An Application of Optimization to Airplane Aileron Design

Kushang Patel and Praveen Ravishankar
Georgia Institute of Technology, Atlanta, GA, 30332

Ailerons on airplanes are a critical control surface, which is required in order to perform the rolling motion during flight. This paper optimizes the design of the aileron in order to minimize the cost of aileron from design to manufacturing to usage by the owner. A deterministic optimization approach as well as a stochastic optimization approach considering uncertainties are utilized. The optimization approaches involved using MATLAB's built-in fmincon() function and Monte-Carlo simulations. The results from different approaches and methods are compared. While both the deterministic and stochastic approaches resulted in feasible solutions, the stochastic approach to design optimization produces the most realistic results due to the implementation of design uncertainty.

Nomenclature

b = Wing Span
b_a = Aileron Span
b_{ai} = Location of Inner Edge of Aileron along the wing span
C = Wing Chord
C_a = Aileron Chord
C_{D_R} = Aircraft Drag Coefficient in a Rolling Motion
C_D = Aircraft Drag Coefficient
C_l = Rolling Moment Coefficient
$C_{L_{\delta_A}}$ = Rolling Moment Coefficient due to aileron deflection derivative
δ_A = Aileron Deflection
δ_{Amax} = Maximum Up and Down Aileron Deflection
ΔL = Incremental Change in Lift due to aileron deflection
ΔD = Incremental Change in Drag due to the rolling speed
I_{xx} = Aircraft Geometrical Moment of Inertia about the x-axis
L_A = Aerodynamics Rolling Moment
L_{cg} = All Rolling Moments applied to System
λ = Wing Taper
P = Roll Rate
P_{ss} = Steady State Roll Rate
Υ = Time Rate of Change of Roll Rate
ϕ_0 = Bank Angle at wing level flight
ϕ_1 = Bank Angle at which aircraft achieves the steady state roll rate
ϕ_2 = Desired Bank Angle
ρ = Density of Air
$\rho_{material}$ = Density of Aileron Material
q = Dynamic Pressure
S = Wing Planform Area
S_a = Aileron Platform Area
S_{ht} = Horizontal Tail Platform Area
S_{vt} = Vertical Tail Platform Area
S_w = Wing Platform Area
t_2 = Time to bank to a desired bank angle

t_{ss}	=	TIme to Reach Steady State Roll Rate
V_R	=	Rolling Linear Speed
V_S	=	Aircraft Stall Speed
V_T	=	Aircraft True Speed
y_A	=	Average Distance between aileron center and the x-axis
y_D	=	Average Distance between rolling drag center and the x-axis
y_o	=	Outboard position of the aileron in relation to the wing
y_i	=	Inboard position of the aileron in relation to the wing

I. Introduction

The two main requisites for comfortable and safe flight are stability and control. In order to achieve stability and control, lifting surfaces and the control surfaces of aircrafts must be effectively designed, which is performed by optimization. Lifting surfaces include the wing, the horizontal stabilizers, and the tail. Essentially, lifting surfaces generate lifting force required to fly the aircraft. Control surfaces such as ailerons, elevators, and rudders control the movement and rotation of the aircraft based on input from a yoke and pedal. In particular, the yaw, pitch, and roll are controlled by the rudder, elevators, and ailerons respectively.

All control surfaces are very important for flight; however, the scope of this study is to optimize the design of ailerons which control the roll or lateral control of an aircraft. Aileron design variables to be optimized include the size of the aileron and the aileron deflection (δ_{Amax}). For the purpose of the example in this study, various design parameters were modeled after the famous Cessna 182 aircraft.

The overall objective of the optimization is to design an aileron that minimizes the cost of the aileron. The cost of the aileron is first associated with how long it takes to achieve a 30 degree bank angle multiplied by the operating cost of the Cessna 182. The second cost associated with the aileron design involves the cost associated with the material of the aileron. Lastly, the cost associated with the design and manufacturing is estimated and added to the final cost. In order to minimize the objective function, multiple constraints were considered. The constraints were set based upon guidelines in order to achieve an optimal design within the flight envelope.

The design optimization is further conducted using deterministic and stochastic approaches, and further validated by Monte Carlo Simulations. The deterministic solution resulted in a feasible solution; however, this results in the same optimal solution every time. This may lead to over-designing the design of the aileron. The stochastic solution takes uncertainty into consideration by introducing different distributions of input variables for designing the aileron. As a result, the stochastic method results in the most realistic solutions.

II. Theory[1,2,3]

A. Total Aerodynamic Rolling Moment - Calculation of Required Time to Execute a Turn

In order to perform lateral movement during flight, a deflection of both ailerons simultaneously and differentially produces a rolling moment. The following diagram showcases all major forces exerted on the aircraft during lateral movement (from the perspective of the front view of the aircraft).

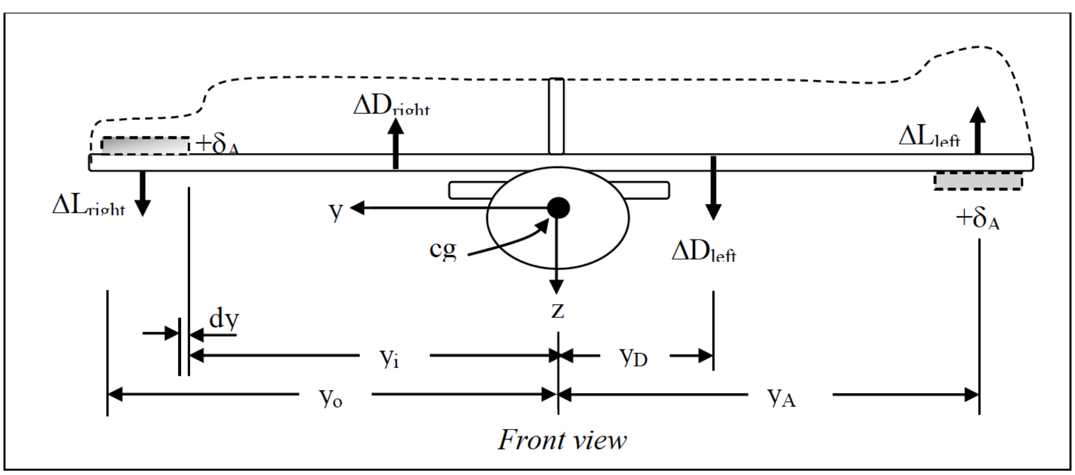

Figure 1: Front View of Aircraft with Moments due to Drag Forces[1]

As indicated in Figure 1 above, the two major moments contributing to the overall rolling moment of the aircraft are the moments generated from the drag forces exerted on the aileron (ΔL) and the simplified moments generated from the rolling drag forces exerted on the aircraft as a whole (ΔD). Additionally, in accordance with the coordinate system established in the above diagram, the rolling moment equals the mass moment of inertia in the x-direction multiplied by the time rate of change of the roll rate in the x-direction (P) of the aircraft:

$$\Sigma L_{cg} = I_{xx} \frac{\partial P}{\partial t}$$

The parameter y_A represents the average distance between the aileron and the x-axis (at which the aircraft's center of mass is located), while the parameter y_d represents the average distance between the centroid of the rolling drag force and the x-axis. Hence, by observing Figure 1 above, the total aerodynamic rolling moment in a rolling motion is calculated to be:

$$\sum L_{cg} = (2\Delta L) * y_a - \Delta D * y_d = L_A - \Delta D * y_d$$

The factor "2" is not multiplied by (ΔD) as the overall rolling drag for the two sides of the plane will be further computed. The lifting moment generated from the ailerons (L_A) can be expressed in terms of the wing area (S), total wingspan (b), and the dynamic pressure (q):

$$L_A = qSC_l b$$

The dynamic pressure can be modeled using the air density (ρ) and the true airspeed of the aircraft (V_T):

$$q = 0.5 * \rho * V_T^2$$

The equation for V_T can be expressed as follows:

$$V_T = \beta V_S$$

$V_S$represents the vehicle's stall speed, while βrepresents the stall speed coefficient, a parameter that is typically between 1.1 and 1.3.

Moreover, the term C_l is a parameter that is dependent upon the aircraft configuration, aileron deflection, sideslip angle, and rudder deflection. However, for the purposes of simplicity, it is assumed that the Cessna 182 will have no sideslip angle or rudder deflection, allowing the parameter C_l to be expressed as:

$$C_l = C_{l_{\delta_A}} \delta_A$$

C_l represents the aileron roll control derivative, which is the derivative of the aircraft's rolling moment coefficient due to aileron deflection. An alternative equation for C_l is presented as:

$$C_l = \frac{2C_{L_{\alpha_w}} \tau \delta_A}{Sb} \int_{y_i}^{y_o} Cy dy$$

The variable $\tau$is the aileron effectiveness parameter, whose value will be computed using a linear regression elaborated in the next section.

Additionally, $C_{l_{\delta_A}}$ is a coefficient representing the aileron roll control power, whose equation is presented as follows:

$$C_{l_{\delta_A}} = \frac{\partial C_l}{\partial \delta_A} = \frac{2C_{L_{\alpha_w}}\tau}{Sb}\int_{y_i}^{y_o} Cydy$$

The equation for the wing chord C is a function of the distance along the wing span (y) and the wing taper ratio λ:

$$C = C_r[1 + \frac{2(\lambda-1)y}{b}]$$

The variable C_r is the wing root chord. Substituting this equation for C into the integral above and performing the integration yields in the following equation for $C_{l_{\delta_A}}$:

$$C_{l_{\delta_A}} = \frac{2C_{L_{\alpha_w}}\tau}{Sb}\left[\frac{y^2}{2} + \frac{2(\lambda-1)y^3}{3b}\right]_{y_i}^{y_o}$$

Next, the rolling drag force exerted on the aircraft is equal to:

$$\Delta D = \Delta D_{left} + \Delta D_{right} = 0.5 * \rho * V_R^2 * S_{tot} * C_{D_g}$$

The speed V_R is the linear rolling speed of the aircraft and can be calculated by:

$$V_R = P * y_D$$

In addition, the value S_{tot} is equal to the sum of the wing planform area, horizontal tail planform area, and vertical planform area:

$$S_{tot} = S_w + S_{ht} + S_{vt}$$

As a result, it can be stated that the parameter y_D is the average of the three distances between the aircraft's center of mass and the centers of masses of the wing, horizontal tail, and vertical tail.

Lastly, the parameter C_{D_g} is the aircraft drag coefficient during rolling motion, which includes the contribution of the drag forces exerted on the fuselage. This value is typically between 0.8 and 1.1, so an average value of 0.95 will be selected for the deterministic optimization aspect of this project.

By substituting the sum of the moments into the equation relating the sum to the derivative of the angular momentum (in the x-direction), the following equation is yielded:

$$L_A + \Delta D * y_D = I_{xx}\frac{\partial P}{\partial t} = I_{xx}\Psi; \Psi = \frac{\partial P}{\partial t}$$

Additionally, since the variable P represents the time rate of change of the lateral angle (Φ) of the aircraft:

$$P = \frac{\partial \Phi}{\partial t}$$

Rearranging, and making further substitutions using the equations reviewed above yields:

$$\Phi = \int_0^{P_{ss}} \frac{P}{\Psi}dP = \int_0^{P_{ss}} \frac{I_{xx}P}{L_A + \Delta D * y_D}dP = \int_0^{P_{ss}} \frac{I_{xx}P}{(0.5*\rho*V_T^2 SC_l b) + (0.5*\rho*V_R^2 * S_{tot} * C_{D_g})}dP$$

The value P_{ss} is the constant steady-state value of the roll rate attained after the initial rolling acceleration, which is achieved by the aircraft once the lateral drag moment is equal in magnitude to the applied rolling moment. The following graphs in figure 2 and figure 3 demonstrates the relationship between the roll rate and the bank angle versus time of the aircraft:

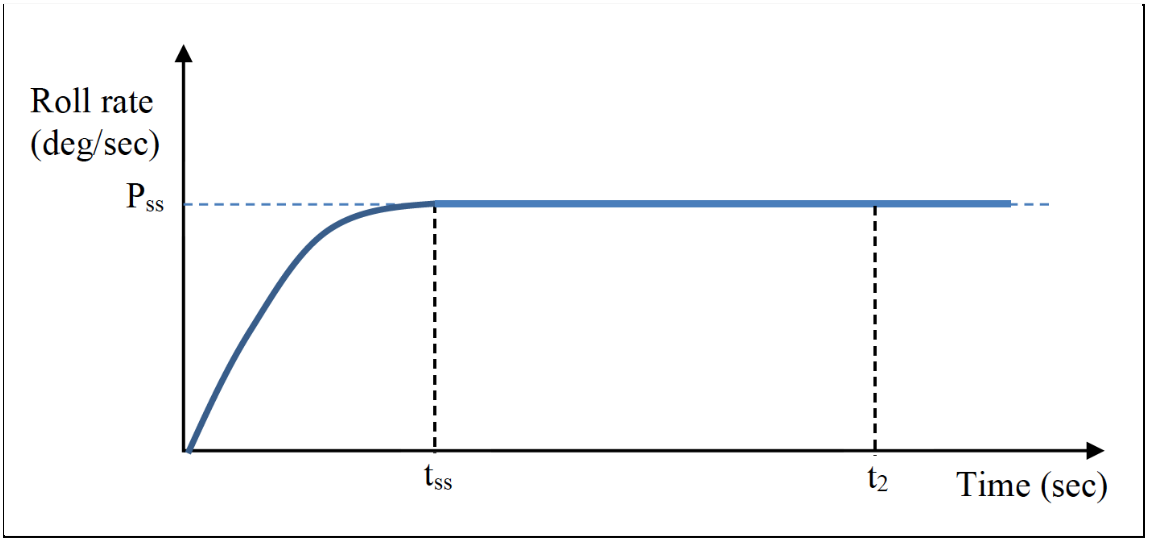

Figure 2: Roll Rate vs. Time[1]

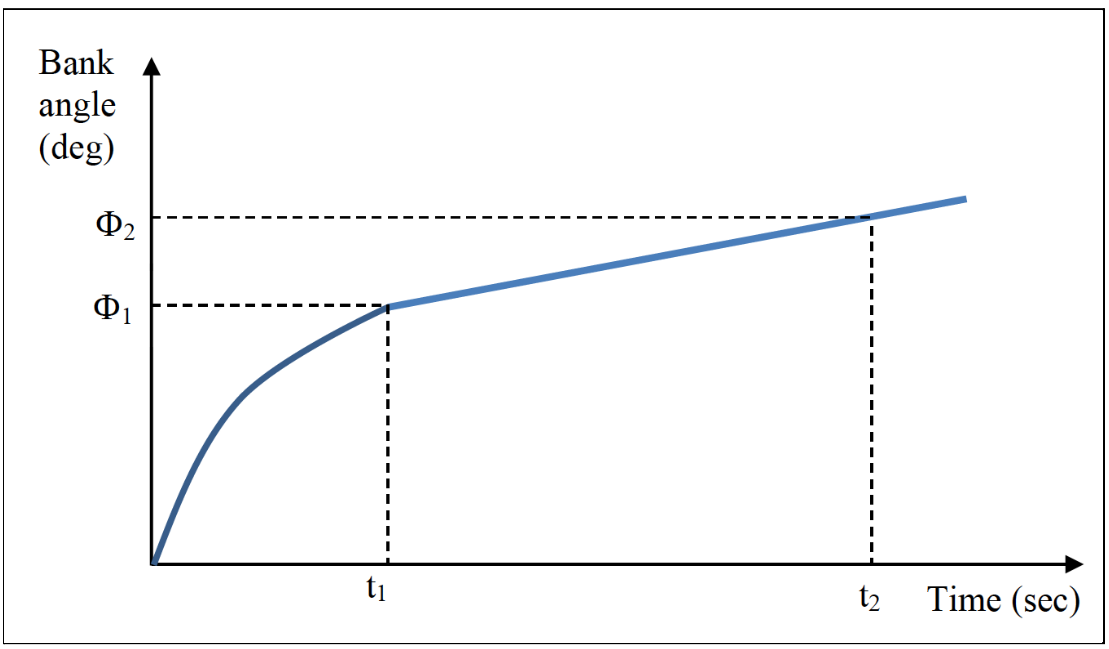

Figure 3: Roll Rate vs. Time[1]

Additionally, the equation solving for the variable P_{ss} (whose derivation is beyond the scope of this project) is as follows:

$$P_{ss} = \sqrt{\frac{2L_A}{\rho S_{tot} C_{D_R} y_D^3}}$$

The time steps t_1 and t_{ss} represent the same time value - the amount of time passed for the aircraft's roll rate to be stabilized. As a result, the variable Φ(with respect to those specified limits of integration) actually represents $\Phi_1$and is integrated to equal:

$$\Phi_1 = \frac{I_{xx}}{\rho y_D^{\ 3}(S_w+S_{ht}+S_{vt})C_{D_R}} ln(P_{ss}^{\ 2})$$

As the steady-state roll rate P_{ss} and the initial acceleration are assumed to be constant, simple kinematic equations can be utilized to determine the relationships of the above variables:

$$\Phi_2 = P_{ss}(t_2 - t_{ss}) + \Phi_1$$

$$t_2 = t_{ss} + \Delta t$$

$$\Phi_2 = P_{ss}\Delta t + \Phi_1$$

$$\Delta t = time\ required\ to\ roll\ from\ \Phi_1\ to\ \Phi_2 = \frac{\Phi_2-\Phi_1}{P_{ss}}$$

The variable t_2 equals the total amount of time to rotate by the desired bank angle (Φ_2), which is given. Assuming the initial angle (Φ_o) and the initial roll rate (P_o) both equal 0:

$$t_{ss} = \sqrt{\frac{2\Phi_1}{\psi}}$$

$$P_{ss}^{\ 2} = 2\Psi\Phi_1$$

Using the equation for P_{ss} displayed above, $\Phi_1$can easily be solved. As a result:

$$\Psi = \frac{P_{ss}^{\ 2}}{2\Phi_1}$$

$$t_2 = \sqrt{\frac{2\Phi_1}{\psi}} + \frac{\Phi_2-\Phi_1}{P_{ss}}$$

$$t_2 = \sqrt{\frac{4\Phi_1^{\ 2}}{P_{ss}^{\ 2}}} + \frac{1}{P_{ss}} * (\Phi_2 - \frac{I_{xx}}{\rho y_D^{\ 3}(S_w+S_{ht}+S_{vt})C_{D_R}} ln(P_{ss}^{\ 2}))$$

However, the above equation assumes that the time required to bank a certain desired angle is greater than the time required to reach the steady-state roll rate P_{ss}. If the converse is true, then t_2 can simply be expressed as:

$$t_2 = \sqrt{\frac{2\Phi_2}{\psi}}$$

Hence, the equation(s) to calculate the amount of time required to bank a certain angle has been obtained.

B. Regression to Obtain Aileron Effectiveness Parameter

In order to accurately predict τ or the aileron effectiveness parameter, data was obtained that related the control surface to lifting surface chord ratio to the aileron effectiveness parameter. The 70 points of data was obtained by magnifying in on data points to obtain this relationship[1]. The effectiveness parameter is applicable to any control surface and thus is obtained experimentally for varying sizes of the lifting surface and the varying sizes of the control surface. The data was plotted with various order polynomials; however, it was determined that a 5th order polynomial was sufficient to represent the data. The following 5th order polynomial was utilized:

$$y = c_6 x^5 + c_5 x^4 + c_4 x^3 + c_3 x^2 + c_2 x + c_1 + e$$
$$y = \hat{c} x + e$$

The pseudo-inverse method was implemented to solve for the coefficients in the 5th order polynomial utilized to fit the data. The following represents the equation from the pseudo-inverse method utilized to obtain the coefficients:

$$\hat{c} = (x^T x)^{-1} x^T y$$

Table 1: Regression Coefficients

C_1	**-0.0072**
C_2	**4.4096**
C_3	**-21.7674**
C_4	**64.6802**
C_5	**-89.9370**
C_6	**46.4205**

Coefficients for the polynomial were obtained via the pseudo-inverse method as seen in Table 1. Figure 4 shows the plotted data along with the 5th order regression. To quantify the fit of the regression with the data, the R^2 value as well as the ANOVA table were calculated. The R^2 value determines if the regression fits the data, while the ANOVA table was used to determine the statistical significance of the regression.

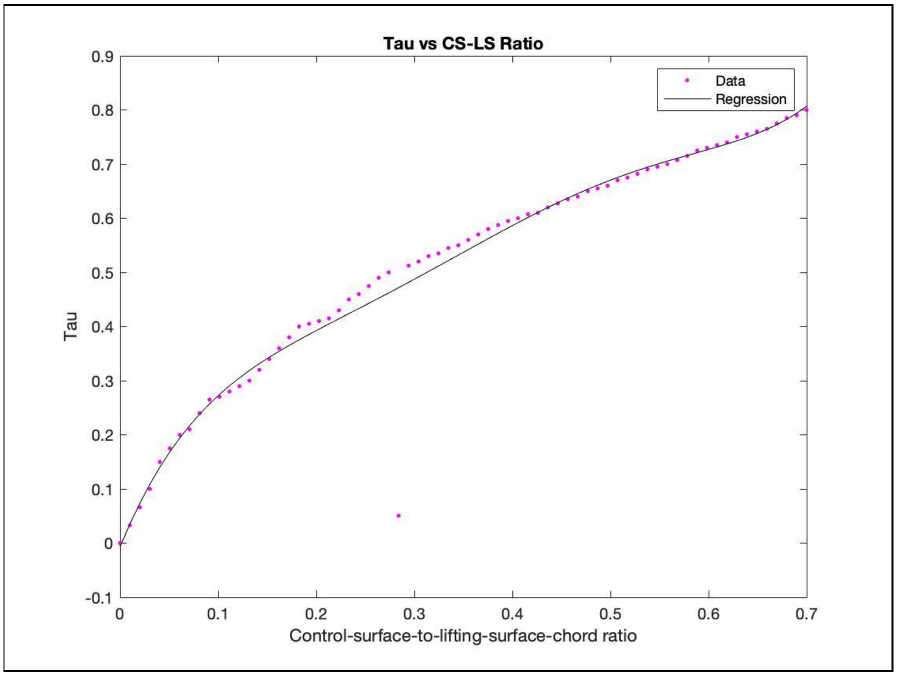

Figure 4: Data and Respective Regression

Table 2 shows the ANOVA table is calculated, while Table 2 calculates the ANOVA table for the control surface effectiveness parameter as a function of the control surface to lifting surface chord ratio.

Table 2. ANOVA Table Template for Calculations

Source of Variance	SS (Sum of Squares)	DF (Degrees of Freedom)	MS (Mean Square)	F_0
Regression	$SS_r=\sum(X^*c)^T(X^*c)$	$df_r=k$	$MS_r=SS_r/k$	$F_0=MS_r/MS_e$
Residual	$SS_e=SS_t-SS_r$	$df_e=j-k$	$MS_e=SS_e/df_e$	
Total	$SS_t=\sum Y^TY$	$df_t=k$		

Table 3. ANOVA Table Calculations for Tau versus Control surface to lifting surface chord ratio

Source of Variance	SS (Sum of Squares)	DF (Degrees of Freedom)	MS (Mean Square)	F_0
Regression	SS_r=20.9939	df_r=6	MS_r=3.4990	F_0=1.1668E3
Residual	SS_e=0.1919	df_e=64	MS_e=0.0030	
Total	SS_t=21.1858	df_t=70		

Based upon the calculations in Table 3, the F_0 is calculated to be 1.1668E3. The calculated F_0 is then compared to the theoretical F_0 value. The theoretical value was calculating my setting α or the percentile of the F distribution equal to 0.05. This can also be interpreted as a 95% Confidence interval. The theoretical F_0 was calculated to be 3.9778 which is much less than the value calculated from the data. As a result, it can be concluded that the regression analysis is statistically significant. Further, to determine if the regression fits the data set, the R^2 value was calculated utilizing the following formula:

$$R^2 = \frac{SS_r}{SS_t}$$

The calculated R^2 value was 0.9909, suggesting that the regression fits the data at a high level of accuracy. With the information from the ANOVA table and the calculated R^2 value, the regression has been concluded to be statistically significant and to be an accurate predictor to estimate the aileron effectiveness parameter (τ).

C. Parameters Utilized for Uncertainty

Significant uncertainty exists for a few parameters involved in the calculation of the objective function. These parameters are as follows: the aircraft's moment of inertia in the x-direction, the stall speed coefficient (which is utilized to calculate the stall speed of the aircraft), and the rolling drag coefficient. Firstly, variations in the aircraft's total mass undoubtedly exist due to differences in the weight of the passengers/pilots and any cargo on board for different flights. Because the total mass thereby affects the aircraft's moment of inertia, uncertainty therefore exists with I_{xx}. The probability density function (PDF) for the moment of inertia used for these calculations is as follows:

$$I_{xx} \sim Weibull(90,\ 1)\ +\ 1285.3$$

The Weibull distribution was selected because the aircraft's moment of inertia definitely cannot be less than that of the plane itself; additionally, as the value of I_{xx} deviates more from the aircraft's true moment of inertia, the probability of attaining that value of I_{xx} decreases, which can be easily represented by a Weibull distribution. A scale parameter of 90 and a shape parameter of 1 were selected to best fit the predicted PDF of I_{xx}.

Furthermore, uncertainty exists in the stall speed coefficient as well, whose values range from 1.1 to 1.3. A normal distribution was selected to represent the PDF of the stall speed coefficient, β:

$$\beta \sim Normal(1.2,\ 0.03)$$

The mean was selected to equal 1.2 since it lies in the middle of the aforementioned range of the stall speed coefficient. Additionally, since the probability of attaining a value outside three standard deviations of the mean is relatively low for a normal distribution a value of 0.03 was selected for the standard deviation for the stall speed coefficient's PDF.

Moreover, the rolling drag coefficient ranges between 0.8 and 1.1. A uniform distribution ranging between these bounds was used to represent the value of C_{D_R}:

$$C_{D_R} = Uniform(0.8,\ 1.1)$$

The uncertainties of these three parameters will be taken into account for the stochastic optimization part of the project.

III. Example Problem

The aileron design problem examined and analyzed in this study will be completed based on the specifications of the Cessna 182 aircraft. The Cessna 182 is one of the most mass-produced airplanes in the aviation industry and is also a very popular training aircraft for all pilots. Important constants for the problem are defined as follows:[4,5]

$$b = 11\,m$$
$$c = 1.472\,m$$
$$S_w = 21\,m^3$$
$$S_{ht} = 5.3\,m^3$$

$$S_{vt} = 4.2\, m^3$$
$$V_s = 55\, knots$$
$$I_{xx} = 1285.3\, kg/m^2$$
$$\lambda = 0.8$$

Additionally, it's vital to properly characterize the quality specifications of the Cessna 182 in terms of its weight and maneuverability requirements, which includes aircraft classes, flight phases, and levels of acceptability. This characterization must be performed in order to properly determine the general function (and therefore the design) of its ailerons. Firstly, four different classes of aircraft exist which are characterized by the weight of the airplane. It is observed that the Cessna 182 fits in the Class I category as the maximum take-off mass is less than 6000 kg. Furthermore, the Cessna 182 corresponds to a Category C aircraft. These categories indicate the type of maneuvers that the airplane is suited to perform. Class C includes take-off, powered-approach, wave-off, and landing. Lastly, the levels of acceptability relate to the intended handling qualities and maneuverability of the aircraft of focus, and falls in three control levels. To ensure certainty of the functionality and safety of the aircraft, of the pilot(s), and the passengers, it's desired to attain an acceptability of Level 1. Therefore, as the Cessna 182 is categorized as a Class I aircraft with flight phase C while having Level 1 acceptability, the aircraft should be able to achieve a 30 degree bank angle in less than 1.3 seconds.[1] As seen in Figure 1, these specifications were set with requirements from the US Military for aircraft design, MIL-F-8785C.

(a) Time to achieve a specified bank angle change for Class I			
Level	Flight phase category		
	A	B	C
	Time to achieve a bank angle of 60° (s)	Time to achieve a bank angle of 45° (s)	Time to achieve a bank angle of 30° (s)
1	1.3	1.7	1.3
2	1.7	2.5	1.8
3	2.6	3.4	2.6

Figure 5. Aircraft Specification set by MIL-F-8785C[1]

A. Objective Function

In order to calculate the objective function, the cost must be calculated for the aileron. The cost to operate for each banking turn, the material cost for the aileron, and the manufacturing and assembly costs of the aileron were all considered when calculating the total cost of the airplane's aileron. The cost to operate for each banking turn is primarily governed by the amount of time required for the airplane with the designed aileron to complete a 30° bank turn. This time to complete the turn is then multiplied by the published average cost to operate a Cessna 182 per hour, which was converted to per second. The Cessna 182 approximately costs $216 per hour to operate which further reduces to 6 cents per second. 6

$$P_{turn} = (0.06) * (t_2)$$

The material cost of the aileron significantly contributes to the cost of the aileron as well. The Cessna 182 is manufactured with Aluminum Alloy 7075, which has an estimated cost of $8.55/kg. As a result, the cost of the aileron increases as the planform area of the aileron as well as the volume of the aileron increase.[7]

$$P_{material} = 8.55\rho_{material}V$$

Lastly, the design and manufacturing costs of the aileron can also be estimated for the Cessna 182 based on prior publications on the manufacturing and assembly costs. The final cost of a Cessna 182 is approximately $515,000, with approximately 27% of the costs being associated with the design and manufacturing of the wings. With the given optimization, the size of the aileron in comparison to the whole wing can be calculated to calculate an approximate cost of the aileron design and manufacturing.[7]

$$P_{design\&mfg} = (515,000) * (0.27) * (\frac{b_a}{b})$$

Therefore, the objective function is defined as the total cost to make a bank angle (which is controlled by the aileron design), the material cost of the aileron (affected by the aileron geometry), and the total cost of the aileron design and manufacturing (also affected by geometry). The objective function for the design of the aileron is:

$$f(\bar{x}) = P_{turn} + P_{material} + P_{design\&mfg}$$

In order to optimize the size of the aileron and the deflection, the following objective function has to be minimized via the optimization algorithm.

B. Optimization Statement and Constraints

The following represents the final optimization statement and constraints in designing the three geometrical characteristics of the ailerons:

$$Minimize\ f(\bar{x}) = P_{turn} + P_{material} + P_{design\&mfg}$$

$$Subject\ to:$$
$$g_1: 0.10 \leq |y_o - y_i| \leq 0.20$$
$$g_2: |\delta_{A,max}| \leq \pm\ 25^o$$
$$g_3: 0.6 \leq y_i \leq 0.9$$
$$g_4: 0.6 \leq y_o \leq 0.99$$
$$g_5: 0.15 \leq C_a \leq 0.25$$

In order to minimize the cost of the aileron, some constraints must be considered to ensure a feasible design of the aileron. If the design variables are outside of the prescribed constraints, the airplane might operate out of the flight envelope causing failure. The constraints are also set in order to set feasible lower and upper bounds of design variables.

Constraint g_1 constrains the difference between the outbound and inbound aileron positions. The difference between the outboard and inboard position must be between 10 and 20 percent of the wing span. Constraint g_2 sets the upper and lower bound of the aileron deflection. The aircraft aileron should not deflect more than 25 degrees in the positive and negative direction. Constraint g_4 and g_5 set the upper and lower bound of the inboard and outboard of the aileron, respectively. In order to account for the flaps and the usage of spoilers in the aircraft, the inboard angle should at least be greater than approximately 50% of the wing span while the outboard angle should be greater than approximately 60% of the wing span. Constrain g_3 examines the upper and lower bound of the aileron chord in relation to the overall wing chord. This constraint was set in coordination to aircraft design principles which suggest a range for the aileron chord in relation to the wing chord.the g_6 constraint sets the aileron chord between 0.15 and 0.25.

IV. Results

Table 4: Results from the Deterministic Optimization Approach

Y_i	Y_o	C_a	$\delta_{A,max}$	f(x)	Iterations
0.8000	0.9000	0.2498	24.9998	178941.8928	10

Table 5: Results from the Stochastic Optimization Approach

Simulation Number	Y_i	Y_o	C_a	$\delta_{A,max}$	f(x)	Iterations
1	0.7620	0.8620	0.1500	22.1069	178942.08	17
2	0.6000	0.7346	0.1664	0.9550	183754.33	12
3	0.6904	0.7903	0.1662	24.9999	178942.01	16
4	0.6000	0.7000	0.2140	24.6038	178941.98	18
5	0.7992	0.8993	0.2432	0.7800	178964.43	8

Table 6: Results from Monte Carlo Optimization

Y_i	Y_o	C_a	$\delta_{A,max}$	f(x)	Iterations
0.6749	0.7749	0.2369	14.1850	178942.13	100000

V. Discussion

A. Deterministic Results

In order to obtain results for the deterministic approach, MATLAB's fmincon() function is utilized. In order to perform this optimization. initial design variables such as Y_i, Y_o, C_a, and $\delta_{A,max}$ were arbitrarily selected. Via the optimization algorithm, the objective function was minimized, and the design variables at that point were determined to be the optimum solution. Results for the deterministic design optimization are shown in Table 4. The deterministic design returns an optimal design in 10 iterations with a minimum objective function value of \$178941.89. The design; however, does not take uncertainty into consideration. As a result, a small variation from the deterministic design variables could lead to failure or a solution that is not feasible. The deterministic design approach produces the same results each time for the specific inputs to the system. As a result, the deterministic design approach may result in over designing the system or in this case, the aileron.

B. Stochastic Results

Similar to the deterministic optimization process, MATLAB's fmincon() function was used for the stochastic optimization process as well. However, unlike the deterministic approach, for the stochastic approach, the parameters possessing significant uncertainty were replaced with a randomly-selected value from their respective PDFs as mentioned previously in this paper. Table 5 displays five simulations of the stochastic optimization approach to account for different optimized values under the uncertainty of the three parameters: I_{xx}, β, C_{D_R}. As indicated by the table, three of these five simulations (Simulation #'s 1, 3, and 4) attained feasible results. Simulation #'s 2 and 5 corresponded to unrealistic results, as it is unlikely that the maximum deflection of the aileron is less than 1^o. Nevertheless, for all five simulations, the values of the inner and outer edges of the designed aileron (expressed in terms of the percentage across the wing span) are approximately 10-20% less than those obtained by the deterministic approach. In addition, the values of the aileron chord C_a are overall less than that obtained from the deterministic optimization. Lastly, the optimized cost function values are approximately around \$178,000-\$179,000 (with the exception of that of Simulation #2), which is similar to the optimized value obtained from the deterministic approach as well.

C. Monte Carlo Simulation Results

To validate the results obtained from the deterministic and stochastic optimization methods, a Monte Carlo Simulation was also performed utilizing the constraints described in Section III B. The simulation yielded results displayed in Table 6 above. By observing the table, it is noticed that the locations for the aileron's inner and outer edges are more similar to those obtained from the stochastic approach and are smaller than those obtained from the deterministic approach. Additionally, the calculated maximum aileron deflection angle is approximately 10^o less than the realistic solutions from both the deterministic and stochastic approaches, while the calculated aileron chord length is similar to that obtained from the deterministic optimization and to two out of the five simulations from the stochastic optimization. In short, it is concluded that the Monte Carlo Simulation achieved results that are consistent with those of the deterministic and stochastic optimization methods.

VI. Conclusion

The three optimization algorithms (deterministic approach using MATLAB's fmincon(), stochastic approach using MATLAB's fmincon(), and the Monte Carlo Simulation) all achieved feasible and realistic solutions, with the exception of two out of the five simulations from the stochastic approach. As a result, the stochastic optimization process should primarily be used since it is the only approach that obtained at least a few unrealistic solutions, prompting for the optimization algorithm to be further modified and improved. One possible solution would be to normalize all inputs to the optimization process, which was not originally done. By performing this process, it is possible that the stochastic optimization algorithm would yield more realistic solutions (particularly those that allow much higher lenience of the maximum aileron deflection angle). In addition, although feasible and theoretically realistic solutions were obtained, those solutions can neither be confirmed or denied to be accurate, truly realistic solutions applied to the Cessna 182 aircraft in the real world. Hence, experimental data should be obtained for the various parameters utilized to compute the time to roll the aircraft by 30º and for the geometrical design parameters of the actual ailerons. Furthermore, more research should be done to determine the true cost function of the aileron's design more accurately, possibly implementing more costs than the time spent to roll, the aileron's material, and the aileron's manufacturing process. Lastly, the design of the aileron is affected by the overall design of the wing itself; this implies that the failure modes of the wing (such as the wing rear spar) is affected by the aileron's design as well. As this project focused solely on the design of the aircraft's aileron, future work may include taking into consideration the effect of the aileron's design on the wing's failure modes as well.

VII. Contribution

For the report's content, Kush primarily worked on the Abstract, Introduction, Regression, Objective Function, Deterministic Approach (Results and Discussion), and the Appendix sections. Praveen primarily worked on the Theory, Parameters Utilized for Uncertainty, Stochastic Approach (Results and Discussion), Monte Carlo Simulation (Results and Discussion), and Conclusion sections. Both Kush and Praveen additionally assisted with reviewing, revising, and formatting all parts of the paper as a whole.

For the MATLAB codes, Kush and Praveen provided equal contributions to the following MATLAB files: "main.m", "confunc.m", and "objective.m". Kush additionally primarily created the "findtau.m" and "get_tau.m" linear regression MATLAB files, while Praveen chiefly created the "objective_st.m" and "objective_mcs.m" MATLAB files.

Appendix

main.m performs the design optimization using objective.m to perform deterministic optimization, objective_st.m to perform stochastic optimization, and objective_mcs.m to perform optimization via Monte Carlo Simulation. confunc.m is utilized for stochastic constraints and deterministic constraints.

In order to solve for the optimal parameter of ailerons the following steps should be completed:

1. In the main.m file, first set the desired starting points for the optimization algorithm.
2. Run the main.m file to obtain the optimal design points and optimal value of the objective function at the optimal design points.
3. To run the Monte Carlo Simulation, set the desired number of iterations in the main.m file. The Monte Carlo Simulation contains 2 layers of iterations. As a result, set the desired value for “n_sim1” and “n_sim2.”
4. After the main.m file is run, “det_opt_vals” in the workspace contains the optimal design parameters for the deterministic approach. “fun_det” in the workspace contains the value of the objective function via the deterministic approach.
5. After the main.m file is run, “stoc_opt_vals” in the workspace contains the optimal design parameters for the stochastic approach. “fun_stoc” in the workspace contains the value of the objective function via the stochastic approach.
6. After the main.m file is run, “iter_vals_min” in the workspace contains the optimal design parameters calculated via the Monte Carlo Method.

References

[1]Sadraey, M. "Design of Control Surfaces," *Aircraft Design: A Systems Engineering Approach,* Wiley, 2013.

[2]Brincklow, J. "Optimization of Aileron Spanwise Size and Shape to Minimize Induced Drag in Roll with Correlating Adverse Yaw," *DigitalCommons@USE,* Utah State University, 2020.

[3]Tulapurkara, E. "Chapter 6 Lateral static stability and control," *Flight Dynamics II - Stability and Control, IIT Madras*.

[4]McIver, Johm B. *Cessna Skyhawk Performance Assessment* . Temporal , 23 Jan. 2003, www.temporal.com.au/c172.pdf.

[5]"Metrics for Various Aircraft ." *Aircraft Mass Properties*, jsbsim.sourceforge.net/MassProps.html.

[6]"CESSNA T182T Price and Operating Costs." *Aircraft Cost Calculator*, www.aircraftcostcalculator.com/AircraftOperatingCosts/336/Cessna+T182T.

[7]Willcox, K. "Cost Analysis," *Aircraft Systems Engineering,* MIT Aerospace Computational Design Laboratory.