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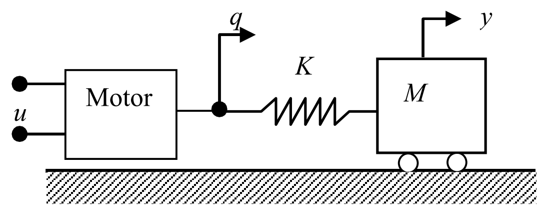
ME6403 Homework Set #7 Due Wednesday 4/14/2021

The figure below depicts a motion control device powered by a D.C. motor with a flexible coupling. This type of system is frequently encountered in machine tool servoing and robotics. The equations of motion for the mass and the motor may be derived as follows: $M\ddot{y} + K(y - q) = 0$, $Ri + K_b\dot{q} = u$, and $F_{motor} = K_f i = K(q - y)$, where u is



the input voltage, i is the motor current, R is the motor armature resistance and K_b and K_f are the motor back-emf and force constants, respectively. Assume the nominal values of M=5 Kg, K=500 N/m, K_b=K_f=10, and R=0.1 Ohms.

1) Derive the continuous-time state equation for the system taking $\mathbf{x}=[y\ q\ dy/dt]^T$ as state vector and u as input. Discretize the state equation assuming a sampling rate of 100 Hz (T=0.01).
2) Design a dynamic (combined state feedback/Reduced order observer) controller assuming availability of q and i such that the step response of y to a step reference input r settles down in less than 0.1 second, has maximum overshoot of 10%, and zero steady-state error. The desired (continuous-time) closed-loop poles may be placed at -25±25j and -25. The integral controller is not necessary for this part! Why?
3) Simulate the performance of your controller in (2) by plotting y and u vs. t assuming a constant desired output y_d=1. Examine the robustness of your controller with respect to variations in mass M of up to 50%.
4) Design a polynomial controller to accomplish the same objectives as those in (2). Repeat the simulations in (3) for the polynomial controller.
5) Add a ZPET Controller to the polynomial controller in (4) to track the following periodic desired trajectory: $y_d(t)=4t^2(3-4t)$, $0\le t\le 0.5$, $y_d(t)=1$, $0.5\le t\le 1$, $y_d(t)=1-y_d(t-1)$, $1\le t\le 2$, and $y_d(t+2)=y_d(t)$.
6) Add a stable repetitive controller to the ZPET controller in (5) to automatically generate the reference input r needed to track y_d specified in (5).
7) Simulate the performance of the tracking controller in (5) and (6). Examine the robustness of the two controllers respect to variations in mass M of up to 50%.