Discrete Mathematics

Section 10: Sequences and Series

Questions

1. Find the 10th term of the sequence defined by $a_n = 3n^2 - 2n + 5$.
2. Given the sequence $a_n = 2^n + 3$, find the sum of the first 5 terms.
3. If the sequence is defined by $a_1 = 4$ and $a_{n+1} = 3a_n + 1$, find a_5.
4. Find the 15th term of the arithmetic sequence where the first term is 7 and the common difference is 3.
5. Determine the common difference and the 8th term of the arithmetic sequence: $a_n = 2, 5, 8, \ldots$
6. Find the sum of the first 10 terms of the arithmetic sequence where the first term is 4 and the common difference is 6.
7. Find the sum of the first 7 terms of the geometric sequence $a_n = 3, 6, 12, \ldots$
8. Determine the 5th term of the geometric sequence defined by $a_n = 5 \cdot 2^{n-1}$.
9. Find the 4th term of the sequence defined by $a_n = (-1)^n \cdot \frac{3n^3 - 2n^2 + 4n - 5}{2}$.
10. Find the sum of the first 6 terms of the sequence where each term is defined by $a_n = 2^n - 1$.
11. Find the sum of the first 10 terms of the geometric sequence where the first term is 5 and the common ratio is 2.
12. Determine the 8th term of the arithmetic sequence where the first term is 12 and the common difference is -4.
13. Given the arithmetic sequence where the first term is 5 and the common difference is 3, find the sum of the first 7 terms.
14. Find the 6th term of the sequence defined by $a_n = 3n^2 - n + 2$.
15. If the first term of a geometric sequence is 3 and the common ratio is $\frac{1}{3}$, find the 4th term.
16. Given that the first term of a geometric sequence is 7 and the 5th term is 112, find the common ratio.
17. Determine the 3rd term and the common difference of the arithmetic sequence where the 2nd term is 9 and the 5th term is 21.
18. Given the sequence defined by $a_n = 5 - 2n$, find the sum of the first 6 terms.
19. Find the sum of the first 10 terms of the sequence defined by $a_n = 4n + 1$.

20. Determine the 8th term of the sequence defined by $a_n = \frac{2n}{n+1}$.

21. Given the arithmetic sequence where the first term is 4 and the common difference is 6, find the 12th term.

22. Find the sum of the first 7 terms of the geometric sequence where the first term is 3 and the common ratio is 2.

23. If the sum of the first 6 terms of an arithmetic sequence is 84 and the first term is 6, find the common difference.

24. Find the sum of the first 6 terms of the geometric sequence where the first term is 8 and the common ratio is $\frac{1}{3}$.

25. Find the sum of the series $2 + 4 + 8 + 16 + \cdots + 512$.

Questions and Answers

1. Find the 10th term of the sequence defined by $a_n = 3n^2 - 2n + 5$.
   Calculate the 10th term:
   $$a_{10} = 3(10)^2 - 2(10) + 5 = 3(100) - 20 + 5 = 300 - 20 + 5 = 285$$

2. Given the sequence $a_n = 2^n + 3$, find the sum of the first 5 terms.
   Calculate each term:
   $$a_1 = 2^1 + 3 = 5, \quad a_2 = 2^2 + 3 = 7, \quad a_3 = 2^3 + 3 = 11, \quad a_4 = 2^4 + 3 = 19, \quad a_5 = 2^5 + 3 = 35$$
   The sum of the first 5 terms is:
   $$\text{Sum} = 5 + 7 + 11 + 19 + 35 = 77$$

3. If the sequence is defined by $a_1 = 4$ and $a_{n+1} = 3a_n + 1$, find a_5.
   Calculate each term:
   $$a_2 = 3a_1 + 1 = 3(4) + 1 = 13$$
   $$a_3 = 3a_2 + 1 = 3(13) + 1 = 40$$
   $$a_4 = 3a_3 + 1 = 3(40) + 1 = 121$$
   $$a_5 = 3a_4 + 1 = 3(121) + 1 = 364$$

4. Find the 15th term of the arithmetic sequence where the first term is 7 and the common difference is 3.
   Use the formula for the nth term:
   $$a_n = a_1 + (n-1)d$$
   Calculate the 15th term:
   $$a_{15} = 7 + (15-1) \cdot 3 = 7 + 42 = 49$$

5. Determine the common difference and the 8th term of the arithmetic sequence: $a_n = 2, 5, 8, \ldots$
   The common difference is $d = 5 - 2 = 3$.
   Use the formula for the nth term:
   $$a_n = a_1 + (n-1)d$$
   The 8th term is:
   $$a_8 = 2 + (8-1) \cdot 3 = 2 + 21 = 23$$

6. Find the sum of the first 10 terms of the arithmetic sequence where the first term is 4 and the common difference is 6.
   Use the formula for the sum of the first n terms:
   $$S_n = \frac{n}{2}(2a_1 + (n-1)d)$$
   Calculate the sum:
   $$S_{10} = \frac{10}{2}(2 \cdot 4 + (10-1) \cdot 6) = 5 \cdot (8 + 54) = 5 \cdot 62 = 310$$

7. Find the sum of the first 7 terms of the geometric sequence $a_n = 3, 6, 12, \ldots$
   The common ratio is $r = \frac{6}{3} = 2$.
   Use the formula for the sum of the first n terms of a geometric sequence:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

   Calculate the sum:

$$S_7 = \frac{3(1 - 2^7)}{1 - 2} = \frac{3(-127)}{-1} = 3 \cdot 127 = 381$$

8. Determine the 5th term of the geometric sequence defined by $a_n = 5 \cdot 2^{n-1}$.
   Calculate the 5th term using the sequence formula:

$$a_5 = 5 \cdot 2^{5-1} = 5 \cdot 2^4 = 5 \cdot 16 = 80$$

9. Find the 4th term of the sequence defined by $a_n = (-1)^n \cdot \frac{3n^3 - 2n^2 + 4n - 5}{2}$.
   Calculate the 4th term:

$$a_4 = (-1)^4 \cdot \frac{3(4)^3 - 2(4)^2 + 4(4) - 5}{2} = 1 \cdot \frac{3 \cdot 64 - 2 \cdot 16 + 16 - 5}{2}$$

$$= \frac{192 - 32 + 16 - 5}{2} = \frac{171}{2} = 85.5$$

10. Find the sum of the first 6 terms of the sequence where each term is defined by $a_n = 2^n - 1$.
    Calculate each term:

$$a_1 = 2^1 - 1 = 1$$
$$a_2 = 2^2 - 1 = 3$$
$$a_3 = 2^3 - 1 = 7$$
$$a_4 = 2^4 - 1 = 15$$
$$a_5 = 2^5 - 1 = 31$$
$$a_6 = 2^6 - 1 = 63$$

    Sum the terms:

$$\text{Sum} = 1 + 3 + 7 + 15 + 31 + 63 = 120$$

11. Find the sum of the first 10 terms of the geometric sequence where the first term is 5 and the common ratio is 2.
    Use the formula for the sum of the first n terms of a geometric sequence:

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

    Calculate the sum:

$$S_{10} = \frac{5(1 - 2^{10})}{1 - 2} = \frac{5(-1023)}{-1} = 5 \cdot 1023 = 5115$$

12. Determine the 8th term of the arithmetic sequence where the first term is 12 and the common difference is -4.
Use the formula for the nth term:
$$a_n = a_1 + (n-1)d$$
Calculate the 8th term:
$$a_8 = 12 + (8-1)(-4) = 12 - 28 = -16$$

13. Given the arithmetic sequence where the first term is 5 and the common difference is 3, find the sum of the first 7 terms.
Use the formula for the sum of the first n terms of an arithmetic sequence:
$$S_n = \frac{n}{2} \cdot (2a_1 + (n-1)d)$$
Calculate the sum:
$$S_7 = \frac{7}{2} \cdot (2 \cdot 5 + (7-1) \cdot 3) = \frac{7}{2} \cdot (10 + 18) = \frac{7}{2} \cdot 28 = 7 \cdot 14 = 98$$

14. Find the 6th term of the sequence defined by $a_n = 3n^2 - n + 2$.
Calculate the 6th term:
$$a_6 = 3(6)^2 - 6 + 2 = 3(36) - 6 + 2 = 108 - 6 + 2 = 104$$

15. If the first term of a geometric sequence is 3 and the common ratio is $\frac{1}{3}$, find the 4th term.
Use the formula for the nth term of a geometric sequence:
$$a_n = a_1 \cdot r^{n-1}$$
Calculate the 4th term:
$$a_4 = 3 \cdot \left(\frac{1}{3}\right)^{4-1} = 3 \cdot \frac{1}{27} = \frac{1}{9}$$

16. Given that the first term of a geometric sequence is 7 and the 5th term is 112, find the common ratio.
Let the common ratio be r. Use the formula for the n-th term of a geometric sequence:
$$a_n = a_1 \cdot r^{n-1}$$
Set up the equation:
$$112 = 7 \cdot r^{5-1} = 7 \cdot r^4$$
Solve for r:
$$r^4 = \frac{112}{7} = 16$$
$$r = \sqrt[4]{16} = 2$$
Thus, the common ratio is $r = 2$.

17. Determine the 3rd term and the common difference of the arithmetic sequence where the 2nd term is 9 and the 5th term is 21.
Let the first term be a and the common difference be d. The equations are:

$$a + d = 9 \quad \text{and} \quad a + 4d = 21$$

Subtract the first equation from the second:

$$3d = 12 \quad \Rightarrow \quad d = 4$$

Substitute d back into $a + d = 9$:

$$a = 5$$

The 3rd term is:

$$a + 2d = 5 + 8 = 13$$

18. Given the sequence defined by $a_n = 5 - 2n$, find the sum of the first 6 terms.
Calculate each term and then sum:

$$a_1 = 5 - 2 \cdot 1 = 3$$
$$a_2 = 5 - 2 \cdot 2 = 1$$
$$a_3 = 5 - 2 \cdot 3 = -1$$
$$a_4 = 5 - 2 \cdot 4 = -3$$
$$a_5 = 5 - 2 \cdot 5 = -5$$
$$a_6 = 5 - 2 \cdot 6 = -7$$
$$\text{Sum} = 3 + 1 - 1 - 3 - 5 - 7 = -12$$

19. Find the sum of the first 10 terms of the sequence defined by $a_n = 4n + 1$.
Calculate the sum of the first 10 terms:

$$S_{10} = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

where $a_1 = 4(1) + 1 = 5$ and $d = 4$:

$$S_{10} = \frac{10}{2}\left(2(5) + (10-1)4\right) = 5\left(10 + 36\right) = 5 \times 46 = 230$$

20. Determine the 8th term of the sequence defined by $a_n = \frac{2n}{n+1}$.
Calculate the 8th term:

$$a_8 = \frac{2 \cdot 8}{8+1} = \frac{16}{9}$$

21. Given the arithmetic sequence where the first term is 4 and the common difference is 6, find the 12th term.
Use the formula for the nth term:

$$a_n = a_1 + (n-1)d$$

Calculate the 12th term:

$$a_{12} = 4 + (12-1) \cdot 6 = 4 + 66 = 70$$

22. Find the sum of the first 7 terms of the geometric sequence where the first term is 3 and the common ratio is 2.
Use the formula for the sum of the first n terms of a geometric series:

$$S_n = \frac{a(1-r^n)}{1-r}$$

Calculate the sum of the first 7 terms:

$$S_7 = \frac{3(1-2^7)}{1-2} = \frac{3(-127)}{-1} = 3 \cdot 127 = 381$$

23. If the sum of the first 6 terms of an arithmetic sequence is 84 and the first term is 6, find the common difference.
Use the sum formula for arithmetic sequences:

$$S_n = \frac{n}{2}\left(2a_1 + (n-1)d\right)$$

Solve for d:

$$84 = \frac{6}{2}\left(2(6) + (6-1)d\right)$$

$$84 = 3\left(12 + 5d\right) \Rightarrow 84 = 36 + 15d \Rightarrow 48 = 15d \Rightarrow d = \frac{48}{15} = 3.2$$

24. Find the sum of the first 6 terms of the geometric sequence where the first term is 8 and the common ratio is $\frac{1}{3}$.
Use the formula for the sum of the first n terms of a geometric series:

$$S_n = \frac{a(1-r^n)}{1-r}$$

Calculate the sum of the first 6 terms:

$$S_6 = \frac{8\left(1-\left(\frac{1}{3}\right)^6\right)}{1-\frac{1}{3}} = \frac{8\left(1-\frac{1}{729}\right)}{\frac{2}{3}} = \frac{8 \times \frac{728}{729}}{\frac{2}{3}} = \frac{8 \times 728 \times 3}{729 \times 2} = \frac{17472}{1458} \approx 11.98$$

25. Find the sum of the series $2 + 4 + 8 + 16 + \cdots + 512$.
This is a geometric series where the first term $a = 2$ and the common ratio $r = 2$. Find the number of terms n:

$$512 = 2 \cdot 2^{n-1} \Rightarrow 2^{n-1} = 256 \Rightarrow 2^{n-1} = 2^8 \Rightarrow n - 1 = 8 \Rightarrow n = 9$$

Use the sum formula for the geometric series:

$$S_n = \frac{a(1-r^n)}{1-r}$$

Calculate the sum:

$$S_9 = \frac{2(1-2^9)}{1-2} = \frac{2(-511)}{-1} = 1022$$

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