P 8

$r = a(t) R$

r: actual distance which increases with time

$a(t)$: scale factor

R: Co moving distance (independent of time)

i) $v = \dot{r} = \dot{a}(t) R$

$d = r = a(t) R$

$\Rightarrow H(t) = \dfrac{\dot{a}}{a}$

ii) $\dot{a} = aH$, H is independent of time

$\ddot{r} = a H^2 R = H^2 d$, $a = a(t)$
$d = aR$

$$\ddot{r} > \frac{Gm}{d^2}$$

$$\Rightarrow \boxed{d > \sqrt[3]{\frac{Gm}{H^2}}}$$

iii) Try substituting the values and comparing the two

iv) Note: $1+z = \frac{f_e}{f_0} = \frac{\lambda_0}{\lambda_e} = \frac{a(t_0)}{a(t_e)}$

$$a(t_e) = \frac{a(t_0)}{1+z} = \frac{1}{1+z}$$

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v) All the wavelengths will be redshifted and radiance power will fall by a factor of $(1+z)^4$. Hence, $T_{obs} = \frac{T}{1+z}$

vi) $4\pi d^2$!

vii) Let d_e be the distance b/w Earth and the body at the time of emission then flux observed at Earth will be:

$$F = \frac{1}{4\pi d_e^2} \frac{L}{(1+z)^4} = \frac{L}{4\pi d^2 (1+z)^2}$$

[Analyse the situation in the frame of comoving coordinates]

viii)

$$\theta = \frac{l}{d/_{1+z}} = \frac{l(1+z)}{d}$$

ix) Eliminate d:

$$\frac{F}{\theta^2} = \frac{L}{4\pi(1+z)^4 l^2}$$

x) $$1+z = \left(\frac{L\theta^2}{4\pi l^2 F}\right)^{1/4}$$

$$d = \frac{l}{\theta}\left(\frac{L\theta^2}{4\pi l^2 F}\right)^{1/4} = \sqrt{\frac{l}{\theta}\sqrt{\frac{L}{4\pi F}}}$$