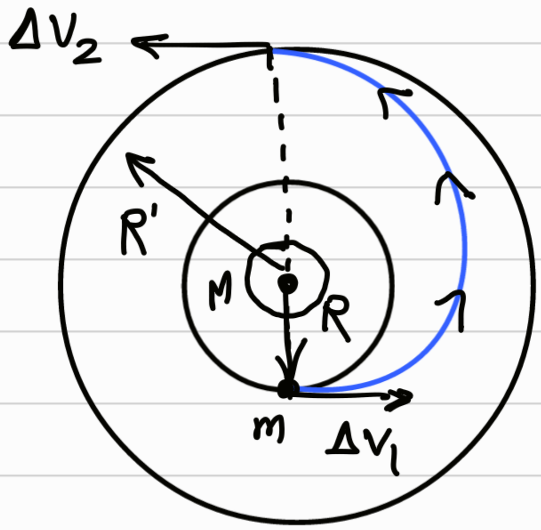
Orbit transfer.

Q5. B) ① (i)



← Hohmann's transfer orbit.

$$P_i = -\frac{GMm}{R} \quad ; \quad K_i = \frac{1}{2} m v_1^2$$

$$P_f = -\frac{GMm}{R'} \quad ; \quad K_f = \frac{1}{2} m v_2^2$$

* Angular momentum will be conserved about plant.

$$\therefore m v_1 \cdot R = m v_2 \cdot R'$$

$$\therefore \boxed{v_1 R = v_2 R'}$$

$$\therefore PE_i + KE_i = PE_f + KE_f$$

conservation of mechanical energy.

$$\therefore \; -\frac{GMm}{R} + \frac{1}{2}mv_1^2 = -\frac{GMm}{R'} + \frac{1}{2}mv_2^2$$

$$\therefore \; \frac{1}{2}m(v_1^2 - v_2^2) = GMm\left(\frac{R' - R}{RR'}\right)$$

$$v_2 = \frac{v_1 R}{R'}$$

$$\frac{1}{2}m\left(\frac{(R')^2 - R^2}{(R')^2}\right)v_1^2 = \frac{GMm(R' - R)}{R \cdot R'}$$

$$\frac{1}{2}\frac{\cancel{(R' - R)}(R' + R)}{R'}v_1^2 = \frac{GM}{R}\cancel{(R' - R)}$$

$$\therefore \; v_1^2 = \frac{2GM}{R}\left(\frac{R'}{R' + R}\right)$$

$$\left\{ \therefore \; v_1 = \sqrt{\frac{2GMR'}{R(R' + R)}} \right\}$$

$$\Delta V_1 = V_1 - V$$

$V \rightarrow$ velocity of satellite while orbiting circle of radius R.

$$\Delta V_1 = \sqrt{\frac{GM(2R')}{R(R'+R)}} - \sqrt{\frac{GM}{R}}$$

$$\therefore \boxed{\Delta V_1 = \sqrt{\frac{GM}{R}}\left(\sqrt{\frac{2R'}{R+R'}} - 1\right)}$$

$$V_2 = \frac{V_1 \cdot R}{R'}$$

$$\therefore V_2 = \sqrt{\frac{2GM \cdot R'}{R(R'+R)}} \cdot \frac{R}{R'} = \sqrt{\frac{2GM \cdot R}{R'(R'+R)}}$$

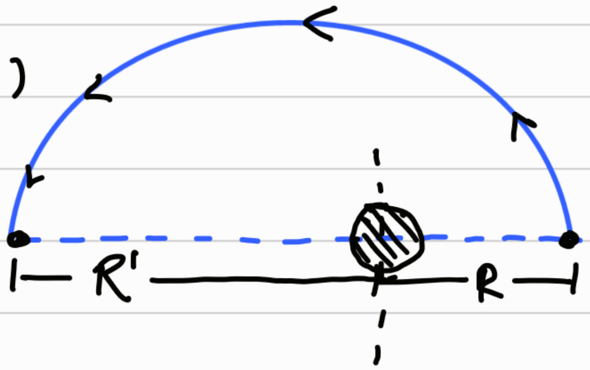
$$\Delta V_2 = V_0 - V_2$$

V_0 – velocity of satellite while orbiting circle of radius R'.

$$\Delta V_2 = \sqrt{\frac{GM}{R'}} - \sqrt{\frac{GM \cdot 2R'}{R'(R'+R)}}$$

$$\boxed{\Delta V_2 = \sqrt{\frac{GM}{R'}}\left(1 - \sqrt{\frac{2R'}{R'+R}}\right)}$$

(ii)



$$a(1-e) = R$$
$$a(1+e) = R'$$

$$\therefore \quad 2a = R + R' \Rightarrow \boxed{a = \frac{R+R'}{2}}$$

$$\therefore \quad 1 - e = \frac{2R}{R+R'}$$

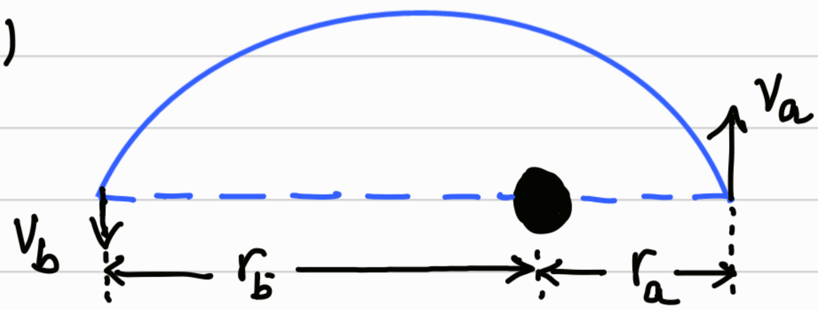
$$\rightarrow \quad \boxed{e = \frac{R'-R}{R'+R}}$$

(iii) According to kepler's third law

$t_H = \pi\sqrt{\frac{R^3}{GM}}$ $\Rightarrow$ R in our case is $a = \frac{R'+R}{2}$

$$\therefore \boxed{t_H = \pi\sqrt{\frac{(R'+R)^3}{8GM}}}$$

(iv)



$$\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = \frac{1}{2}mv_b^2 - \frac{GMm}{r_b}$$

$\therefore$ so as we have calculated in (i) question

$$\frac{1}{2}v_a^2 = \frac{GM r_b}{r_a(r_a+r_b)}$$

$$\frac{1}{2}v_a^2 = \frac{GM(2a-r_a)}{r_a(2a)}$$

$$\therefore \frac{v_a^2}{2} = \frac{GM}{r_a} - \frac{GM}{2a}$$

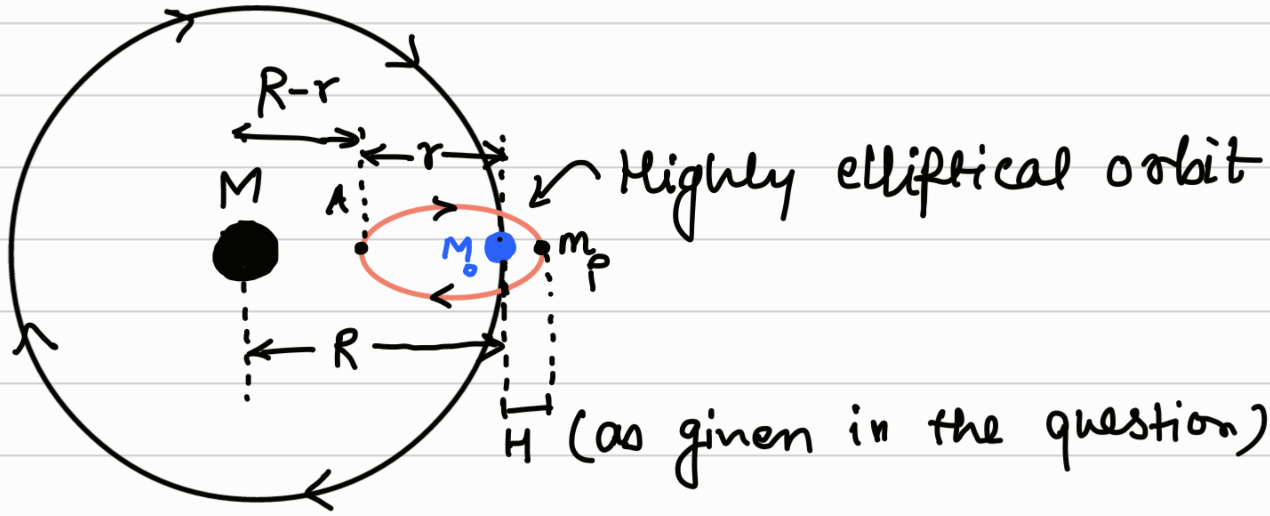
$$\frac{v_a^2}{2} = GM\left(\frac{1}{r_a} - \frac{1}{2a}\right)$$

$\therefore$ In general we can write

$$\boxed{v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

→ It's also called vis-viva equation.

(iii)



→ F.B.D of satellite at apogee.

$$F_p \longleftarrow \bullet \longrightarrow F_m$$

$$\longrightarrow a_c$$

$$F_m - F_p = m\, a_c$$

$$a_c = \underbrace{\omega_m^2}_{\text{angular velocity of moon}} \cdot r$$

$$\therefore \quad \frac{GMm}{(R-r)^2} - \frac{GM_0 m}{r^2} = m\omega_m^2 (R-r)$$

$$\omega_m = ??$$

$$\Rightarrow \frac{GMM_0}{R^2} = M_0 \omega^2 \cdot R$$

$$\therefore \omega = \sqrt{\frac{GM}{R^3}}$$

$$\therefore \frac{\cancel{G}M}{(R-r)^2} - \frac{\cancel{G}M_0}{r^2} = \frac{\cancel{G}M(R-r)}{R^3}$$

$$\frac{M}{R^2\left(1-\frac{r}{R}\right)^2} - \frac{M_0}{r^2} = \frac{M(R-r)}{R^3}$$

$$\frac{M}{R^2}\left(1-\frac{r}{R}\right)^{-2} - \frac{M_0}{r^2} = \frac{M(R-r)}{R^3}$$

$$R >> r$$

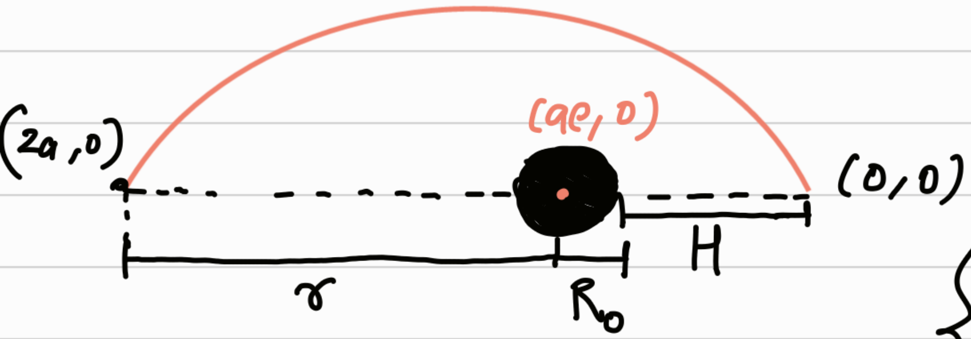
$$\therefore \frac{M}{R^2}\underbrace{\left(1+\frac{2r}{R}\right)}_{\text{binomial approx}} - \frac{M_0}{r^2} = \frac{M}{R^2} - \frac{Mr^2}{R^3}$$

$$\frac{2Mr}{R^3} + \frac{Mr}{R^3} = \frac{M_0}{r^2}$$

$$\frac{3Mr}{R^3} = \frac{M_0}{r^2}$$

$$\therefore \boxed{r = \sqrt[3]{\frac{M_0}{3M}} \cdot R}$$ (apogee distance)

so called radius of hill's sphere of the moon



$$\left\{ e = \frac{r - (R_0 + H)}{r + (R_0 + H)} \right\}$$

as earlier mentioned in solution.

$$\therefore \boxed{e = \frac{\sqrt[3]{\frac{M_0}{3M}}R - (R_0 + H)}{\sqrt[3]{\frac{M_0}{3M}}R + (R_0 + H)}}$$