Summary of the application of Kalman filtering for Phasor Measurement

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Abstract—**This report essentially focuses on the results of the implementation of a recursive Kalman filter for signal amplitude and phase measurement. The recursive Kalman filter block was developed in *MATLAB 5.3* using the *Simulink 3.0* toolbox. The M Programming Language was used to develop the *kfamp* and *KalmanParameters* S-functions, which form the fundamental building blocks for the recursive Kalman filter. The sensitivity of this algorithm on various parameters was compared with the Fourier method, and the results are presented. The software along with its library serves as a tool for researchers and students to experiment on. The design of the basic Kalman filter has also been extended to include decaying dc components.**

Index Terms—**Kalman filter, steady state, DC offset, harmonics, frequency response.**

I. Introduction

THIS report is a supplement of the one submitted on completion of Phase I of this project. In the Phase II, a 'Recursive Kalman Filter' block was developed using the MATLAB environment and the Simulink toolbox. This software works on all MATLAB versions higher than 5.3. The Simulink block is described by S-functions, *kfamp* and *KalmanParameters* developed in MATLAB.

When a fault occurs in a power transmission line, it is necessary to estimate the fundamental frequency components of the steady-state post fault currents and voltages from the corrupted voltage and current signals. To achieve this, the transient components are superimposed on the steady-state signals immediately after the fault, so that the transients become the corrupting noise in the problem. So the basic problem is to estimate the steady-state components of the sending-end voltages and currents in the presence of transient components.

The signal to be measured is supplied to the data acquisition board, which converts the analog signal into the digital signal. The digital signal (Z_s) is input to the recursive Kalman filter block. The output of the block is a real number giving the estimated amplitude and phase of the fundamental signal (60 Hz in the case of the voltage signal).

The signal may contain odd or even harmonics and dc offset that distort the signal. Thus it is not always simple to estimate the amplitude of the fundamental. We thus observe the usefulness of the Kalman filter in the presence of harmonics, dc offset, and even measurement noise. The measurement noise may be a random white noise sequence that occurs because of analog to digital converters or other sensing devices connected to the system.

Some of the algorithms that have been proposed in literature are essentially non-recursive digital filters. These filters use finite length windows, and their outputs essentially depend on the data contained within this window. The short window technique computes the voltage and current phasors from sampled data and then estimates the impedance as seen from the relay locations. The Kalman filter on the other hand is a recursive filter whose output depends on the present inputs as well as on all previous inputs. The Kalman filter differs from these filtering algorithms used in Power systems in that its gain coefficients vary with time. The advantages of this method over the Fourier method for Phasor measurement is shown in Section V.

This paper consists of six sections. In section II, the various components used in our block and test setup is described. In Section III we describe the algorithm, and in Section IV, the test setup and results are discussed. In Section V these results are compared with the Fourier method for changes in sampling rate and cut-off frequency. Section VI summarizes the outcome of the study and experimentation. The appendix lists the various notations and conventions used, and their associations.

II. Component Description

The various components used in the library and the test examples are described in detail in this section.

A. Signal Generator

1) Description:

The signal generator block is used to construct voltage or current signals of the desired frequency and amplitude.

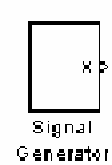

Fig. 1. Recursive Kalman Filter Block.

2) Inputs

The input dialog box is used for entering the amplitude and the frequency of the signals. For example, if the signal to be

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generated is a 60 Hz signal of amplitude 1 Volt, with third harmonic (3x60 Hz) of amplitude 0.5V and fifth harmonic (5x60Hz) of amplitude 0.3V, starting at time 0 seconds, then the input to the dialog box will be as shown in figure 2.

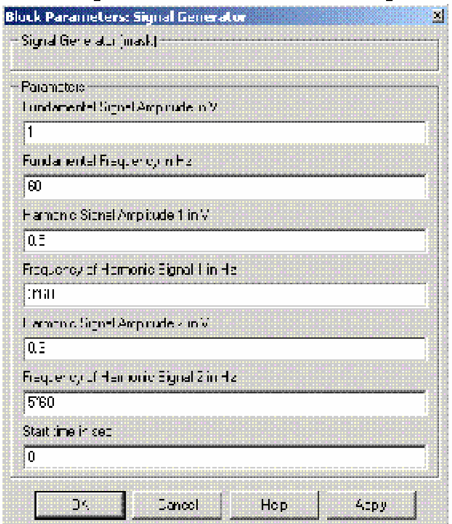

Fig. 2. Dialog Box for Signal Generator Block

B. *Data Acquisition Board (DAB) Block*

1) *Description*

The dab block simulates a data acquisition board and includes an anti-aliasing butterworth filter, vertical resolution tuner, amplifier, and an A/D converter.

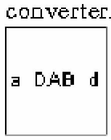

Fig. 3. Data Acquisition Board Block

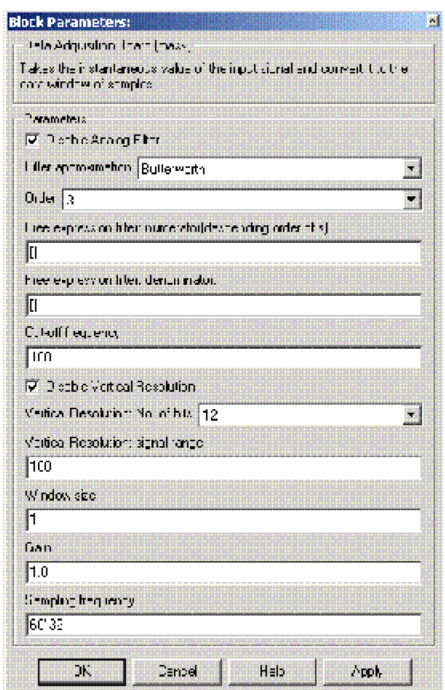

Fig. 4. DAB input dialog box.

2) *Inputs*

The dialog box is used for entering the order, coefficients, and the cutoff frequency for the butterworth filter. The vertical resolution of the signal can be 8, 12, or 16 bit. But, the sampling frequency of the A/D converter should be atleast 2 times higher than the highest frequency of the signal and its harmonics. In the example given, the butterworth filter is disabled, the vertical resolution is 16 bit, gain is 1.0, and the sampling frequency is 32 times the fundamental frequency (if the fundamental is the dominant signal).

C. *Recursive Kalman Filter Block*

1) *Description*

The recursive Kalman filter block is essentially used for estimating the amplitude of the fundamental signal. It also has Phase and DC offset estimation as additional features.

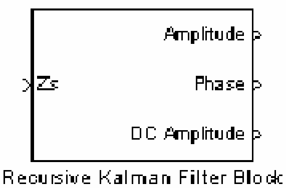

Fig. 5. Recursive Kalman Filter Block.

2) *Inputs*

The dialog box is used for entering the parameters for the Kalman filter.

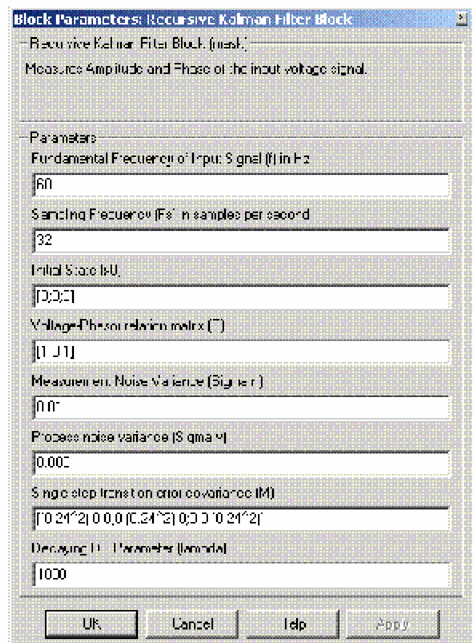

Fig. 6. Dialog Box for recursive Kalman filter Block

The various parameters used and their significance is tabulated in table 5 and table 6 of Appendix A. If the DC offset is negligible, the DC parameter (?) should have a very low value (? <0.1). A high variance would give the best result in the case of a signal with DC offset. Similar, rules apply for process noise and measurement noise.

III. ALGORITHM DESCRIPTION

A three state model was used as described in [1] for the implementation of the recursive Kalman filter. The three state model assumes a fundamental dominant signal and some DC offset.

1) Recursive Kalman filterBlock

The algorithm for the recursive Kalman filter is as shown in the flowchart of figure 7.

The recursive Kalman filter block invokes two S-functions, *KalmanParameters* and *kfamp*. The execution of the block proceeds in the sequence as shown in the flow chart. The *KalmanParameters* function computes the parameters required for the Kalman filter algorithm, and the *kfamp* function estimates the phasor of the input signal, and the DC offset.

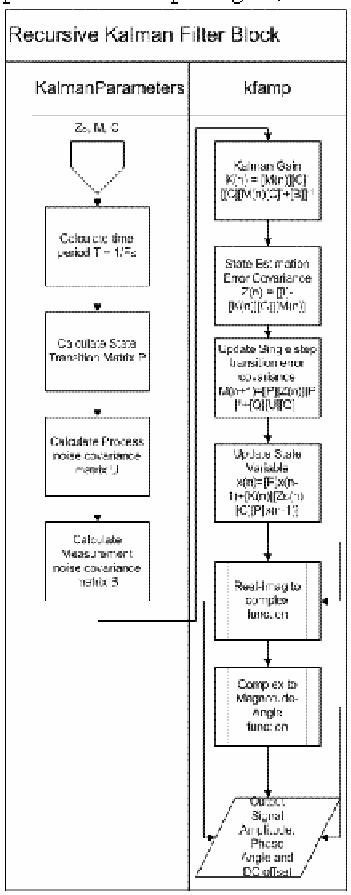

Fig. 7. Flow diagram for the recursive Kalman filter

2) Example 1

Example 1 is used for demonstrating the recursive Kalman filter block. This integrates all the blocks that were discussed in section II. The flow of execution for this example is shown in figure 8.

3) Example 2

Example 2 demonstrates the recursive Kalman filter in the presence of DC offset. The only difference in the parameters entered before the execution of the program is the DC offset parameter (?). This parameter should be set to a very high value than in example 1. The flowchart for this example is as shown in figure 9.

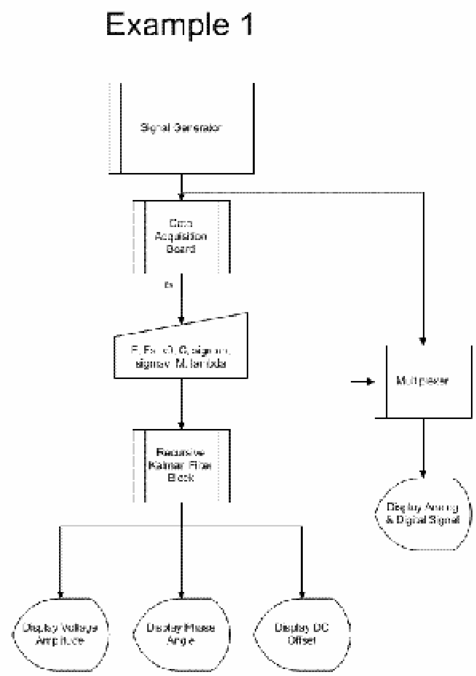

Fig. 8. Flow diagram for the Example 1.

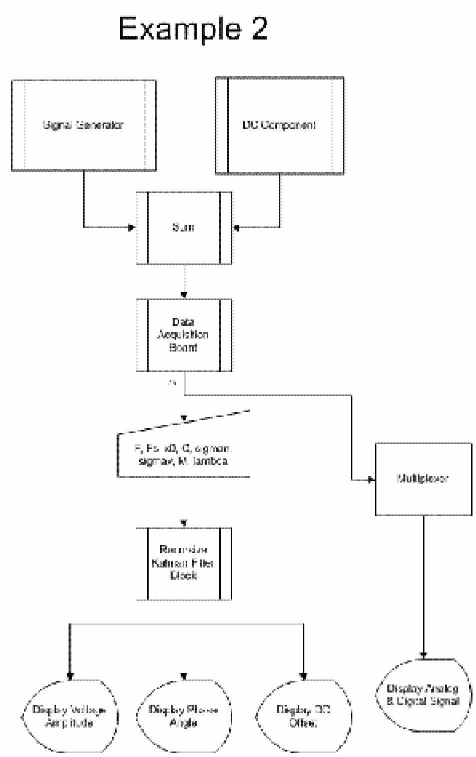

Fig. 9. Flow diagram for the Example 2.

4) Example 3

Example 3 demonstrates the frequency response of the filter. In the example shown, the filter is excited with a chirp signal and the fft of the output gives the frequency response of the filter. The flow chart for this example is shown in figure 10.

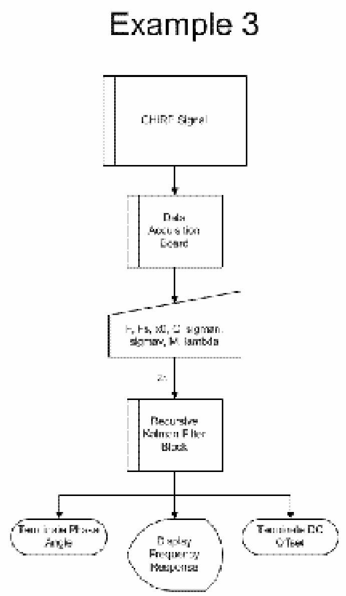

Fig. 10. Flow diagram for the Example 3.

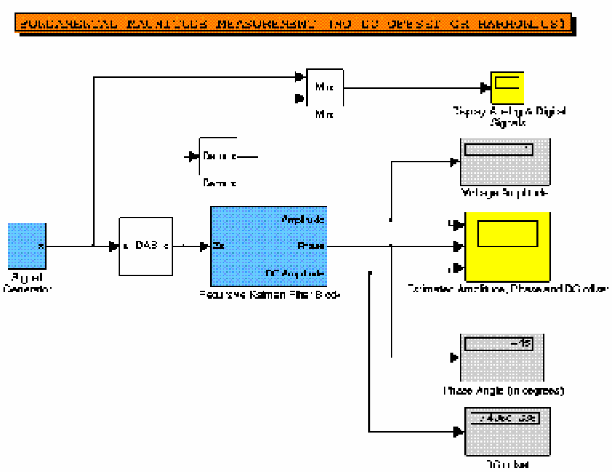

Fig. 11. Test setup for fundamental amplitude measurement with no DC offset and harmonics.

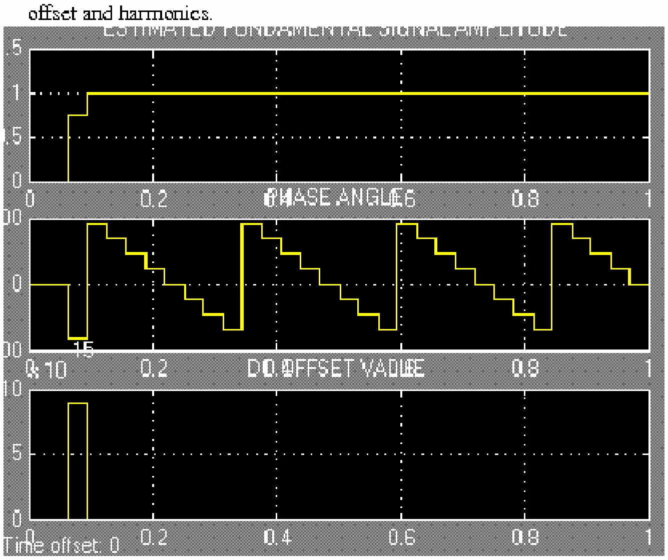

Fig. 12. Test results for fundamental amplitude measurement with no DC offset and harmonics. (a) Estimated Amplitude, (b) Phase Angle, (c) DC offset.

5) Kalman Parameters

The parameters used in the examples cited, are as shown in Table 1.

f	60 Hz
Fs	1920 Hz
$X0$	$[0\ 0\ 0]^T$
$[P]=[Q]$	$\begin{bmatrix} \cos(\omega\Delta T) & -\sin(\omega\Delta T) & 0 \\ \sin(\omega\Delta T) & \cos(\omega\Delta T) & 0 \\ 0 & 0 & e^{-\lambda\Delta T} \end{bmatrix}$
$[C]$	[1 0 1]
s_n	0.01
s_v	0.005
$[M(n)]$	$\begin{bmatrix} 0.24^2 & 0 & 0 \\ 0 & 0.24^2 & 0 \\ 0 & 0 & 0.24^2 \end{bmatrix}$
? (with(out) dc offset)	1000(0.01)

Table 1: Parameters used as input to the recursive Kalman filter.

where, ?=2?f, ?T=1/Fs.

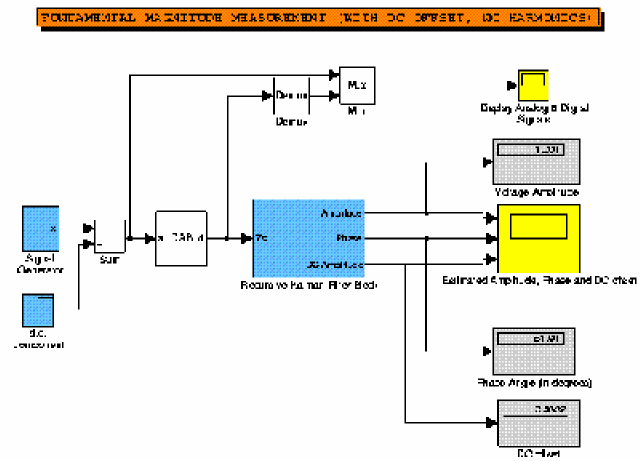

Fig. 13. Test setup for fundamental amplitude measurement with DC offset and no harmonics.

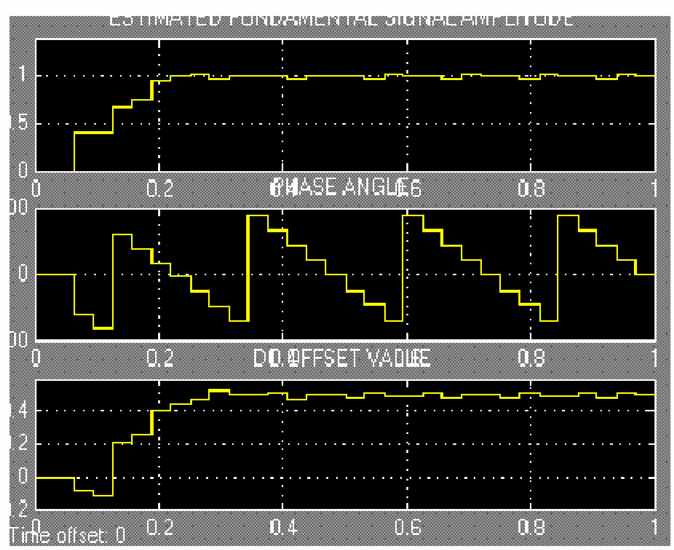

Fig. 14. Test results for fundamental amplitude measurement with DC offset and no harmonics. (a) Estimated amplitude, (b) Phase Angle, (c) DC offset.

IV. TEST SETUP AND RESULTS

The recursive Kalman filter block was tested using the set up as shown in figure 11 and figure 13 for harmonics, and dc offset. The set up shown in figure 16 was used to find the frequency response of the filter. The results of the simulation are as shown in figure 12, 14, and 17.

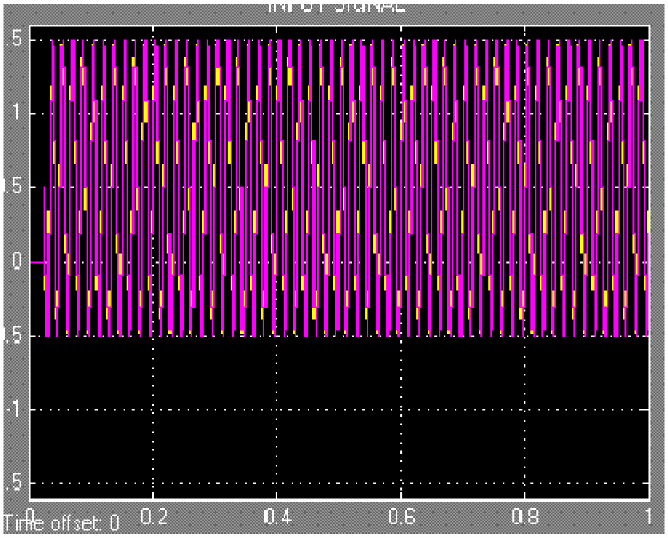

Fig. 15. DC Offset of 0.5V in the input signal

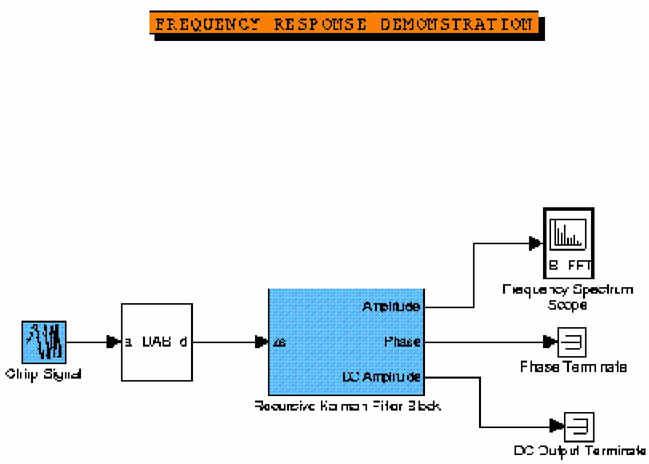

Fig. 16. Test setup for frequency response of the filter.

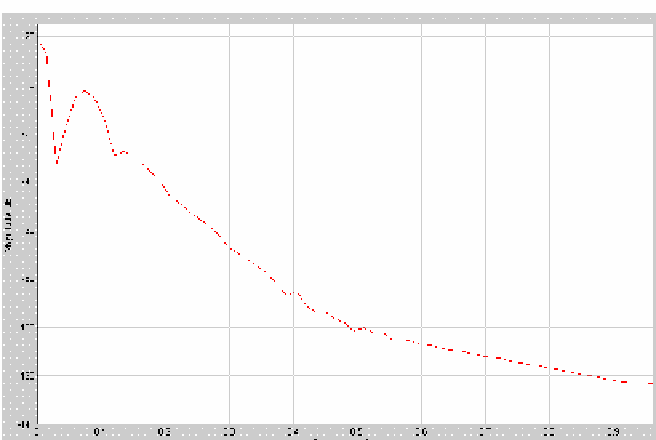

Fig. 17. DC Offset of 0.5V in the input signal

V. PERFORMANCE COMPARISON

The performance of the algorithm is compared with that of the Fourier-based method. The inputs to both the blocks were the same, and the outputs were recorded. The sensitivity of the algorithm was studied for:

a. same input
b. changes in sampling rate
c. changes in low-pass filter cut-off frequency

A. *Same Input*

The results that are given in table 2, compare the Kalman filter output with the Fourier method output. All the three test examples that was explained in the earlier section was used for the purpose.

Parameters\Algorithm	***Recursive Kalman Filter Block Output***	***Fourier Block Ouput***
Actual Fundamental Amplitude and no DC offset	1.0/1.0	1.0/0.7702
	0.0/(7.41e-036)	0.0/NA
Actual Fundamental Amplitude, third and fifth harmonics	1.0/1.0	1.0/0.734
	0.5/NA	0.5/NA
	0.5/NA	0.5/NA
Actual Fundamental Amplitude and DC offset	1.0/1.001	1.0/1.0
	0.5/0.4832	0.5/NA

Table 2: Performance comparison against the Fourier algorithm.

B. *Changes in Sampling Rate*

Any decrease in the sampling rate would reduce the efficiency of the filter in estimating the actual amplitude. This test was carried out for sampling rates that are 8, 16, 32, 64, and 128 times the dominant frequency in the signal. This is listed in table 3.

Sampling Frequency Fs (samples per second)	**Recursive Kalman Filter Block Output**	**Fourier Block Ouput**
480	1.0/0.9124	1.0/0.2899
960	1.0/0.972	1.0/0.9444
1920	1.0/1.0	1.0/0.7702
3840	1.0/1.0	1.0/0.2277
7680	1.0/1.0	1.0/0.205

Table 3: Performance comparison for changes in sampling frequency.

C. *Changes in cut-off frequency*

The dab block has a butterworth low pass filter that is used as an antialiasing filter. The sampling frequency (Fs) was kept constant at 960 Hz. The testing was carried out for various cut-off frequencies and this is tabulated below:

Cut-off Frequency Fc (Hz)	**Recursive Kalman Filter Block Output**	**Fourier Block Ouput**
70	1.0/0.8269	1.0/0.6819
120	1.0/0.972	1.0/0.958
240	1.0/0.972	1.0/1.001
450	1.0/0.972	1.0/0.9823

Table 4: Performance comparison for changes in cut-off frequency.

VI. Conclusion

A method to estimate the voltage and current phasors from the sampled signal based on Kalman filter was developed as Phase II of the project. Estimation of the post-fault voltage and currents can be useful for fault identification.

A method for including decaying dc and harmonic components in the filter was described. The three state model reaches the correct estimates within 0.42 of a cycle . It was also noted that the results of the Kalman filter algorithm out performed that of the Fourier method under normal conditions or for low sampling frequencies. However, the output saturates after a particular change in the anitaliasing cutoff frequency. If the signal is corrupted by some random white noise, the estimate provided by the Kalman filter is more accurate (0.9907V) than that of Fourier based method (0.7702V). The amount of DC offset can also be measured with high accuracy. The main drawback and advantage with the Kalman filter for estimation is that the parameters are to be continually tuned to obtain the accurate estimate.

Appendix A

1. *Index to matrices and variables*

The terms used in the Section II are defined as follows:

[x(n)]	State vector composed of the real and imaginary components of the signal phasor at the instant n?T
$x_r(n)$	Real component of signal
$x_i(n)$	Imaginary component of signal phasor.
[P]	State transition matrix
[Q]	Driving function matrix that advances the changes [?x(n)] by one sampling interval

Table 5 State Transition matrix parameters

$z_1(n)$	Measured signal at the time instant n?T
[C]	Defines the relationship between instantaneous sampled voltage and the phasor representation.
b(n)	Measurement noise modeled as a random variable.
[B]	Covariance of the noise model
[U]	The covariance matrix for [?x(n)] inputs.
[K(n)]	Kalman gain vector.
[Z(n)]	State estimation error covariance.
[M(n)]	Single step transition errors covariance matrix.
[I]	Identity matrix

Table 6: Kalman Gain calculation parameters.

2. *Covariance matrix*

The covariance matrix [D] of a vector [d] = [d11 d21]^T is defined as:

$$[\mathrm{D}] = \begin{bmatrix} E\{\mathrm{d}_{11}^2\} & E\{\mathrm{d}_{11}\,\mathrm{d}_{21}\} \\ E\{\mathrm{d}_{21}\,\mathrm{d}_{11}\} & E\{\mathrm{d}_{21}^2\} \end{bmatrix}$$

where, E denotes the expectation operator, and if the expected value $E\{\mathrm{d}_{11}\ \mathrm{d}_{12}\} = 0$, then d_{11} and d_{12} are orthogonal to each other (uncorrelated).

References

[1] Praveen Pankajakshan, "Phasor Measurement using Kalman Filtering", pp 1-5, Feb. 2003.

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