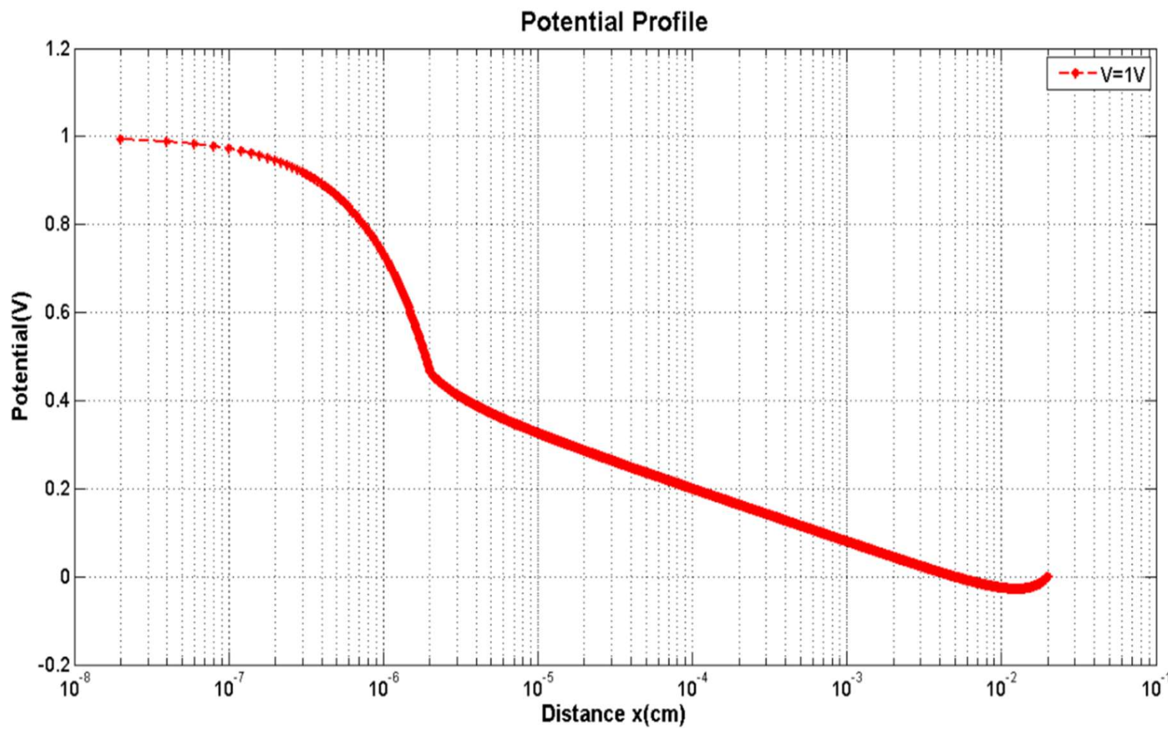
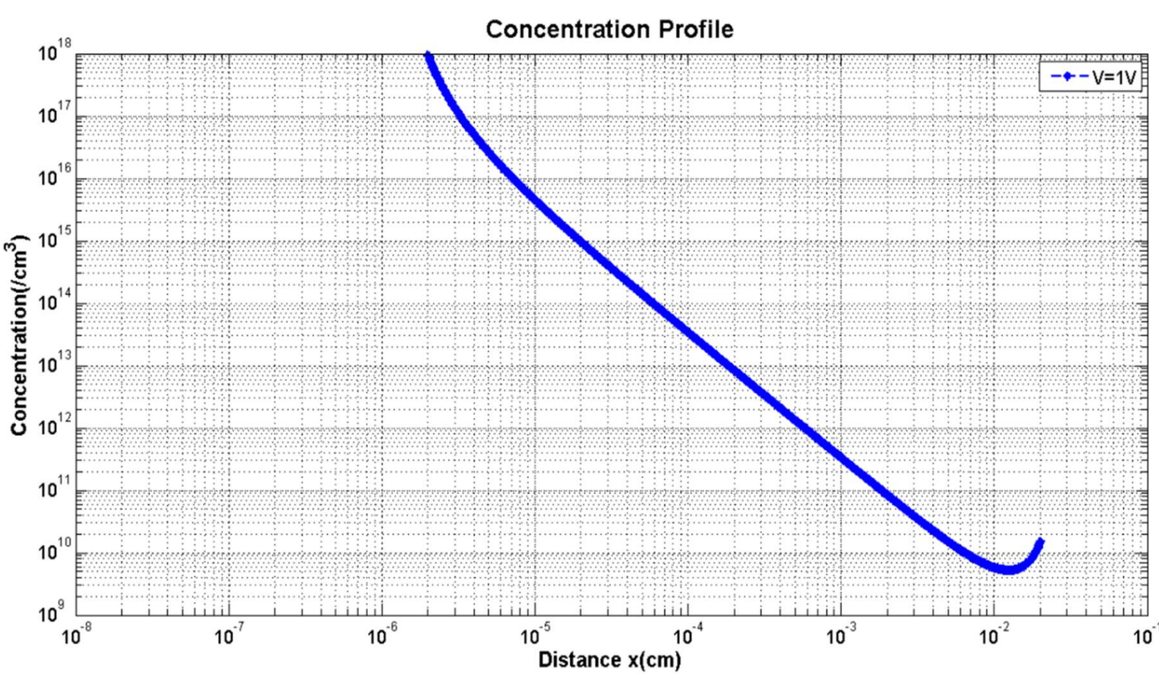
<u>Solution of problem sheet 5</u>

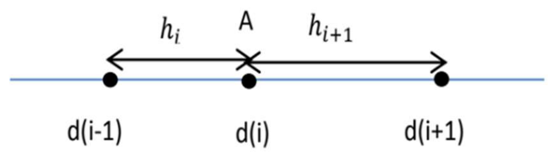
Solution f





In this solution we have consider a uniform grid points in oxide and non-uniform grid point spacing in semiconductor.

At point ‘A’,



Where d[i], d[i-1], d[i+1]= Position of grid points

Taylor’s Series expansion of any function is

$$f(x+h_{i+1}) = f(x) + \frac{f'(x)}{1!}(h_{i+1}) + \frac{f''(x)}{2!}h_{i+1}^{2} \dots\dots so\ on \quad \dots\dots\dots\dots\dots\dots(1)$$

$$f(x-h_i) = f(x) - \frac{f'(x)}{1!}(h_i) + \frac{f''(x)}{2!}h_i^{2} \dots\dots so\ on \quad \dots\dots\dots\dots\dots\dots(2)$$

Neglecting higher order terms

Multiply equation (1) and (2) by and respectively and adding them, gives us

$$f''(x) = \frac{f(x+h_{i+1}) * h_i - (h_i + h_{i+1})f(x) + f(x-h_i) * h_{i+1}}{({h_{i+1}}^2 * h_i + {h_i}^2 * h_{i+1})/2}$$

In numerical form,

$$f'' = \frac{f_{i+1} * h_i - f_i * (h_i + h_{i+1}) + f_{i-1} * h_{i+1}}{({h_{i+1}}^2 * h_i + {h_i}^2 * h_{i+1})/2}$$

Now, the Poisson’s equation from previous questions is

$$\frac{\partial^2 V_i}{\partial x^2} = \frac{q}{\epsilon}\left(n_i \exp\left(\frac{V_i}{kT}\right)\right)$$

The finite difference form would be

$$\frac{V_{i+1} * h_i - V_i * (h_i + h_{i+1}) + V_{i-1} * h_{i+1}}{({h_{i+1}}^2 * h_i + {h_i}^2 * h_{i+1})/2} = \frac{q}{\epsilon}\left(n_i \exp\left(\frac{V_i}{kT}\right)\right)$$

$$\boxed{V_{i+1} * h_i - V_i * (h_i + h_{i+1}) + V_{i-1} * h_{i+1} = \frac{q}{\epsilon} * \frac{({h_{i+1}}^2 * h_i + {h_i}^2 * h_{i+1})}{2} * \left(n_i \exp\left(\frac{V_i}{kT}\right)\right)}$$

For n cases, the system of equations would look like

$$V_1 = V_a$$

$$\boxed{V_3 * h_2 - V_2 * (h_2 + h_3) + V_1 * h_3 = \frac{q}{\epsilon} * \frac{({h_3}^2 * h_2 + {h_2}^2 * h_3)}{2} * \left(n_i \exp\left(\frac{V_2}{kT}\right)\right)}$$

$$V_4 * h_3 - V_3 * (h_3 + h_4) + V_2 * h_4 = \frac{q}{\epsilon} * \frac{({h_4}^2 * h_3 + {h_3}^2 * h_4)}{2} * \left(n_i \exp\left(\frac{V_3}{kT}\right)\right)$$

$$\dots\dots..$$

$$V_n = V_b$$

Where are boundary conditions.

Because of 20nm oxide on the semiconductor the function and its jacobian is changed as

following:

f(V1,V2,....VN)=

$$
\begin{bmatrix}
V_1 - V_a \\
\epsilon_{ox} * V_3 - 2\,\epsilon_{ox} * V_2 + \epsilon_{ox} * V_1 \\
\epsilon_{ox} * V_4 - 2\,\epsilon_{ox} * V_3 + \epsilon_{ox} * V_2 \\
\vdots \\
\vdots \\
\epsilon_{ox} * V_{N1} - 2\,\epsilon_{ox} * V_{N1-1} + \epsilon_{ox} * V_{N1-2} \\
\epsilon_{Si} * V_{N1+1} - (\epsilon_{ox} + \epsilon_{Si}) * V_{N1} + \epsilon_{ox} * V_{N1-1} - \frac{dx^2 * q * n_i * e^{\frac{qV_i}{KT}}}{\epsilon_0} \\
V_{N1+2} * h_{N1} - V_{N1+1} * (h_{N1} + h_{N1+1}) + V_{N1} * h_{N1+1} - \frac{q}{\epsilon} * \frac{\left(h_{N1+1}{}^2 * h_{N1} + h_{N1}{}^2 * h_{N1+1}\right)}{2} * \left(n_i \exp\left(\frac{V_3}{kT}\right)\right) \\
\vdots \\
V_{i+1} * h_i - V_i * (h_i + h_{i+1}) + V_{i-1} * h_{i+1} - \frac{q}{\epsilon} * \frac{\left(h_{i+1}{}^2 * h_i + h_i{}^2 * h_{i+1}\right)}{2} * \left(n_i \exp\left(\frac{V_i}{kT}\right)\right) \\
V_N - V_b
\end{bmatrix}
$$

Oxide-semiconductor boundary equation

So dx= distance between the two grid point in oxide region.

Here N1 grid point is boundary point of oxide layer and semiconductor.

Oxide-semiconductor boundary equation

$$
Df = \begin{bmatrix}
1 & 0 & 0 & 0 & \ldots\ldots & 0 \\
\epsilon\,ox & -2 * \epsilon\,ox & \epsilon\,ox & 0 & \ldots\ldots & 0 \\
0 & \epsilon\,ox & -2 * \epsilon\,ox & \epsilon\,ox & \ldots\ldots & 0 \\
0 & 0 & \epsilon\,ox & G(N1) & \epsilon\,si & \ldots\ldots \\
\ldots\ldots & \ldots & \ldots\ldots & \epsilon\,si * h(N) & G(N-1) & \epsilon\,si * h(N-1) \\
0 & 0 & \ldots\ldots & \ldots\ldots & 0 & 1
\end{bmatrix}
$$

Where, $G(N1) = -(\epsilon_{ox} + \epsilon_{Si}) - \frac{dx^2 * q^2 * n_i * e^{\frac{qV_i}{KT}}}{\epsilon_0 * kT}$

$$G(N-1) = -\epsilon_{Si} * \left(h_{N-1} + h_N\right) - \frac{\left(h_{i+1}{}^2 * h_i + h_i{}^2 * h_{i+1}\right)}{2} * \frac{q^2 * n_i * e^{\frac{qV_{N-1}}{kT}}}{\epsilon_0 * kT}$$