Discrete Mathematics

Section 4: Proofs

Questions

1. Prove that the square of any odd integer is always 1 more than a multiple of 4.
2. Prove that the product of any three consecutive integers is divisible by 6, for all integers $n \geq 1$. Use the fact that among any three consecutive integers greater than or equal to 1, at least one is divisible by 3.
3. Prove that for any integer $n \geq 1$, $n^3 - n$ is divisible by 6, using the result from the previous question. Verify this for $n = 2$ and $n = 4$.
4. Prove that the sum of the squares of any two odd integers is always even.
5. Prove that for any positive integer n, $2^n - 1$ is odd.
6. Prove that the difference of the squares of any two consecutive integers is equal to the sum of those integers. Then, verify the result for $n = 5$ and $n = 10$.
7. Prove that for any two odd integers a and b, the sum of their squares, $a^2 + b^2$, is never divisible by 4. Provide reasoning and examples.
8. For two consecutive integers n and $n+1$, determine whether the sum of their cubes is divisible by their product. If not, provide the correct conclusion.
9. Prove by contrapositive that if n^2 is even, then n is even.
10. Prove that for any two distinct primes p and q, $p^2 - q^2$ is divisible by $p - q$.
11. Prove by contradiction that if a and b are integers and $a^2 = 4b + 1$, then a is odd.
12. Prove by contradiction that if n is an integer and $3n + 2$ is even, then n is even.
13. Prove by contradiction that the sum of an irrational number and a rational number is irrational.
14. Prove by contradiction that there is no largest prime number.
15. Prove by exhaustion that if n is an integer such that $1 \leq n \leq 4$, then $n^2 - n + 1$ is always odd.
16. Prove by exhaustion that for any two distinct integers a and b where $1 \leq a \leq 3$ and $1 \leq b \leq 3$, the product ab is always less than 10.
17. Prove that the square root of any prime number is irrational.
18. Prove that there exists a pair of distinct prime numbers p and q such that $p+q$ is also a prime number.
19. Prove that there exists a unique positive integer solution to the equation $3x + 2y = 5$.
20. Prove by induction that for all integers $n \geq 1$, $2^{2n} - 1$ is divisible by 3.
21. Prove by induction that for all integers $n \geq 1$, the sum of the squares of the first n natural numbers is given by $\frac{n(n+1)(2n+1)}{6}$.

22. Prove by induction that for all integers $n \geq 1$, $3^n - 1$ is divisible by 2.
23. Prove by induction that for all integers $n \geq 5$, $n^2 < 2^n$.
24. Prove by induction that for all integers $n \geq 4$, $n! > 2^n$.
25. Prove by induction that for all integers $n \geq 1$, the difference between 7^n and 2^n is divisible by 5.

Questions and Answers

1. Prove that the square of any odd integer is always 1 more than a multiple of 4.
   Let n be an odd integer. Then $n = 2k + 1$ for some integer k. The square of n is $(2k + 1)^2$.

   Expanding this, we get:
   $$(2k + 1)^2 = 4k^2 + 4k + 1.$$

   Notice that $4k^2 + 4k$ is a multiple of 4, since both terms are divisible by 4.

   Therefore, we can write:
   $$(2k + 1)^2 = 4(k^2 + k) + 1.$$

   Since k^2+k is an integer, it follows that $4(k^2+k)$ is always a multiple of 4. Consequently, the expression $(2k + 1)^2$ is 1 more than a multiple of 4, confirming the desired result. ✓

2. Prove that the product of any three consecutive integers is divisible by 6, for all integers $n \geq 1$. Use the fact that among any three consecutive integers greater than or equal to 1, at least one is divisible by 3.
   Let the three consecutive integers be n, $n + 1$, and $n + 2$, where $n \geq 1$. We will consider two cases based on whether n is even or odd.

   Case 1: Suppose n is even. Then n is divisible by 2. Notice that $n + 1$ is odd, and $n + 2$ is even. Since $n + 2$ is also divisible by 2, we already know the product is divisible by 2. Now, $n + 1$ must be divisible by 3 since, out of every three consecutive integers greater than or equal to 1, at least one is divisible by 3. Therefore, the product $n(n + 1)(n + 2)$ is divisible by both 2 and 3, and thus by 6.

   Case 2: Suppose n is odd. Then $n + 1$ is even, and $n + 2$ is odd. Since $n + 1$ is divisible by 2, the product is divisible by 2. Additionally, among n and $n + 2$, at least one is divisible by 3. Therefore, the product $n(n + 1)(n + 2)$ is divisible by both 2 and 3, and thus by 6. ✓

3. Prove that for any integer $n \geq 1$, $n^3 - n$ is divisible by 6, using the result from the previous question. Verify this for $n = 2$ and $n = 4$.
   For any integer $n \geq 1$, consider the expression $n^3 - n$. We can factor this as:
   $$n^3 - n = n(n - 1)(n + 1).$$

   Notice that n, $n-1$, and $n+1$ are three consecutive integers. From the result in the previous question, we know that the product of any three consecutive integers is divisible by 6 for all integers $n \geq 1$. Therefore, $n^3 - n$ is divisible by 6.

   Now, let's verify this for specific values of n. For $n = 2$:
   $$2^3 - 2 = 8 - 2 = 6, \text{ which is divisible by 6.}$$

   Similarly, for $n = 4$:
   $$4^3 - 4 = 64 - 4 = 60, \text{ which is also divisible by 6.}$$

   Thus, the result holds for these values and for all integers $n \geq 1$. ✓

4. Prove that the sum of the squares of any two odd integers is always even.
   Let a and b be odd integers. Then $a = 2k + 1$ and $b = 2m + 1$ for some integers k and m. The squares are:
   $$a^2 = (2k + 1)^2 = 4k^2 + 4k + 1,$$
   $$b^2 = (2m + 1)^2 = 4m^2 + 4m + 1.$$

   The sum is:
   $$a^2 + b^2 = 4k^2 + 4k + 1 + 4m^2 + 4m + 1 = 4k^2 + 4k + 4m^2 + 4m + 2.$$

   Factoring out a 2, we get:
   $$a^2 + b^2 = 2(2k^2 + 2k + 2m^2 + 2m + 1),$$

   which is even. ✓

5. Prove that for any positive integer n, $2^n - 1$ is odd.
We will prove that $2^n - 1$ is odd for any positive integer n by using basic properties of even and odd numbers.

Base Case: Consider $n = 1$. Then:
$$2^1 - 1 = 1$$
which is odd, so the statement holds for $n = 1$.

Inductive Step: Suppose $2^n - 1$ is odd for some integer n. We need to prove that $2^{n+1} - 1$ is also odd.

Consider $2^{n+1} - 1$:
$$2^{n+1} - 1 = 2 \times 2^n - 1$$
Since 2^n is an integer, multiplying it by 2 makes it even. Subtracting 1 from an even number results in an odd number. Therefore, $2^{n+1} - 1$ is odd.

Thus, by mathematical induction, $2^n - 1$ is odd for all positive integers n. ✓

6. Prove that the difference of the squares of any two consecutive integers is equal to the sum of those integers. Then, verify the result for $n = 5$ and $n = 10$.
Let the two consecutive integers be n and $n+1$. Their squares are n^2 and $(n+1)^2$, respectively. The difference is:
$$(n+1)^2 - n^2 = (n^2 + 2n + 1) - n^2 = 2n + 1,$$
which is the sum of the two consecutive integers:
$$n + (n+1) = 2n + 1.$$
Now, let's verify this result for specific values of n.

For $n = 5$:
$$(5+1)^2 - 5^2 = 36 - 25 = 11, \quad \text{and the sum is} \quad 5 + 6 = 11.$$
For $n = 10$:
$$(10+1)^2 - 10^2 = 121 - 100 = 21, \quad \text{and the sum is} \quad 10 + 11 = 21.$$
Thus, the result holds for both cases . ✓

7. Prove that for any two odd integers a and b, the sum of their squares, $a^2 + b^2$, is never divisible by 4. Provide reasoning and examples.
Let a and b be two odd integers. Since any odd integer can be written in the form $2k + 1$ for some integer k, we have:
$$a = 2m + 1 \quad \text{and} \quad b = 2n + 1$$
for some integers m and n. Now, calculate the squares:
$$a^2 = (2m+1)^2 = 4m^2 + 4m + 1$$
$$b^2 = (2n+1)^2 = 4n^2 + 4n + 1$$
The sum of the squares is:
$$a^2 + b^2 = (4m^2 + 4m + 1) + (4n^2 + 4n + 1) = 4(m^2 + m + n^2 + n) + 2$$
This expression simplifies to $4k + 2$ for some integer k, which is not divisible by 4. Thus, the sum of the squares of any two odd integers is never divisible by 4. For example, let $a = 3$ and $b = 5$:
$$a^2 + b^2 = 3^2 + 5^2 = 9 + 25 = 34$$
34 is not divisible by 4, aligning with the proof. ✓

8. For two consecutive integers n and $n+1$, determine whether the sum of their cubes is divisible by their product. If not, provide the correct conclusion.
Let the two consecutive integers be n and $n+1$. Their cubes are n^3 and $(n+1)^3$. The sum of their cubes is:
$$n^3 + (n+1)^3 = n^3 + (n^3 + 3n^2 + 3n + 1) = 2n^3 + 3n^2 + 3n + 1.$$
Their product is:
$$n(n+1) = n^2 + n.$$
To determine if the sum of their cubes is divisible by their product, divide the sum by the product:
$$\frac{2n^3 + 3n^2 + 3n + 1}{n^2 + n}.$$
After performing the division, the result is:
$$2n + 1 + \frac{2n+1}{n^2+n}.$$
Since the division does not yield a whole number, the sum of the cubes is not divisible by the product of the two consecutive integers. Therefore, the correct conclusion is that the sum of the cubes of two consecutive integers is not divisible by their product. ✓

9. Prove by contrapositive that if n^2 is even, then n is even.
Contrapositive: If n is odd, then n^2 is odd. An odd number can be written as $2k+1$ for some integer k. Squaring it, we get:
$$(2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1,$$
which is odd. Therefore, if n is odd, n^2 is odd, proving the contrapositive. ✓

10. Prove that for any two distinct primes p and q, $p^2 - q^2$ is divisible by $p - q$.
We are asked to prove that for any two distinct primes p and q, the expression $p^2 - q^2$ is divisible by $p - q$.
We begin by recalling the difference of squares identity:
$$p^2 - q^2 = (p-q)(p+q).$$
Since $p^2 - q^2$ can be factored as $(p-q)(p+q)$, it is clear that $p^2 - q^2$ is divisible by $p - q$, because $(p-q)$ is a factor of the expression.
Example: Let $p = 7$ and $q = 3$. Then:
$$p^2 - q^2 = 7^2 - 3^2 = 49 - 9 = 40,$$
and
$$p - q = 7 - 3 = 4.$$
40 is divisible by 4, and $40 \div 4 = 10$.
Therefore, $p^2 - q^2$ is divisible by $p - q$ for any distinct primes p and q. ✓

11. Prove by contradiction that if a and b are integers and $a^2 = 4b + 1$, then a is odd.
Assume the opposite, that a is even. Then $a = 2k$ for some integer k. Substituting this into the equation, we get:
$$(2k)^2 = 4b + 1, \quad \text{or} \quad 4k^2 = 4b + 1.$$
Dividing both sides by 4, we get:
$$k^2 = b + \frac{1}{4},$$
which implies that $b + \frac{1}{4}$ is an integer. However, $\frac{1}{4}$ is not an integer, leading to a contradiction.
Therefore, our assumption that a is even is incorrect, and a must be odd. ✓

12. Prove by contradiction that if n is an integer and $3n+2$ is even, then n is even.
    We will prove this by contradiction.

    Assume the opposite, that n is odd. Then $n = 2k+1$ for some integer k. Substituting this into $3n+2$, we get:

    $$3(2k+1)+2 = 6k+3+2 = 6k+5.$$

    Now, observe that $6k$ is even, because any integer multiplied by 6 is even. Adding 5 to this gives $6k+5$, which is odd because adding an odd number (5) to an even number results in an odd number.

    Since $6k+5$ is odd, this contradicts the given condition that $3n+2$ is even. Therefore, our assumption that n is odd must be false, and thus, n must be even. ✓

13. Prove by contradiction that the sum of an irrational number and a rational number is irrational.
    Assume the opposite, that the sum of an irrational number x and a rational number r is rational. Let $x + r = q$, where q is rational. Then:

    $$x = q - r,$$

    which is the difference of two rational numbers, hence rational. This contradicts the assumption that x is irrational. Therefore, the sum of an irrational number and a rational number is irrational. ✓

14. Prove by contradiction that there is no largest prime number.
    We will prove this by contradiction.

    Assume the opposite, that there is a largest prime number, p_n. Let $p_1, p_2, \ldots, p_n$ represent all the prime numbers, with p_n being the largest prime. Now, consider the number:

    $$P = p_1 \cdot p_2 \cdot \ldots \cdot p_n + 1.$$

    This number P is greater than p_n and is not divisible by any of the primes $p_1, p_2, \ldots, p_n$ because dividing P by any of these primes leaves a remainder of 1. Therefore, P is either prime itself or divisible by a prime not in the list of primes up to p_n. This contradicts the assumption that p_n is the largest prime. Therefore, there is no largest prime number. ✓

15. Prove by exhaustion that if n is an integer such that $1 \leq n \leq 4$, then $n^2 - n + 1$ is always odd.
    We calculate $n^2 - n + 1$ for each integer n where $1 \leq n \leq 4$ and check if the result is odd.

    - For $n = 1$: $1^2 - 1 + 1 = 1$ (odd)
    - For $n = 2$: $2^2 - 2 + 1 = 3$ (odd)
    - For $n = 3$: $3^2 - 3 + 1 = 7$ (odd)
    - For $n = 4$: $4^2 - 4 + 1 = 13$ (odd)

    Since $n^2 - n + 1$ is odd for all n in the range $1 \leq n \leq 4$, the statement is proven by exhaustion. ✓

16. Prove by exhaustion that for any two distinct integers a and b where $1 \leq a \leq 3$ and $1 \leq b \leq 3$, the product ab is always less than 10.
    The distinct pairs (a, b) where $1 \leq a \leq 3$, $1 \leq b \leq 3$, and $a \neq b$ are: $(1, 2)$, $(1, 3)$, $(2, 1)$, $(2, 3)$, $(3, 1)$, and $(3, 2)$. Now, calculate the product for each pair:

    - For $(1, 2)$: $1 \times 2 = 2$ (less than 10)
    - For $(1, 3)$: $1 \times 3 = 3$ (less than 10)
    - For $(2, 1)$: $2 \times 1 = 2$ (less than 10)
    - For $(2, 3)$: $2 \times 3 = 6$ (less than 10)
    - For $(3, 1)$: $3 \times 1 = 3$ (less than 10)
    - For $(3, 2)$: $3 \times 2 = 6$ (less than 10)

    Since ab is less than 10 for all distinct pairs (a, b), where $1 \leq a \leq 3$ and $1 \leq b \leq 3$, the statement is proven by exhaustion. ✓

17. Prove that the square root of any prime number is irrational.
We will prove that the square root of any prime number is irrational by contradiction.

Assume the opposite, that $\sqrt{p}$, where p is a prime number, is rational. Then we can express $\sqrt{p}$ as a fraction of two integers in lowest terms:

$$\sqrt{p} = \frac{a}{b}, \quad \text{where } \gcd(a, b) = 1.$$

Squaring both sides gives:

$$p = \frac{a^2}{b^2}.$$

Multiplying both sides by b^2 gives:

$$pb^2 = a^2.$$

This equation implies that a^2 is divisible by p.

Since p is prime and divides a^2, it must also divide a. Therefore, a can be written as $a = pk$ for some integer k. Substituting this expression for a into the equation $pb^2 = a^2$ gives:

$$pb^2 = (pk)^2 = p^2k^2.$$

Dividing both sides of this equation by p results in:

$$b^2 = pk^2.$$

This shows that b^2 is divisible by p, which means that b must also be divisible by p (since p is prime).

Now, both a and b are divisible by p. But this contradicts our original assumption that $\gcd(a, b) = 1$ (i.e., a and b have no common divisors). Therefore, our assumption that $\sqrt{p}$ is rational must be false, meaning that $\sqrt{p}$ is irrational. ✓

18. Prove that there exists a pair of distinct prime numbers p and q such that $p+q$ is also a prime number.
Consider the primes $p = 2$ and $q = 3$. Their sum is $2 + 3 = 5$, which is also a prime number. ✓

19. Prove that there exists a unique positive integer solution to the equation $3x + 2y = 5$.
The equation $3x + 2y = 5$ has a positive integer solution. Solving for x and y, we find $x = 1$ and $y = 1$ to be one of the positive integer solution. ✓

20. Prove by induction that for all integers $n \geq 1$, $2^{2n} - 1$ is divisible by 3.
Base case: For $n = 1$, the expression is $2^{2(1)} - 1 = 4 - 1 = 3$, which is divisible by 3. So, the base case holds.

Inductive step: Assume the statement holds for some integer $k \geq 1$, i.e., $2^{2k} - 1$ is divisible by 3. This means that $2^{2k} - 1 = 3m$ for some integer m.
Now, we need to prove that $2^{2(k+1)} - 1$ is divisible by 3.
Consider $2^{2(k+1)} - 1$. We can express it as:

$$2^{2(k+1)} - 1 = 2^{2k+2} - 1 = 4 \times 2^{2k} - 1.$$

We can factor out 4 from the expression:

$$4 \times 2^{2k} - 1 = (4 \times 2^{2k} - 4) + 3 = 4 \times (2^{2k} - 1) + 3.$$

Since $2^{2k} - 1$ is divisible by 3 by the inductive hypothesis, say $2^{2k} - 1 = 3m$, we have:

$$4 \times (2^{2k} - 1) + 3 = 4 \times 3m + 3 = 3(4m + 1).$$

This shows that $2^{2(k+1)} - 1$ is also divisible by 3.

Therefore, by induction, the statement is true for all $n \geq 1$. ✓

21. Prove by induction that for all integers $n \geq 1$, the sum of the squares of the first n natural numbers is given by $\frac{n(n+1)(2n+1)}{6}$.

Base case: For $n = 1$, the sum of the squares of the first 1 natural number is $1^2 = 1$, and the right-hand side is:

$$\frac{1(1+1)(2(1)+1)}{6} = \frac{1 \times 2 \times 3}{6} = 1.$$

So, the base case holds.

Inductive step: Assume the statement holds for some integer $k \geq 1$, i.e., the sum of the squares of the first k natural numbers is given by:

$$1^2 + 2^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}.$$

We need to prove that the statement holds for $k+1$.

Consider the sum of the squares of the first $k+1$ natural numbers:

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2.$$

By the inductive hypothesis, the sum of the squares of the first k natural numbers is:

$$\frac{k(k+1)(2k+1)}{6}.$$

So, the left-hand side is:

$$\text{Left-hand side} = 1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2.$$

Next, simplify the expression:

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}.$$

Factor out $(k+1)$ from both terms:

$$= \frac{(k+1)\left[k(2k+1) + 6(k+1)\right]}{6}.$$

Simplify further:

$$= \frac{(k+1)\left[2k^2 + k + 6k + 6\right]}{6} = \frac{(k+1)(2k^2 + 7k + 6)}{6}.$$

Factor the quadratic expression $2k^2 + 7k + 6$:

$$= \frac{(k+1)(k+2)(2k+3)}{6}.$$

Now, substitute $k+1$ into the original formula for the right-hand side:

$$\text{Right-hand side} = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}.$$

Simplify:

$$= \frac{(k+1)(k+2)(2k+3)}{6}.$$

Thus, the left-hand side equals the right-hand side:

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}.$$

Therefore, the statement holds for $k+1$.

By the principle of mathematical induction, the statement is true for all $n \geq 1$. ✓

22. Prove by induction that for all integers $n \geq 1$, $3^n - 1$ is divisible by 2.
Base case: For $n = 1$, $3^1 - 1 = 2$, which is divisible by 2. So, the base case holds.

Inductive step: Assume the statement holds for some integer $k \geq 1$, i.e., $3^k - 1$ is divisible by 2. This means that $3^k - 1 = 2m$ for some integer m. We need to prove that $3^{k+1} - 1$ is divisible by 2. Consider $3^{k+1} - 1$. We can write this as:

$$3^{k+1} - 1 = 3 \times 3^k - 1 = (3 \times 3^k - 3) + 2.$$

Factor out 3:

$$3(3^k - 1) + 2.$$

By the inductive hypothesis, $3^k - 1 = 2m$ for some integer m, so:

$$3(2m) + 2 = 6m + 2.$$

$6m + 2$ is divisible by 2.

Therefore, by the principle of mathematical induction, the statement is true for all $n \geq 1$. ✓

23. Prove by induction that for all integers $n \geq 5$, $n^2 < 2^n$.
Base case: For $n = 5$, we have $5^2 = 25$ and $2^5 = 32$. Since $25 < 32$, the base case holds.

Inductive step: Let k be any integer with $k \geq 5$, and suppose $k^2 < 2^k$. This is our inductive hypothesis. We need to show that $(k+1)^2 < 2^{k+1}$.
Expanding the left-hand side:

$$(k+1)^2 = k^2 + 2k + 1.$$

By the inductive hypothesis, we have $k^2 < 2^k$. Therefore, it follows that:

$$k^2 + 2k + 1 < 2^k + 2k + 1.$$

Now, we know that $2k + 1 < 2^k$ for $k \geq 5$. Adding this to the previous inequality gives:

$$k^2 + 2k + 1 < 2^k + 2k + 1 < 2^k + 2^k = 2^{k+1}.$$

Thus, $(k+1)^2 < 2^{k+1}$, as required.

By the principle of mathematical induction, the statement is true for all integers $n \geq 5$. ✓

24. Prove by induction that for all integers $n \geq 4$, $n! > 2^n$.
Base case: For $n = 4$, we have $4! = 24$ and $2^4 = 16$. Since $24 > 16$, the base case holds.

Inductive step: Assume the statement holds for some integer $k \geq 4$, i.e., $k! > 2^k$. We need to prove that $(k+1)! > 2^{k+1}$.
Consider:

$$(k+1)! = (k+1) \cdot k!.$$

By the inductive hypothesis, $k! > 2^k$, so:

$$(k+1)! = (k+1) \cdot k! > (k+1) \cdot 2^k.$$

Now, we need to show that:

$$(k+1) \cdot 2^k > 2^{k+1}.$$

This simplifies to:

$$(k+1) \cdot 2^k > 2 \cdot 2^k \iff k+1 > 2.$$

This inequality holds for all integers $k \geq 2$, and since we are starting from $k \geq 4$, it is certainly true. Therefore:

$$(k+1)! > 2^{k+1}.$$

By the principle of mathematical induction, the statement is true for all integers $n \geq 4$. ✓

25. Prove by induction that for all integers $n \geq 1$, the difference between 7^n and 2^n is divisible by 5.

We will prove that $7^n - 2^n$ is divisible by 5 for all integers $n \geq 1$ by induction.

Base case: For $n = 1$, we calculate:

$$7^1 - 2^1 = 7 - 2 = 5,$$

which is divisible by 5. Thus, the base case holds.

Inductive step: Assume the statement holds for some integer $k \geq 1$, i.e.,

$$7^k - 2^k \text{ is divisible by 5.}$$

This means that:

$$7^k - 2^k = 5m \quad \text{for some integer } m.$$

We need to prove that $7^{k+1} - 2^{k+1}$ is also divisible by 5.

Now, consider:

$$7^{k+1} - 2^{k+1} = 7 \cdot 7^k - 2 \cdot 2^k.$$

Using the inductive hypothesis, we know that $7^k - 2^k = 5m$. Thus, substitute this into the expression:

$$7^{k+1} - 2^{k+1} = 7(7^k) - 2(2^k) = 7(5m + 2^k) - 2(2^k).$$

Expanding this, we get:

$$7 \times 5m + 7 \times 2^k - 2 \times 2^k = 35m + (7 \times 2^k - 2 \times 2^k).$$

Simplifying the terms:

$$35m + 5 \times 2^k.$$

Both terms, $35m$ and 5×2^k, are clearly divisible by 5. Therefore, we can factor out 5:

$$7^{k+1} - 2^{k+1} = 5(7m + 2^k),$$

which shows that $7^{k+1} - 2^{k+1}$ is divisible by 5.

Thus, by the principle of mathematical induction, the statement holds for all integers $n \geq 1$. ✓

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