

# Distilling Reasoning Capabilities into Smaller Language Models

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- generating step by step CoT rationales and using it to induce reasoning capabilities in small language models

the approach feels similar to least to most prompting in LLMs where a given problem is broken into many subproblems and each subproblems are solved to solve the bigger problem

## Abstract

Step-by-step reasoning approaches like chain of thought (CoT) have proved to be very effective in inducing reasoning capabilities in large language models. However, the success of the CoT approach is fundamentally tied to the model size, and billion parameter-scale models are often needed to get CoT to work. In this paper, we propose a knowledge distillation approach that leverages the step-by-step CoT reasoning capabilities of larger models and distills these abilities into smaller models.

In this work, we propose an alternative reasoning scheme, SOCRATIC CoT that learns a decomposition of the original problem into a sequence of subproblems and uses it to guide the intermediate reasoning steps. We use SOCRATIC CoT to train a combination of two small distilled models: a *problem decomposer* and a *subproblem solver*. In practice, given a new problem, the two distilled models work in sync to decompose and solve complex problems. On multiple reasoning datasets (GSM8K, StrategyQA, and SVAMP), our proposed distillation strategies boosts the performance of smaller models over 70% compared to the baselines. Finally, we investigate when SOCRATIC CoT is an effective alternative to CoT, demonstrating cases where a much smaller model (GPT-2 large) can outperform a 10X larger model (GPT-3 6B). Our code is available [here](#).

## 1 Introduction

Large language models (LLMs) have demonstrated strong performance on a variety of reasoning tasks (Brown et al., 2020; Hoffmann et al., 2022; Chowdhery et al., 2022, *inter alia*). One particularly interesting strategy for prompting these models is chain-of-thought (CoT), which has been shown to elicit reasoning abilities in LLMs by asking the model to incorporate intermediate reasoning steps while

solving a problem (Nye et al., 2021; Wei et al., 2022b; Wang et al., 2022). However, CoT has been shown to work primarily on models with hundreds of billions of parameters (Wei et al., 2022b,a) or those tuned to a wide range of tasks (Chung et al., 2022; Iyer et al., 2022).

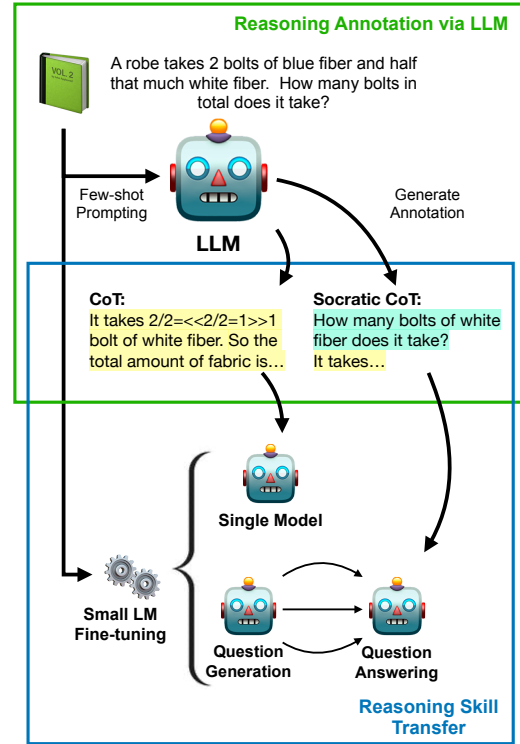


Figure 1: Illustration of the proposed framework. First, an LLM is prompted to decompose a multi-step problem providing annotation for the intermediate steps leading to the final solution. Then, the generated annotation is used to provide additional supervision when fine-tuning smaller models.

Due to the significant computational resources or expensive API calls required to access CoT-capable LLMs, we ask whether it is possible to elicit such reasoning capabilities in smaller models.<sup>1</sup>

<sup>1</sup>Following Li et al. (2022), we argue that *small* and *large* models are relative terms and context-dependent. We consider

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Small-sized, non-fine-tuned language models are known to be poor reasoners (Stolfo et al., 2022). Therefore, a possible approach to induce CoT-like reasoning abilities in smaller models would be fine-tuning them on step-by-step examples.

In our work, we propose a framework for leveraging the reasoning capabilities of LLMs to supervise the training of smaller models. This approach can be thought of as a form of *knowledge distillation* (Hinton et al., 2015), where a larger teacher model transfers knowledge to a smaller student model. However, unlike standard knowledge distillation, our method transfers the reasoning abilities of the teacher model only using its generated solutions as a proxy, i.e., we do not assume access to the teacher model parameters. Our approach consists of prompting an LLM to produce step-by-step annotations leading to the answer for a set of problems. This annotation is then used as supervision to fine-tune the student model. A high-level illustration of the process is provided in Figure 1.

Within this framework, we study three different types of *annotation structure* for supervising our distillation approach: (i) We consider fine-tuning on the *gold step-by-step* solution procedure for datasets where the step-by-step solutions are available. (ii) We study whether *procedural supervision*, coming from the *chain of thought* (CoT) of the teacher model can improve upon the baseline. (iii) We propose a third type of supervision structure, which we call **SOCRATIC CoT**. This approach relies on learning a semantic decomposition of the original problem into a sequence of subproblem-solution pairs using two models – a) a question generator that learns to decompose the problem into a sequence of subproblems, and b) a question-answering model that solves the various generated subproblems (more details are in section 3.2). This approach can be thought of as an extension of the typical chain of thought reasoning where, unlike CoT, the intermediate steps are now decomposed into subquestion-solution pairs; the subquestions guide the generation of intermediate steps that lead to the final answer to the problem.

We train distilled student models with various annotation structures mentioned above. Depending on the annotation available for the given data, we use the teacher model to generate either a CoT-like solution to a problem or, if the step-by-step anno-

tation is available, a set of subquestions leading to the solution of the problem, or both (examples of different annotations are shown in Figure 2).

We perform our analyses on three multi-step reasoning datasets: **GSM8K** (Cobbe et al., 2021), **StrategyQA** (Geva et al., 2021), and **SVAMP** (Patel et al., 2021). We consider data with various types of annotation to cover a range of realistic data scenarios. Our results show that supervision by CoT-decomposed examples helps smaller models perform better, and subquestioning introduced by SOCRATIC CoT can provide further improvement. We observe performance gains of up to 40% with LLM-generated step-by-step annotations – this validates the effectiveness of our distillation framework (detailed analysis in Section 5).

## 2 Related Work

### Decomposing Multi-Step Reasoning Tasks

Solving multi-step reasoning tasks like MWPs has been a popular area of research for the last couple of years (Kushman et al., 2014; Hosseini et al., 2014; Roy et al., 2015; Amini et al., 2019; Zhang et al., 2020; Shridhar et al., 2022; Opedal et al., 2023). However, the majority of the modern approaches for these problems are shifting towards using large language models, often relying on approaches involving prompting or in-context learning (Cobbe et al., 2021; Kojima et al., 2022; Wei et al., 2022b; Chowdhery et al., 2022; Lewkowycz et al., 2022; Srivastava et al., 2022). One such prompting approach is the chain of thought prompting (Wei et al., 2022b), which prompts the language model to generate a series of intermediate steps that improve the reasoning capabilities in LLMs. Wang et al. (2022) took another step forward and sampled multiple reasoning paths and selected the most relevant output using majority voting. Huang et al. (2022) used the most voted outputs to further fine-tune the model for better performance. Kojima et al. (2022) further improved the reasoning of LLM in a zero-shot manner by appending “Let’s think step by step” to the prompt. In contrast, our work does not propose prompting solutions; instead, we explicitly guide the student model reasoning using sub-questions at each step. Most similar to our work is the work by Zhou et al. (2022) which decomposes questions into sub-questions and asks the language model to solve each sub-question sequentially. However, this work is also restricted

-exactly the same approach as in least to most complex prompting

<https://arxiv.org/abs/2205.10625>

models with billions of parameters to be large, and models with millions of parameters to be small.

to prompting and only works with LLMs with billions of parameters.

**Knowledge Distillation** Our approach is reminiscent of knowledge distillation (Ba and Caruana, 2014; Hinton et al., 2015) in that we use a student network to mimic the large teacher language model. Snell et al. (2022) demonstrated the usefulness of providing instruction that can help models achieve better reasoning skills. Similar to our hypothesis, Eisenstein et al. (2022) argued that question-answering systems should focus not only on the final answer, but also on the rationale that justifies their reasoning, to help them reason better. We go beyond this; in our work, in addition to the question-answering system, we also focus on what questions need to be asked at each step that can help to learn that reasoning step better. Finally, similar to our hypothesis of injecting reasoning capabilities into smaller models, Li et al. (2022) used CoT-like reasoning from LLMs to train smaller models on a joint task of generating the solution and explaining the generated solution. We, on the other hand, use the LLM to generate subquestions and solution pairs and use them together to inject reasoning capabilities into smaller models.

**Subquestioning as supervision** The idea of inquiring or asking information-seeking questions for discovery learning has been studied well in the past (Bruner, 1961). Rao and Daumé III generated clarification questions based on Stack Exchange questions as supervision, Klein and Nabi (2019) used a joint question answering model to ask questions from a given span of text and later answer them, and (Rajani et al., 2019; Shwartz et al., 2020) asked questions to improve common sense QA models. In contrast, our work focuses on multistep reasoning tasks where intermediate clarifying questions and reasoning steps may not always be available and may need to be extracted from a teacher model.

### 3 Methodology

The setting we consider consists of a data set  $\mathcal{D}$ , where each problem  $P_i$  is accompanied by a final answer  $a_i$  that can be reached by several steps of reasoning. The task of solving the problem using a model  $\psi$  is to predict an answer  $\hat{a} = \psi(P)$  such that  $\hat{a} = a$ . We consider different data scenarios where intermediate annotations of the solution may be available in different forms (e.g., step-by-step, as a semantic decomposition by subquestions) or

#### Reasoning Problem

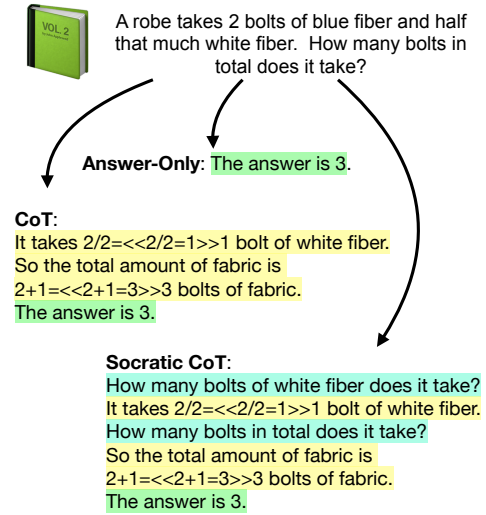


Figure 2: Illustration of the three different kinds of annotation structure. Our proposed approach, SOCRATIC CoT, augments the typical chain-of-thought step-by-step solution with subquestioning.

may not be present. Depending on the availability of annotations, we propose different approaches to augment the training of a small model on  $\mathcal{D}$  by using LLMs.

#### 3.1 Distilling step-by-step reasoning via CoT

A data set may present an annotation that contains intermediate reasoning steps that lead to the answer  $a_i$  (i.e., a chain-of-thought annotation). This intermediate annotation can be used directly to fine-tune a small model. However, in cases where such step-by-step information is not available, we use a LLM to generate the reasoning steps that might improve the performance of the small model.

To achieve this, we consider a small subset of the dataset  $\mathcal{D}$  and decompose each problem  $P_i$  into  $n_i$  intermediate reasoning steps. We construct these intermediate reasoning steps manually, since we only need a few examples as prompts (examples are provided in Appendix Table 6).

For each remaining problem  $P \in \mathcal{D}$ , we then prompt a large language model  $\mathcal{M}$  to generate the intermediate reasoning steps. We make sure that the chain of reasoning steps is meaningful by checking whether the last solution matches the ground truth answer, i.e. whether  $a_i^{(n_i)} = a_i$ , where  $a_i^{(n_i)}$  represents the answer corresponding to the last reasoning step. If this is not the case, we discard the problem and sample a new chain by prompting the model again (for a maximum of 3 times). In this

- annotation of training data using LLM can be done via two options - 1. CoT 2. SOCRATIC CoT - in CoT - there are possibilities that human annotated step by step solution to a problem might be available. if not we use few shot CoT technique to prompt the LLM model and annotate the data

way, we obtain an augmented dataset  $\mathcal{D}^*$  in which a subset of problems is paired with a sequence of reasoning steps leading to the correct result. Finally, we can distill the reasoning capabilities into smaller models by fine-tuning them with the generated intermediate steps.

### 3.2 Distilling step-by-step reasoning through SOCRATIC CoT

In this section, we describe how CoT can be enhanced through subquestioning. An illustration of our approach is shown in Figure 3.

#### 3.2.1 Extracting the Reasoning Capability from the Teacher

In Section 3.1, we detailed how an LLM can be used to generate the intermediate annotation of a problem  $P_i$  as a chain of steps leading to the answer  $a_i$ . We now extend this procedure to include a subquestion at each step of the solution. Following a similar procedure as described in Section 3.1, we prompt the LLM with few exemplars of problems decomposed as a set of intermediate subquestion-solution pairs (the prompts are reported in Appendix Table 6). This way, we obtain an intermediate annotation that includes subquestioning. In particular, each of the  $n_i$  steps constituting the overall solution is a subquestion-solution pair, denoted  $q_i^{(j)}, s_i^{(j)}$ ,  $j \in \{1, \dots, n_i\}$  (an example is shown in Figure 2). We refer to the ordered list of subquestion-solution pairs for problem  $P_i$  as  $(q_i^{(1)}, s_i^{(1)}), \dots, (q_i^{(n_i)}, s_i^{(n_i)})$ .

#### 3.2.2 Transferring the Reasoning Capability into the Student

We present **two strategies to distill the reasoning annotation** provided by the LLM into smaller models.

In the **first strategy, a single unified student** is trained to generate the subquestion-solution pairs simultaneously, while in the second strategy, the question generation and question-answering tasks are assigned to two separate models. We call this **second strategy iterative** because the question-answering model is trained to solve each subquestion iteratively.

**Unified.** Using the problems in  $\mathcal{D}$  that contain the chain of intermediate questions and solutions, we train a *unified* student model  $\mathcal{M}_{uni}$  that learns to generate the sequence of subquestion-solution pairs  $\{(q^{(1)}, s^{(1)}), (q^{(2)}, s^{(2)}), \dots\}$  that lead to the

solution of a given problem. We use a pre-trained transformer-based model (Vaswani et al., 2017) and train it on the chain of subquestion-solution pairs for each problem  $P$ . Given a step  $j$  of problem  $P$  (i.e., the concatenation of  $q^{(j)}$  and  $s^{(j)}$ ) consisting of a sequence of  $m_j$  tokens  $\{x_j^{(1)}, \dots, x_j^{(m_j)}\}$ , we use a typical auto-regressive language modeling loss,  $\mathcal{L}$ :

$$\mathcal{L}_j(P) = - \sum_{k=1}^{m_j} \log \mathbb{P}_{uni}(x_j^{(k)} | x_j^{:(k-1)}, P) \quad (1)$$

where  $\mathbb{P}_{uni}(x|c)$  is the probability assigned by  $\mathcal{M}_{uni}$  to token  $x$  given context  $c$ , and  $x^{:(y)}$  indicates the sequence  $\{x^{(1)}, \dots, x^{(y)}\}$ . The loss  $\mathcal{L}_j$  is computed for each problem  $P_i$  and for each pair  $(q^{(j)}, s^{(j)})$  leading to the final answer  $a_i$ .

**Iterative.** The *iterative* version of the student separates the tasks of generating the subquestions and providing an intermediate answer to each subquestion into two distinct models: a question generation (QG) model and a question answering (QA) model. Both the QG and QA models are implemented using a Transformer-based language model (Vaswani et al., 2017). In particular, the QA model  $\mathcal{M}_{qa}$  is iteratively trained to answer the teacher-generated sub-questions. The learning objective is computed at the token level for each intermediate solution:

$$\mathcal{L}(P, s^{(j)}) = - \sum_{k=1}^{l_j} \log \mathbb{P}_{QA}(y_j^{(k)} | y_j^{:(k-1)}, q^{(j)}, s^{:(j-1)}, P)$$

where  $l_j$  and the  $y_j$ 's represent, respectively, the length and the tokens of the intermediate solution  $s^{(j)}$ .  $s^{:(j-1)}$  consists of the previous solution generated by the QA model iteratively in the past iterations.

Similarly, the QG model is trained to acquire the ability of the teacher model to decompose the problem's main question into a series of sub-steps, each of which corresponds to a subquestion. The loss for this model is analogous to Equation 1, with the only difference being that the intermediate solutions are not considered for the QG model. During training, the previous intermediate solutions generated by the QA model are replaced with the teacher-generated solutions using teacher forcing (Cho et al., 2014). However, the intermediate solutions generated by the model are used at inference time.

- In SOCRATIC CoT - instead of just using rationale to annotate a problem, here we decompose a problem into multiple subquestions and annotate the subquestions. all these pairs of subquestions and solutions are used to arrive at the final solution. few shots of such decomposed and answered examples are used to prompt the LLM this is exactly similar to least to most prompting



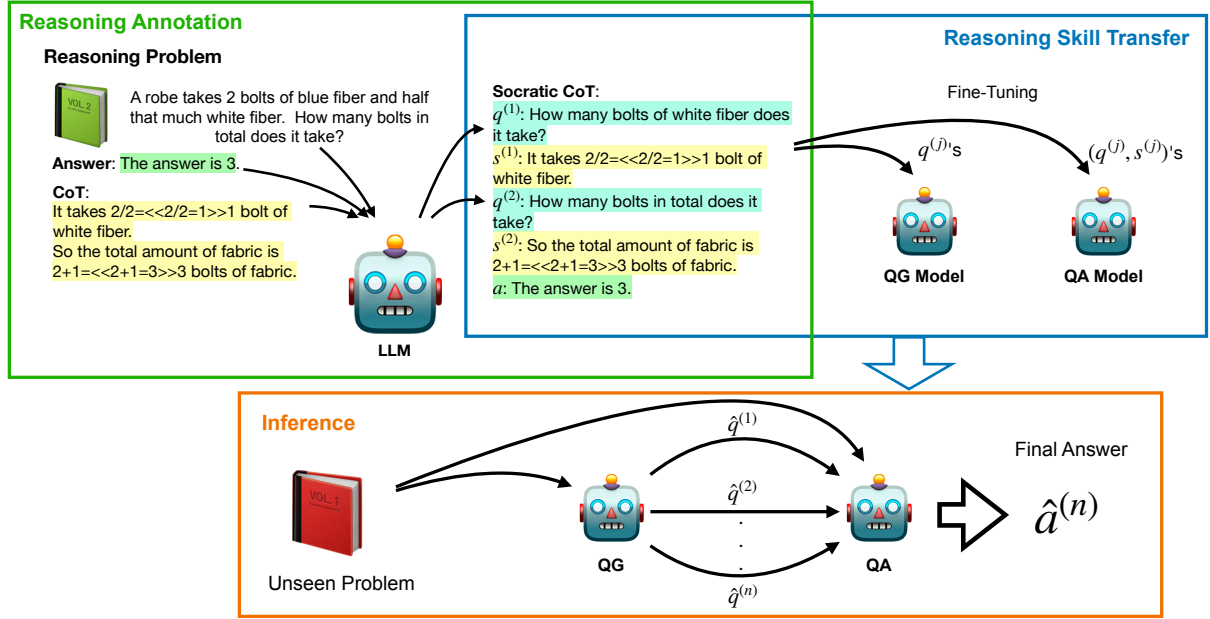


Figure 3: Detailed explanation of our framework. First, a LLM is prompted to decompose the input problem  $P$  into a series of subquestion-solution pairs  $(q_i^{(j)}, s_i^{(j)})$ ,  $j \in \{1, \dots, n_i\}$  with an answer at each step  $a_i^{(j)}$ . The generated subquestions-solutions are used to train two student models: a) the QG model which learns to mimic sub questioning capability of the LLM and b) the QA model, which learns to solve each subquestion. At the bottom, the inference process is depicted for an unseen problem and no LLM is involved. The QG model breaks the unseen problem into simpler subquestions and the QA model solves each one of them eventually leading to the final answer  $a_i^{(n_i)}$ .

### 3.3 Inference-time Predictions

Given an unseen problem  $P$ , the unified student model can directly predict a solution as a sequence of subquestions and answers. In the iterative approach, we first generate the subquestions conditioning the generation of the QG model on  $P$ . After these questions are generated, they are provided to the QA model one by one, decoding the intermediate solution  $\hat{s}^{(j)}$  at step  $j$  token by token according to the model’s probability distribution over its vocabulary:

$$\mathbb{P}_{QA}(y_j^{(k)} | y_j^{(k-1)}, \hat{q}^{(j)}, \hat{s}^{(j-1)}, P), \quad (2)$$

where  $y_j^{(k)}$  is the  $k$ -th token being decoded in greedy fashion.

After the last solution  $\hat{s}^{(n)}$  has been generated, the numerical prediction  $\hat{a}^{(n)}$  is parsed from the text using simple heuristics.

## 4 Empirical Analysis

### 4.1 Datasets

We study how smaller models can learn to reason better on three multi-step reasoning datasets: GSM8K (Cobbe et al., 2021), StrategyQA (Geva

et al., 2021), and SVAMP (Patel et al., 2021). GSM8K consists of 8.5K grade school math word problems, each requiring 2 to 8 steps of reasoning to solve. The solutions primarily involve a sequence of elementary calculations using basic arithmetic operations ( $+$ ,  $-$ ,  $\times$ ,  $\div$ ). The dataset is divided into 7.5K training problems and 1K test problems. To evaluate the model on SVAMP, we train the model on 761 multi-step math word problems taken from the ASDiv (Miao et al., 2020) training set and evaluate it on 237 multi-step SVAMP problems. For StrategyQA, the test set with facts is not available, so we split the data into 80% training, 10% as validation data, and the last 10% as test data. We do not shuffle the data to maintain reproducibility.

### 4.2 Experimental Setup

We use three kinds of annotation, corresponding to the three datasets that we consider.

**Step-by-step solution** : The GSM8K dataset falls into this category and includes a Socratic version where intermediate subquestion-solution pairs are provided for each MWP. While the intermediate step-by-step solutions were manually anno-

Unified	
Input: A robe takes 2 bolts of blue fiber and half that much white fiber. How many bolts in total does it take?	Output: How many bolts of white fiber does it take? It takes $2/2 = \langle\langle 2/2=1 \rangle\rangle$ 1 bolt of white fiber. How many bolts in total does it take? So the total amount of fabric is $2+1 = \langle\langle 2+1=3 \rangle\rangle$ 3 bolts of fabric. The answer is 3.
Iterative	
Iteration 1	
Input: A robe takes 2 bolts of blue fiber and half that much white fiber. How many bolts in total does it take?	Output: <b>QG:</b> How many bolts of white fiber does it take? <b>QA:</b> It takes $2/2 = \langle\langle 2/2=1 \rangle\rangle$ 1 bolt of white fiber.
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Iteration 2	
Input: A robe takes 2 bolts of blue fiber and half that much white fiber. How many bolts in total does it take? How many bolts of white fiber does it take? It takes $2/2 = \langle\langle 2/2=1 \rangle\rangle$ 1 bolt of white fiber.	Output: <b>QG:</b> How many bolts in total does it take? <b>QA:</b> So the total amount of fabric is $2+1 = \langle\langle 2+1=3 \rangle\rangle$ 3 bolts of fabric. The answer is 3.

Table 1: Example demonstraing the input-output format for unified vs iterative setup. QG represents the question generation model and QA is the question answerer mdoel. Note that QA model uses the QG output to answer it as shown in Figure 3.

tated, the authors report that the subquestions were generated by prompting GPT-3. We reproduced a subset of these subquestions using a GPT-3 model with prompts, and we observed a high similarity between the questions provided and the ones generated by us (BERT  $F_1$  score of 95%). For SOCRATIC CoT, we thus use the subquestioning annotation already provided.

**Supporting facts** : We study the StrategyQA dataset, which falls in this category. Strategy QA consists of a factual question with binary True/False as the final answer. Additional supporting facts and decomposed questions are provided. However, the set of facts and the decomposed questions provided with a given question are not always aligned (i.e., a fact is not necessarily the answer to one subquestion). Therefore, having a setup similar to the one for GSM8K is not possible. We thus consider two versions of the data. One in which the supporting facts are used as CoT and the corresponding questions are generated by prompting a GPT-3 model, and a second in which we take the provided questions and generate the facts (this time aligned with the questions) using GPT-3.

**Final answers only** : AsDiv/SVAMP falls in this category and for training, we use GPT-3 to generate both intermediate subquestions and solutions. Intermediate solutions are used as CoT and the generated subquestion-solution pairs for SOCRATIC CoT.

### 4.3 Implementation Details

We use GPT-2 variants (Radford et al., 2019) as student models. GPT-3 175B (Brown et al., 2020) served as the teacher model for decomposing complex problems into a series of simpler substeps (we report the prompts used in Appendix Table 6).

All models were trained using the Huggingface library (Wolf et al., 2020) on an NVIDIA Tesla A100 GPU with 40 GB of memory. Each experiment was run for the same number of iterations to ensure fairness with periodic evaluation over the validation set. Teacher forcing was used during training to replace the generated responses with ground truth answers from the training dataset.

**Evaluation Metric.** To evaluate the question-answering performance on the GSM8K, SVAMP, and StrategyQA datasets, we compute the accuracy based on the final answer provided by the student model.

## 5 Results and Discussion

**Can our framework improve the reasoning capabilities of smaller models?** Table 2 demonstrates that leveraging LLMs reasoning capabilities using our framework can improve the reasoning results for all dataset types.

**Step-by-Step Solution.** When human-annotated step-by-step solutions are available, training smaller models with LLM-generated CoT is not advantageous, as shown on GSM8K. This is to

Dataset	Model	Answer Only	GT Steps	GT Facts	Iterative			Unified
					CoT	Soc <sub>CoT</sub>	Soc <sub>GT</sub>	
GSM8K	Small (124M)	1.45	5.05	-	4.70	5.98	<b>6.44</b> (↑ 20%)	5.10
	Medium (355M)	2.90	7.88	-	7.10	11.57	<b>12.74</b> (↑ 38%)	7.90
	Large (774M)	4.62	14.10	-	12.85	17.89	<b>21.08</b> (↑ 33%)	13.25
	GPT-3 (6B)	-	21.00	-	-	-	-	-
StrategyQA	Medium (355M)	54.10	-	52.02	55.01	52.05	<b>60.31</b> (↑ 13%)	52.05
	Large (774M)	61.10	-	62.80	55.90	61.32	<b>66.40</b> (↑ 5%)	59.45
	XL (1.5B)	60.51	-	<b>66.30</b>	58.07	62.30	63.56 (↓ 4%)	62.05
SVAMP	Small (124M)	2.15	-	-	5.35	<b>6.79</b>	-	5.82
	Medium (355M)	4.80	-	-	17.30	<b>18.99</b>	-	17.62
	Large (774M)	7.40	-	-	<b>23.60</b>	18.14	-	17.45

Table 2: Accuracy comparison (in %) on the three considered datasets. We consider three human-annotated baselines: final answers only (Answer Only), ground-truth step-by-step solution (GT Steps), and supporting facts (GT Facts). We compare the different supervision strategies for fine-tuning the small models: **CoT** represents the case where the chain of intermediate reasoning steps is generated by GPT-3, **Soc<sub>CoT</sub>** represents the case where both the chain of intermediate solutions and the subquestions are generated by LLM and used to fine-tune small models. **Soc<sub>GT</sub>** represents the case where GT solutions/facts are used when prompting GPT-3 to generate the subquestions. Iterative and Unified represent the two **Soc<sub>CoT</sub>** strategies described above. All models are GPT-2 versions and their size is reported within parentheses. All experiments were run at least 3 times and the average is reported. GPT-3 6B results are taken from Cobbe et al. (2021).

be expected since the annotation generated by an LLM is likely to be noisier and of lower quality than human-annotated data. However, the ground-truth step-by-step annotation can be leveraged to prompt an LLM to generate subquestions for the SOCRATIC CoT approach, giving a performance boost of up to 38% when the LLM-generated subquestions are used at inference time. **When the subquestions are learned by the QG model (Iterative Soc<sub>CoT</sub>), the accuracy of the student model decreases slightly but still improves over the step-by-step annotation without subquestions (17.89 vs. 14.10).** Figure 5 shows a comparison of predictions generated by Soc<sub>CoT</sub> models and a model trained on the GT step-by-step annotation. Unified SOCRATIC CoT performs similarly to training with the step-wise ground-truth annotation. We additionally include the score produced by GTP-3 6B to show that training with SOCRATIC CoT can help a small model (GPT-2 large with 774M parameters) perform as well as a nearly 10x larger model fine-tuned with human annotated data.

**Supporting facts.** On StrategyQA, we observe that the inclusion of ground-truth supporting facts in the fine-tuning procedure improves the performance of the small models. However, surprisingly, when the supporting facts are generated by GPT-3, their inclusion actually hurts performance (58.07 vs 60.51 for GPT-2 Large). We hypothesize that

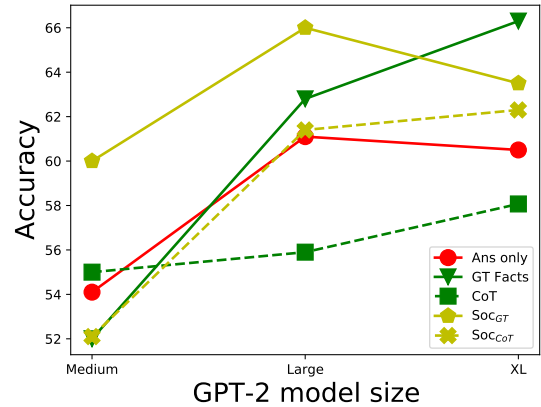


Figure 4: Accuracy comparison for different supervision strategies on StrategyQA. The baseline method consists of fine-tuning on final answers only (Ans only), and it is compared to fine-tuning with: ground-truth supporting facts (GT Facts), GPT-3-generated supporting facts (CoT), ground-truth supporting facts with GPT-3-generated subquestions (**Soc<sub>CoT</sub>**), and LLM-generated facts with human-annotated subquestions (**Soc<sub>GT</sub>**).

this is likely due to the imperfect factual knowledge provided by the LLM, which mars the quality of the supervision. We have observed that the GT supporting facts provided often do not represent a logical sequence of propositions leading to the final answer. This is likely the reason why decomposing the problem through subquestions based on such facts actually harms accuracy (see **Soc<sub>CoT</sub>** column

- this shows that solving a problem by decomposing them into simpler problems is essential rather than just thinking step by step

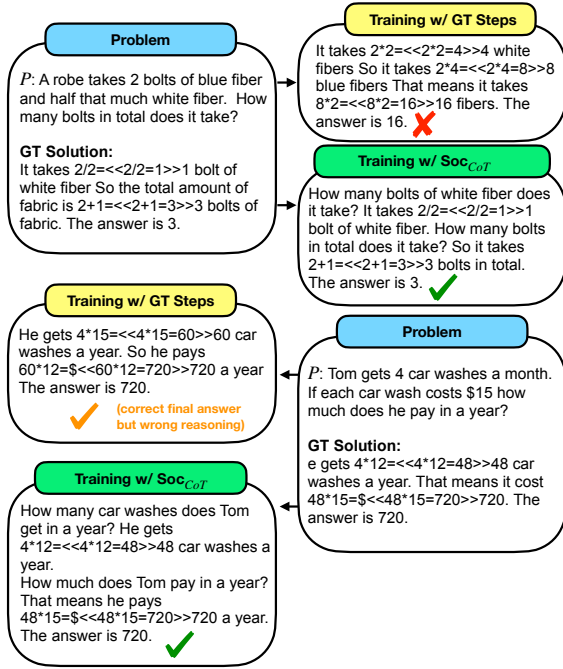


Figure 5: Example of predictions generated by a GPT-2 Large model fine-tuned with GT steps and SOCRATIC CoT on GSM8K dataset.

in Table 2). Instead, using the provided subquestions and using an LLM to generate the answers (representing coherent facts leading to the final answer) proves to be an effective strategy (60.31 vs. 52.02 for GPT-2 Medium). A more detailed comparison between our proposed approaches is presented in Figure 4. However, GPT-2 XL models perform well when trained on facts as unlike smaller models, larger models can encode more facts at once in their parameters, which assists in answering a factual question.

**Answers only.** On the SVAMP dataset, which includes only final answers and no intermediate annotation, LLMs can be used to generate both the intermediate steps and the subquestions. Both the consideration of intermediate solutions without subquestions (**CoT**) and the consideration of intermediate solutions with subquestions (**SocCoT**) lead to an improvement in performance. The trend here is similar to what was observed for StrategyQA, with SOCRATIC CoT being more effective for the two smaller models but falling back to **CoT** for the larger model.

**Can SOCRATIC CoT be used as a prompting strategy?** We experimented with SOCRATIC CoT as a prompting strategy. First, we prompted GPT-3 (175B) to decompose the main problem into

Models	Methodology	Accuracy
GPT-3 (1-shot)	CoT	27.5
(175B)	Sub-ques	47.1 (↑ 41%)

Table 3: Accuracy comparison (in %) of using CoT vs SOCRATIC CoT (Sub-ques) on the GSM8K dataset for GPT-3 model with prompting.

simpler steps by formulating subquestions. Then, GPT-3 is used again to solve the sequence of subproblems in a single-shot setting with a problem decomposed into intermediate subquestions and solutions included in the prompt. The introduction of subquestioning boosts accuracy by over 40% compared to standard CoT prompting (Table 3). Other work (e.g., Wei et al. 2022b) has used a larger number of exemplars in the few-shot prompt, achieving higher overall accuracy. We limited our experiments to single-shot prompts due to budget constraints.

## 6 Ablation Studies

In this Section, we describe additional analyses regarding specific components of the framework we propose, as well as negative results that we obtained with alternative strategies.

**How good are the sub-questioning capabilities of a smaller model?** We investigate in more detail the ability of a small model to decompose a problem by generating meaningful subquestions. We fine-tuned GPT-2 Large on the GPT-3 generated subquestions provided in the GSM8K dataset. We then evaluated the quality of the generated questions in terms of BLEU score (Post, 2018), BERT F<sub>1</sub> score (Zhang et al., 2019), and by measuring for how many problems the number of questions generated by GPT-2 (#Q) matches the number of GPT-3 annotated questions for a given problem.

We found that the fine-tuned GPT-2 predicted an incorrect number of subquestions for the majority of problems (see Table 4, first row). Thus, following previous work on subquestion generation (Shridhar et al., 2022), we introduced a *guidance mechanism* that conditions the generation of subquestions for a problem  $P$  on the equations describing the intermediate solutions of  $P$ . This strategy improved the quality of the generated questions for all three metrics considered (Table 4, second row). To avoid the dependence on the step-by-step annotation of the equations for each problem  $P$



Methodology	BLEU	BERT $F_1$	# Q
No-guidance	51.5	0.78	0.42
Guidance	<b>58.8</b>	<b>0.81</b>	<b>0.80</b>

Table 4: BLEU, BERT  $F_1$  and the number of questions (# Q) comparison between the question generator model and the Socratic subquestions present in the GSM8K dataset using GPT2-large model.

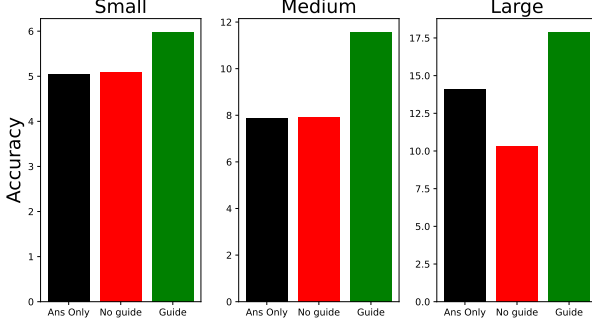


Figure 6: Accuracy of student models (QA + QG) when the question generation is conditioned using the guidance model (Guide) and with non-guided question generation (No guide). Ans only represents the baseline. All models are GPT-2 versions.

at inference time, we train an additional sequence-to-sequence model to predict, given  $P$ , the set of equations that lead to the solution of the problem. At inference time, the predictions for the guidance model are used to condition the generation by the QG model. Although the predicted equations often do not lead to the correct solution of the problem, they help the QG model to generate more meaningful sub-questions. Figure 6 shows the overall accuracy of the GPT-2 student models (QA + QG) fine-tuned with SOCRATIC CoT on the GSM8K data with and without equation conditioning provided by the guide model. We have extended this guidance mechanism to StrategyQA and SVAMP, where the generation of subquestions is conditioned on the number of facts (StrategyQA) or steps (SVAMP) needed to answer the problem.

### Eliminating the need for a subquestion module.

We have experimented with an alternative training solution that does not involve a question-generation model. This strategy aims to improve the supervision for fine-tuning a small model through subquestioning, but without relying on the presence of subquestions at test time. The procedure consists of training the student model to generate the entire chain of steps leading to an intermediate answer. That is, when the sub-question

GPT-2	No SubQ	SubQ with QG
Small	2.70	<b>5.98</b>
Medium	7.20	<b>11.57</b>
Large	8.18	<b>17.89</b>

Table 5: Accuracy comparison (in %) of student models trained with (SubQ with QG) and without (No SubQ) question generation model on GSM8K.

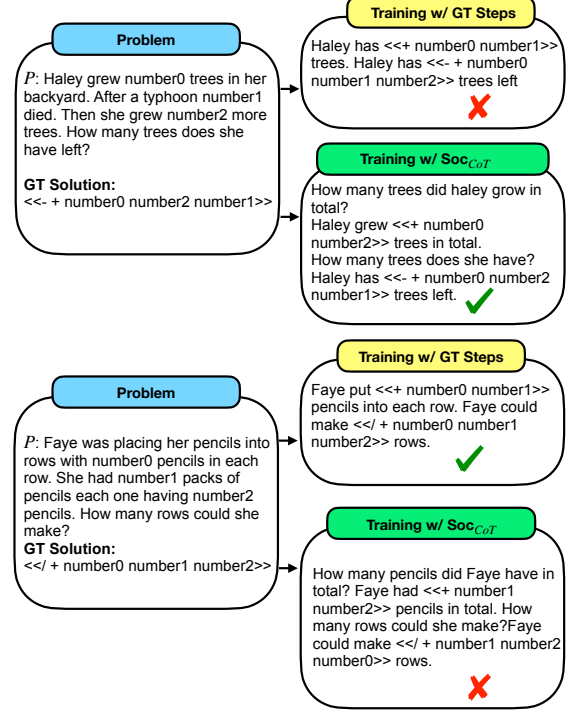


Figure 7: Example of predictions generated by a GPT-2 Medium model fine-tuned with GT steps and SOCRATIC CoT on the SVAMP dataset.

$q^{(1)}$  is asked, the model is trained to generate the answer  $s^{(1)}$ , but when  $q^{(j)}$  is asked, the model is trained to generate the chain of thought reasoning  $\{s^{(1)}, s^{(2)}, \dots, s^{(j)}\}$  (instead of just  $s^{(j)}$ ). This eliminates the need for the intermediate subquestions at inference time, as the model is trained to *implicitly* decompose the main problem into smaller reasoning steps. However, this method leads to significant performance degradation (results are reported in Table 5), highlighting the need for subquestions at inference time.

**Example outputs** In Figures 5 and 7, we report example outputs predicted by GPT-2 models for a set of GSM8K and SVAMP problems.

## 7 Conclusion

The chain-of-thought style of step-by-step reasoning has proven to be very effective for reasoning

in LLMs. In this work, we propose ways to distill these reasoning capabilities into smaller models and suggest ways to further improve them by explicitly asking stepwise questions. We demonstrate the effectiveness of our proposed methodology on three popular multi-step reasoning datasets, and discuss cases where one method should be preferred over the other for different datasets.

## Limitations

In our work, we use only one solution from the LLM to distill information into the student model, and according to Wang et al. (2022), multiple subquestion-solution pairs can be sampled, and using majority voting, all pairs leading to the most frequent answer can be used to distill knowledge into the student models. Also, due to computational budget, we used a single prompt to compare the CoT and SOCRATIC CoT and using more prompts (up to 8) might lead to a fairer comparison and better results (Wei et al., 2022b). We leave these experiments for the future.

## Ethical Considerations

Although this work improves the reasoning capabilities of smaller models, the models are still not powerful enough to be used in sensitive settings such as education. We plan to release our code and model checkpoints, but the models must be used carefully by users, as many generative models, including ours, are prone to hallucination.

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Let's generate sub-questions for these problems. Use exactly one operation per step.

—  
Q: Zoe was unboxing some of her old winter clothes . She found number0 boxes of clothing and inside each box there were number1 scarves and number2 mittens . How many pieces of winter clothing did Zoe have total ?

SQ1: How many pieces of winter clothing did Zoe have in each box?

A1: Zoe had  $\ll + \text{number1} \text{ number2} \gg$  pieces of winter clothing in each box.

SQ2: How many pieces of winter clothing did Zoe have total ?

A2: Zoe had  $\ll * \text{number0} + \text{number1} \text{ number2} \gg$  pieces of winter clothing in total.

—  
Q: Katie picked number0 tulips and number1 roses to make flower bouquets . If she only used number2 of the flowers though , how many extra flowers did Katie pick ?

SQ1: How many flowers did Katie pick in total?

A1: Katie picked  $\ll + \text{number0} \text{ number1} \gg$  flowers in total.

SQ2: How many extra flowers did Katie pick ?

A2: Katie picked  $\ll - + \text{number0} \text{ number1} \text{ number2} \gg$  extra flowers.

—  
Q: Conner has number0 dollars in his bank account . Every month he spends number1 dollars . He does not add money to the account . How much money will Conner have in his account after number2 months ?,

SQ1: How much money does Conner spend in total? A1: Conner spends  $\ll * \text{number1} \text{ number2} \gg$  dollars.

SQ2: How much money will Conner have in his account after 8.0 months ? A2: After 8.0 months, Conner will have  $\ll - \text{number0} * \text{number1} \text{ number2} \gg$  dollars.

---

For each of the following topics, generate intermediate answers to the subquestions leading to the final answer.

—  
Topic: Albany, Georgia (City in Georgia, United States)

Will the Albany in Georgia reach a hundred thousand occupants before the one in New York?

Albany, GA has around 75,000 people.

Albany, NY has almost 100,000 people.

The difference is  $100,000 - 75,000 = 25,000$

The difference is  $100,000 - 100,000 = 0$

No, 25,000 is not smaller than 0.

The final answer is NO.

—  
Topic: The Police (English rock band)

Could the members of The Police perform lawful arrests?

Only law enforcement officers can perform lawful arrests.

No, the members of The Police (rock band) are not law enforcement officers.

The final answer is NO.

—  
Topic: Wonder Woman (2017 film) (American superhero film directed by Patty Jenkins) Is a Boeing 737 cost covered by Wonder Woman (2017 film) box office receipts?

The average cost of a US Boeing 737 plane is 1.6 million dollars.

Wonder Woman (2017 film) grossed over 800 million dollars at the box office.

Yes, 800 is larger than 1.6.

The final answer is YES.

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Table 6: Exemplars included in the few-shot prompt for the decomposition of the problems from the ASDiv (upper row) and StrategyQA (lower row) datasets.