The George W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology

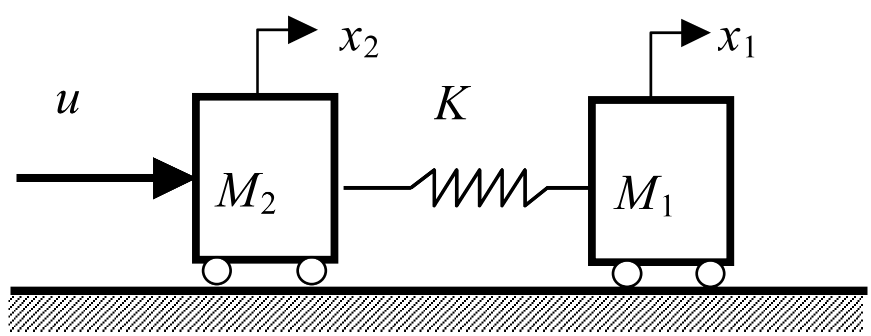
ME6401 **HW SET #6** Due Monday 11/2/2020

1. A projectile is following a constant velocity profile with an evasive sinusidal perturbation of frequency $\omega$ along each direction (i.e., $\dot{x} = c_1 + c_2 \sin \omega t$) in the $xyz$ plane. It is easily seen that $xyz$ satisfy

$$\begin{aligned} \dddot{x} + \omega^2 \dot{x} &= 0 \\ \dddot{y} + \omega^2 \dot{y} &= 0 \\ \dddot{z} + \omega^2 \dot{z} &= 0 \end{aligned}$$

Design an observer that can estimate the state (i.e., position, velcity, and acceleration) in real time from postion $(x, y, z)$ measurement so that it can be intercepted.

2. Consider the speed control of the 2–mass spring system shown below.




The state equation of the system taking the velocities of the two masses and the relative displacement as the states is given by

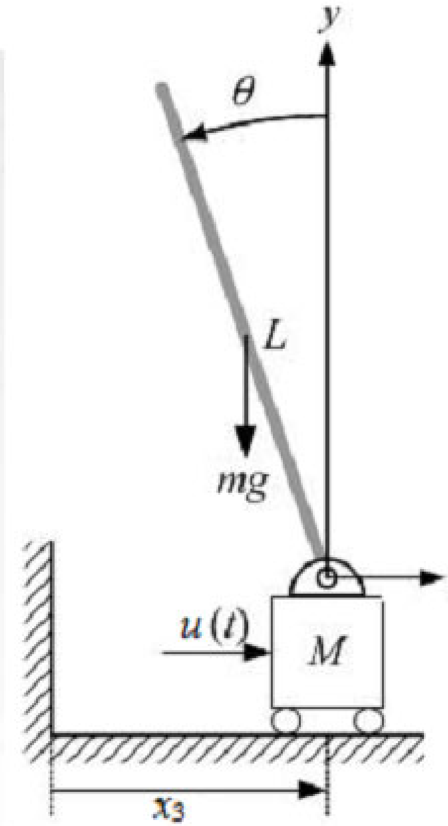
$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & \frac{-K}{M_1} \\ 0 & 0 & \frac{K}{M_2} \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M_2} \\ 0 \end{bmatrix} u, \; y = x_1$$

Assume the nominal values of $M_1 = M_2 = 1$ Kg and let $\omega = \sqrt{2K}$ denote the natural freuqncy of the system.
a) Design a state feedback controller that places the closed loop poles of the system at $-\omega$ and $-\omega \pm \omega j$. Find the resulting closed loop transfer function of the system between the reference $r$ and output $y$ (recall that $u = -Kx + r$)
b) Find the reference $r$ such that $y$ approaches $y_d = 1 - \cos \omega t$ asymptotically.
c) Design an observer to estimate the states needed in the feedback controller in (a). Place the observer poles at $-10\omega$ and $-10\omega \pm 10\omega j$ (somewhat faster than the controller poles).
d) Design a reduced order (second order) observer instead of the full order observer in (c). Place the observer poles at $10(-\omega \pm \omega j)$.
e) Show that the combined controller/reduced order observer can be represented with the block diagram in the Dynamic Compensation slides slides. Find $F_1$ and $F_2$ transfer functions.

3. An inverted pendulum on a driven cart is modeled as

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 30 & 0 & 0 & 20 \\ 0 & 0 & 0 & 1 \\ -2 & 0 & 0 & -10 \end{bmatrix} x + \begin{bmatrix} 0 \\ -25 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = \begin{bmatrix} 40 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix} x$$




where the state vector $x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T$ consists of, respectively, the pendulum angle, the pendulum's angular velocity, the cart position, and the cart's velocity. The input represents the voltage applied to the wheels on the cart.

a) Design a stabilizing state-feedback controller with a full-order observer and a reduced-order (first order) observer. Compare the performance of each using Matlab or Simulink simulations.

b) Can the system be observed through only one of the outputs? Which one?