George W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology

ME6401B,Q **HW SET #7** Due Monday 11/16/2020

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1) Redo Problem 2.d & 2.e (Combined Reduced Order Observer and Controller) of Homework #6 using the direct Polynomial Control design. Compare your results to those obtained previously.

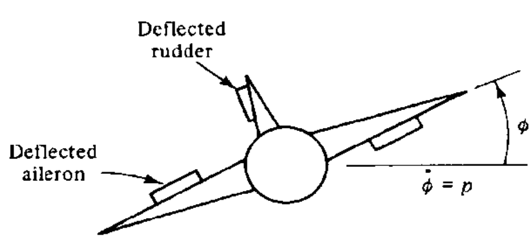
2) Find the steady-state LQR optimal controller for the first order system $\dot{x} = ax + u$ with performance index $J = \int_0^\infty x^2 + ru^2 \, dt$. Discuss your solution for a=0, a>0, a<0, and 0<r<∞.

3) Consider a 2nd order system given by the input-output transfer function $\frac{Y(s)}{U(s)} = \frac{s+2}{s^2+2s+5}$. We wish to find u(t) to minimize $J = y(5)^2 + \int_0^5 ru^2(t)dt$ for some $r>0$. Set up the solution to this problem as a standard LQ optimal controller by finding A, B, Q, and R matrices, and the corresponding Riccati equation. Solve the problem numerically and plot the resulting optimal input and output trajectories for $r=10^{-6}$, 0.01, and 10. Explain the effects of r on your results. Note that you cannot use the ARE (steady-state solution for this problem).

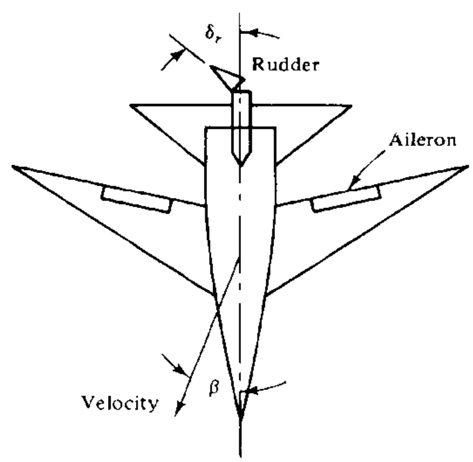
4) Let $J = \int_0^\infty y(t)^2 + ru^2(t)dt$ for the system in problem 3. Plot the closed-loop poles (root-locus) resulting from the LQR controller (i.e., eigenvalues of A-BK) as r varies from 0 to ∞.

5) Consider the lateral dynamics of an aircraft discussed in the lecture earlier. Find an LQR state feedback controller u=-Kx to minimize $J = \int_0^\infty (\phi^2 + \beta^2 + \delta_a^2 + \delta_r^2)dt$.

$$\frac{d}{dt}\begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} = \begin{bmatrix} -10 & 0 & -10 & 0 \\ 0 & -0.7 & 9 & 0 \\ 0 & -1 & -0.7 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} p \\ r \\ \beta \\ \phi \end{bmatrix} + \begin{bmatrix} 20 & 2.8 \\ 0 & -3.13 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}$$



Front view



Top view

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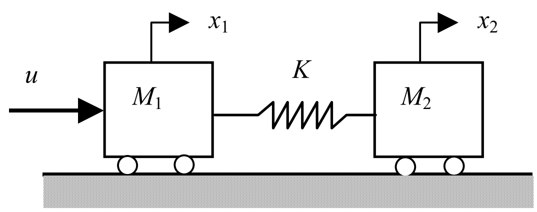
<u>Optional Problems</u>: *You do not have to turn them in.*

1) Find the steady-state Kalman filter (in closed form) for the first order system $\dot{x} = ax + v$, $y = x + w$ where v and w are independent white random processes with covariance's 1 and r>0, respectively. Discuss your solution for a=0, a>0, a<0, and 0<r<∞.

2) Many position control devices in machine tools and robots can be modeled as a 2 mass-spring system shown in the figure. Assume the nominal values of $M_1 = M_2 = 1$ Kg and $K = 2 \times 10^4$ N/m.



Suppose a linear optical encoder and an accelerometer are used to measure x_1 and $\ddot{x}_2$ (note that $\ddot{x}_2$ can be expressed as an output of the system in terms of the state variables from the equations of motion). Assume a white Gaussian input noise of zero mean and standard deviation σ (i.e., V=σ^2) and measurement noise covariance W=$\begin{bmatrix} 0.01 & 0 \\ 0 & 1 \end{bmatrix}$.

Design an LQG controller to minimize the expected value of $J = \int_0^\infty x_2^{\,2} + ru^2 \, dt$ for some *r>0*. Starting with nominal values of σ=1 and r=1, fine tune your design to achieve the fastest step response for the nominal plant parameters while maintain stability for the load mass $0.5 \le M_2 \le 2$. Plot you responses for M_2=0.5, 1, and 2.