Addition of Two's Complement Numbers

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1 Radix Complement

The r's complement of an n-digit number N in base r is defined as $r^n - N$ for $N \neq 0$ and as 0 for $N = 0$. Comparing with the (r - 1)'s complement, we note that the r's complement is obtained by adding 1 to the (r - 1)'s complement, since $r^n - N = [(r^n - 1) - N] + 1$. Thus, the 10's complement of decimal $2389\ is\ 7610 + 1 = 7611$ and is obtained by adding 1 to the 9's complement value. The 2's complement of binary $101100\ is\ 010011 + 1 = 010100$ and is obtained by adding 1 to the 1's-complement value.

The following problems provides the brief idea to perform the Two's complement of a number and Addition of the two's complement numbers.

Two's complement of number
The process of representation as follows,

- Consider the Positive number,
- calculate the 1's complement,
- Add 1 at LSB.

Example Represent -19 in Two's complement form

$$\begin{aligned} Given\ number &= -19 \\ -19 &= -(010011) \\ 1's\ complement\ (010011) &= 101100 \\ Add\ 1\ at\ LSB &= 101101 \\ -19 &= 101101 \end{aligned}$$

HINT
Alternative method to calculate the twos complement of number.

- Find the binary 1 position from LSB side of a number.
- If any digit equal to binary 1, then keep the bit as binary 1 and do the ones complement of the remaining numbers after the binary 1 bit position.

By using above process the two's complement of -19 as follows,

$$-19 = -(010011)$$

here the LSB bit is 1, then keep the LSB bit as 1 and do the one's complement of remaining numbers.

$$-19 = -(010011)$$
$$-19 = 101101$$

Example: Represent the following binary number in 2's complement form 01100011000
By using above hint, first find the binary 1 digit at LSB side, if any binary 1 is present then keep the bit as it is and do the 1'scomplement process to the remaining bits after that bit. The calculation of 2's complement as shown figure 1.

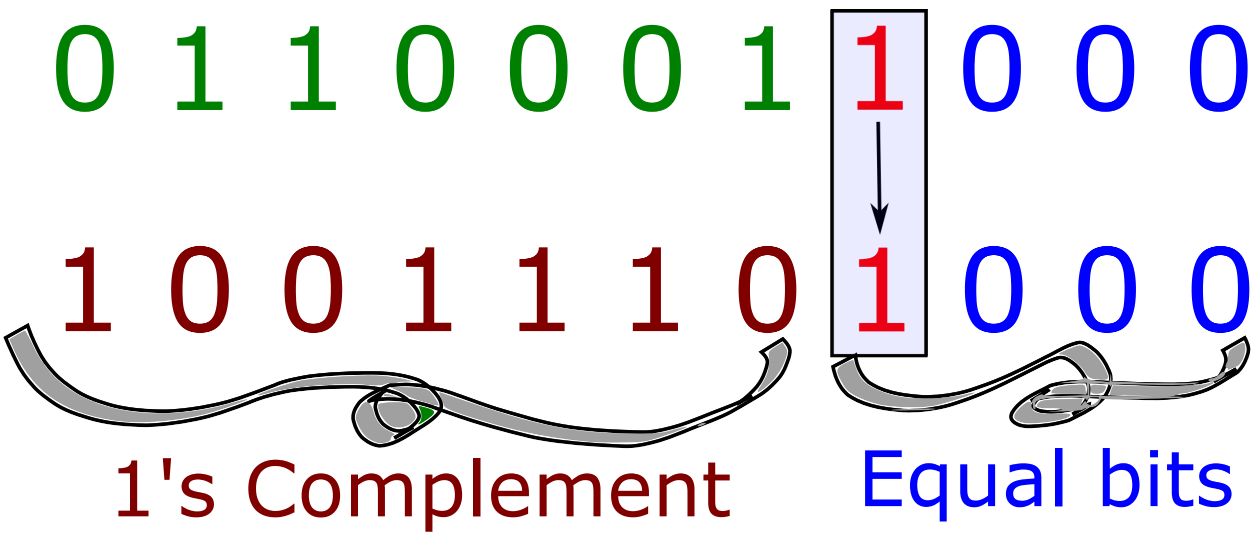

Figure 1: 2's Complement of 01100011000

2 Addition Problems

2.1 ADD -19 and -7

$$-19 = -(010011) = 101101 \tag{1}$$
$$-7 = -(00111) = 11001 \tag{2}$$

when performing the addition of any numbers first one can calculate the size of the final result which is equals to maximum operand size plus one. Here the maximum input size is 6 bit, then the final result will be 7 bit wide.

In general if the operands A and B of size M and N, then the size of the addition of A and B is equal to **maximum size {M,N}+1**.

The addition of equations 1 and 2 as follows,

$$-19 = 101101 \tag{3}$$
$$-7 = 11001 \tag{4}$$

The size of the inputs are not equal and to avoid the overflow problem we need to extend the MSB bit in both operands until the size reaches to size of the final result, here the final resultant size is 7bit.

In the above figure red colour digits indicates the extension of the sign bit.

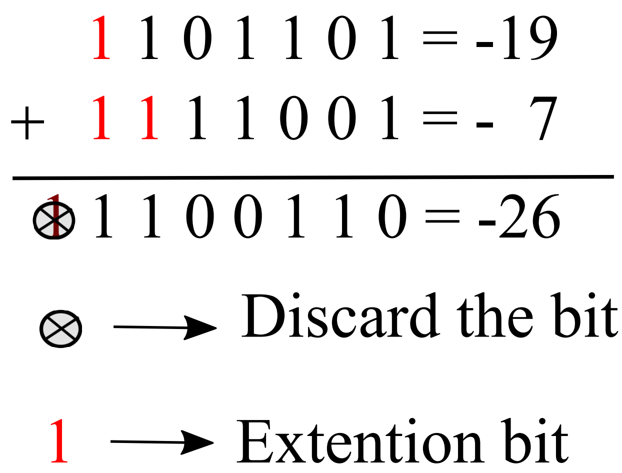

Figure 2: Addition of -19 and -7

Verification

$$\begin{aligned} 1100110 &= -1 \times 2^6 + 1 \times 2^5 + 1 \times 2^2 + 1 \times 2^1 && (5)\\ &= -64 + 32 + 4 + 2 && (6)\\ &= -26 && (7) \end{aligned}$$

2.2 ADD 1001 and 10101

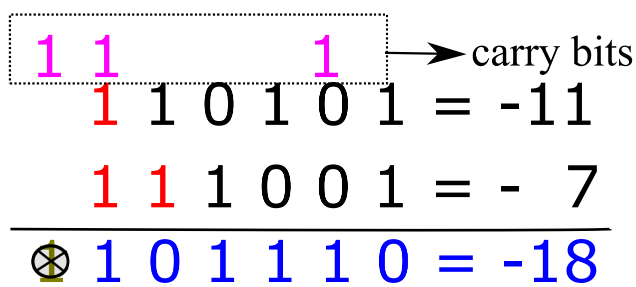

Figure 3: Addition of -11 and -7

2.3 ADD -86 and -14

The figure 4 shows the addition of -86 & -14. The verification of the final result as shown in below.

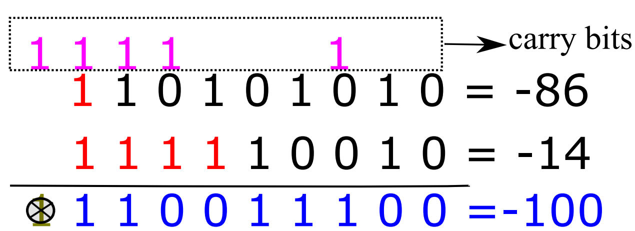

Figure 4: Addition of -86 and -14

Verification

$$\begin{aligned}
110011100 &= -1 \times 2^8 + 1 \times 2^7 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 && (8)\\
&= -256 + 128 + 16 + 8 + 4 && (9)\\
&= -256 + 156 && (10)\\
&= -100 && (11)
\end{aligned}$$

2.4 ADD -14 and -15

The figure 5 shows the addition of $-14 \& -15$.

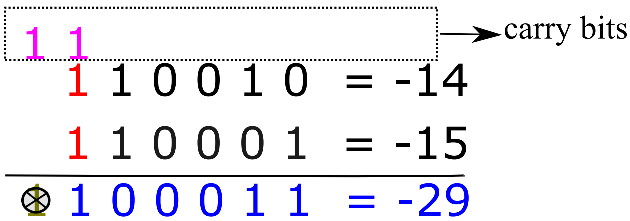

Figure 5: Addition of -14 and -15

2.5 ADD -46 and -85

The figure 6 shows the addition of $-46 \& -85$.

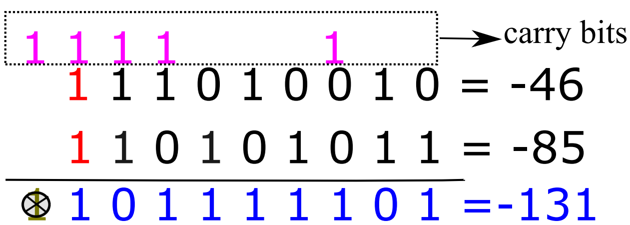

Figure 6: Addition of -46 and -85