Topic: Confidence interval for the proportion

Question: A study shows that 78% of patients who try a new medication for migraines feel better within 30 minutes of taking the medicine. If the study involved 120 patients, construct a 95% confidence interval for the proportion of patients who feel better within 30 minutes of taking the medicine.

Answer choices:

A $(a, b) \approx (0.73, 0.83)$

B $(a, b) \approx (0.72, 0.84)$

C $(a, b) \approx (0.71, 0.85)$

D $(a, b) \approx (0.68, 0.88)$

Solution: C

We know that the sample proportion is $\hat{p} = 0.78$, and that the confidence level is 95%. The critical value for this confidence level is $z^* = 1.96$. The sample size is $n = 120$. So we can plug these values into the confidence interval formula.

$$(a, b) = \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$(a, b) = 0.78 \pm 1.96 \sqrt{\frac{0.78(1-0.78)}{120}}$$

$$(a, b) \approx 0.78 \pm 0.0741$$

$$(a, b) \approx (0.78 - 0.0741, 0.78 + 0.0741)$$

$$(a, b) \approx (0.71, 0.85)$$

This means that we're 95% confident that the proportion of patients who feel better within 30 minutes of taking the medicine is between 71% and 85%.

Topic: Confidence interval for the proportion

Question: A study shows that 243 of 500 randomly selected households were using a family member to care for their young children. Build a 90% confidence interval for the proportion of households using a family member to care for young children.

Answer choices:

A $(a, b) \approx (0.45, 0.52)$

B $(a, b) \approx (0.44, 0.53)$

C $(a, b) \approx (0.43, 0.54)$

D $(a, b) \approx (0.42, 0.55)$

Solution: A

The sample proportion is

$$\hat{p} = \frac{243}{500} = 0.486$$

With $\hat{p} = 0.486$, a confidence level of 90% and therefore a critical value of $z^* = 1.65$, and a sample size of $n = 500$, the confidence interval will be

$$(a, b) = \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$(a, b) = 0.486 \pm 1.65 \sqrt{\frac{0.486(1-0.486)}{500}}$$

$$(a, b) \approx 0.486 \pm 0.0369$$

$$(a, b) \approx (0.486 - 0.0369, 0.486 + 0.0369)$$

$$(a, b) \approx (0.45, 0.52)$$

This means that we're 90% confident that the proportion of households using a family member to care for young children is between 45% and 52%.

Topic: Confidence interval for the proportion

Question: Bentley is a dog who helps police find drugs, but he has a low success rate. In Bentley's time on the job, it's estimated that he correctly identified drugs 59% of the time. How many different trials would the police need to put him through to verify that this is his actual success rate, at a 95% confidence level and with a margin of error of 0.05?

Answer choices:

A $n = 372$

B $n = 385$

C $n = 413$

D $n = 645$

Solution: B

The minimum sample size n that gives a fixed margin of error is given by

$$n = \left(\frac{z^*\sqrt{\hat{p}(1-\hat{p})}}{ME}\right)^2$$

Since we're looking for the smallest possible sample size that maintains a specific margin of error, we need to optimize $\hat{p}(1-\hat{p})$, which we do using $\hat{p} = 0.5$. The confidence level is $95\,\%$, and the critical value for this confidence level is $z^* = 1.96$. The margin of error is $ME = 0.05$.

Plugging these values into the formula, we can find the minimum number of trials we need to verify Bentley's success rate.

$$n = \left(\frac{1.96\sqrt{0.5(1-0.5)}}{0.05}\right)^2$$

$$n = \left(\frac{1.96\sqrt{0.5(0.5)}}{0.05}\right)^2$$

$$n = \left(\frac{1.96(0.5)}{0.05}\right)^2$$

$$n = 384.16$$

Since we need more than 384 trials to test Bentley, we have to round up to 385 trials, so we can say $n = 385$.