - C
- ch1 dps(m) ch2 → case - D

Two Cases:

(i) $ch1 \neq ch2$

(ii) $ch1 == ch2$

① ch1 ≠ ch2

$$\underset{ch1}{a}\ \underset{m}{[\,d\ \ c\ \ a\,]}\ \underset{ch2}{d}$$

-dbs(m)- case-A	case-B -dps(m) ch2	case-C ch1 dps(m)-	case-D ch1 dps(m) ch2
_ _ _ _ _ (m)	_ _ _ _ d ✓	a _ _ _ _	a _ _ _ d
_ _ _ a _	_ _ _ a d X	a _ _ a _	a _ _ a d
_ _ c _ _	_ _ c _ d X	a _ c _ _ X	a _ c _ d
_ _ c a _ X	_ _ c a d X	a _ c a _	a _ c a d
_ d _ _ _ ✓	_ d _ _ d	a d _ _ _ X	a d _ _ d
_ d _ a _ X	_ d _ a d	a d _ a _	a d _ a d
_ d c _ _ X	_ d c _ d	a d c _ _ X	a d c _ d
_ d c a _ X	_ d c a d X	a d c a _ X	a d c a d
Set → S1 → marked s.s.	Set → S2 → marked ss	Set → S3 → marked ss	~~Set → S4~~ = 0

Case-D: X

becose ch1 & ch2 are froezed and also not equal ∴ 0

so we have to find.

$$\boxed{S1 \cup S2 \cup S3} = S1 + S2 + S3 - S1 \cap S2 - S1 \cap S3 - \underbrace{\cancel{S2 \cap S3}}_{0} + \underbrace{S1 \cap S2 \cancel{\cap S3}}_{0} + \underbrace{S1 - S1}_{\text{total impact} = 0}$$

$$= S1 + S2 - S1 \cap S2 + S1 + S3 - S1 \cap S3 - S1$$

$$= \underbrace{S1 + S2 - S1 \cap S2} + \underbrace{S1 + S3 - S1 \cap S3} - S1$$

dps(m ch2) — dps(m) - S1 ; dps(m) ch2 S2 + dps(ch1 m) — - dps(m) S1 ; ch1 dps(m) S3 - dps(m) S1

Reoccurence Relation →

$$\boxed{\begin{array}{l} \text{dps(ch1 m ch2)} \quad \text{and} \quad ch1 \neq ch2 \\ \quad \llcorner \ \text{dps(m ch2)} + \text{dps(ch1 m)} - \text{dps(m)} \end{array}}$$

— 1

② If ch1 == ch2

↳ If we have to create recurrence relation then we have to split middle string conditionally.

ch1 == ch2

- ch |———| ch — ch is absent in middle part → **Case 2.1**
- ch |—ch—| ch — single occurence of ch in middle part → case 2.2
- ch |—ch—ch—| ch — more than one occurence of ch in middle part → case 2.3

Case 2.1 ch1 == ch2, No availability of ch in middle part, eg- a b c b a

- m -	ch1 m -	- m ch2	ch1 m ch2
- - - - - x	a - - - -	- - - - a	a - - - a
- - - b -	a - - b - x	- - - b a x	a - - b a
- - c - -	a - c - - x	- - c - a x	a - c - a
- - c b - x	a - c b - x	- - c b a x	a - c b a x
- b - - -	a b - - - x	- b - - a x	a b - - a
- b - b -	a b - b - x	- b - b a ✓	a b - b a
- b c - - x	a b c - - x	- b c - a x	a b c - a x
- b c b -	a b c b - x	- b c b a x	a b c b a
Set → S1	Set → S2	Set → S3	Set → S4

$$S1 \cup S2 \cup S3 \cup S4 = S1 + \cancelto{1}{S2} + \cancelto{1}{S3} + S4 - \cancelto{0}{S1 \cap S2} - \cancelto{0}{S1 \cap S3} - \cancelto{0}{S1 \cap S4} - \cancelto{1}{S2 \cap S3} - \cancelto{0}{S2 \cap S4}$$

$$- \cancelto{0}{S3 \cap S4} + \cancelto{0}{S1 \cap S2 \cap S3} + \cancelto{0}{S1 \cap S2 \cap S4} + \cancelto{0}{S1 \cap S3 \cap S4}$$

$$+ \cancelto{0}{S2 \cap S3 \cap S4} - \cancelto{0}{S1 \cap S2 \cap S3 \cap S4}$$

$$S1 \cup S2 \cup S3 \cup S4 = S1 + 1 + \cancel{1} + S4 \cancel{-1}$$

Relation b/w $S1$ & $S4$

$\underline{S4 \rightarrow S1 + 1}$

$$= S1 + 1 + S4$$

$$= S1 + 1 + S1 + 1$$

$$= 2[S1] + 2$$

<u>Case 2.1</u> →

dps(ch1 m ch2) && ch1 == ch2 & no ch available in middlestring.

$$= 2\,dps(m) + 2$$

Case. 2.2. ch1 == ch2 and single occurence of ch in middle part →

Eg → $\underbrace{a}_{ch1}\ \underbrace{b\ a\ c}_{m}\ \underbrace{a}_{ch2}$

- m -	ch1 m -	- m ch2	ch1 m ch2
- - - - - → x	a - - - - ✓	- - - - a ✓	a - - - a ✓
- - - c - x	a - - c - x	- - - c a x	a - - c a ✓
- - a - - ✓	a - a - - ✓	- - a - a ✓	a - a - a
- - a c - x	a - a c - x	- - a c a ✓	a - a c a x
- b - - -	a b - - - x	- b - - a x	a b - - a ✓
- b - c - x	a b - c - x	- b - c a x	a b - c a x
- b a - - x	a b a - - ✓	- b a - a x	a b a - a x
- b a c - x	a b a c - x	- b a c a x	a b a c a x
Set → S1	Set → S2	Set → S3	Set → S4

$$S1 \cup S2 \cup S3 \cup S4 = S1 + S2 + S3 + S4 - S1 \cap S2 - S1 \cap S3 - S1 \cap S4 - S2 \cap S3 - S2 \cap S4$$

$$- S3 \cap S4 + S1 \cap S2 \cap S3 + S1 \cap S2 \cap S4 + S1 \cap S3 \cap S4$$

$$+ S2 \cap S3 \cap S4 - S1 \cap S2 \cap S3 \cap S4$$

observation.

1. $S1 \cap S4 = 0$

2. $S4 = S1 + 1$

3. $S2 = S1 \cap S2 + S2 \cap S4$

4. $S3 = S1 \cap S3 + S3 \cap S4$

5. $S2 \cap S3 = S1 \cap S2 \cap S3 + S2 \cap S3 \cap S4$.

$$S1 \cup S2 \cup S3 \cup S4 = S1 + \cancel{S2} + \cancel{S3} + \overset{S1+1}{\cancel{S4}} - \cancel{S1 \cap S2} - \cancel{S1 \cap S3} - \overset{0}{\cancel{S1 \cap S4}} - \cancel{S2 \cap S3} - \cancel{S2 \cap S4}$$
$$- \cancel{S3 \cap S4} + \cancel{S1 \cap S2 \cap S3} + \overset{0}{\cancel{S1 \cap S2 \cap S4}} + \overset{0}{\cancel{S1 \cap S3 \cap S4}}$$
$$+ \cancel{S2 \cap S3 \cap S4} - \overset{0}{\cancel{S1 \cap S2 \cap S3 \cap S4}}$$

$$S1 \cup S2 \cup S3 \cup S4 = S1 + S1 + 1$$
$$= 2S1 + 1$$

case. 2.2.

dps(ch1 m ch2), if ch1 == ch2 & one ch in middle part

dps(ch1 m ch2) = 2 dps(m) + 1

case 2.3

If ch1 == ch2 and more than one ch available in middle part

next Right ch   m'   next left ch

a   b   a   {c}   a   d   a

ch1   m   ch2

i   i'   j'   j

Bigger problem $ch1_i \; m \; ch2_j$

Smaller problem $ch1_{i'} \; m' \; ch2_{j'}$

$$\boxed{dps(ch1 \; m \; ch2) = 2\,dps(m) - dps(m')}$$

Single ch

$dps(ch1 \; m \; ch2) = 2\,dps(m) - 1$ upgrade for more than one ch

a   b   a   c   a   d   a.

i   i'   m'   j'   j

m

$$dps(ch1 \; m \; ch_2) = 2\,dps(m) - dps(m')$$

cases for all distinct palindromic Subseq :→

String → ch1 m ch2.

$ch1 \neq ch2$.

$dps[ch1\ m\ ch2] = dps[\underline{m ch2}] + dps[\underline{ch1\ m}] - dps(\underline{m})$

$ch1 = ch2$

→ No Repitition

$dps(ch1\ m\ ch2) = \underline{2}\ \underline{dps(m)} + \underline{2}$

→ Single Repitition

$dps(ch1\ m\ ch2) = 2 * \underline{dps}(m) + 1$

→ More than one Repition.

$dps(ch1\ m\ ch2) = 2dps(m) - dps(m')$

(1, 9) → (2, 7)

a	b	a	c	d	a	s	a	p	a
0	1	2	3	4	5	6	7	8	9
i		j, i'					j, j'		j

gap=0 gap=1 gap=2 Ending gap gap=4

Start

	a_0	d_1	a_2	a_3	d_4
a_0	1	2	4	0→3 5	8
d_1	x	1	2	2+2-1 = 3	6
a_2	x	x	1	2	3
a_3	x	x	x	1	2
d_4	x	x	x	x	1

next index of character → next[ ]

previous index of character → prev[ ]

logic + DP

String →

a	b	d	a	c	a	b	d	a	e	b	a
0	1	2	3	4	5	6	7	8	9	10	11

HashMap<Character, Integer>

left to Right (set prev)

0 <	1	2	0	4	3	1	2	5	9	6	8
0	1	2	3	4	5	6	7	8	9	10	11

a → ~~0~~ ~~3~~ ~~5~~ ~~8~~ 11

b → ~~1~~ ~~6~~ 10

d → ~~2~~ 7

c → 4

e → 9

Similarly set → Next