Discrete Mathematics

Section 2: Logic

Questions

1. Determine if the following sentences are statements. If they are statements, state whether they are true or false.
   (a) The sky is blue.
   (b) $2 + 2 = 5$.
   (c) Do your homework.
   (d) $\forall x \in \mathbb{R}, x^2 \geq 0$.

2. Write the negation of the following statements:
   (a) All swans are white.
   (b) There exists a natural number that is not a positive integer.
   (c) If it rains, then the ground will be wet.

3. Determine the truth value of the following compound statements:
   (a) $(2 < 3) \wedge (4 > 5)$
   (b) $(1 + 1 = 2) \vee (2 + 2 = 5)$
   (c) $\neg(0 = 1)$

4. Express the following sentences as compound statements:
   (a) If it rains, then the ground will be wet and it will be cloudy.
   (b) Either the sky is clear or it is raining, but it is not both clear and raining.
   (c) If it is hot, then we will go swimming or stay indoors.

5. Convert the following statements into compound statements using logical connectives:
   (a) The sun is shining or it is raining, and it is not cold.
   (b) If I study, then I will pass the exam, or I will get extra credit.
   (c) It is either Monday or Tuesday, and I have a meeting only if it is Monday.

6. Determine the truth value of the following compound statements:
   (a) $(5 > 3) \wedge (2 = 2)$
   (b) $(4 + 1 = 5) \vee (7 - 3 = 1)$
   (c) $\neg(6 \leq 6)$

7. Construct the truth table for the compound statement $(p \wedge q) \vee \neg r$.

8. Construct the truth table for the compound statement $\neg((p \vee \neg q) \wedge (r \to s))$.

9. Construct the truth table for the compound statement $\neg((p \to q) \vee (\neg r \wedge s))$.
10. Determine whether the following pairs of statements are logically equivalent. Provide detailed explanations for your answers.
    (a) $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$
    (b) $\neg(p \vee q)$ and $\neg p \wedge \neg q$
11. See if the statement, $\neg(p \to (q \wedge r))$ and $\neg(p) \vee \neg(q \wedge r)$ are logically equivalent using a truth table.
12. Determine if the following statements are tautology or contradiction. Use truth table if necessary:
    (a) $(p \wedge (p \to q)) \to q$
    (b) $(p \wedge \neg p) \vee (q \wedge \neg q)$
13. Consider the statement: "All even numbers are divisible by 2."
    (a) Express this statement using a universal quantifier.
    (b) Determine the contrapositive of this statement.
    (c) Determine the negation of this statement.
14. For the given logic circuit, determine the output Z based on the inputs A and B:

    $$Z = \text{NOR}(\text{NAND}(A, B), \text{NOR}(A, B))$$

    Construct the truth table to determine the output Z for all possible input combinations of A and B. Additionally, determine the value of Z when $A = 0$ and $B = 1$.
15. For the given logic circuit, determine the output Z based on the inputs A, B, and C:

    $$Z = \text{NOR}(\text{NAND}(A, B), \text{NOR}(A, \text{NOR}(C, C)))$$

    Construct the truth table to determine the output Z for all possible input combinations of A, B, and C. Additionally, determine the value of Z when $A = 1$, $B = 0$, and $C = 1$.
16. For the given logic circuit, determine the output Z based on the inputs A, B, and C:

    $$Z = \text{NOR}(\text{NAND}(A,\ \neg(B)), \text{NOR}(\text{NAND}(B, C),\ \neg(A)))$$

    Construct the truth table to determine the output Z for all possible input combinations of A, B, and C. Additionally, determine the value of Z when $A = 1$, $B = 0$, and $C = 1$.
17. Determine if the following statements are tautologies or contradictions. Construct a truth table to justify your answer:
    (a) $A = ((p \wedge (p \to q)) \to (q \vee \neg r)) \vee (r \wedge \neg q)$
    (b) $B = (\neg(p \wedge q) \vee (r \to q)) \wedge (p \to (\neg q \vee r))$
18. Consider the statement: "All even numbers greater than 4 are divisible by 2 and can be expressed as the sum of two primes."
    (a) Express this statement using a universal quantifier and appropriate predicates.
    (b) Determine the contrapositive of this statement.
    (c) Determine the negation of this statement and explain its meaning.
19. Construct the truth table to determine if the biconditional statement $(p \leftrightarrow q) \wedge (r \leftrightarrow s)$ is logically equivalent to $Z = ((p \wedge q) \vee (\neg p \wedge \neg q)) \wedge ((r \wedge s) \vee (\neg r \wedge \neg s))$.

20. Convert the statement: *If it rains and it is windy, then the ground will be wet or it will be cloudy* into a compound statement using logical connectives and constructing its truth table. Additionally, determine its contrapositive and negation.

21. For the given logic circuit, determine the output Z based on the inputs A, B, and C:

$$Z = \text{AND}(\text{NAND}(A, B), \text{OR}(C, \text{NOR}(A, B)))$$

    Construct the truth table to determine the output Z for all possible input combinations of A, B, and C. Additionally, determine whether the circuit implements a tautology, contradiction, or neither. Provide a detailed explanation.

22. Construct the truth table for the compound statement $A = ((p \to q) \land (\neg r \lor s)) \to (\neg p \lor q)$. Then, determine if A is a tautology, contradiction, or neither. Finally, verify the logical equivalence of this statement with its contrapositive and provide a detailed explanation.

23. For the given statement: *"If the system is either not functioning or not on, then the output is either incorrect or the input is not received."*, express this statement as a compound logical statement, construct its truth table, determine its contrapositive, and provide its negation.

24. Consider the statement: "All integers are either even or odd."

    (a) Express this statement using a universal quantifier.
    (b) Determine the contrapositive of this statement.
    (c) Determine the negation of this statement.

25. Verify the logical equivalence of the following compound statements by constructing their truth tables and providing a detailed explanation:

    (a) $(p \to (q \land r)) \equiv ((p \to q) \land (p \to r))$
    (b) $(\neg(p \lor q) \land (r \lor s)) \equiv ((\neg p \land \neg q) \land (r \lor s))$

Qustions and Answers

1. Determine if the following sentences are statements. If they are statements, state whether they are true or false.
   (a) The sky is blue.
   (Statement, True)
   (b) $2 + 2 = 5$.
   (Statement, False)
   (c) Do your homework.
   (Not a statement)
   (d) $\forall x \in \mathbb{R}, x^2 \geq 0$.
   (Statement, True)

2. Write the negation of the following statements:
   (a) All swans are white.
   Negation: Not all swans are white (or There exists a swan that is not white).
   (b) There exists a natural number that is not a positive integer.
   Negation: Every natural number is a positive integer.
   (c) If it rains, then the ground will be wet.
   Negation: It rains and the ground is not wet.

3. Determine the truth value of the following compound statements:
   (a) $(2 < 3) \wedge (4 > 5)$
   (False, since $4 > 5$ is False)
   (b) $(1 + 1 = 2) \vee (2 + 2 = 5)$
   (True, since $1 + 1 = 2$ is True)
   (c) $\neg(0 = 1)$
   (True, since $0 = 1$ is False)

4. Express the following sentences as compound statements:
   (a) If it rains, then the ground will be wet and it will be cloudy.
   Let p be "it rains," q be "the ground will be wet," and r be "it will be cloudy."
   The compound statement is $p \to (q \wedge r)$.
   (b) Either the sky is clear or it is raining, but it is not both clear and raining.
   Let p be "the sky is clear," and q be "it is raining."
   The compound statement is $(p \vee q) \wedge \neg(p \wedge q)$.
   (c) If it is hot, then we will go swimming or stay indoors.
   Let p be "it is hot," q be "we will go swimming," and r be "we will stay indoors."
   The compound statement is $p \to (q \vee r)$.

5. Convert the following statements into compound statements using logical connectives:
   (a) The sun is shining or it is raining, and it is not cold.
   Let p be "the sun is shining," q be "it is raining," and r be "it is cold."
   The compound statement is $(p \vee q) \wedge \neg r$.
   (b) If I study, then I will pass the exam, or I will get extra credit.
   Let p be "I study," q be "I will pass the exam," and r be "I will get extra credit."
   The compound statement is $p \to (q \vee r)$.
   (c) It is either Monday or Tuesday, and I have a meeting only if it is Monday.
   Let p be "it is Monday," q be "it is Tuesday," and r be "I have a meeting."
   The compound statement is $(p \vee q) \wedge (r \to p)$.

6. Determine the truth value of the following compound statements:

   (a) $(5 > 3) \wedge (2 = 2)$
   (True, since both $5 > 3$ and $2 = 2$ are True)

   (b) $(4 + 1 = 5) \vee (7 - 3 = 1)$
   (True, since $4 + 1 = 5$ is True)

   (c) $\neg(6 \leq 6)$
   (False, since $6 \leq 6$ is True)

7. Construct the truth table for the compound statement $(p \wedge q) \vee \neg r$.
   Truth table for $(p \wedge q) \vee \neg r$:

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \vee \neg r$
T	T	T	T	F	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

8. Construct the truth table for the compound statement $\neg((p \vee \neg q) \wedge (r \to s))$.
   Truth table for $\neg((p \vee \neg q) \wedge (r \to s))$:

p	q	r	s	$\neg q$	$p \vee \neg q$	$r \to s$	$(p \vee \neg q) \wedge (r \to s)$	$\neg((p \vee \neg q) \wedge (r \to s))$
T	T	T	T	F	T	T	T	F
T	T	T	F	F	T	F	F	T
T	T	F	T	F	T	T	T	F
T	T	F	F	F	T	T	T	F
T	F	T	T	T	T	T	T	F
T	F	T	F	T	T	F	F	T
T	F	F	T	T	T	T	T	F
T	F	F	F	T	T	T	T	F
F	T	T	T	F	F	T	F	T
F	T	T	F	F	F	F	F	T
F	T	F	T	F	F	T	F	T
F	T	F	F	F	F	T	F	T
F	F	T	T	T	T	T	T	F
F	F	T	F	T	T	F	F	T
F	F	F	T	T	T	T	T	F
F	F	F	F	T	T	T	T	F

9. Construct the truth table for the compound statement $\neg((p \to q) \vee (\neg r \wedge s))$.
Truth table for $\neg((p \to q) \vee (\neg r \wedge s))$:

p	q	r	s	$\neg r$	$p \to q$	$\neg r \wedge s$	$(p \to q) \vee (\neg r \wedge s)$	$\neg((p \to q) \vee (\neg r \wedge s))$
T	T	T	T	F	T	F	T	F
T	T	T	F	F	T	F	T	F
T	T	F	T	T	T	T	T	F
T	T	F	F	T	T	F	T	F
T	F	T	T	F	F	F	F	T
T	F	T	F	F	F	F	F	T
T	F	F	T	T	F	T	T	F
T	F	F	F	T	F	F	F	T
F	T	T	T	F	T	F	T	F
F	T	T	F	F	T	F	T	F
F	T	F	T	T	T	T	T	F
F	T	F	F	T	T	F	T	F
F	F	T	T	F	T	F	T	F
F	F	T	F	F	T	F	T	F
F	F	F	T	T	T	T	T	F
F	F	F	F	T	T	F	T	F

10. Determine whether the following pairs of statements are logically equivalent. Provide detailed explanations for your answers.

(a) $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$
Explanation: These statements are logically equivalent. This can be shown using the distributive property of logical connectives. The expression $p \wedge (q \vee r)$ can be distributed as $(p \wedge q) \vee (p \wedge r)$. Both statements will have the same truth value for any values of p, q, and r.

(b) $\neg(p \vee q)$ and $\neg p \wedge \neg q$
Explanation: These statements are logically equivalent. This is an application of De Morgan's laws, which state that the negation of a disjunction is the conjunction of the negations. Hence, $\neg(p \vee q) \equiv \neg p \wedge \neg q$. Both statements will have the same truth value for any values of p and q.

11. See if the statement, $\neg(p \to (q \wedge r))$ and $\neg(p) \vee \neg(q \wedge r)$ are logically equivalent using a truth table.
Truth table for $\neg(p \to (q \wedge r))$ and $\neg(p) \vee \neg(q \wedge r)$:

p	q	r	$q \wedge r$	$\neg(q \wedge r)$	$p \to (q \wedge r)$	$\neg(p)$	$\neg(p) \vee \neg(q \wedge r)$	$\neg(p \to (q \wedge r))$
T	T	T	T	F	T	F	F	F
T	T	F	F	T	F	F	T	T
T	F	T	F	T	F	F	T	T
T	F	F	F	T	F	F	T	T
F	T	T	T	F	T	T	T	F
F	T	F	F	T	T	T	T	F
F	F	T	F	T	T	T	T	F
F	F	F	F	T	T	T	T	F

From the truth table, we can see that $\neg(p \to (q \wedge r))$ does not have the same truth values as $\neg(p) \vee \neg(q \wedge r)$ in all cases. Hence, they are not logically equivalent.

12. Determine if the following statements are tautology or contradiction. Use truth table if necessary:

(a) $(p \wedge (p \to q)) \to q$
Truth table for $(p \wedge (p \to q)) \to q$:

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since the final column is all True, the statement is a tautology.

(b) $(p \wedge \neg p) \vee (q \wedge \neg q)$
Truth table for $(p \wedge \neg p) \vee (q \wedge \neg q)$:

p	q	$\neg p$	$\neg q$	$p \wedge \neg p$	$q \wedge \neg q$	$(p \wedge \neg p) \vee (q \wedge \neg q)$
T	T	F	F	F	F	F
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	F	F

Since the final column is all False, the statement is a contradiction.

13. Consider the statement: "All even numbers are divisible by 2."

(a) Express this statement using a universal quantifier.
$\forall x \in \mathbb{Z}$, (if x is even, then x is divisible by 2).
or
$\forall x \in \mathbb{Z}, (E(x) \to D(x))$.
where $E(x)$ denotes "x is even" and $D(x)$ denotes "x is divisible by 2."

(b) Determine the contrapositive of this statement.
$\forall x \in \mathbb{Z}$, (if x is not divisible by 2, then x is not even).
or
$\forall x \in \mathbb{Z}, (\neg D(x) \to \neg E(x))$.

(c) Determine the negation of this statement.
$\exists x \in \mathbb{Z}$ such that x is even and x is not divisible by 2.
or
$\exists x \in \mathbb{Z}, (E(x) \wedge \neg D(x))$.

14. For the given logic circuit, determine the output Z based on the inputs A and B:

$$Z = \text{NOR}(\text{NAND}(A, B), \text{NOR}(A, B))$$

Construct the truth table to determine the output Z for all possible input combinations of A and B. Additionally, determine the value of Z when $A = 0$ and $B = 1$.
Truth table for $Z = \text{NOR}(\text{NAND}(A, B), \text{NOR}(A, B))$:

A	B	$\text{NAND}(A, B)$	$\text{NOR}(A, B)$	$\text{NOR}(\text{NAND}(A, B), \text{NOR}(A, B))$	Z
0	0	1	1	0	0
0	1	1	0	0	0
1	0	1	0	0	0
1	1	0	0	1	1

The final column shows the output Z for all possible input combinations of A and B. When $A = 0$ and $B = 1$, the output Z is 0.

15. For the given logic circuit, determine the output Z based on the inputs A, B, and C:

$$Z = \text{NOR}(\text{NAND}(A, B), \text{NOR}(A, \text{NOR}(C, C)))$$

Construct the truth table to determine the output Z for all possible input combinations of A, B, and C. Additionally, determine the value of Z when $A = 1$, $B = 0$, and $C = 1$.
Truth table for $Z = \text{NOR}(\text{NAND}(A, B), \text{NOR}(A, \text{NOR}(C, C)))$:

A	B	C	$\text{NAND}(A, B)$	$\text{NOR}(C, C)$	$\text{NOR}(A, \text{NOR}(C, C))$	Z
0	0	0	1	1	0	0
0	0	1	1	0	1	0
0	1	0	1	1	0	0
0	1	1	1	0	1	0
1	0	0	1	1	0	0
1	0	1	1	0	1	0
1	1	0	0	1	0	1
1	1	1	0	0	1	1

The final column shows the output Z for all possible input combinations of A, B, and C. When $A = 1$, $B = 0$, and $C = 1$, the output Z is 0.

16. For the given logic circuit, determine the output Z based on the inputs A, B, and C:

$$Z = \text{NOR}(\text{NAND}(A, \neg(B)), \text{NOR}(\text{NAND}(B, C), \neg(A)))$$

Construct the truth table to determine the output Z for all possible input combinations of A, B, and C. Additionally, determine the value of Z when $A = 1$, $B = 0$, and $C = 1$.
Truth table for $Z = \text{NOR}(\text{NAND}(A, \neg(B)), \text{NOR}(\text{NAND}(B, C), \neg(A)))$:

A	B	C	$\neg(B)$	$\text{NAND}(A, \neg(B))$	$\text{NAND}(B, C)$	$\neg(A)$	$\text{NOR}(\text{NAND}(B, C), \neg(A))$	Z
0	0	0	1	1	1	1	0	0
0	0	1	1	1	1	1	0	0
0	1	0	0	1	1	1	0	0
0	1	1	0	1	0	1	0	0
1	0	0	1	0	1	0	0	1
1	0	1	1	0	1	0	0	1
1	1	0	0	1	1	0	0	0
1	1	1	0	1	0	0	1	0

The final column shows the output Z for all possible input combinations of A, B, and C. When $A = 1$, $B = 0$, and $C = 1$, the output Z is 1.

17. Determine if the following statements are tautologies or contradictions. Construct a truth table to justify your answer:

   (a) $A = ((p \land (p \to q)) \to (q \lor \neg r)) \lor (r \land \neg q)$
   Truth table for $A = ((p \land (p \to q)) \to (q \lor \neg r)) \lor (r \land \neg q)$:

p	q	r	$p \to q$	$p \land (p \to q)$	$\neg r$	$q \lor \neg r$	$r \land \neg q$	$(q \lor \neg r) \lor (r \land \neg q)$	A
T	T	T	T	T	F	T	F	T	T
T	T	F	T	T	T	T	F	T	T
T	F	T	F	F	F	F	T	F	T
T	F	F	F	F	T	F	T	T	T
F	T	T	T	F	F	T	F	T	T
F	T	F	T	F	T	T	F	T	T
F	F	T	T	F	F	F	T	F	T
F	F	F	T	F	T	T	T	T	T

   Since the final column is all True, the statement is a tautology.

   (b) $B = (\neg(p \land q) \lor (r \to q)) \land (p \to (\neg q \lor r))$
   Truth table for $B = (\neg(p \land q) \lor (r \to q)) \land (p \to (\neg q \lor r))$:

p	q	r	$p \land q$	$\neg(p \land q)$	$r \to q$	$\neg(p \land q) \lor (r \to q)$	$\neg q$	$\neg q \lor r$	$p \to (\neg q \lor r)$	B
T	T	T	T	F	T	T	F	T	T	T
T	T	F	T	F	T	T	F	F	F	F
T	F	T	F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T	T	T	T
F	T	T	F	T	T	T	F	T	T	T
F	T	F	F	T	T	T	F	F	T	T
F	F	T	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T	T

   Since the final column has both True and False values, the statement is neither a tautology nor a contradiction.

18. Consider the statement: "All even numbers greater than 4 are divisible by 2 and can be expressed as the sum of two primes."

   (a) Express this statement using a universal quantifier and appropriate predicates.
   $\forall x \in \mathbb{Z}, (x > 4 \land E(x) \to (D(x) \land P(x)))$,
   where $E(x)$ denotes "x is even," $D(x)$ denotes "x is divisible by 2," and $P(x)$ denotes "x can be expressed as the sum of two primes."

   (b) Determine the contrapositive of this statement.
   $\forall x \in \mathbb{Z}, (\neg(D(x) \land P(x)) \to (x \leq 4 \lor \neg E(x)))$.
   or
   If x is not divisible by 2 or cannot be expressed as the sum of two primes, then $x \leq 4$ or x is not even.

   (c) Determine the negation of this statement and explain its meaning.
   $\exists x \in \mathbb{Z}$ such that $(x > 4 \land E(x) \land (\neg D(x) \lor \neg P(x)))$.
   This means that there exists an even number greater than 4 that is either not divisible by 2 or cannot be expressed as the sum of two primes.

19. Construct the truth table to determine if the biconditional statement $(p \leftrightarrow q) \wedge (r \leftrightarrow s)$ is logically equivalent to $Z = ((p \wedge q) \vee (\neg p \wedge \neg q)) \wedge ((r \wedge s) \vee (\neg r \wedge \neg s))$.

Truth table for $(p \leftrightarrow q) \wedge (r \leftrightarrow s)$:

p	q	r	s	$p \leftrightarrow q$	$r \leftrightarrow s$	$(p \leftrightarrow q) \wedge (r \leftrightarrow s)$
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	F
T	F	T	F	F	F	F
T	F	F	T	F	F	F
T	F	F	F	F	T	F
F	T	T	T	F	T	F
F	T	T	F	F	F	F
F	T	F	T	F	F	F
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	F
F	F	F	F	T	T	T

Truth table for $Z = ((p \wedge q) \vee (\neg p \wedge \neg q)) \wedge ((r \wedge s) \vee (\neg r \wedge \neg s))$:

p	q	r	s	$(p \wedge q) \vee (\neg p \wedge \neg q)$	$(r \wedge s) \vee (\neg r \wedge \neg s)$	Z
T	T	T	T	T	T	T
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	T	F	F	T	T	T
T	F	T	T	F	T	F
T	F	T	F	F	F	F
T	F	F	T	F	F	F
T	F	F	F	F	T	F
F	T	T	T	F	T	F
F	T	T	F	F	F	F
F	T	F	T	F	F	F
F	T	F	F	F	T	F
F	F	T	T	T	T	T
F	F	T	F	T	F	F
F	F	F	T	T	F	F
F	F	F	F	T	T	T

Since the last columns for $(p \leftrightarrow q) \wedge (r \leftrightarrow s)$ and $((p \wedge q) \vee (\neg p \wedge \neg q)) \wedge ((r \wedge s) \vee (\neg r \wedge \neg s))$ are identical, the statements are logically equivalent.

20. Convert the statement: *If it rains and it is windy, then the ground will be wet or it will be cloudy* into a compound statement using logical connectives and constructing its truth table. Additionally, determine its contrapositive and negation.
Let R be "it rains", W be "it is windy", G be "the ground will be wet", and C be "it will be cloudy". The compound statement is $(R \wedge W) \rightarrow (G \vee C)$.
Truth table for $(R \wedge W) \rightarrow (G \vee C)$:

R	W	G	C	$R \wedge W$	$G \vee C$	$(R \wedge W) \rightarrow (G \vee C)$
T	T	T	T	T	T	T
T	T	T	F	T	T	T
T	T	F	T	T	T	T
T	T	F	F	T	F	F
T	F	T	T	F	T	T
T	F	T	F	F	T	T
T	F	F	T	F	T	T
T	F	F	F	F	F	T
F	T	T	T	F	T	T
F	T	T	F	F	T	T
F	T	F	T	F	T	T
F	T	F	F	F	F	T
F	F	T	T	F	T	T
F	F	T	F	F	T	T
F	F	F	T	F	T	T
F	F	F	F	F	F	T

Contrapositive: $(\neg(G \vee C)) \rightarrow (\neg(R \wedge W))$. Negation: $(R \wedge W) \wedge \neg(G \vee C)$.

21. For the given logic circuit, determine the output Z based on the inputs A, B, and C:

$$Z = \text{AND}(\text{NAND}(A, B), \text{OR}(C, \text{NOR}(A, B)))$$

Construct the truth table to determine the output Z for all possible input combinations of A, B, and C. Additionally, determine whether the circuit implements a tautology, contradiction, or neither. Provide a detailed explanation.
Truth table for $Z = \text{AND}(\text{NAND}(A, B), \text{OR}(C, \text{NOR}(A, B)))$:

A	B	C	NAND(A, B)	NOR(A, B)	OR$(C, \text{NOR}(A, B))$	Z
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	0	0	0
0	1	1	1	0	1	1
1	0	0	1	0	0	0
1	0	1	1	0	1	1
1	1	0	0	0	0	0
1	1	1	0	0	1	0

Since the output Z is not consistently true or false for all possible input combinations, it is neither a tautology nor a contradiction.

22. Construct the truth table for the compound statement $A = ((p \to q) \land (\neg r \lor s)) \to (\neg p \lor q)$. Then, determine if A is a tautology, contradiction, or neither. Finally, verify the logical equivalence of this statement with its contrapositive and provide a detailed explanation.

Truth table for $((p \to q) \land (\neg r \lor s)) \to (\neg p \lor q)$:

p	q	r	s	$p \to q$	$\neg r$	$\neg r \lor s$	$(p \to q) \land (\neg r \lor s)$	$\neg p$	$\neg p \lor q$	A
T	T	T	T	T	F	T	T	F	T	T
T	T	T	F	T	F	F	F	F	T	T
T	T	F	T	T	T	T	T	F	T	T
T	T	F	F	T	T	F	F	F	T	T
T	F	T	T	F	F	T	F	F	F	T
T	F	T	F	F	F	F	F	F	F	T
T	F	F	T	F	T	T	F	F	F	T
T	F	F	F	F	T	T	F	F	F	T
F	T	T	T	T	F	T	T	T	T	T
F	T	T	F	T	F	F	F	T	T	T
F	T	F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T	T	T
F	F	T	T	T	F	T	T	T	F	T
F	F	T	F	T	F	F	F	T	F	T
F	F	F	T	T	T	T	T	T	F	T
F	F	F	F	T	T	T	T	T	F	T

The statement $((p \to q) \land (\neg r \lor s)) \to (\neg p \lor q)$ is a tautology, as it is true for all possible truth values of p, q, r, and s. The contrapositive of $((p \to q) \land (\neg r \lor s)) \to (\neg p \lor q)$ is $\neg(\neg p \lor q) \to \neg((p \to q) \land (\neg r \lor s))$. This simplifies to $(p \land \neg q) \to ((p \land \neg q) \lor (r \land \neg s)$

Truth table for $(p \land \neg q) \to (p \land \neg q) \lor (r \land \neg s)$:

p	q	r	s	$\neg q$	$\neg s$	$p \land \neg q$	$r \land \neg s$	$(p \land \neg q) \lor (r \land \neg s)$	$(p \land \neg q) \to ((p \land \neg q) \lor (r \land \neg s))$
T	T	T	T	F	F	F	F	F	T
T	T	T	F	F	T	F	T	T	T
T	T	F	T	F	F	F	F	F	T
T	T	F	F	F	T	F	F	F	T
T	F	T	T	T	F	T	F	T	T
T	F	T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	F	T	T
T	F	F	F	T	T	T	T	T	T
F	T	T	T	F	F	F	F	F	T
F	T	T	F	F	T	F	T	T	T
F	T	F	T	F	F	F	F	F	T
F	T	F	F	F	T	F	F	F	T
F	F	T	T	T	F	F	F	F	T
F	F	T	F	T	T	F	T	T	T
F	F	F	T	T	F	F	F	F	T
F	F	F	F	T	T	F	T	T	T

Since both the original statement and its contrapositive yield the same truth table results (always true), they are logically equivalent.

23. For the given statement: *"If the system is either not functioning or not on, then the output is either incorrect or the input is not received."*, express this statement as a compound logical statement, construct its truth table, determine its contrapositive, and provide its negation.
Let A be "the system is functioning", O be "the system is on", C be "the output is correct", and I be "the input is received". The compound statement is $(\neg A \vee \neg O) \to (\neg C \vee \neg I)$.
Truth table for $(\neg A \vee \neg O) \to (\neg C \vee \neg I)$:

A	O	C	I	$\neg A$	$\neg O$	$\neg C$	$\neg I$	$\neg A \vee \neg O$	$\neg C \vee \neg I$	$(\neg A \vee \neg O) \to (\neg C \vee \neg I)$
T	T	T	T	F	F	F	F	F	F	T
T	T	T	F	F	F	F	T	F	T	T
T	T	F	T	F	F	T	F	F	T	T
T	T	F	F	F	F	T	T	F	T	T
T	F	T	T	F	T	F	F	T	F	F
T	F	T	F	F	T	F	T	T	T	T
T	F	F	T	F	T	T	F	T	T	T
T	F	F	F	F	T	T	T	T	T	T
F	T	T	T	T	F	F	F	T	F	F
F	T	T	F	T	F	F	T	T	T	T
F	T	F	T	T	F	T	F	T	T	T
F	T	F	F	T	F	T	T	T	T	T
F	F	T	T	T	T	F	F	T	F	F
F	F	T	F	T	T	F	T	T	T	T
F	F	F	T	T	T	T	F	T	T	T
F	F	F	F	T	T	T	T	T	T	T

Contrapositive: $\neg(I \wedge \neg O) \to \neg(\neg A \wedge O \wedge \neg C)$. This simplifies to $(\neg I \vee O) \to (A \vee \neg O \vee C)$.
Negation: $\neg((\neg A \wedge O \wedge \neg C) \to (I \wedge \neg O))$. This simplifies to $(\neg A \wedge O \wedge \neg C) \wedge \neg(I \wedge \neg O)$.

24. Consider the statement: "All integers are either even or odd."

   (a) Express this statement using a universal quantifier.
   This statement means that for every integer x, x is either even or odd.
   This can be expressed as: $\forall x \in \mathbb{Z}, (x \text{ is even}) \vee (x \text{ is odd})$

   (b) Determine the contrapositive of this statement.
   The contrapositive of a statement $P \to Q$ is $\neg Q \to \neg P$.
   In this case: $\forall x \in \mathbb{Z}, \neg((x \text{ is even}) \vee (x \text{ is odd})) \to \neg(x \in \mathbb{Z}.)$
   This means that if an integer x is not even or odd, then x is not an integer.

   (c) Determine the negation of this statement.
   The negation of a universal statement $\forall x, P(x)$ is $\exists x, \neg P(x)$.
   In this case: $\exists x \in \mathbb{Z}, \neg((x \text{ is even}) \vee (x \text{ is odd})$
   This means that there exists an integer x that is neither even nor odd.

25. Verify the logical equivalence of the following compound statements by constructing their truth tables and providing a detailed explanation:

(a) $(p \to (q \land r)) \equiv ((p \to q) \land (p \to r))$

Truth table for $(p \to (q \land r)) \equiv ((p \to q) \land (p \to r))$:

p	q	r	$q \land r$	$p \to (q \land r)$	$p \to q$	$p \to r$	$(p \to q) \land (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

The truth table shows that $(p \to (q \land r))$ and $((p \to q) \land (p \to r))$ have identical truth values for all possible combinations of p, q, and r. Therefore, the statements are logically equivalent.

(b) $(\neg(p \lor q) \land (r \lor s)) \equiv ((\neg p \land \neg q) \land (r \lor s))$

Truth table for $(\neg(p \lor q) \land (r \lor s)) \equiv ((\neg p \land \neg q) \land (r \lor s))$:

p	q	r	s	$p \lor q$	$\neg(p \lor q)$	$r \lor s$	$\neg p \land \neg q$	$(\neg p \land \neg q) \land (r \lor s)$	$(\neg(p \lor q) \land (r \lor s))$
T	T	T	T	T	F	T	F	F	F
T	T	T	F	T	F	T	F	F	F
T	T	F	T	T	F	T	F	F	F
T	T	F	F	T	F	F	F	F	F
T	F	T	T	T	F	T	F	F	F
T	F	T	F	T	F	T	F	F	F
T	F	F	T	T	F	T	F	F	F
T	F	F	F	T	F	F	F	F	F
F	T	T	T	T	F	T	F	F	F
F	T	T	F	T	F	T	F	F	F
F	T	F	T	T	F	T	F	F	F
F	T	F	F	T	F	F	F	F	F
F	F	T	T	F	T	T	T	T	T
F	F	T	F	F	T	T	T	T	T
F	F	F	T	F	T	T	T	T	T
F	F	F	F	F	T	F	T	F	F

The truth table shows that $(\neg(p \lor q) \land (r \lor s))$ and $((\neg p \land \neg q) \land (r \lor s))$ have identical truth values for all possible combinations of p, q, r, and s. Therefore, the statements are logically equivalent.

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