

count all distinct Palindromic Subseq:

string str =  $a_0 d_1 a_2 a_3 d_4$

Discussion →

$ds(adcad) \rightarrow ds(ch1 \text{ m } ch2)$

brute force → Generate all subseq. pick unique palindrome using HashSet or HashMap.

$a_0 d_1 a_2 \rightarrow ada$   
 $a_0 d_1 a_3 \rightarrow ada$   
 $d_1 a_2 d_4 \rightarrow dad$   
 $d_1 a_3 d_4 \rightarrow dad$   
 $d_1 a_2 a_3 d_4 \rightarrow daad$   
 $d_1 d_4 \rightarrow d$   
 $\vdots$   
 $\vdots$   
 $\vdots$

distinct from Repetition

Total 6 answers

$ds(adcad) \rightarrow ds(ch1 \text{ m } ch2)$   
 $\begin{cases} - s(m) - \\ ch1 \text{ } s(m) - \\ - s(m) \text{ } ch2 \\ ch1 \text{ } s(m) \text{ } ch2 \end{cases} \rightarrow \textcircled{\text{I}}$

$s(st) \rightarrow s(ch1 \text{ m } m)$   
 $\begin{cases} - s(m) \\ ch1 \text{ } s(m) \end{cases}$

↓ opposite

$s(st) \rightarrow s(m \text{ } ch2)$   
 $\begin{cases} s(m) - \\ s(m) \text{ } ch2 \end{cases}$

→  $\textcircled{\text{II}}$

$\begin{cases} - - - \\ - b - \\ a - - \\ a b - \end{cases} \textcircled{1}$

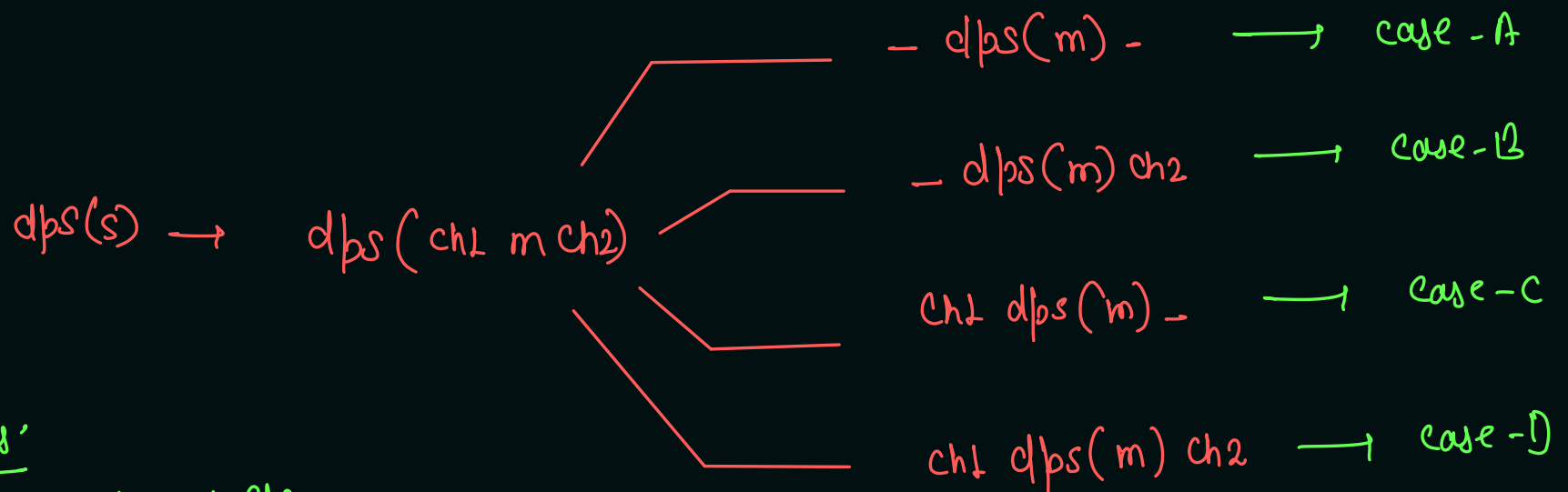
$\begin{cases} - - c \\ - b c \\ a - c \\ a b c \end{cases} \textcircled{2}$

Required set and Union :-

$$(1) \quad S(S_1 \cup S_2) = S(S_1) + S(S_2) - S(S_1 \cap S_2)$$

$$(2) \quad S(S_1 \cup S_2 \cup S_3) = S(S_1) + S(S_2) + S(S_3) - S(S_1 \cap S_2) - S(S_2 \cap S_3) - S(S_1 \cap S_3) + S(S_1 \cap S_2 \cap S_3)$$

$$(3) \quad S(S_1 \cup S_2 \cup S_3 \cup S_4) = S(S_1) + S(S_2) + S(S_3) + S(S_4) - S(S_1 \cap S_2) - S(S_1 \cap S_3) - S(S_1 \cap S_4) - S(S_2 \cap S_3) - S(S_2 \cap S_4) - S(S_3 \cap S_4) + S(S_1 \cap S_2 \cap S_3) + S(S_1 \cap S_2 \cap S_4) + S(S_1 \cap S_3 \cap S_4) + S(S_2 \cap S_3 \cap S_4) - S(S_1 \cap S_2 \cap S_3 \cap S_4)$$



Two Cases:

(i)  $ch_1 \neq ch_2$

(ii)  $ch_1 == ch_2$

① ch1  $\neq$  ch2

a  $\begin{bmatrix} d & c & a \end{bmatrix}$  d  
ch1 m ch2

-dbs(m)- case-A

- --- -  
- - - a -  
- - c - -  
- - c a -  
- d - - -  
- d - a -  
- d c -  
- d c a -

Set  $\rightarrow$  S1  $\rightarrow$  marked s-s.

so we have to find.

case-B - dps(m) ch2

- --- d  
- - - a d  
- - c - d  
- - c a d  
- d - - d  
- d - a d  
- d c - d  
- d c a d

Set  $\rightarrow$  S2  $\rightarrow$  marked ss

case-C ch1.dps(m)-

a --- -  
a - - a -  
a - c - -  
a - c a -  
a d - - -  
a d - a -  
a d c - -  
a d c a -

Set  $\rightarrow$  S3  $\rightarrow$  marked ss

ch1 dps(m) ch2  
case-D

a --- d  
a - - a d  
a - c - d  
a - c a d  
a d - - d  
a d - a d  
a d c - d  
a d c a d

~~Set  $\rightarrow$  S4 = 0~~

because ch1 & ch2 are frozen and also not equal  $\Rightarrow$  0

$$\boxed{S1 \cup S2 \cup S3} = (S1) + (S2) + (S3) - (S1 \cap S2) - (S1 \cap S3) - (S2 \cap S3) + (S1 \cap S2 \cap S3)$$

$$= S1 + S2 - S1 \cap S2 + S1 + S3 - S1 \cap S3 - S1$$

~~$+ S1 \cap S2 \cap S3$~~   $+ (S1) - S1$   
total impact  $\Rightarrow$  0

$$= \underbrace{s_1 + s_2 - s_1 \cap s_2}_{\text{dps}(m, ch_2)} + \underbrace{s_1 + s_3 - s_1 \cap s_3}_{\text{dps}(ch_1, m)} - \underbrace{s_1}_{\text{dps}(m)}$$

Diagram illustrating the recurrence relation for the intersection of sets \$s\_1, s\_2, s\_3\$:

- The first term,  $s_1 + s_2 - s_1 \cap s_2$ , is labeled  $\text{dps}(m, ch_2)$ . It branches into:
  - $\text{dps}(m) - s_1$  (labeled  $s_1$ )
  - $\text{dps}(m) \cap s_2$  (labeled  $s_2$ )
- The second term,  $s_1 + s_3 - s_1 \cap s_3$ , is labeled  $\text{dps}(ch_1, m)$ . It branches into:
  - $s_1$  (labeled  $s_1$ )
  - $s_3$  (labeled  $s_3$ )
- The third term,  $-s_1$ , is labeled  $\text{dps}(m)$ .

Recurrence Relation  $\rightarrow$

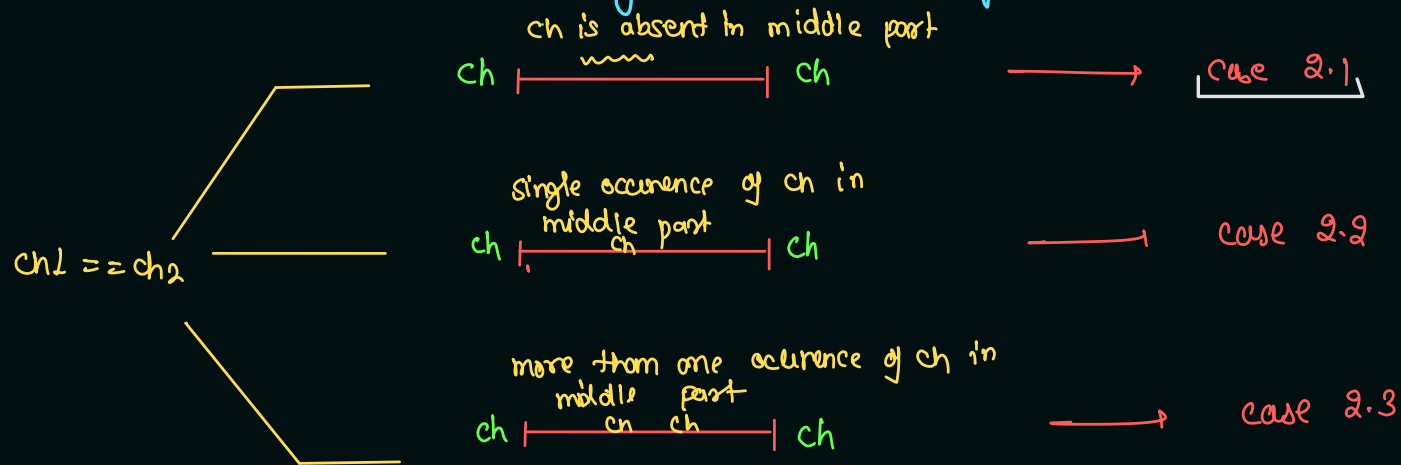
$$\text{dps}(ch_1, m, ch_2) \quad \text{and} \quad ch_1 \neq ch_2$$

$$\hookrightarrow \text{dps}(m, ch_2) + \text{dps}(ch_1, m) - \text{dps}(m)$$

$\rightarrow$  ①

② If  $ch1 == ch2$

↳ If we have to create recurrence relation then we have to split middle string conditionally.



Case 2.1  $ch1 == ch2$ , No availability of  $ch$  in middle part, eg-  $a b c b a$

Set  $\rightarrow S1$

Diagram showing possible splits for  $a b c b a$  where  $ch$  is absent in the middle part:

- $a - - - a$  (marked with X)
- $a - - b -$
- $a - c -$
- $a - c b -$  (marked with X)
- $a - b - -$
- $a - b - b -$
- $a - b c -$  (marked with X)
- $a - b c b -$

Set  $\rightarrow S2$

Diagram showing possible splits for  $a b c b a$  where  $ch$  is present in the middle part:

- $a - - - a$  (marked with X)
- $a - - b -$  (marked with X)
- $a - c -$  (marked with X)
- $a - c b -$  (marked with X)
- $a - b - -$  (marked with X)
- $a - b - b -$  (marked with X)
- $a - b c -$  (marked with X)
- $a - b c b -$  (marked with X)

Set  $\rightarrow S3$

Diagram showing possible splits for  $a b c b a$  where  $ch$  is present in the middle part:

- $a - - - a$
- $a - - b a$  (marked with X)
- $a - c - a$  (marked with X)
- $a - c b a$  (marked with X)
- $a - b - - a$  (marked with X)
- $a - b - b a$  (marked with X)
- $a - b c - a$  (marked with X)
- $a - b c b a$  (marked with X)

Set  $\rightarrow S4$

Diagram showing possible splits for  $a b c b a$  where  $ch$  is present in the middle part:

- $a - - - a$
- $a - - b a$
- $a - c - a$
- $a - c b a$  (marked with X)
- $a - b - - a$
- $a - b - b a$
- $a - b c - a$  (marked with X)
- $a - b c b a$

$$\begin{aligned}
 S1 \cup S2 \cup S3 \cup S4 = & S1 + S2 + S3 + S4 - S1 \cap S2 - S1 \cap S3 - S1 \cap S4 - S2 \cap S3 - S2 \cap S4 \\
 & - S3 \cap S4 + S1 \cap S2 \cap S3 + S1 \cap S2 \cap S4 + S1 \cap S3 \cap S4 \\
 & + S2 \cap S3 \cap S4 - S1 \cap S2 \cap S3 \cap S4
 \end{aligned}$$

$$S1 \cup S2 \cup S3 \cup S4 = S1 + 1 + 1 + S4 + 1$$

$$= S1 + 1 + S4$$

$$= S1 + 1 + S1 + 1$$

$$= 2[S1] + 2$$

Relation b/w  $S1$  &  $S4$

$$S4 \rightarrow S1 + 1$$

Case 2.1  $\rightarrow$

$dp_s(ch1, m, ch2)$  . & &  $ch1 == ch2$  & no ch available in middle string.

$$= 2 dp_s(m) + 2$$

Case-2.2.

$ch1 == ch2$

and

Single occurrence of

ch

in

middle part  $\rightarrow$

Eg  $\rightarrow$

$\underbrace{a}_{ch1} \underbrace{b a c}_m \underbrace{a}_{ch2}$

$\underbrace{\quad m \quad}_{\text{middle part}}$   
 $\text{---} \text{---} \text{---} \rightarrow x$   
 $\text{---} \text{---} c \text{---} x$   
 $\text{---} a \text{---} \rightarrow$   
 $\text{---} a c \text{---} x$   
 $\text{---} b \text{---} \text{---} x$   
 $\text{---} b \text{---} c \text{---} x$   
 $\text{---} b a \text{---} x$   
 $\text{---} b a c \text{---} x$

Set  $\rightarrow S1$

$\underbrace{\quad ch1 \quad m \quad}_{\text{middle part}}$   
 $\boxed{a} \text{---} \text{---} \text{---} \rightarrow$   
 $a \text{---} \text{---} c \text{---} x$   
 $a \text{---} a \text{---} \rightarrow$   
 $a \text{---} a c \text{---} x$   
 $a b \text{---} \text{---} x$   
 $a b \text{---} c \text{---} x$   
 $a b a \text{---} \rightarrow$   
 $a b a c \text{---} x$

Set  $\rightarrow S2$

$\text{---} m \quad ch2$   
 $\text{---} \text{---} \text{---} a \rightarrow$   
 $\text{---} \text{---} c a x$   
 $\text{---} a \text{---} a \rightarrow$   
 $\text{---} a c a \rightarrow$   
 $\text{---} b \text{---} a x$   
 $\text{---} b \text{---} c a x$   
 $\text{---} b a \text{---} a x$   
 $\text{---} b a c a x$

Set  $\rightarrow S3$

$\underbrace{\quad ch1 \quad m \quad ch2}_{\text{middle part}}$   
 $\text{---} a \text{---} \text{---} a \rightarrow$   
 $a \text{---} \text{---} c a \rightarrow$   
 $a \text{---} a \text{---} a \rightarrow$   
 $a \text{---} a c a x$   
 $a b \text{---} \text{---} a \rightarrow$   
 $a b \text{---} c a x$   
 $a b a \text{---} a x$   
 $a b a c a x$

Set  $\rightarrow S4$

$$\begin{aligned}
 S1 \cup S2 \cup S3 \cup S4 &= S1 + S2 + S3 + S4 - S1 \cap S2 - S1 \cap S3 - S1 \cap S4 - S2 \cap S3 - S2 \cap S4 \\
 &\quad - S3 \cap S4 + S1 \cap S2 \cap S3 + S1 \cap S2 \cap S4 + S1 \cap S3 \cap S4 \\
 &\quad + S2 \cap S3 \cap S4 - S1 \cap S2 \cap S3 \cap S4
 \end{aligned}$$

observation:

$$① \quad S1 \cap S4 = 0$$

$$② \quad S4 = S1 + 1$$

$$③ \quad S2 = S1 \cap S2 + S2 \cap S4$$

$$④ \quad S3 = S2 \cap S3 + S3 \cap S4$$

$$⑤ \quad S2 \cap S3 = S1 \cap S2 \cap S3 + S2 \cap S3 \cap S4$$

$$\begin{aligned} S1 \cup S2 \cup S3 \cup S4 &= S1 + \cancel{S2} + \cancel{S3} + \cancel{S4} - \cancel{S1 \cap S2} - \cancel{S1 \cap S3} - \cancel{S1 \cap S4} - \cancel{S2 \cap S3} - \cancel{S2 \cap S4} \\ &\quad - \cancel{S3 \cap S4} + \cancel{S1 \cap S2 \cap S3} + \cancel{S1 \cap S2 \cap S4} + \cancel{S1 \cap S3 \cap S4} \\ &\quad + \cancel{S2 \cap S3 \cap S4} - \cancel{S1 \cap S2 \cap S3 \cap S4} \end{aligned}$$

$$\begin{aligned} S1 \cup S2 \cup S3 \cup S4 &= S1 + S1 + 1 \\ &= 2S1 + 1 \end{aligned}$$

case 2.2.

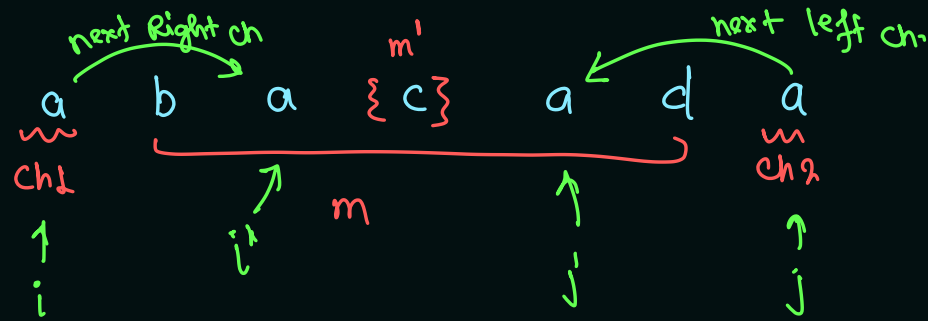
$dpS(ch1 \text{ m } ch2)$ , &  $ch1 = 2 \text{ } ch2$  & one ch in middle part

$$dpS(ch1 \text{ m } ch2) = 2 dpS(m) + 1$$



# case 2.3

if  $ch_1 == ch_2$  and more than one ch available in middle part



Bigger problem  $ch_1; m; ch_2;$

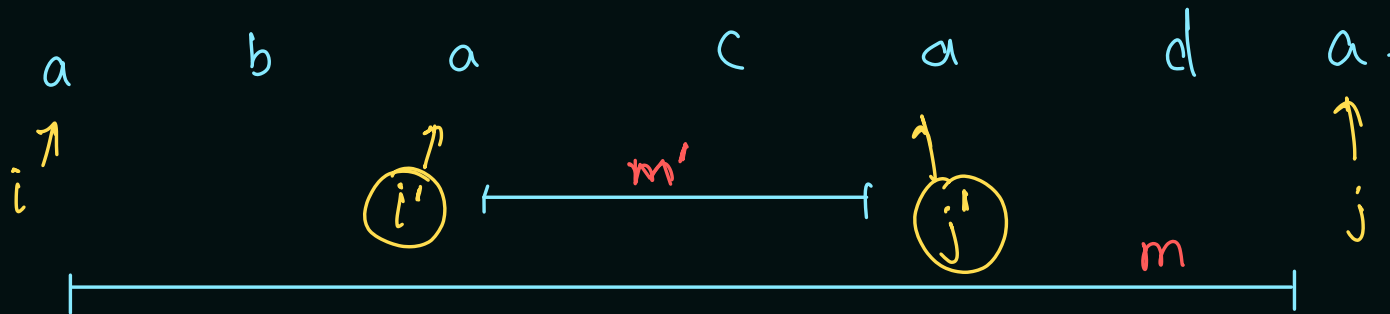
Smaller problem  $ch_1; m'; ch_2;$

$$dps(ch_1 \ m \ ch_2) = 2 \ dps(m) - \underbrace{dps(m')}$$

Single ch

$$dps(ch_1 \ m \ ch_2) = 2 \ dps(m) - 1$$

↑ upgrade for more than one ch.



$$dps(ch_1 \ m \ ch_2) = 2 \ dps(m) - dps(m')$$

cases for all distinct palindromic subseq<sup>o</sup> →

String →  $ch_1 \ m \ ch_2$ .

$ch_1 \neq ch_2$ .

$$dps[ch_1 \ m \ ch_2] = \underline{dps[m \ ch_2]} + \underline{dps[ch_1 \ m]} - \underline{dps(m)}$$

$ch_1 = ch_2$

→ No Repetition

$$dps(ch_1 \ m \ ch_2) = \underline{2 \ dps(m)} + \underline{2}$$

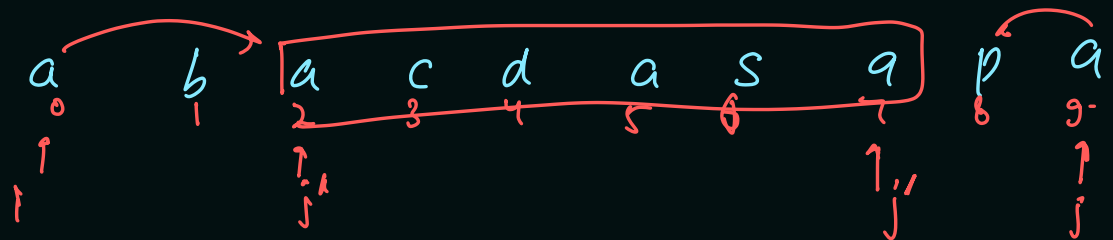
→ Single Repetition

$$dps(ch_1 \ m \ ch_2) = 2 * \underline{dps(m)} + 1$$

→ More than one Repetition.

$$dps(ch_1 \ m \ ch_2) = 2dps(m) - dps(m')$$

(1,2) → (2,2)



gap=0

gap=1

gap=2

Ending gap

gap=4

Start

a <sub>0</sub>	1	2	4	5	8
d <sub>1</sub>	x	1	2	3	6
a <sub>2</sub>	x	x	1	2	3
a <sub>3</sub>	x	x	x	1	2
d <sub>4</sub>	x	x	x	x	1

next index of character → next[]  
 previous index of character → prev[]

logic + DP

String  $\rightarrow$

a	b	d	a	c	a	b	d	a	e	b	a
0	1	2	3	4	5	6	7	8	9	10	11
			$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

HashMap < Character, Integer >

left to Right (set prev)

0	1	2	3	4	5	6	7	8	9	10	11
0	1	2	3	4	5	6	7	8	9	10	11

a  $\rightarrow$  ~~0~~ ~~3~~ ~~8~~ 11

b  $\rightarrow$  ~~1~~ ~~6~~ 10

d  $\rightarrow$  ~~2~~ 7

c  $\rightarrow$  4

e  $\rightarrow$  9

Similarly set  $\rightarrow$  Next