

- ① The small signal model for this problem is shown on the right.

Performing nodal analysis

$$-\frac{v_{out}}{r_L} = g_{mn}(v_{in} - v_s) + \frac{v_{out} - v_s}{r_{on}} = \frac{v_s}{r_{op}} + g_{mp} v_s$$

$$\Rightarrow (g_{mn} + g_{mp} + \frac{1}{r_{on}} + \frac{1}{r_{op}}) v_s = g_{mn} v_{in} + \frac{v_{out}}{r_{on}}$$

$$\Rightarrow v_s = \frac{g_{mn} v_{in} + \frac{v_{out}}{r_{on}}}{g_{mn} + g_{mp} + \frac{1}{r_{on}} + \frac{1}{r_{op}}} = \frac{-\frac{v_{out}}{r_L}}{g_{mp} + \frac{1}{r_{op}}} \Rightarrow v_{out} \left[ \frac{g_{mn} + g_{mp} + \frac{1}{r_{on}} + \frac{1}{r_{op}}}{r_L} + \frac{g_{mp} + \frac{1}{r_{op}}}{r_{on}} \right] + g_{mn} v_{in} (g_{mp} + \frac{1}{r_{op}}) = 0$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = \frac{-g_{mn}(g_{mp} + \frac{1}{r_{op}})}{(g_{mn} + g_{mp} + \frac{1}{r_{on}} + \frac{1}{r_{op}}) \frac{1}{r_L} + (g_{mp} + \frac{1}{r_{op}}) \cdot \frac{1}{r_{on}}} = \boxed{-\frac{100}{26}}$$

For the output impedance, set  $v_{in} = 0$ , remove  $r_L$  and apply a test current:

$$i_{test} = -(g_{mn} + \frac{1}{r_{on}}) v_s + \frac{v_{out}}{r_{on}} = (g_{mp} + \frac{1}{r_{op}}) v_s \Rightarrow v_s = \frac{\frac{v_{out}}{r_{on}}}{g_{mp} + g_{mp} + \frac{1}{r_{on}} + \frac{1}{r_{op}}}$$

$$\Rightarrow i_{test} = \frac{v_{out}(g_{mp} + \frac{1}{r_{op}}) \frac{1}{r_{on}}}{g_{mn} + g_{mp} + \frac{1}{r_{on}} + \frac{1}{r_{op}}} \Rightarrow r_{out} = \frac{v_{out}}{i_{test}} = \frac{g_{mn} + g_{mp} + \frac{1}{r_{on}} + \frac{1}{r_{op}} \cdot r_{on}}{g_{mp} + \frac{1}{r_{op}}} = \boxed{125 k\Omega}$$

- ② The small signal model for this problem is shown to the right.

Performing nodal analysis,  $-\frac{v_{out}}{r_L} = g_m(v_{in} - v_s) + \frac{v_{out} - v_s}{r_o} - g_s v_s = \frac{v_s}{r_s}$

$$\Rightarrow v_s = -\frac{r_s}{r_L} v_{out}, g_m v_{in} + v_{out} \left[ \frac{1}{r_o} + \frac{1}{r_L} + (g_m + \frac{1}{r_o}) \frac{r_s}{r_L} \right] = 0 \Rightarrow \frac{v_{out}}{v_{in}} = \frac{-g_m r_o r_L}{r_o + r_L + [1 + (g_m + g_s) r_o] r_s} = \boxed{-\frac{10^4}{377}}$$

For the output impedance,  $i_{test} = -g_m v_s + \frac{v_{out} - v_s}{r_o} - g_s v_s = \frac{v_s}{r_s}$

$$\Rightarrow i_{test} [1 + (g_m + g_s + \frac{1}{r_o}) r_s] = \frac{v_{out}}{r_o}$$

$$\Rightarrow \frac{v_{out}}{i_{test}} = \frac{r_o + r_s + [(g_m + g_s) r_o] r_s}{1} = \boxed{177 k\Omega}$$

