2023 EE530L, Homework Assignment 2

Assume the following unless specifically mentioned otherwise.

1. Room temperature $T = 300$ K.
2. Thermal voltage, $kT/q = 26$ mV.
3. Intrinsic carrier concentration at $T = 300$ K, $n_i = 1.1 \times 10^{16}/\text{m}^3$.
4. Electron mobility, $\mu_n = 0.135$ m^2/Vs.
5. Hole mobility, $\mu_p = 0.048$ m^2/Vs.
6. MOS Oxide capacitance per unit area, $C_{ox} = 2.7$ fF/sq-μm.

1. A pure silicon crystal is doped with Boron (Z = 5) to a concentration of $N_A = 10^{22}$ atoms/m^3. At a temperature of 0° C, what is the concentration of free electrons and holes in the doped crystal?
2. What is the room temperature zero-bias contact potential V_o of a diode doped with Boron upto $N_A = 10^{22}$ atoms/m^3 on the p-side and Phosphorus (Z = 15) upto $N_D = 10^{20}$ atoms/m^3 on the n-side? If the diode is observed to leak a reverse current 1 pA with a 3 V reverse-bias, what is the voltage V_f that would appear across it when a forward current of 1 mA is forced across it? What is the small-signal conductance g? If the junction area is $A = 100$ sq-μm and the average minority carrier lifetime is $\tau_t = 100$ ps, what is the associated small-signal capacitance C_s with a forward current of 1 mA?
3. An n-channel MOSFET has $W/L = 20\ \mu\text{m}/2\ \mu\text{m}$, applied gate overdrive is $V_{GST} = 0.2$ V, and channel-length-modulation parameter $\lambda = 1/10\ \text{V}^{-1}$ remains constant. Find the drain current I_D, the small signal parameters $g_m = \partial I_D/\partial V_{GST}$ and $1/r_o = \partial I_D/\partial V_{DS}$ at $V_{DS} = V_{GST}/10$, $V_{DS} = V_{GST}$ and $V_{DS} = 10V_{GST}$.
4. Find two drain-to-source voltages, V_1 and V_2, both lesser than V_0, such that the small-signal transconductance g_m of a MOSFET with effective gate-to-source voltage $V_{GST} = V_0$ and $V_{DS} = V_1$ is the same as the output conductance $1/r_o$ of the same MOSFET biased at the same effective gate-to-source voltage $V_{GST} = V_0$, but at the drainto-source voltage $V_{DS} = V_2$. That is, find V_1 and V_2 in terms of V_0 such that $V_1 \leq V_0$, $V_2 \leq V_0$, and such that, for any unknown λ,
$$\left(\frac{\partial I_D}{\partial V_{GS}}\right)\Bigg|_{V_{GST}=V_0, V_{DS}=V_1} = \left(\frac{\partial I_D}{\partial V_{DS}}\right)\Bigg|_{V_{GST}=V_0, V_{DS}=V_2}.$$

Useful formulae:

$$pn = n_i^2 = 4\left(\frac{2\pi kT}{h^2}\right)^3 (m_n^* m_p^*)^{3/2} \exp\left(-\frac{E_g}{kT}\right),$$

$$V_{contact} = (kT/q)\ln(N_A N_D/n_i^2),$$

$$I_{diode} = I_{rs}\left[\exp\left(\frac{qV_{fwd}}{kT}\right) - 1\right],$$

$$g_{diode} = \frac{\partial I_{diode}}{\partial V_{fwd}} \approx \frac{qI_{diode}}{kT},$$

$$C_{diode} = C_{depl} + C_{diff} = A\sqrt{\frac{q\epsilon}{2(V_0 - V_{fwd})}\frac{N_A N_D}{N_A + N_D}} + \tau_T g_{diode}$$

$$I_D = \begin{cases} \mu C_{ox}\frac{W}{L}\left(V_{GST}V_{DS} - \frac{V_{DS}^2}{2}\right)(1 + \lambda(V_{DS} - V_{GST})), & 0 \leq V_{DS} \leq V_{GST}, \\ \mu C_{ox}\frac{W}{L}\frac{V_{GST}^2}{2}(1 + \lambda(V_{DS} - V_{GST})), & 0 \leq V_{GST} \leq V_{DS}. \end{cases}$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \begin{cases} \mu C_{ox}\frac{W}{L}V_{DS}\left[1 - \lambda\left(2V_{GST} - 3V_{DS}/2\right)\right], & 0 \leq V_{DS} \leq V_{GST}, \\ \mu C_{ox}\frac{W}{L}\left[V_{GST} - \lambda V_{GST}\left(3V_{GST}/2 - V_{DS}\right)\right], & 0 \leq V_{GST} \leq V_{DS}. \end{cases}$$

$$\frac{1}{r_o} = \frac{\partial I_D}{\partial V_{DS}} = \begin{cases} \mu C_{ox}\frac{W}{L}\left[V_{GST} - V_{DS} - \lambda\left(V_{GST} - V_{DS}\right)^2 + \lambda V_{DS}(V_{GST} - V_{DS}/2)\right], & 0 \leq V_{DS} \leq V_{GST}, \\ \mu C_{ox}\frac{W}{L}\frac{V_{GST}^2}{2}\lambda, & 0 \leq V_{GST} \leq V_{DS}. \end{cases}$$