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KONERU LAKSHMAIAH EDUCATION FOUNDATION
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Department of Mathematics

DISCRETE MATHEMATICS

I/IV-B.Tech-(I/II Sem), Academic Year: 2025-2026

Session-9(Modelling Functions: Basic Concepts, Injective and Bijective Functions,**)**

Instructional Objective:

1. To understand and modelling Functions, of the real word problems.
2. To solve the Injective and Bijective Functions, problems.

Learning Outcomes:

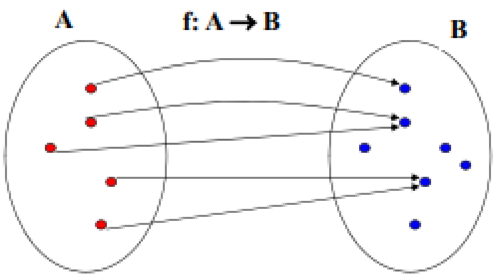
1. Able to be model the Functions, of the real word problems.
2. Able to solve the Injective and Bijective Functions, problems.

Introduction: In this chapter we focus our attention on a special type of functions, that play an important role in mathematics, computer science, and many applications. We also define some functions used in computer science and examine the growth of functions.

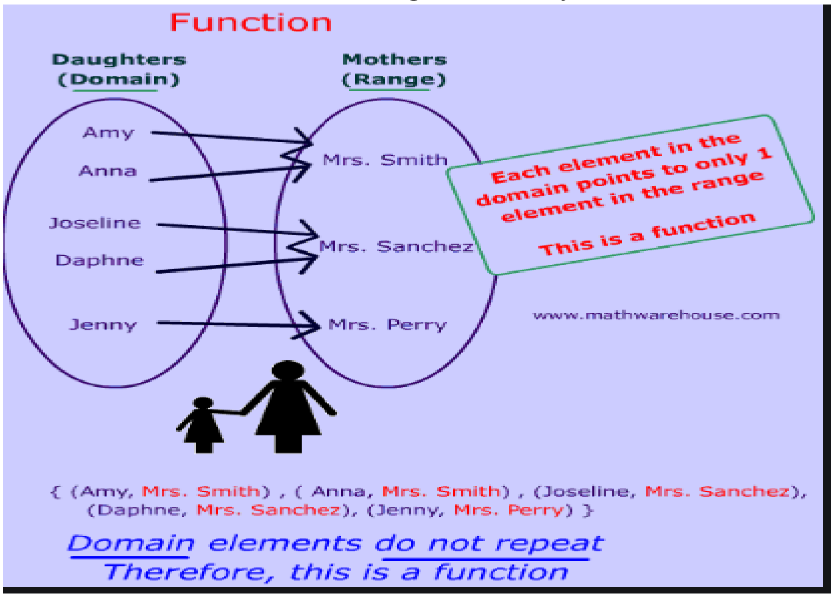
Explanation:

FUNCTIONS

Definition: A function f from a set A to a set B assigns each element of A to exactly one element of B and is denoted by $f : A \to B$. We write f(a) = b to denote the assignment of b of B to an element a of A by the function f.



Here A is called domain of f, B is called co domain of f and the set of images of all elements of A is called the range of f denoted by R_f.Clearly $R_f \subseteq B$.



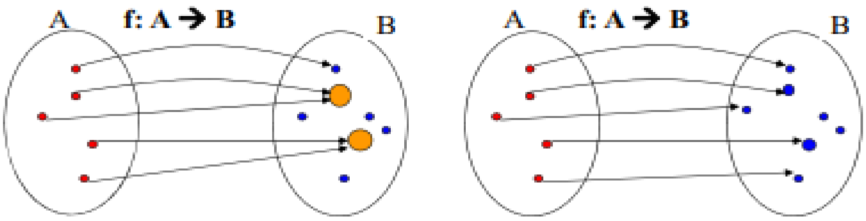
Problem 1. Suppose that each student in a discrete mathematics class is assigned a letter grade from the set {A, B, C, D, F}. And suppose that the grades are A for Adams, C for Chou, B for Good friend, A for Rodriguez, and F for Stevens. What is the domain, co domain, and range of the function that assigns grades to students?

Solution: Let G be the function that assigns a grade to a student in our discrete mathematics class. Note that G(Adams) = A, for instance. The domain of G is the set {Adams, Chou, Good friend, Rodriguez, Stevens}, and the co domain is the set {A, B, C, D, F}. The range of G is the set {A, B, C, F}, because each grade except D is assigned to some student.

Problem 2. Let f be the function that assigns the last two bits of a bit string of length 2 or greater to that string. For example, f (11010) =10. Then, the domain of f is the set of all bit strings of length 2 or greater, and both the co domain and range are the set {00,01,10,11}.

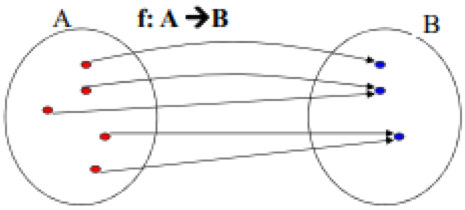
Types of Functions

One-to-one or Injective function: A function f is said to be one-to-one, or injective, if distinct elements of A are mapped into distinct elements of B, i.e., if $f(a) = f(b)$ implies $a = b$ for all $a, b \in A$.



Not injective function **Injective function**

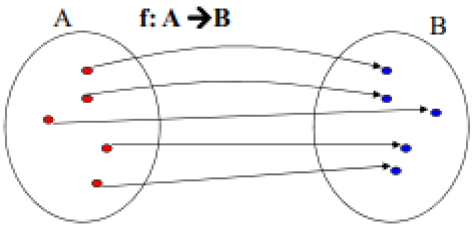
Onto function or Surjective function: A function f from A to B is called onto or surjective if for every $b \in B$ there is an element $a \in A$ such that $f(a) = b$.



Problem: Suppose that each worker in a group of employees is assigned a job from a set of possible jobs, each to be done by a single worker. In this situation, the function f that assigns a job to each worker is one-to-one. To see this, note that if x and y are two different workers, then $f(x) \neq f(y)$ because the two workers x and y must be assigned different jobs.

The function f is onto if for every job there a worker is assigned this job. The function f is not onto when there is at least one job that has no worker assigned it.

Bijective function: A function f is called a bijection if it is both one-to-one and onto.



Example 1:

- Let A = {1,2,3} and B = {a,b,c}
  - Define f as
    - $1 \rightarrow c$
    - $2 \rightarrow a$
    - $3 \rightarrow b$
- Is f a bijection?
- **Yes.** It is both one-to-one and onto.

Problem 1:

The total cost of airfare on a given route is comprised of the base cost C and the fuel surcharge S in rupees. Both C and S are functions of the mileage m; C(m) = 0.4m + 50 and S(m) = 0.03m. Determine a function for the total cost of a ticket in terms of the mileage and find the airfare for flying 1600 miles.

Solution:

Total cost $= C(m) + S(m) = 0.4m + 50 + 0.03m$

$$T(m) = 0.43\ m + 50$$

To find the airfare for flying 1600 miles, we have to apply 1600 instead of m.

$T(m) = 0.43\ (1600) + 50 = 688 + 50 = 738$

So, the total cost of airfare for flying 1600 miles is 738.

Problem 2:

A salesperson whose annual earnings can be represented by function $A(x) = 30,\ 000 + 0.04x$, where x is the rupee value of the merchandise he sells. His son is also in sales and his earnings are represented by the function $S(x) = 25,\ 000 + 0.05x$. Find $(A + S)\ (x)$ and determine the total family income if they each sell Rupees 1, 50, 00, 000 worth of merchandise.

Solution:$(A + S)\ (x) = A(x) + S(x)$

$A(x) = 30,\ 000 + 0.04x$ -------(1)

$S(x) = 25,\ 000 + 0.05x$ -------(2)

$(1) + (2) = 30,\ 000 + 0.04x + 25,\ 000 + 0.05x$

$A(x) + S(x) = 55000 + 0.09x$. Here $x = 15000000$

$A(x) + S(x) = 55000 + 0.09(15000000) = 55000 + 1350000$

Total income = 1405000. So, the required income is 1405000.

Problem 3: The function for exchanging American dollars for Singapore Dollar on a given day is $f(x) = 1.23x$, where x represents the number of American dollars. On the same day the function for exchanging Singapore Dollar to Indian Rupee is $g(y) = 50.50y$, where y represents the number of Singapore dollars. Write a function which will give the exchange rate of American dollars in terms of Indian rupee.

Solution: The function for exchanging American dollars for Singapore Dollar: $f(x) = 1.23x$

S.D = 1.23 (A.D). Here "x" stands for American dollar and f(x) stands for Singapore dollar.

A.D = S.D/1.23 ----(1)

Exchanging Singapore Dollar to Indian Rupee is

$g(y) = 50.50y$, I.R = 50.50 (S.D), S.D = I.R/50.50

Applying S.D = I.R/50.50 in (1), we get A.D = (I.R/50.50)/1.23, A.D = I.R/62.115

I.R = 62.115 AD. So, the exchange rate of American dollars in terms of Indian rupee is I.R = 62.115 AD.

Problem 4: The owner of a small restaurant can prepare a particular meal at a cost of Rupees 100. He estimates that if the menu price of the meal is x rupees, then the number of customers who will order that meal at that price in an evening is given by the function $D(x) = 200-x$. Express his day revenue, total cost, and profit on this meal as functions of x.

Solution: Cost price of the meal = 100, Selling price = x

Number of customers = 200 - x

1 day revenue = No of customers · x = (200 - x) · x,

1 day revenue = 200 x - x^2

Total cost = Cost of meal · No of customers = 100 · (200 - x) = 20000 - 100x

Profit = Total cost - 1 day revenue (200 - x) · x - 100 · (200 - x)

5. Which of the following relations are functions? Give reasons and also find the domain and range of the function.

(i) f = {(1, 3), (1, 5), (2, 3), (2, 5)}

(ii) g = {(2, 1), (5, 1), (8, 1), (11, 1)}

Solution: (i) f = {(1, 3), (1, 5), (2, 3), (2, 5)}

Here, the elements 1 and 2 have more than one f-images, namely 3 and 5.

Hence, f is not a function.

(ii) g = {(2, 1), (5, 1), (8, 1), (11, 1)}

Here, each first element of the ordered pair has a unique image which is the second coordinate.

Hence, g is a function.

Domain of g = {2, 5, 8, 11} and range of g = {1}

6. Let A = {1, 2} and B = {3, 6} and f and g be functions from A to B, defined by $f(x) = 3x$ and $g(x) = x^2 + 2$. Show that f = g.

Solution: Since both f and g are defined from set A.

Therefore, dom(f) = dom(g)

Now, for the co-domain of f

f(1) = 3. 1 = 3

f(2) = 3. 2 = 6

Co-domain of f ={3, 6}

And the co-domain of g

$g(1) = 1^2 + 2 = 3$

$g(2) = 2^2 + 2 = 6$

Co-domain of g ={3, 6}

Co-domain of f = Co-domain of g

Also for each x ∈ A, f(x) = g(x)

Hence, f = g.

Two functions f and g are said to be equal functions if

- **both functions are defined on the same domain**
- **both the functions have the same co-domain and range**
- **for every element in their domain, both functions have the same image**

7. Find the domain and the range of the real function, f(x) = 1/(x + 3).

Solution: We have f(x) = 1/(x + 3)

Clearly, f is not defined for x = -3

Therefore, dom(f) = R – {-3}

Let y = f(x). Then,

y = 1/(x + 3) ⇒ x = (1/y) – 3 …….(i)

Clearly, (i) is not defined for y = 0

Therefore, range(f) = R – {0}

8. Let f: N → N: f(x) = 2x for all x in N. Check whether f is a bijection.

Solution: To prove f(x) is a bijection, we need to prove that f is one-one as well as onto.

f is one-one:

Let x_1, x_2 be any arbitrary element in N

$f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$

Therefore f is one-one

f is onto:

Let f(x) = y in co-domain of f

Then, $2x = y \Rightarrow x = y/2$

For every y in N, $y/2 \notin N$

Therefore, f is not onto

Hence, f is not a bijection.

9. Let f: R →R: f(x) = x^2 and g: R →R: g(x) = 2x + 1. Find (f + g) (x).

Solution: Here, dom(f) = R = dom(g)

Therefore, dom(f) ∩ dom(g) = R

Then, $(f + g)(x) = f(x) + g(x) = x^2 + 2x + 1$

10. Let f(x) = √x and g(x) = x be two functions defined over the set of non-negative real numbers. Find (f/g) (x).

Solution: Both functions are defined on the domain of non-zero real numbers.

Then, $\mathrm{dom}(f/g) = \mathrm{dom}(f) \cap \mathrm{dom}(g) - \{x: g(x) = 0\} = [0, \infty) \cap [0, \infty) - \{0\} = (0, \infty)$

So, $(f/g)(x): (0, \infty) \rightarrow R$ is given by.

$(f/g)(x) = f(x)/g(x) = \sqrt{x}/x = 1/\sqrt{x}$; x is not equal to zero.

11. Write the domain of the following real functions

(i) $f(x) = \dfrac{2x+1}{x-9}$ (ii) $p(x) = \dfrac{-5}{4x^2+1}$

(iii) $g(x) = \sqrt{x-2}$

(iv) $h(x) = x + 6$

Solution:

i) $f(x) = \dfrac{2x+1}{x-9}$

Domain = R – {9}

HINT

If $x = 9$

$f(x) = \dfrac{2(9)+1}{9-9}$

$= \dfrac{18+1}{0}$

= Not defined

ii) $p(x) = \dfrac{-5}{4x^2+1}$

Domain = R

iii) $g(x) = \sqrt{x-2}$

Domain = {2, 3, 4, 5..........}

HINT

If x = 0 and less than 0

$g(0) = \sqrt{0-2} = \sqrt{2} \notin R$

iv) $h(x) = x + 6$

Domain = R

12. Show that the function f: **N** → **N** defined by $f(x) = 2x - 1$ is one-one but not onto.

Solution:

$f(x) = 2x - 1$

$f(1) = 2(1) - 1 = 2 - 1 = 1$

$f(2) = 2(2) - 1 = 4 - 1 = 3$

$f(3) = 2(3) - 1 = 6 - 1 = 5$

$f(4) = 2(4) - 1 = 8 - 1 = 7$ .... etc

co-domain = {1, 2, 3, 4, 5................}

Range = {1, 3, 5, 7................}

It is one – one because distinct elements of first set have distinct images in 2nd set. It is not onto because the co-domain and the range are not same.

13. Show that the function f: **N** → **N** defined by $f(m) = m2 + m + 3$ is one-one function.

Solution:

Given $f(m) = m^2 + m + 3$

$f(1) = (1)^2 + 1 + 3 = 5$

$f(2) = (2)^2 + 2 + 3 = 9$

$f(3) = (3)^2 + 3 + 3 = 15$...................etc

So the function f is one–one. Since every element in 1st set have distinct image in 2nd set.

14. Let $A = \{1, 2, 3, 4\}$ and $B = \mathbf{N}$ Let $f: A \to B$ be defined by $f(x) = x^3$ then,

(i) find the range of f

(ii) identify the type of function

Solution:

Given $f(x) = x^3$

$f(1) = 1^3 = 1$

$f(2) = 2^3 = 8$

$f(3) = 3^3 = 27$

$f(4) = 4^3 = 64$

i) Range = $\{1, 8, 27, 64\}$

ii) Type of function is one-one and into function

15. In each of the following cases state whether the function is bijective or not. Justify your answer.

(i) $f: \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 2x + 1$

(ii) $f: \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 3 - 4x^2$

Solution:

i) Given $f(x) = 2(x) + 1$

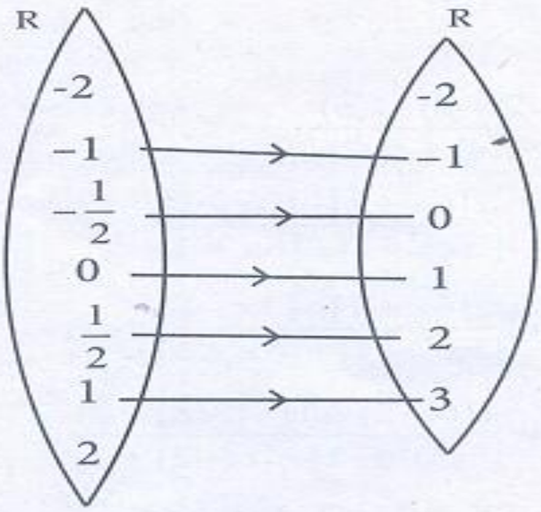
$f(0) = 2(0) + 1 = 1$

$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right) + 1 = 2$

$f\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right) + 1 = 0$

$f(1) = 2(1) + 1 = 3$

$f(-1) = 2(-1) + 1 = -1$



So it is a bijective function.

ii) $f : R \rightarrow R$ defined by $f(x) = 3 - 4x^2$

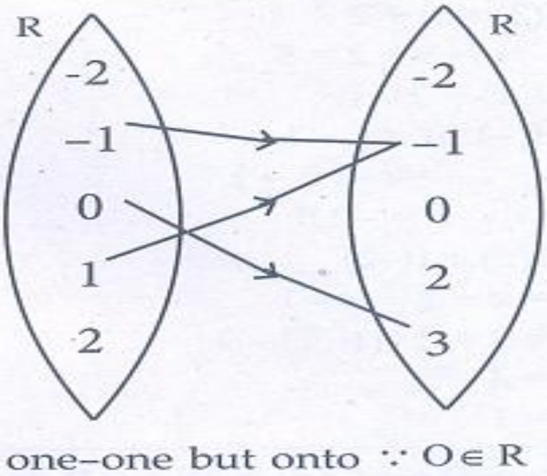
Solution:

Given $f(x) = 3 - 4x2$

$f(0) = 3 - 4(0) = 3$

$f(1) = 3 - 4(1)^2 = -1$

$f(-1) = 3 - 4(-1)^2 = -1$



So it is one–one but onto $\because O \in R$ has no pre image in first set

So it is not bijective.

Session-10 (Functions: Inverse and Composite Functions)

Instructional Objective:

3. To understand and model Inverse and Composite Functions, of the real word problems.
4. To solve the Inverse and Composite Functions problems.

Learning Outcomes:

3. Able to model the Inverse and composite Functions, of the real word problems.
4. Able to solve the Inverse and Composite Functions, problems.

Introduction: In this chapter we focus our attention on a special type of Inverse and Composite function that play an important role in mathematics, computer science, and many applications. We also define some functions used in computer science and examine the growth of functions.

Explanation: (Functions Continuation)

Identity function: The function $f : A \to A$ is called identity function if $f(x) = x$, for all $x \in A$.

Composition of functions: If $f: A \to B$ and $g: B \to C$, then the composition of f and g is a new function from A to C denoted by $g \circ f$, is given by $(g \circ f)(x) = g\big(f(x)\big)$, for all $x \in A$.

Note: 1. $g \circ f$ is defined, where as $f \circ g$ need not be defined. So, the composition of functions is not commutative.

2.Composition of functions is associative. i.e., if $f: A \to B$, $g: B \to C$ and $h: C \to D$ are functions, then $h \circ (g \circ f) = (h \circ g) \circ f$.

3. If $f: A \to B$ and $g: B \to C$, then the composition $g \circ f: A \to C$ is one-to-one or onto or bijection according as f and g are one-to-one or onto or bijection

Inverse of a function: If $f: A \to B$ and $g: B \to A$, then the function g is called the inverse of the function f if $g \circ f = I_A$and $f \circ g = I_B$.

Note: 1. The inverse of g is f.

Properties:

1. The inverse of the function if exists is unique.

2. The necessary and sufficient condition for the function f to be invertible is that f is one-to-one and onto.

3. If f and g are invertible functions then $g \circ f$ is also invertible and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

Note: If a set A has m elements and a set B has n elements, then

a) the number of functions from A to B is n^m

b) the number of one-to-one functions from A to B is n_{P_m} if m<n, otherwise 0.

c) the number of onto functions from A to B is $2^m - 2$ if $m \geq 2$, otherwise 0.

d) the number of bijective functions from A to B is $n!$ if n = m, otherwise 0.

Problem 1: The formula for converting from Fahrenheit to Celsius temperatures is $y = (5x/9) - (160/9)$. Find the inverse of this function and determine whether the inverse is also a function.

Solution:

$y = (5x - 160)/9$

$9y = 5x - 160$

$5x = 9y + 160$

$x = (1/5)(9y + 160)$

$f^{-1}(x) = (1/5)(9x + 160)$

$f^{-1}(x) = (9x/5) + (160/5)$

$f^{-1}(x) = (9x/5) + 32$

Inverse is also a function.

Problem 2:

A simple cipher takes a number and codes it, using the function $f(x) = 3x-4$. Find the inverse of this function, determine whether the inverse is also a function and verify the symmetrical property about the line $y = x$ (by drawing the lines).

Solution:

$$f(x) = 3x-4$$

$$\text{Let } y = 3x - 4$$

$$3x = y + 4$$

$$x = (y + 4)/3$$

$f^{-1}(x) = (x + 4)/3$ Inverse is also a function.

3. Let f: R →R : $f(x) = x^2 + 1$. Find $f^{-1}(10)$.

Solution: Let $f^{-1}(10) = x$.

$\Rightarrow f(x) = 10$

$\Rightarrow x^2 + 1 = 10$

$\Rightarrow x = \pm 3$

Therefore, $f^{-1}(10) = \{-3, 3\}$

The inverse of a Function

Let f: X → Y be a function and let for any x in X, f(x) = y ∈ Y. Then the inverse of f is defined by $f^{-1}(y) = \{x \in X: f(x) = y\}$.

Composition of Functions

Let f: A → B and g: B → C be two given functions. Then, the composition of f and g is represented by g o f which is the function, defined by

(g o f): f: A → C: (g o f)(x) = g{f (x)} for every x in A

4. Let R be the set of all real numbers. Let f: R →R : f(x) = cos x and let g: R →R : g(x) = $3x^2$. Show that (g o f) ≠ (f o g).

Solution: Let x be an arbitrary real number. Then,

(g o f)(x) = g{f(x)} = g{ cos x} = 3 $\cos^2 x$

(f o g)(x) = f{g(x)} = f{$3x^2$} = cos $3x^2$

Clearly, (g o f) ≠ (f o g).

5. Let f: R →R: f(x) = 4x + 3 for all x ∈ R. Show that f is invertible and find f^{-1}.

Solution: Let x_1, x_2 be in R

$f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3 \Rightarrow x_1 = x_2$

Therefore, f is one-one.

Let f(x) = y in co-domain R of f. Then,

∃ x = (y – 3)/4 in R, for every y in co-domain R of f

Therefore, f is onto

⇒ f is bijection

⇒ f^{-1} exists

Now, f(x) = y ⇒ $f^{-1}(y)$ = x = (y – 3)/4

Thus inverse of f is defined by

f^{-1}: R → R: $f^{-1}(y)$ = (y – 3)/4 for all y in R

A function f is invertible (inverse of f exists) if and only if f is a bijection.

6. Let A = {2, 3, 4, 5} and B = {7, 9, 11, 13} and let f = {(2, 7), (3, 9), (4, 11), (5, 13)}. Show that f is invertible and find f^{-1}.

Solution: We have f is a function such that f: A → B: f(x) = {(2, 7), (3, 9), (4, 11), (5, 13)}.

Clearly, each element in the co-domain set has a unique pre-image in the domain set. Therefore, f is one-one.

Now, range(f) = co-domain(f)

⇒ f is onto

⇒ f is a bijection

⇒ f is invertible

Hence, f^{-1} = {(7, 2), (9, 3), (11, 4), (13, 5)}

6. If $f(x) = x^2$, $g(x) = 3x$ and $h(x) = x - 2$, Prove that $(f \text{ o } g) \text{ o } h = f \text{ o } (g \text{ o } h)$

Solution:

$$\begin{aligned}
\text{fog}(x) &= \text{f}(\text{g}(x) = \text{f}(3x) \\
&= (3x)^2 \\
&= 9x^2 \\
(\text{fog})\text{oh}(x) &= \text{fog}(\text{h}(x) \\
&= \text{fog}(x-2) \\
&= 9\,(x-2)^2 \\
&= 9[x^2 - 4x + 4] \\
&= 9x^2 - 36x + 36 \text{ ——— } (1) \\
\text{goh}(x) &= \text{g}(\text{h}(x) = \text{g}(x-2) \\
&= 3(x-2) \\
&= 3x - 6 \\
\text{fo}(\text{goh})(x) &= \text{fo}(3x - 6) \\
&= (3x - 6)2 \\
&= 9x2 - 36x + 36 \text{ ——— } (2)
\end{aligned}$$

from (1) and (2) we get

(fog)oh = fo(goh)

8. If f (x) = $[x-1]/[x+1]$, $x\ {}^{1}\ 1$ show that $f(f(x)) = -1/x$ provided $x \neq 0$

Solution:

Given $f(x) = \dfrac{x-1}{x+1}$

$$f(x) = \left(\frac{x-1}{x+1}\right)$$

$$= \frac{\dfrac{x-1}{x+1} - 1}{\dfrac{x-1}{x+1} + 1}$$

$$= \frac{\dfrac{x-1-(x+1)}{x+1}}{\dfrac{x-1+x+1}{x+1}}$$

$$= \frac{-2}{-2x} \Rightarrow = \frac{-1}{x} \text{ proved}$$

Session-11: Functions for Computer Science: Ceiling Function, Floor Function, Boolean Function.

❖ Instructional Objective

This session is designed

1. To identify Ceiling function
2. To identify Floor function
3. To identify Boolean function

❖ Learning Outcomes

At the end of this session student is

1. Able to identify Ceiling function
2. Able to identify Floor function
3. Able to identify Boolean function

Introduction

❖ Special Types of functions

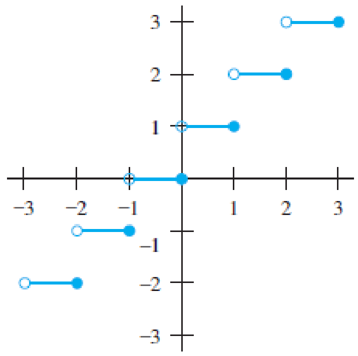
Here we introduce some special types of functions which are use full in computer science engineering.

1. The Ceiling function

The Ceiling function assigns to the real number ‘x’ the smallest integer that is greater than or equal to ‘x’. The value of the ceiling function at ‘x’ is denoted by$\lceil x \rceil$.

Example: $\left\lceil \frac{1}{2} \right\rceil = 1$, $\lceil 3.2 \rceil$.=4

In the following figure displays the graph of the ceiling function$\lceil x \rceil$.This function has the same value throughout the interval (n, n+1] namely ‘n+1’ and jumps to ‘n+2’ when ‘x’ is little greater than ‘n+1’.



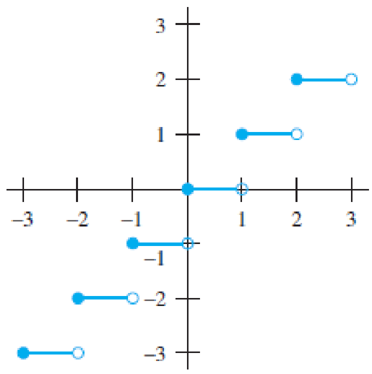
Graph of ceiling function y=$\lceil x \rceil$

2. The floor function

The floor function assigns the real number ‘x’ the largest integer that is less than or equal to ‘x’. The value of the *floor* function at ‘x’ is denoted by $\lfloor x \rfloor$

Example: $\left\lfloor \frac{1}{2} \right\rfloor = 0$, $\lfloor 3.2 \rfloor = 3$

The following figure displays the graph of floor function$\lfloor x \rfloor$. This function has the same value throughout the interval [n, n+1), namely n, and then it jumps to “n+1’when ‘x=n+1’



Graph of ceiling function y=$\lfloor x \rfloor$

3. *The Boolean function*

The circuits in computers and other electronic devices have inputs, each of which is either a 0 or a 1, and produce outputs that are also 0s and 1s. Circuits can be constructed using any basic element that has two different states. Such elements include switches that can be in either the on or the off position and optical devices that can be either lit or unlit. In 1938 Claude Shannon showed how the basic rules of logic, first given by **George Boolean** in 1854 in his The Laws of Thought, could be used to design circuits. These rules form the basis for Boolean algebra.

Boolean algebra provides the operations and the rules for working with the set {0, 1}. Electronic and optical switches can be studied using this set and the rules of Boolean algebra. The three operations in Boolean algebra that we will use most are complementation, the Boolean sum, and the Boolean product. The complement of an element, denoted with a bar, is defined by $\bar{o}$=1 and $\bar{1} = 0$the Boolean sum, denoted by + or by OR, has the following values: 1 + 1 = 1, 1 + 0 = 1, 0 + 1 = 1, 0 + 0 = 0. The Boolean product, denoted by · or by AND, has the following values: 1 · 1 = 1, 1 · 0 = 0, 0 · 1 = 0, 0 · 0 = 0. When there is no danger of confusion, the symbol · can be deleted, just as in writing algebraic products. Unless parentheses are used, the rules of precedence for Boolean operators are: first, all complements are computed, followed by all Boolean products, followed by all Boolean sums.

Example: Find the value of $1.0 + \overline{(0 + 1)}$

Solution: Using the definitions of complementation, the Boolean sum, and the Boolean product,

$$1.0 + \overline{(0 + 1)} = 0 + \bar{1} = 0 + 0 = 0$$

Boolean function: Let B = {0, 1}. Then $B^n = \{(x_1, x_2,\ldots,x_n) \mid x_i \in B \text{ for } 1 \leq i \leq n\}$ is the set of all possible n-tuples of 0s and 1s. The variable x is called a Boolean variable if it assumes values only from B, that is, if its only possible values are 0 and 1. A function from B^n to B is called a Boolean function of degree n.

The function F (x, y) = $x\bar{y}$ from the set of ordered pairs of Boolean variables to the set {0, 1} is a Boolean function of degree 2 with F (1, 1) = 0, F (1, 0) = 1, F (0, 1) = 0, and F (0, 0) = 0. We display these values of F in Table 1.

TABLE 1		
x	y	$F(x, y)$
1	1	0
1	0	1
0	1	0
0	0	0

Review questions:

1. Find the following values

a) $\lceil 4.5 \rceil$

b) $\lceil 7.8 \rceil$

c) $\lfloor 0.5 \rfloor$

d) $\lceil 10.5 \rceil$

Summary:

In this session graphs of functions and different types of graphs and their applications were clearly explained.

Self-assessment questions:

1. Find the following values

a) $\lceil 55/2 \rceil$

b) $\lceil 46/3 \rceil$

c) $\lfloor 50/4 \rfloor$

d) $\lceil 102/9 \rceil$

CLASSROOM DELIVARY PROBLEMS

1. Evaluate the following values

   a) $\lceil 29.5 \rceil$

   b) $\lceil -101.7 \rceil$

   c) $\lceil 201.2 \rceil$

   d) $\lfloor 98.8 \rfloor$

   e) $\lfloor 78.9 \rfloor$

2. Evaluate the following values

   a) $\lceil -30.4 \rceil$

   b) $\lceil 99.6 \rceil$

c) $\lceil 225.4 \rceil$
d) $\lfloor 98.1 \rfloor$
e) $\lfloor -78.5 \rfloor$

2. Find the values of the Boolean function represented by $F(x, y, z) = xy + \bar{z}$.

3. Find the values of the Boolean function represented by $f(x, y, z) = x\bar{y} + z\bar{x} + y$.

TUTORIAL PROBLEMS

1. Evaluate the following values
   a) $\lceil 10.4 \rceil$
   b) $\lceil 100.6 \rceil$
   c) $\lceil 226.4 \rceil$
   d) $\lfloor 97.1 \rfloor$
   e) $\lfloor 74.5 \rfloor$
2. Find the values of the Boolean function represented by $f(x, y, z) = \bar{x} + \bar{y} + \bar{z}$.

HOME ASSIGNMENT PROBLEMS

1. Evaluate the following values
   a) $\lceil 49.4 \rceil$
   b) $\lceil 1000.6 \rceil$
   c) $\lceil 625.4 \rceil$
   d) $\lfloor 700.1 \rfloor$
   e) $\lfloor 25.5 \rfloor$

2. Find the values of the Boolean function represented by $f(x, y, z) = \bar{x}x + \bar{y}y + \bar{z}z$.

Session-12 (PROPOSITIONAL LOGIC)

Instructional Objective:

1. To learn basic concepts of propositions
2. To understand types of logical operators

Learning Outcomes:

1. Able to understand the difference b/w sentence and statement.
2. Able to know application of usage of different types of logical operators.

Introduction

- The development of propositional logic came only much later with the advent of symbolic logic in the work of logicians developed by Augustus and DeMorgan (in 1806-1871).
- Later, George Boole (in 1815-1864) was developed and replaced a mathematical-style in "algebra" by Aristotelian syllogistic logic by employing the numeral "1" for the universal class, the numeral "0" for the empty class.
- Propositional Logic (statement Logic) is the branch of Logic
- It is used to understand and symbolized complicated statements which can bear the truth values of natural languages.
- It is helpful in developing text in algorithms.
- It is helpful to deduct the classical truth-functional propositional logic into Modal propositional logic in system understandable Languages.

Explanation:

Sentence: A sentence is usually collection of words

Proposition: A proposition is usually a declarative sentence for which we can assign truth values i.e. either "true" or "false" but not both.

The words Proposition/ statement / premise, all are having same meaning.

Note: all propositions are sentences but all sentences need not be a proposition.

Examples – Propositions

1	Sachin is a cricket player.	True	Proposition
2	2+2=10	False	Proposition
3	New Delhi is the capital of India	True	Proposition

Operator Name	Symbol	General usage
Conjuction	∧	**"and"**
Disjunction	∨	**"Or"**
Conditional or implication	→	**"If-then"**
Bi-conditional	↔	**" If and only if"**
Negation	⌉ or ~	**"Not"**

Compound statements:

If one or more statements relate to Logical operators, then resultant statement is known as compound statement.

Examples:

p: I study hard   q: I get Grade A

$p \wedge q$ = I study hard and I get Grade A

P V q = I study hard or I get Grade A

$p \rightarrow q$ = If I study hard then I get Grade A

$P \leftrightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$

(If I study hard then I get Grade A) and

(If I get Grade A then I study hard)

~p = I don't study hard

Truth tables :

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Problem-1 : Construct the truth tables for the following $[(p \vee q) \wedge (\sim r)] \leftrightarrow q$

p	q	r	pVq	~r	[(pVq) Λ(~r)]	[(pVq) Λ(~r)] ↔q
T	T	T	T	F	F	F
T	T	F	T	T	T	T
T	F	T	T	F	F	T
T	F	F	T	T	T	F
F	T	T	T	F	F	F
F	T	F	T	T	T	T
F	F	T	F	F	F	T
F	F	F	F	T	F	T

Problem-2 : Construct Truth table for $(p \vee \sim q) \rightarrow (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

1. If the truth value of a compound proposition is "TRUE" for all possible truth values of the propositions present in the compound proposition, then that compound proposition is known as a Tautology.
   Example: PV~P
2. If the truth value of a compound proposition is "FALSE" for all possible truth values of the propositions present in the compound proposition, then that compound proposition is known as a Contradiction.
   Example: PΛ~P
3. If the truth value of a compound proposition is either "TRUE" or "FALSE" for all possible truth values of the propositions present in the compound proposition, then that compound proposition is known as a Contingency.
   Example:
   (pVq)→p

Review Questions:

1. What is a Proposition?
2. What is a Tautology?

Summary:

Student will be able to learn construction of truth tables of the given compound propositions. Also, they verify that whether it is a tautology or not?

Self-assessment questions:

1. If the truth value of a compound proposition is either “TRUE” or “FALSE” for all possible truth values, then it is -----------
   (a) Contingency
   (b) Contradiction
   (c) Tautology
   (d) Proposition

   Ans: a
2. If the truth value of a compound proposition is either “TRUE” for all possible truth values, then it is -----------
   (a) Contingency
   (b) Contradiction
   (c) Tautology
   (d) Proposition

   Ans: c

3. **Terminal questions**

Classroom delivery problems

1. Identify which of the following are sentences or propositions?
   a) New Delhi is the capital of India.
   b) **for x=2,** $x + 7 = 5$.
   c) $x + 3 = 5$
   d) Did you understand?
2. Let p and q be the propositions.
   p: I study hard.
   q: My father will gift me a bike

   Express each of the following propositions in words.

   1) $\neg p$ **2)** $p \vee q$ **3)** $p \to q$

   4) $p \wedge q$ **5)** $p \leftrightarrow q$ **6)** $\neg p \to \neg q$

   7) $\neg p \wedge \neg q$ **8)** $\neg p \vee (p \wedge q)$

3. Let the following propositions be:
   P: The train has arrived.
   Q: The platform is crowded.

   Write the following statements in symbolic form using P and Q:
   a) If the train has not arrived, then the platform is not crowded.
   b) The train has arrived if and only if the platform is crowded.
   c) The train has arrived, and the platform is crowded.
   d) The platform is not crowded.

4. Use a truth table to determine whether $(P \to Q) \leftrightarrow (\neg Q \to \neg P)$ is a tautology.
5. Construct the truth table for $[\,(p \vee q) \wedge (\sim r)\,] \leftrightarrow (q \to r)$.
6. Construct the truth table for $\mathbf{[(P \wedge Q) \vee (\neg P \wedge \neg Q)]}$. Is it a tautology, contradiction, or contingency? Justify.
7. Prove that the statement $\mathbf{[(P \to Q) \wedge (P \wedge \neg Q)]}$ is a contradiction.
8. Show that $\mathbf{(P \wedge Q) \vee (\neg P \wedge Q)}$ is a contingency using a truth table.
9. Examine $\neg(p \to q) \to p$ is a tautology or not. If so, justify your answer.
10. Construct the truth table for $(p \wedge q) \vee (\sim p \wedge r)\} \vee (q \wedge r)$. Is it a tautology, contradiction, or contingency? Justify.

Home Assignment Problems

1. **Identify which of the following are sentences or propositions. Justify your answer.**

   a) The Sun rises in the east.
   b) $x^2 + 5 = 14$
   c) What time is it?
   d) For $x = 3$, $x^2 - 2x = 3$
   e) Please sit down.

2. **Let *p* and *q* be the propositions.**

   p: I complete my homework. q: I can play video games.

   Express each of the following propositions in words.

   a) $\neg p$ **b)** $p \wedge q$ **c)** $p \to q$ **d)** $p \vee \neg q$

   e) $p \leftrightarrow q$ **f)** $\neg p \to q$ **g)** $\neg p \wedge q$ **h)** $(p \vee q) \wedge \neg q$

3. Examine $(p \wedge q) \to p$ is a tautology or not. If so, justify your answer.
4. Show that $(p \vee \sim q) \to (p \wedge q)$ is a contingency.
5. Verify $[q \wedge \sim p) \leftrightarrow (q \to p)$ is Contradiction or not.
6. Construct the truth tables for $(p \to q) \vee ((\sim p \wedge \sim q)$. Is it a tautology, contradiction, or contingency? Justify your answer.

Tutorial Problems

1. Identify which of the following are sentences or propositions. Justify your answer.

   a) Water boils at 100°C.

   b) $x > 10$

   c) Who are you?

   d) For $x = 5$, $x^2 = 25$

   e) Open the window.

2. Let P: The student has completed the assignment.
   Q: The student passes the course.
   Write the following statement in symbolic form
   "If the student has not completed the assignment or has not passed the course, then they must repeat the subject."
3. Construct the truth table for $[(p \vee q) \wedge (\sim r)] \leftrightarrow q$.
4. Examine $p \rightarrow (p \vee q)$ is a tautology or not. If so, justify your answer.
5. Construct the truth tables for $(p \vee q) \wedge ((\sim p) \vee (\sim r))$. Is it a tautology, contradiction, or contingency? Justify your answer.
6. Verify $\neg p \rightarrow (p \rightarrow q)$ is a tautology or not.
7. **Prove the following are tautologies**
   a. $\neg(p \rightarrow q) \rightarrow \neg q$
   b. $(p \wedge q) \rightarrow (p \rightarrow q)$

Session-13 (Propositional Logical Equivalences)

Instructional Objective:

1. To learn logical equivalent propositions of the compound statements
2. To learn how to construct truth table of compound statement.

Learning Outcomes:

1. To learn equivalent propositions of the given statements using truth tables

Introduction:

Explanation:

Converse , Inverse, Contra Positive Of a Conditional Statement:

Converse of $p \rightarrow q$ is $q \rightarrow p$

Inverse of $p \rightarrow q$ *is* $\neg p \rightarrow \neg q$

Contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$.

Note:

1) $p \rightarrow q \equiv \neg q \rightarrow \neg p$.

Means $p \rightarrow q$ and $\neg q \rightarrow \neg p$ *are* logical equivalent

2) $q \rightarrow p \equiv \neg p \rightarrow \neg q$

Means $q \rightarrow p$ and $\neg p \rightarrow \neg q$ *are* logical equivalent

p	q	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$.	$q \rightarrow p$	$\neg p \rightarrow \neg q$
T	T	T	F	F	T	T	T
T	F	F	T	F	F	T	T
F	T	T	F	T	T	F	F
F	F	T	T	T	T	T	T

Problem: Express the contrapositive, converse and the inverse of the given conditional statement

"The home team wins whenever it is raining".

We understand it as "q whenever p"

So, we represent it as $p \to q$

Consider **P: it is raining** **q: the home team wins**

the original statement can be rewritten as:

"If it is raining, then the home team wins."

Converse($q \to p$): If the home team wins, then it is raining

Inverse($\neg p \to \neg q$): If it is not raining, then the home team does not win."

Contrapositive($\neg q \to \neg p$): "If the home team does not win, then it is not raining."

TABLE 7 Logical Equivalences Involving Conditional Statements.
$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

TABLE 8 Logical Equivalences Involving Biconditional Statements.
$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Review Questions:

1. What are the De morgan laws?
2. What is converse of the $p \to q$?

Summary: Student will be able to learn equivalent propositions of the given statements using truth tables

Self-assessment questions:

1. What is inverse of $p \to q$?
   a) $\neg p \to \neg q$
   b) $\neg p \to q$
   c) $p \to \neg q$
   d) $\neg q \to \neg p$

Ans: a

2. What is logically equivalent of $p \to q$?
   a) $\neg p \to \neg q$
   b) $\neg p \vee q$
   c) $p \to \neg q$
   d) $\neg q \to \neg p$

Ans: b

Terminal Questions:

Classroom Delivery Problems

1. Express the contrapositive, converse and the inverse form of the given conditional statement "***I wear sunglasses whenever it is sunny.***"
2. Verify $\neg(p \vee q)$ and $(\neg p \wedge \neg q)$ are logically equivalent or not.
3. Show that $p \to q \equiv \neg p \vee q$
4. Examine that $p \to (q \to r) \equiv (p \wedge q) \to r$

Home Assignment Problems

1. Express the contrapositive, converse and the inverse form of the given conditional statement "***The dog barks if it sees a stranger.***"
2. Show that contrapositive of a given conditional statement and conditional statements are logically equivalent using truth tables.
3. Show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.
4. Use Truth table to test the logical equivalence of $(p \vee q) \wedge (p \vee \neg q) \equiv p$.

Tutorial Problems

1. Express the contrapositive, converse and the inverse form of the given conditional statement "***If it is raining, then I take an umbrella.***"
2. Show that converse and inverse of a given conditional statement are logically equivalent using truth tables.
3. Illustrate De Morgan's law. Demonstrate their logical equivalence using the truth table.
4. Show that $\neg(p \to q) \equiv (p \wedge \neg q)$
5. Use Truth table to test the logical equivalence of $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.

Rules of Inference

Later in this chapter we will study proofs. Proofs in mathematics are valid arguments that establish the truth of mathematical statements. By an **argument**, we mean a sequence of statements that end with a conclusion. By **valid**, we mean that the conclusion, or final statement of the argument, must follow from the truth of the preceding statements, or **premises**, of the argument. That is, an argument is valid if and only if it is impossible for all the premises to be true and the conclusion to be false. To deduce new statements from statements we already have, we use rules of inference which are templates for constructing valid arguments. Rules of inference are our basic tools for establishing the truth of statements. Before we study mathematical proofs, we will look at arguments that involve only compound propositions. We will define what it means for an argument involving compound propositions to be valid. Then we will introduce a collection of rules of inference in propositional logic. These rules of inference are among the most important ingredients in producing valid arguments. After we illustrate how rules of inference are used to produce valid arguments, we will describe some common forms of incorrect reasoning, called **fallacies**, which lead to invalid arguments.

After studying rules of inference in propositional logic, we will introduce rules of inference for quantified statements. We will describe how these rules of inference can be used to produce valid arguments. These rules of inference for statements involving existential and universal quantifiers play an important role in proofs in computer science and mathematics, although they are often used without being explicitly mentioned. Finally, we will show how rules of inference for propositions and for quantified statements can be combined. These combinations of rule of inference are often used together in complicated arguments.

Definition:

An ***argument*** in propositional logic is a **sequence of propositions**. All but the final proposition in the argument are called *premises* and the final proposition is called the *conclusion*. An argument is ***valid*** if the truth of all its premises implies that the conclusion is true. An *argument form* in propositional logic is a sequence of compound propositions involving propositional variables. An argument form is *valid* no matter which particular propositions are substituted for the propositional variables in its premises, the conclusion is true if the premises are all true.

Rules of Inference for Propositional Logic:

We can always use a truth table to show that an argument form is valid.We do this by showing that whenever the premises are true, the conclusion must also be true. However, this can be a tedious approach.

Ex:- when an argument form involves 10 different propositional variables, to use a truth table to show this argument form is valid requires $2^{10} = 1024$ different rows. Fortunately, we do not have to resort to truth tables. Instead, we can first establish the validity of some relatively simple argument forms, called **rules of inference**. These rules of inference can be used as building blocks to construct more complicated valid argument forms. We will now introduce the most important rules of inference in propositional logic. The tautology $(p \wedge (p \rightarrow q)) \rightarrow q$ is the basis of the rule of inference called **modus ponens**, or the **law of detachment**. (Modus ponens is Latin for *mode that affirms*.) This tautology leads to the following valid argument form, which we have already seen in our initial discussion about arguments (where, as before, the symbol ∴ denotes “therefore”):

$$\begin{array}{rl} & p \\ & p \rightarrow q \\ \hline \therefore & q \end{array}$$

Using this notation, the hypotheses are written in a column, followed by a horizontal bar, followed by a line that begins with the therefore symbol and ends with the conclusion. In particular, modus ponens tells us that if a conditional statement and the hypothesis of this conditional statement are both true, then the conclusion must also be true. Example 1 illustrates the use of modus ponens.

TABLE 1 Rules of Inference.

Rule of Inference	*Tautology*	*Name*
p $p \to q$ $\therefore q$	$(p \land (p \to q)) \to q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	$(\neg q \land (p \to q)) \to \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$	Hypothetical syllogism
$p \lor q$ $\neg p$ $\therefore q$	$((p \lor q) \land \neg p) \to q$	Disjunctive syllogism
p $\therefore p \lor q$	$p \to (p \lor q)$	Addition
$p \land q$ $\therefore p$	$(p \land q) \to p$	Simplification
p q $\therefore p \land q$	$((p) \land (q)) \to (p \land q)$	Conjunction
$p \lor q$ $\neg p \lor r$ $\therefore q \lor r$	$((p \lor q) \land (\neg p \lor r)) \to (q \lor r)$	Resolution

Using Rules of Inference to Build Arguments:

When there are many premises, several rules of inference are often needed to show that an argument is valid.

Ex: Let p be the proposition "It is sunny this afternoon", q the proposition "It is colder than yesterday", r the proposition "We will go swimming", s *be* the proposition "We will take a canoe trip" and t *be* the proposition "We will be home by sunset".

Then the premises become $\neg p \land q$, $r \to p$, $\neg r \to s$, and $s \to t$. The conclusion is simply t.

We need to give a valid argument with premises $\neg p \land q$, $r \to p$, $\neg r \to s$, and $s \to t$ and conclusion t.

We construct an argument to show that our premises lead to the desired conclusion as follows.

Step	**Rule**
1. $\neg p \land q$	Premise
2. $\neg p$	Simplification using (1)
3. $r \to p$	Premise
4. $\neg r$	Modus tollens using (2) and (3)
5. $\neg r \to s$	Premise

6. s Modus ponens using (4) and (5)
7. $s \to t$ Premise
8. t Modus ponens using (6) and (7)

Note that we could have used a truth table to show that whenever each of the four hypotheses is true, the conclusion is also true. However, because we are working with five propositional variables, p, q, r, s, and t such a truth table would have 32 rows.

Practice Problems:

1. Use resolution to show the hypotheses "Allen is a bad boy or Hillary is a good girl" and "Allen is a good boy or David is happy" imply the conclusion "Hillary is a good girl or David is happy."
2. Determine whether the conclusion C, follows logically from the premises H1: $\sim p \vee q$, H2 : $\sim(q \wedge \sim r)$, H3: $\sim r$, C: $\sim p$
3. Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."
4. Use resolution to show that the compound proposition $(p \vee q) \wedge (\neg p \vee q) \wedge (p \vee \neg q) \wedge (\neg p \vee \neg q)$ is not satisfiable.
5. *Verify* $r \vee s$ *follows logically from the premises or not*
   $c \vee d$, $(c \vee d) \to \neg h$, $\neg h \to (a \wedge \neg b)$, and
   $(a \wedge \neg b) \to (r \vee s)$

"."

Session-15 (Predicates and Quantifiers, Negation of Quantified Statements)

Instructional Objective:

3. To learn predicates and quantifiers, and negation of quantified statements

4. To learn the validity of the given premises by using connectives, logical equivalences predicates and quantifiers

Learning Outcomes:

2. To learn predicates and quantifiers, and negation of quantified statements
3. To check the validity of the given premises by using connectives, logical equivalences predicates and quantifiers

Introduction:

- The propositional logic does not allow us to represent many of the statements that we use in mathematics, computer science.
- Predicate calculus is a generalization of propositional calculus, which contains all propositional variables and constants.

- A part of declarative sentence which is describing about the properties of an object is called ***Predicate.***

Consider the statement involving the variable 'x'

"$x > 3$"

The above statement "x is greater than 3" has two parts, the first part variable x, is the subject of the statement. The second part the ***predicate***, "is greater than 3" refers to a property that the subject of the statement can have. So we denote the statement "x is greater than 3" by P(x), where P(x) denotes the ***predicate*** "is greater than 3" and "x" is the variable. The statement P(x) is also said to be the value of the propositional function P at x.

Example: Let P(x) denote the statement "$x > 3$" What are the truth values of P (4) and P(2)?

- We obtain the statement P(4) by setting x=4 in the statement "$x > 3$".
- Hence "4>3" is true
- Similarly"2>3" is false

❖ Quantifiers

When the variables in a propositional function are assigned values, the resulting statement become a proposition with certain truth value. However there is another important way ***quantification. Quantification*** expresses the extent to which a predicate is true over a range of elements.

In English, the words ***all, some, many, none and few*** are used in quantification.

❖ The universal quantifier

The ***universal quantification*** of P(x) is the statement

"P(x) for all values of 'x' in the domain."

The notation $\forall \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$ denotes the ***universal quantification*** of P(x). Here $\forall$ is called the ***universal quantifier.*** We read $\forall \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$ **as "for all xP(x)"**
or "for every xP(x)"

***Example*:** Let P(x) be the statement "x+1>x."
Here P(x) is true for all real numbers, therefore the quantification $\forall x P(x)$ is true

❖ The Existential quantifier

The existential quantification of P(x) is the proposition

"There exist an element 'x' in the domain such that P(x)"

The notation $\exists \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$ denotes `the existential quantifier of P(x). Here $\exists$ is called ***existential quantifier.***

The existential quantification $\exists \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$ is read as

"There is an 'x' such that P(x)"

"There is at least one x such that P(x)"

Or

"For some x P(x)."

Example1: Let P(x) denote the statement "x>3."What is the truth value of the quantification $\exists \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$,where the domain consists of all real numbers

Because "x>3." is sometimes true for x=4

Therefore $\exists \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$ is true.

Example 2: Let P(x) denote the statement "x=x+1."What is the truth value of the quantification $\exists \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$,where the domain consists of all real numbers.

- Because P(x) is false for every real number, the existential quantification $\exists \boldsymbol{x} \boldsymbol{P}(\boldsymbol{x})$ is false.

❖ Negation of quantified expressions

We will often want to consider the negation of a quantified expressions, for example consider the statement
"Every student in your class has taken a course in Discrete Mathematics"
This statement is universal quantification namely $\forall x P(x)$
Where P(x) is the statement ***"x has taken a course in Discrete Mathematics"*** and the domain consists of the students in your class.
The negation of the above statement is
"There is student in your class who has not taken a course in Discrete Mathematics"
Simply this is existential quantification of the negation of original propositional function, namely $\exists x \neg P(x)$
Therefore we can have the logical equivalence

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

Suppose we wish to negate an existential quantification. For instance consider the proposition "There is a student in this class who has takena course in ***Discrete Mathematics".*** This is the existential quantification. This is the existential quantification
$\exists x Q(x)$
Where $Q(x)$ is the statement "x has taken a course in Discrete Mathematics". The negation of this statement is the proposition
"It is not the case there a student in this class who has taken a course in Discrete Mathematics". Which is just the universal quantification of the negation of the original propositional function $\forall x \neg Q(x)$

$$\neg \exists x Q(x) \equiv \forall x \neg Q(x$$

Review Questions:

3. What in universal quantifier, give example?
4. What is what is existential quantifier, give example

Summary: Student will be able to learn Predicates and Quantifiers, Negation of Quantified Statements

Self-assessment questions:

1. Let P(x) be the statement "$x=x^2$" if the domain consists of all integers, what are the truth values of i)P(0) ii) P(1) iii)P(2)
i) P (0) is true
ii)P(1) is true
iii) P(2) is false
2. Translate the statements into English, where C(x) is "x is a comedian"
And F(x) is "x is funny" and the domain consists of all people
i))$\forall x(C(x) \rightarrow F(x))$
ii))$\forall x(C(x) \wedge F(x))$
Answer i) Every comedian is funny
ii) Every person is funny comedian

Terminal Questions:

Classroom Delivery Problems

C: Some fierce creatures do not drink coffee 1. Let A (x) denote the statement "Computer 'x' is under attack by an intruder". Suppose that of computers on campus, only C2, Math1 are currently under attack by intruders. What are truth values of A (C1), A (C2) and A (Math1)

2. Let Q(x, y) denote the statement "x is capital of y" what are these truth values
i) Q (London, United States of America)
ii) Q (Oslo, Norway)
iii) Q (Cairo, Egypt)
iv) (Vatican City, Vatican City)
v) (Beijing, Canada)

3. Let N(x) be the statement "x has visited Kashmir", where the domain consist of the students in your class. Express each of these quantifications in English
i)$\exists x N(x)$
ii)$\forall x N(x)$
ii)$\neg\exists x N(x)$
iv)$\exists x \neg$N(x)
v) $\neg\forall x N(x)$
vi) $\forall x \neg N(x)$

4. Let C(x) be the statement "x has a cat," let D(x) be the statement "x has a dog," and let F(x) be the statement "x has a ferret." Express each of these statement in terms of C(x), D(x), F(x), quantifiers, and logical connectives. Let the domain consists of all students in your class.
a) A student in your class has a cat, a dog, and a ferret
b) All students in your class have a cat, a dog, or a ferret
c) Some students in your class has a cat and a ferret, but not a dog.
d) No student in your class has a cat, a dog, or a ferret
e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet

5. Translate these system specifications into English where the predicate *S(x, y)* is "*x* is in state *y*" and where the domain for *x* and *y* consists of all systems and all possible states, respectively.
a) ∃xS*(x,* open)
b) ∀*x(S(x,* malfunctioning) ∨ *S(x,* diagnostic))
c) ∃*xS(x,* open) ∨ ∃*xS(x,* diagnostic)
d) ∃*x* ¬*S(x,* available)
e) ∀*x* ¬*S(x,* working)
6. Consider the following argument consisting of two Premises P1 and P2, and a conclusion C, check whether the C is valid or not?
P1: All lions are fierce
P2: Some lions do not drink coffee

Home Assignment Problems

1. Let P(x) be the statement "the word x contains the letter a." what are these truth values
a) P(orange)
b) P (lemon)

c) P (true)
d) P (false)

2. Use predicates and quantifiers to express the system specifications “Every mail message larger than one megabyte will be compressed” and “If a user is active, at least one network link will be available

3. Translate these statements into English, where R(x) is “x is a rabbit” and H(x) is “x hops” and the domain consists of all animals.

a) $\forall x\big(R(x) \rightarrow H(x)\big)$
b) $\forall x\big(R(x) \wedge H(x)\big)$
c) $\exists x\big(R(x) \rightarrow H(x)\big)$
d) $\exists x\big(R(x) \wedge H(x)\big)$

4. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives

a) No one is perfect
b) Not everyone is perfect
c) All your friends are perfect
d) At least one of your friends is perfect
e) Everyone is your friend and is perfect
f) Not everybody is your friend or someone is not perfect

5. Consider the following argument consisting of three Premises P1 ,P2&P3 and a conclusion C, check whether the C is valid or not?
P1: All hummingbirds are richly colored
P2: No large birds live on honey
P3: Birds that do not live on honey are dull in color
C: Humming birds are small

Tutorial Problems

1. Let Q(x, y) denote the statement ‘x=y+3”. “What are the truth values of the proposition Q(1,2) and Q(3,0)
2. Express the statements “Every student in this class has visited Mexico”, and “Every student in this class has visited either Canada or Mexico” using predicates and quantifiers

3. Let P(x) be the statement “x spends more than five hours every weekdays in class,” where the domine for ‘x’ consists of all students. Express each of these quantifications in English
a) $\exists x P(x)$
b) $\forall x P(x)$
c) $\exists x \neg P(x)$
d) $\forall x \neg P(x)$

4. Translate these specifications into English where *F(p)* is “Printer *p* is out of Service,” *B(p)* is “Printer *p* is busy,” *L(j)* is “Print job *j* is lost,” and *Q(j)* is “Print job *j* is queued.”
a) $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$

b) $\forall p B(p) \rightarrow \exists j Q(j)$
c) $\exists j\ (Q(j) \wedge L(j)) \rightarrow \exists p F(p)$
d) $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$

5. Express each of these system specifications using predicates, quantifiers, and logical connectives.

- **a)** When there is less than 30 megabytes free on the hard disk, a warning message is sent to all users.
- **b)** No directories in the file system can be opened and no files can be closed when system errors have been detected.
- **c)** The file system cannot be backed up if there is a user currently logged on.