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ME6403 **HW SET #6** Due 04/05/2021

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1) Prove that a system in
a) Controllable Canonical Form (CCF) is always controllable.
b) Observable Canonical Form (OCF) is always observable.

2) Consider a single input single output system $x^+=Gx+Hu$, $y=Cx$ with a diagonal G matrix. Under what condition is the system controllable and observable? Prove your assertion.

3) Consider the system $x^+=Gx+Hu$ given by $G=\begin{bmatrix}0.5 & 1\\ 0.5 & 0.7\end{bmatrix}$ and $H=\begin{bmatrix}0.2\\ 0.1\end{bmatrix}$.
a) Is this system stable or unstable? why?
b) Design a state feedback controller $u=-Kx+r$ that places the closed-poles at $0.4\pm0.3j$.
c) Find the reference input r so that $y=x_1$ reaches the desired output $y_d=1$.
d) Design a feedback controller that that accomplishes the tracking objective in (c) in spite of an unknown but constant disturbance d influencing the system according to $x^+=Gx+Hu+Fd$, $F=\begin{bmatrix}0.1\\ -0.1\end{bmatrix}$. You may place the closed loop poles at $0.4\pm0.3j$ and 0.5.
e) Simulate the response of the system in (d) for $y_d=1$ and $d=1$. Plot x(t) and u(t) vs.t.

4) Consider a 3rd order discrete-time system in Observable Canonical form: $x^+=Gx+Hu$, $y=Cx$, where $G=\begin{bmatrix}a_1 & 1 & 0\\ a_2 & 0 & 1\\ a_3 & 0 & 0\end{bmatrix}$, $H=\begin{bmatrix}b_1\\ b_2\\ b_3\end{bmatrix}$, $C=[1\ 0\ 0]$.
a) Design a full-order dead-beat observer of the form $\hat{x}^+=G_o\hat{x}+Hu+Ly$ by finding G_o and L.
b) Design a reduced-order dead-beat observer of the form $\mathbf{z}^+=G_r\mathbf{z}+H_uu+H_yy$, $\hat{x}_b=\mathbf{z}+\mathbf{L}y$ by finding all the associated matrices G_r, H_u, …