

Topic: Covariance

Question: Calculate the covariance of the sample.

X	2	3	4	5	6
Y	3	5	7	9	11

Answer choices:

A $s_{XY} = 8$

B $s_{XY} = 7$

C $s_{XY} = 5$

D $s_{XY} = 4$



Solution: C

Find the mean of X ,

$$\bar{X} = \frac{2 + 3 + 4 + 5 + 6}{5}$$

$$\bar{X} = \frac{20}{5}$$

$$\bar{X} = 4$$

and then the mean of Y .

$$\bar{Y} = \frac{3 + 5 + 7 + 9 + 11}{5}$$

$$\bar{Y} = \frac{35}{5}$$

$$\bar{Y} = 7$$

Now use the means to find the sample covariance.

$$s_{XY} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = (2 - 4)(3 - 7) + (3 - 4)(5 - 7)$$

$$+ (4 - 4)(7 - 7) + (5 - 4)(9 - 7) + (6 - 4)(11 - 7)$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = -2(-4) - 1(-2) + 0(0) + 1(2) + 2(4)$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 8 + 2 + 2 + 8$$



$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 20$$

$$s_{XY} = \frac{20}{5 - 1}$$

$$s_{XY} = 5$$



Topic: Covariance

Question: If X takes on the sample values $\{2, 4, 6, 8, 10, 15\}$, and Y takes on the sample values $\{12, 17, 23, 25, 33, 40\}$, find the covariance of X and Y .

Answer choices:

- A 39.2
- B 46
- C 47
- D 56



Solution: C

Find the mean of X ,

$$\bar{X} = \frac{2 + 4 + 6 + 8 + 10 + 15}{6}$$

$$\bar{X} = \frac{45}{6}$$

$$\bar{X} = 7.5$$

and then the mean of Y .

$$\bar{Y} = \frac{12 + 17 + 23 + 25 + 33 + 40}{6}$$

$$\bar{Y} = \frac{150}{6}$$

$$\bar{Y} = 25$$

Now use the means to find the sample covariance.

$$s_{XY} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = (2 - 7.5)(12 - 25) + (4 - 7.5)(17 - 25)$$

$$+ (6 - 7.5)(23 - 25) + (8 - 7.5)(25 - 25)$$

$$+ (10 - 7.5)(33 - 25) + (15 - 7.5)(40 - 25)$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 235$$



$$s_{XY} = \frac{235}{6 - 1}$$

$$s_{XY} = 47$$



Topic: Covariance

Question: Two corporations record their stock returns between 2010 and 2014. From the sample, calculate the covariance of their stock returns.

	2010	2011	2012	2013	2014
X	2%	1%	-2%	4%	-1%
Y	3%	0%	1%	2%	1%

Answer choices:

- A $0.95 \%^2$
- B $1.08 \%^2$
- C $1.35 \%^2$
- D $1.49 \%^2$



Solution: C

Find the mean of X ,

$$\bar{X} = \frac{2 + 1 + (-2) + 4 + (-1)}{5}$$

$$\bar{X} = \frac{4}{5}$$

$$\bar{X} = 0.8$$

and then the mean of Y .

$$\bar{Y} = \frac{3 + 0 + 1 + 2 + 1}{5}$$

$$\bar{Y} = \frac{7}{5}$$

$$\bar{Y} = 1.4$$

Now use the means to find the sample covariance.

$$s_{XY} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{n - 1}$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = (2 - 0.8)(3 - 1.4) + (1 - 0.8)(0 - 1.4)$$

$$+ (-2 - 0.8)(1 - 1.4) + (4 - 0.8)(2 - 1.4) + (-1 - 0.8)(1 - 1.4)$$

$$\sum (X_i - \bar{X})(Y_i - \bar{Y}) = 5.4$$

$$s_{XY} = \frac{5.4}{5 - 1}$$



$$s_{XY} = 1.35$$

