EE530L, HW2

① $N_A = 10^{22}/m^3$. Free hole concentration, $p = N_A = 10^{22}/m^3$.

$$\frac{n_i^2(273K)}{n_i^2(300K)} = \left(\frac{273}{300}\right)^3 \cdot \exp\left(\frac{(1.11)(1.6\times10^{-19})}{(1.38\times10^{23})}\left(\frac{1}{300} - \frac{1}{273}\right)\right) = 0.0108.$$

Free Electron concentration, $n = \frac{n_i^2}{p} = \frac{n_i^2}{N_A} = \frac{0.0108\times1.21\times10^{32}}{10^{22}} /m^3 = \boxed{1.31\times10^8/m^3}$.

② $V_0 = \frac{kT}{q}\ln\frac{N_A N_D}{n_i^2} = \frac{(1.38\times10^{-23}\times300)}{(1.602\times10^{-19})}\ln\left(\frac{10^{22}\cdot10^{20}}{1.21\cdot10^{32}}\right) = \boxed{590mV}$.

$C = A\sqrt{\frac{q\epsilon}{2(V_0+V_R)}\cdot N_A N_D} = 10^{-10}\cdot\sqrt{\frac{1.602\times10^{-19}\times11.7\times8.854\times10^{-12}}{2(0.89)}\cdot\frac{100}{101}\cdot10^{20}}\ F = \boxed{3.04fF}$.

③ $V_{fwd} = \frac{kT}{q}\left[\ln\left(\frac{I_{fwd}+I_{rs}}{I_{rs}}\right)\right] = \boxed{536mV}$, $g = \frac{Iq}{kT} = \boxed{38.7\ mS}$, $C_s = g\tau_T = \boxed{(3.87)pF}$.

③
$$I_D = \begin{cases} \mu C_{ox}\frac{W}{L}(V_{GST}V_{DS} - V_{DS}^2/2)(1+\lambda(V_{DS}-V_{GST})), & 0 \le V_{DS} \le V_{GST}, \\ \mu C_{ox}\frac{W}{L}\frac{V_{GST}^2}{2}(1+\lambda(V_{DS}-V_{GST})), & 0 \le V_{GST} \le V_{DS}. \end{cases}$$

$$g_m = \begin{cases} \mu C_{ox}\frac{W}{L}\left[V_{DS}(1+\lambda(V_{DS}-V_{GST})) - (V_{GST}V_{DS}-V_{DS}^2/2)\lambda\right], & 0 \le V_{DS} \le V_{GST}, \\ \mu C_{ox}\frac{W}{L}\left[V_{GST}(1+\lambda(V_{DS}-V_{GST})) - V_{GST}^2\lambda/2\right], & 0 \le V_{GST} \le V_{DS}. \end{cases}$$

$$\frac{1}{r_o} = \begin{cases} \mu C_{ox}\frac{W}{L}\left[(V_{GST}-V_{DS})(1+\lambda(V_{DS}-V_{GST})) + (V_{GST}V_{DS}-V_{DS}^2/2)\lambda\right], & 0 \le V_{DS} \le V_{GST}, \\ \mu C_{ox}\frac{W}{L}\cdot\frac{V_{GST}^2}{2}\cdot\lambda, & 0 \le V_{GST} \le V_{DS}. \end{cases}$$

Using the above expressions, we obtain for $V_{GST} = 0.2V$,

(i) At $V_{DS} = V_{GST}/10$, $I_D = \boxed{13.6\mu A}$, $g_m = \boxed{70.2\ \mu S}$, $\frac{1}{r_o} = \boxed{643\mu S}$.

(ii) At $V_{DS} = V_{GST}$, $I_D = \boxed{72.9\mu A}$, $g_m = \boxed{722\ \mu S}$, $\frac{1}{r_o} = \boxed{7.29\ \mu S}$.

(iii) At $V_{DS} = 10V_{GST}$, $I_D = \boxed{86.0\ \mu A}$, $g_m = \boxed{853\mu S}$, $\frac{1}{r_o} = \boxed{7.29\mu S}$.

⑤ ~~When $V_{DS} = V_{GST}/2$, $\frac{1}{r_o} = \mu C_{ox}\frac{W}{L}\left[\frac{V_{GST}}{2}\left(1-\lambda\frac{V_{GST}}{2}\right) + \frac{3V_{GST}^2\lambda}{8}\right]$~~

~~$= \left(\mu C_{ox}\frac{W}{L}V_{GST}^2\right.$~~

(4) In triode region, $g_m = \mu C_{ox}\frac{W}{L}V_{DS}\left[1-\lambda\left(2V_{GST}-\frac{3V_{DS}}{2}\right)\right]$,

$$\frac{1}{r_o} = \mu C_{ox}\frac{W}{L}\left[V_{GST}-V_{DS}+\lambda V_{DS}(V_{GST}-V_{DS}/2)-\lambda(V_{GST}-V_{DS})^2\right].$$

We want $g_m\big|_{V_{DS}=V_1} = \left(\frac{1}{r_o}\right)\Big|_{V_{DS}=V_2}$ at $V_{GST}=V_0$, for any λ.

$\Rightarrow V_1\left[1-\lambda\left(2V_0-\frac{3V_1}{2}\right)\right] = V_0-V_2+\lambda V_2\left(V_0-\frac{V_2}{2}\right)-\lambda(V_0-V_2)^2$, for any λ.

$$= V_0-V_2-\lambda\left[V_0^2+\frac{3V_2^2}{2}-3V_0V_2\right].$$

$\Rightarrow V_1=V_0-V_2$, and $2V_0V_1-\frac{3V_1^2}{2} = V_0^2+\frac{3V_2^2}{2}-3V_0V_2$.

$\Rightarrow 2V_0^2-2V_0V_2-\frac{3}{2}(V_0-V_2)^2 = V_0^2+\frac{3V_2^2}{2}-3V_0V_2$

$\Rightarrow \frac{V_0^2}{2}+3V_2^2-4V_0V_2=0 \Rightarrow \frac{V_2}{V_0} = \frac{2}{3}\pm\frac{\sqrt{5}}{3\sqrt{2}}$, $\frac{V_1}{V_0} = \frac{1}{3}\mp\frac{\sqrt{5}}{3\sqrt{2}}$.

$V_2\approx 0, V_1\approx V_0 \Rightarrow \frac{V_2}{V_0} = \boxed{\frac{2}{3}-\frac{\sqrt{5}}{3\sqrt{2}}\approx 0.140}$, $\frac{V_1}{V_0}\approx\boxed{0.86}$. $\left(\frac{\sqrt{5}}{3\sqrt{2}}\approx 0.527\right)$.

i.e; the MOSFET biased at V_{GST} has the same transconductance at $V_{DS}\approx 0.86\,V_{GST}$, as the output conductance at $V_{DS}\approx 0.14\,V_{GST}$.