Phasor Measurement using Kalman filtering

Praveen Pankajakshan

Abstract—**This paper essentially focuses on the design and implementation of a Kalman filter for digital distance protection scheme. A brief review of voltage and current Kalman filters is followed by sensitivity analysis due to assumption of incorrect model parameters. The dependency of these parameters on the signal characteristics has been tabulated. We assume a rotating phasor model for the voltage and current. Sample designs are also presented for voltage and current estimation during transients. The design of the basic Kalman filter has been extended to include decaying dc and harmonic frequency components.**

***Index Terms*—Fault location, Kalman filter, parameter estimation, power system transients, state equation.**

I. Introduction

POWER system protection using digital processors has been an active area of research and development for many years. When a fault occurs on a three-phase transmission line, it is desirable first to sense the problem and then take the appropriate relaying action to protect the remainder of the system. The protection systems that are currently available rely on the use of advanced digital processing of the transient signals accompanying the fault to arrive at a trip decision.

This paper focuses on the application of one such technique, the Kalman filtering method in Power System relaying. New applications of Kalman filtering keep appearing regularly, and many of these are now outside the original application area of control and navigation. The important classes of theoretical and statistical problems in communication and control that overlap with those experienced in power systems are:

1. Prediction of random signals
2. Separation of these random signals from random noise
3. Detection of signals of known form (like pulse, sinusoids, etc.) in the presence of random noise.

When a fault occurs in a power transmission line, it is necessary to estimate the fundamental frequency components of the steady-state post fault currents and voltages from the corrupted voltage and current signals. To achieve this, the transient components are superimposed on the steady-state signals immediately after the fault, so that the transients become the corrupting noise in the problem. So the basic problem is to estimate the steady-state components of the sending-end voltages and currents in the presence of transient components. For distance relaying, these components are used to determine the apparent impedance to the fault and hence the fault location.

Some of the algorithms that have been proposed in literature are essentially non-recursive digital filters. These filters use finite length windows, and their outputs essentially depend on the data contained within this window. The short window technique computes the voltage and current phasors from sampled data and then estimates the impedance as seen from the relay locations. The Kalman filter on the other hand is a recursive filter whose output depends on the present inputs as well as on all previous inputs. The Kalman filter differs from these filtering algorithms used in Power systems in that its gain coefficients vary with time.

This paper consists of five sections. In section II, the state space model for the voltage and current phasor is developed along with the output equations. The dependency of the Kalman filter parameters on the signal type is also tabulated. Section III contains the implementation aspects of the algorithm with an example illustration to highlight this. The appendix lists the various notation conventions used and their associations.

II. State Space Model of Signal as Phasors

One of the functions of digital relays for protection of power systems is to estimate voltage and current phasors from sampled data. The samples contain information, noise, and measurement errors. A filtering technique is therefore required to condition the data before estimating the phasors.

A. *Signal representation as Phasors*

The noise-free current or voltage signal can be expressed by the real or imaginary part of

$$s(t) = A\, e^{(j\omega_o t+\varphi)} \tag{1}$$

where A is the amplitude and Φ is the phase angle at t = 0. Consider a Rayleigh distribution for the amplitude and a uniform distribution for the phase angle, the noise-free current or voltage following the fault occurrence is expressible as [2]:

$$s(t) = C_1 s_r + C_2 s_i \tag{2}$$

where s_r and s_i are statistically independent, zero mean, Gaussian random variables. The variance of s_1 or s_2 is designated as σ_v^2 for the voltage signals and σ_i^2 for the current signals.

B. *State Space Model*

The Kalman filtering is applied to a state space model of the system. The model includes state transition and output equations. If the signal is sampled with a frequency of F_s then, the time interval between one sample and the other is ΔT the

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estimation of the voltage at any time (n+1) ΔT in terms of the phasors at time nΔT are as follows:

$$\begin{bmatrix} x_r(n+1) \\ x_i(n+1) \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} x_r(n) \\ x_i(n) \end{bmatrix} + \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \begin{bmatrix} \Delta x_r(n) \\ \Delta x_i(n) \end{bmatrix} \quad (3)$$

The state transition equation can also be written in vector notation as:

$$[x(n+1)] = [P][x(n)] + [Q][\Delta x(n)] \quad (4)$$

The sampled value of the signal is related to the signal phasor. This relation is called the output equation and is as follows:

$$z_s(n) = [C_{11} \quad C_{12}] \begin{bmatrix} x_r(n) \\ x_i(n) \end{bmatrix} + b(n) \quad (5)$$

$$[z_s(n)] = [C][x(n)] + [b(n)] \quad (6)$$

The definitions of the terms used and their implications are explained in the Appendix at the end of this paper.

The measurement noise b(n) and the change in voltage Δv due to varying loads are included in the Kalman filter using principles of statistics. They can be considered as a white sequence with an exponentially decreasing variance [2]. It is also assumed that there does not exist any cross relationship between the two. This leads us to a covariance matrix having only diagonal terms. Since these are random variables, and they obey Gaussian distribution, it is possible to estimate reasonable values of the elements of the covariance matrices. The diagonal elements of these matrices are the variances of the matrix. The current noise signal on the other hand would appear as a random exponential process plus a white noise sequence with a decreasing variance. The value of the initial variance was found nearly equal to the mean square of the sending end current.

C. Recursive Kalman filter

The design of the Kalman filter is based on the statistical properties of the signal that is to be processed. The time varying filter coefficients are calculated to minimize the square of the expected errors between the values of the actual and estimated system states. The Kalman gain can be calculated by recursively solving equations (7), (8) and (9).

$$[K(n)] = [M(n)][C]^T \left[[C][M(n)][C]^T + [B]\right]^{-1} \quad (7)$$

Computing the state estimation error covariance:

$$[Z(n)] = \left[[I] - [K(n)][C]\right][M(n)] \quad (8)$$

Now projecting the single step transition error covariance:

$$[M(n+1)] = [P][Z(n)][P]^T + [Q][U][Q]^T \quad (9)$$

Equation (7), (8) and (9) do not include any system measurements. Therefore, the Kalman gains can be calculated in advance and stored for use during the real time calculations. This procedure reduces real-time computations.

D. Filter Implementation

The Kalman filter equation, which estimates new values of the state variables, is as follows:

$$\left[\hat{x}(n)\right] = [P]\left[\hat{x}(n-1)\right] + [K(n)] \left[z_s(n) - [C][P]\left[\hat{x}(n-1)\right]\right] \quad (10)$$

The symbol ^ indicates that the signal is the calculated estimate. The implementation equation is given as:

$$\begin{bmatrix} \hat{x}_r(n) \\ \hat{x}_i(n) \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_r(n-1) \\ \hat{x}_i(n-1) \end{bmatrix} + \begin{bmatrix} K_{11}(n) \\ K_{21}(n) \end{bmatrix} \left[x_s(n) - [C_{11} \quad C_{12}] \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_r(n-1) \\ \hat{x}_i(n-1) \end{bmatrix} \right] \quad (11)$$

Since the state transition matrix and the driving function matrix perform the same operation, namely rotating the phasors through an angle corresponding to one sampling interval, they are of the form:

$$[P] = [Q] = \begin{bmatrix} \cos(\omega\Delta T) & -\sin(\omega\Delta T) \\ \sin(\omega\Delta T) & \cos(\omega\Delta T) \end{bmatrix}$$

The estimate to the state vector is obtained by minimizing the difference between the measured and estimated data.

III. DC AND HARMONIC COMPONENT MODELS

In reality, the power system signal is composed of not only the fundamental component but also a decaying dc component, components of harmonic and non-harmonic frequencies, and noise. Each frequency component can be considered as a phasor composed of a real and imaginary component, which should be included among the system states. As the number of states increases the Kalman parameters gets augmented accordingly. Decaying dc terms can be modeled as a single state in which the values of the state are reduced to $e^{-\lambda\Delta T}$ times the previous value. Three specific models are introduced in this section.

A. Three state model

To include a dc term decaying according to $e^{-\lambda\Delta T}$, the P and Q matrix should be augmented as:

[P]=[Q]	$\begin{bmatrix} \cos(\omega\Delta T) & -\sin(\omega\Delta T) & 0 \\ \sin(\omega\Delta T) & \cos(\omega\Delta T) & 0 \\ 0 & 0 & e^{-\lambda\Delta T} \end{bmatrix}$
[C]	[1 0 1]
[ΔI(n)]	$[\Delta I_r(n)\ \Delta I_i(n)\ \Delta I_{dc}(n)]$
[U]	$\begin{bmatrix} U & 0 \\ 0 & U_{33} \end{bmatrix}$ U_{33} is the variance of the unexpected changes in the decaying dc component
[M(n)]	Initial covariance matrix includes an additional term.

B. Four state model

This section discusses the inclusion of a second frequency in the model. Assume that the pre fault currents are of the fundamental

[P]=[Q]	$[P]=[Q]=\begin{bmatrix} \cos(\omega\Delta T) & -\sin(\omega\Delta T) & 0 & 0 \\ \sin(\omega\Delta T) & \cos(\omega\Delta T) & 0 & 0 \\ 0 & 0 & \cos(2\omega\Delta T) & -\sin(2\omega\Delta T) \\ 0 & 0 & \sin(2\omega\Delta T) & \cos(2\omega\Delta T) \end{bmatrix}$
[C]	[1 0 1 0]
[ΔI(n)]	$[\Delta I_{1r}(n)\ \Delta I_{1i}(n)\ \Delta I_{2r}(n)\ \Delta I_{2i}(n)]$
[U]	$\begin{bmatrix} U & 0 & 0 \\ 0 & U_{33} & 0 \\ 0 & 0 & U_{44} \end{bmatrix}$ U_{33} and U_{44} is the variance of the unexpected changes in the in-phase and quadrature components
[M(n)]	Initial covariance matrix includes two additional term.

C. Higher state model

The highest frequency of a signal that can be recovered from a sampled signal should be at least one half of the sampling frequency. The anti-aliasing filter removes components of higher frequencies substantially lesser than the sampling frequency. The filter model considered in this paper assumes a 11 state model. The ten states correspond to the in-phase and quadrature components of the fundamental and the higher order frequencies. The last state is the decaying dc component.

IV. DISTANCE PROTECTION SCHEME

The Kalman filter models described in the last section are used in a digital distance protection scheme. It is considered that seven continuous, three line currents, residual current, and three phase voltages, are supplied by current and voltage transducers to the scheme. It is assumed that the seven analog signals are sampled simultaneously, held and converted into digital signals by seven independent channels of analog-to-digital converters. Upon the completion of the digitizing process, the fault processing program is initiated. This program consists of four major stages namely, fault detection, fault type classification and optimal estimation of the voltage and current phasors, fault zone computation, and fault location calculations.

A. Fault Detection and Classification

The samples of the seven quantities for two cycles are stored in the memory. The fault detection algorithm works by comparing the latest samples of the line currents with corresponding cycle stored in memory. A change of 0.05 p.u. corresponds to an abnormal condition. Upon detecting a disturbance, the largest of the three compensated fault currents is used greater than K_1, this particular phase is faulted. On the other hand, if this ratio is less than K_2, this particular phase is unfaulted. A value between 0.7 and 0.85 for K_1 and between 0.25 and 0.4 for K2 were found to be reasonable. Also for secure classification of the fault, the fault has to be classified as the same type for three consecutive computational steps.

As soon as the fault is classified, the algorithm selects the appropriate voltage and current pairs to compute the apparent impedance.

B. Zone Computation

The apparent impedance as seen from relay location is calculated depending on the type of fault.

For single line to ground fault (say Phase A to ground), the apparent impedance is simply the ratio of selected voltage to the selected current [2]. Representing the selected voltage and selected current pair as complex quantities in the form:

$$V_{selected} = V_1 + jV_2$$

$$I_{selected} = I_{s1} + jI_{s2} = IA + KI_o$$

I_o is the zero sequence current that contains both the in-phase and quadrature component of the zero sequence current. The apparent impedance may then be computed as:

$$Z_{app} = R_{app} + jX_{app} = \frac{(V_1 + jV_2)}{(I_{s1} + jI_{s2})}$$

For line to line fault (say Phase A and Phase B), the selected voltage is the line to line voltage of the faulted phases and the selected current is the difference between the currents of the faulted phases. To correct for the fault resistance, the current in the fault is assumed to be proportional to the difference between the compensated currents of the faulted phases at the relay location. Therefore, for a line to line fault between phases A and B, the apparent impedance is first computed by equation () using the following selected voltage and current:

$$V_{selected} = V_A - V_B$$

$$I_{selected} = I_A - I_B$$

Then, to compute the fault location and correct for the fault resistance the expression for the apparent impedance is modified to be:

$$Z_{app} = R_{app} + jX_{app} = (R_1 + jX_1)x + \frac{R_F K_F (I_d + jI_q)}{(I_{s1} + jI_{s2})}$$

Equation () is used to calculate the fault location for line to line faults.

V. FILTER IMPLEMENTATION

The developed system is tested for some typical Kalman parameters and then modifications were made to this to check the sensitivity. We assume that the input signal has a fundamental frequency of 60 Hz and the sampling frequency used is around 720Hz.

A. State transition and driving function matrix

From Appendix B it is clear that the state transition matrix rotates the phasor by an angle 30^o corresponding to one sample interval. Is:

$$[P]=[Q]=\begin{bmatrix} 0.8660 & -0.5000 \\ 0.5000 & 0.8660 \end{bmatrix}$$

B. Noise and voltage covariance matrix

As explained in Appendix B, the covariance matrix for noise and voltage can be written as:

$$[B]=\begin{bmatrix} \sigma_n^2 & 0 \\ 0 & \sigma_n^2 \end{bmatrix}, \ \sigma_n = 0.01 \text{ p.u}$$

$$[U]=\begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}, \ \sigma_v = 0.005 \text{ p.u}$$

C. Single step transition errors covariance matrix

The matrix [M(n)] is composed of the variance of the single step prediction errors. These terms are estimates of the errors due to the propagation of incorrect estimate of the states. The inverse of the initial covariance matrix reflects the degree of knowledge of how close the initial estimate is to the unknown states to be estimated. The matrix is initialized to the covariance matrix of the transitions from the prefault to the postfault states. As the Kalman gain equations are recursively solved, the numerical values of the diagonal terms converge to the steady state values.

In the absence of further knowledge, a reasonable estimate for the ration of the mean post fault magnitude to the pre fault magnitude would be 0.7. As well, the standard deviation of the transition may be assumed to be 0.24 of the pre fault value, so the estimate for the covariance matrix [M(1)] would be:

$$M(1)=\begin{bmatrix} 0.24^2 & 0 \\ 0 & 0.24^2 \end{bmatrix}$$

VI. SENSITIVITY STUDIES

The sensitivity test is carried out by varying the various model parameters and noting the effects of these on the estimated value. In the two-state Kalman filter, the initial covariance, the initial value of the noise variance and the noise variance rate of decrease were changed by more than ±50 percent. The magnitude of the voltage estimate was obtained using the above different parameters and compared with the exact value. In addition, the effect of deleting the third state on the three-state Kalman filter was examined. Finally changing the sampling rate from 8 samples per cycle to 64 samples per cycle was tested. As can be seen from Fig. 1 and 2, the three-state Kalman filter model is not sensitive to ±50 percent changes in the noise variance parameters, or, ±50 50 percent change in the initial covariance matrix.

Deleting the third state in the three-state Kalman filter model reduces the rate of convergence and increased the steady state error as shown in Fig. 3.

In the three-state Kalman filter, changing the sampling rate

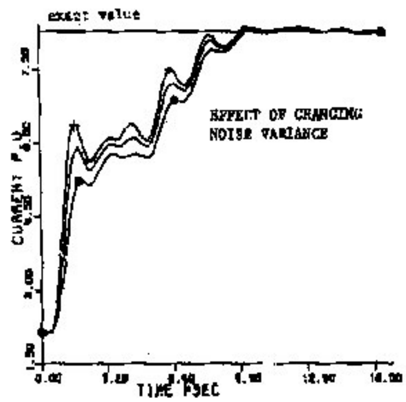

Fig. 1. Effect of changing the initial variance of the current noise in estimating the magnitude of the current.

from 64 to 32 samples per cycle did not change the estimated values appreciably. However a change was noticed at a

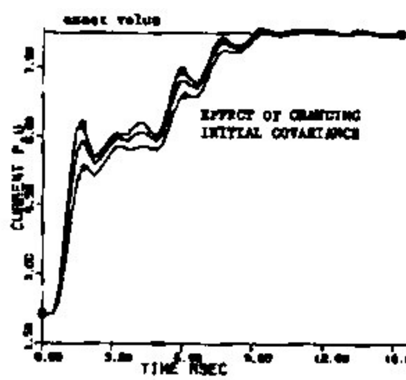

Fig. 2. Effect of changing the initial covariance matrix in estimating the magnitude of the current.

sampling rate of 8 samples per cycle as shown in Fig. 5.

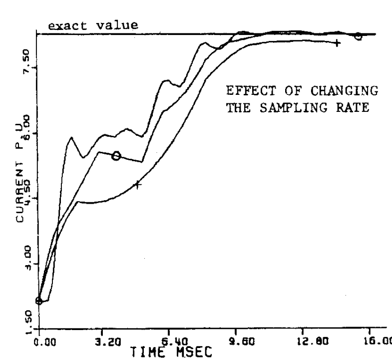

Fig. 3. Effect of changing the sampling rate in estimating the magnitude of the current.

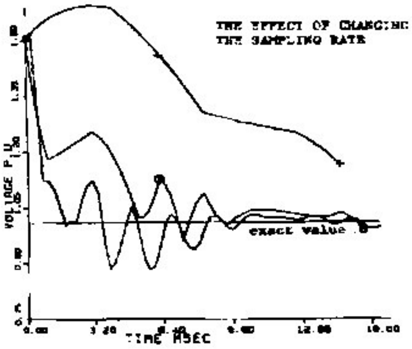

Fig. 3. Effect of changing the sampling rate in estimating the magnitude of the voltage.

VII. Conclusion

A method to estimate the voltage and current phasors from the sampled signal based on Kalman filter was developed. Estimation of the post-fault voltage and currents was used for fault identification and classification. The Kalman filter is also suitable for estimating several variables other than voltage or current. A method for including decaying dc and harmonic components in the filter was described. For a sinusoidal input, a two state model reaches the correct estimates within two sample periods. However, this model does not provide acceptable results if non-fundamental frequency components are present in the input. The Kalman filter theory requires that the [Δv(n)] and [b(n)] components are white sequence. For inputs that satisfy the assumption, the Kalman gain will give the optimal result. But in real systems the noise are band limited and are therefore not perfectly white. However the Kalman filter can still be applied but the design will not be optimal. Also elements of the system which are present, but have not been modeled may seriously affect the accuracy of the output. All anticipated states should therefore be considered for inclusion in the system model.

Appendix A

1. *Index to matrices and variables*

The terms used in the state transition equations (3) and (4) are defined as follows:

[x(n)]	State vector composed of the real and imaginary components of the signal phasor at the instant nΔT
$x_r(n)$	Real component of signal
$x_i(n)$	Imaginary component of signal phasor.
[P]	State transition matrix
[Q]	Driving function matrix that advances the changes [Δx(n)] by one sampling interval

The notations used in the output equations (5)-(6) and the Kalman gain calculations (7)-(9) are:

$z_s(n)$	Measured signal at the time instant nΔT
[C]	Defines the relationship between instantaneous sampled voltage and the phasor representation.
b(n)	Measurement noise modeled as a random variable.
[B]	Covariance of the noise model
[U]	The covariance matrix for [Δx(n)] inputs.
[K(n)]	Kalman gain vector.
[Z(n)]	State estimation error covariance.
[M(n)]	Single step transition errors covariance matrix.
[I]	Identity matrix

2. *Covariance matrix*

The covariance matrix [D] of a vector [d] = $[d11\ d21]^T$ is defined as:

$$[\mathrm{D}] = \begin{bmatrix} E\{\mathrm{d}_{11}^2\} & E\{\mathrm{d}_{11}\ \mathrm{d}_{21}\} \\ E\{\mathrm{d}_{21}\ \mathrm{d}_{11}\} & E\{\mathrm{d}_{21}^2\} \end{bmatrix}$$

where, *E* denotes the expectation operator, and if the expected value $E\{\mathrm{d}_{11}\ \mathrm{d}_{12}\} = 0$, then d_{11} and d_{12} are orthogonal to each other (uncorrelated).

Appendix B

1. *Signal characteristics:*

Fundamental frequency (f)	60 Hz
Sampling frequency (F_s)	720 Hz
Highest frequency (f_h)	< 360 Hz
Time interval between samples (ΔT)	≈1.389 ms
Variation of voltage/current to loads (σ_v)	0.005 p.u.

The voltage variation is statistically considered to be white sequence, uncorrelated, with zero mean and Gaussian distribution.

2. *Noise characteristics:*

We assume a very similar model as the voltage variation for the measurement noise. These may be due to random noise, errors due to transducer non-linearity, truncations and other considerations. The standard deviation for the noise is (σ_n) 0.01 p.u.

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