Spooky Quizzes: Objective Round

"We are at the very beginning of time for the human race. It is not unreasonable that we grapple with problems. But there are tens of thousands of years in the future. Our responsibility is to do what we can, learn what we can, improve the solutions and pass them on."

Single option correct

Q.1. A hemispherical metal bowl insulated from its surroundings of thickness t and radius R exists in vacuum. $t \ll R$. A particle with charge Q is placed at the centre of the sphere as shown in figure 1. How much energy should be imparted to the charge so it escapes to infinity?

a. $\dfrac{Q^2 t}{4\pi\epsilon_0 R^2}$

b. $\dfrac{Q^2 t^2}{8\pi\epsilon_0 R^3}$

c. $\dfrac{Q^2 t}{16\pi\epsilon_0 R^2}$

d. $\dfrac{Q^2 t^2}{32\pi\epsilon_0 R^3}$

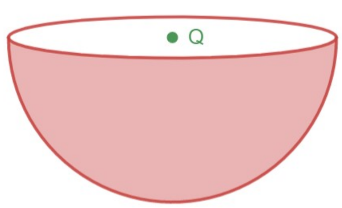

Figure 1: Hemispherical bowl and charge

Q.2.* A very long uniform perfectly elastic cord is hanging freely under a uniform downward gravitational field with one end attached to a rigid support. At the top, the ratio of the speed of a longitudinal wave to the speed of a transverse wave is 20:1. The ratio of times taken by a longitudinal wave and a transverse wave to reach the bottom from the top?
a. $1 : 40$
b. $1 : 30$
c. $1 : 20$
d. $1 : 15$

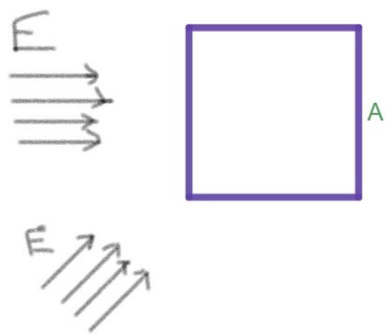

Figure 2: Frame of rods

Q.3. A square frame is made by four conducting rods as shown in figure 2. The rod is exposed to a uniform electric field of strength E, first along its sides and then along its diagonal as shown. The total charge on rod A during these exposures is Q_1 and Q_2 respectively. What is the ratio Q_1:Q_2?
a. $1 : 1$
b. $1 : \sqrt{2}$
c. $\sqrt{2} : 1$
d. $\sqrt{2} - 1 : 1$

Q.4. A regular tetrahedron has two ideal capacitors of capacitance C connected along one pair of opposite edges, two ideal inductors of inductance L connected along another pair of opposite edges, and perfectly conducting wires the remaining pair of opposite edges. (See figure 3.) Ignoring the mutual inductance between the inductors, what is the natural frequency (as cycles per unit time) of electromagnetic oscillations in the circuit?

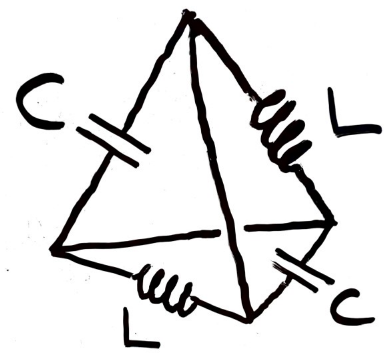

Figure 3: Electro-tetrahedron

a. $\dfrac{1}{\pi\sqrt{2LC}}$
b. $\dfrac{1}{2\pi\sqrt{LC}}$
c. $\dfrac{1}{2\pi\sqrt{2LC}}$
d. $\dfrac{1}{\pi\sqrt{LC}}$

Q.5. Two circular disks of radii R_1 and R_2 are held horizontally at a height h above the flat ground during the day. If $h \ll R_1 \sim R_2$, what is the ratio of the areas of the penumbra region of their shadows?
a. $R_1^2 : R_2^2$
b. $\sqrt{R_1} : \sqrt{R_2}$
c. $1 : 1$
d. $R_1 : R_2$

Q.6. The muon, a heavier cousin of the electron has identical properties such as charge, spin, g-factor. However, it is about 207 times more massive than the electron. What can you say about the magnetic dipole moment of the muon?
a. It is 207 times that of an electron.
b. It is the same as that of an electron.
c. It is 0.00483 times than of an electron.
d. It is 0.135 times that of an electron.

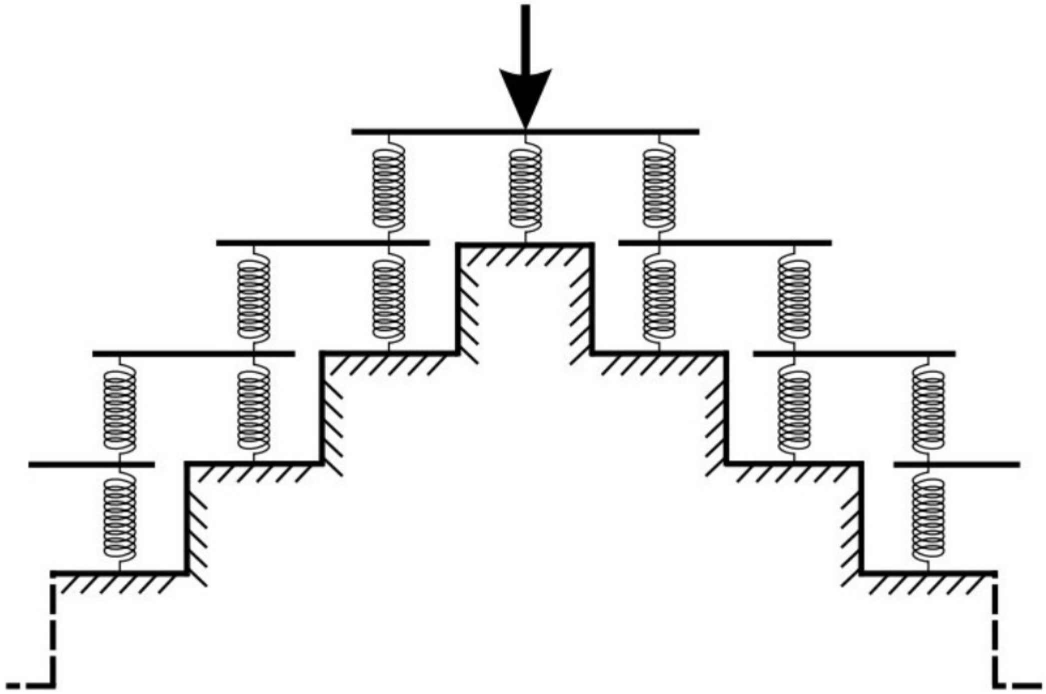

Figure 4: Infinite springs and platforms

Q.7.* Consider an infinite spring platform system shown in figure 4. Suppose all platforms are massless and must stay horizontal always, and all springs are identical, with a spring constant $2N/m$. The force is required to lower the top platform by distance $10cm$ is closest to which of the following values?
a. $0.30N$
b. $0.35N$
c. $0.40N$
d. $0.45N$

Q.8. If the Earth-Sun system is to be treated as two point masses in a non-relativistic, weak gravity quantum framework, i.e. by the Schrodinger equation, then the Earth exists in a superposition of several orbitals, with the same principal quantum number n. Then, what is n?
Take our orbit to be roughly circular.
Hint: Use the fact that the Bohr model gives the right energies of orbitals. Earth-Sun distance $= 1.5 \times 10^{11} m$

Mass of the Earth $= 6 \times 10^{24} kg$
Mass of the Sun $= 2 \times 10^{30} kg$
a. 1.0×10^{16}
b. 3.5×10^{51}
c. 2.5×10^{74}
d. 4.0×10^{20}

Q.9. A transparent disk of thickness t and radius R, $t \ll R$, has varying refractive index given by:

$$\eta(r) = 2\left(1 - \frac{r^2}{2R^2}\right)$$

where r is the distance from the centre of the disk. Such a disk acts as a converging lens for paraxial rays. Find the focal length.
a. $\sqrt{2Rt}$
b. $\frac{}{2t + R}$
c. $\frac{R^2}{2t}$
d. $\frac{R^3}{3t^2}$

Q.10.* Consider hot ionized gas confined in a sphere at some pressure. The sphere is always a perfect blackbody, and is allowed to uniformly expand or contract. The variations in radius are assumed as adiabatic since the loss of heat due to radiation is much slower than the rate at which changes in radius happen. What the relation between the luminosity and radius of the sphere?
a. $L \propto r^{-8}$
b. $L \propto r^{-6}$
c. $L \propto r^{-4}$
d. $L \propto r^{-2}$

Q.11. A plank of wood is floating in a pond. There is a piece of ice at the centre of the plank. As the ice melts, the water flows into the pond. When the piece of ice has completely melted, what is the change in the level of water?
a. The water level reduces.
b. The water level stays the same.
c. The water level increases if mass of ice is greater than mass of wood.
d. The water level increases always.

Q.12. A system of unknown composition is taken through a cyclic process shown as A-B-C-A in the following T-S (Temperature-Entropy) diagram. Find the work done by the system per cycle in this process.

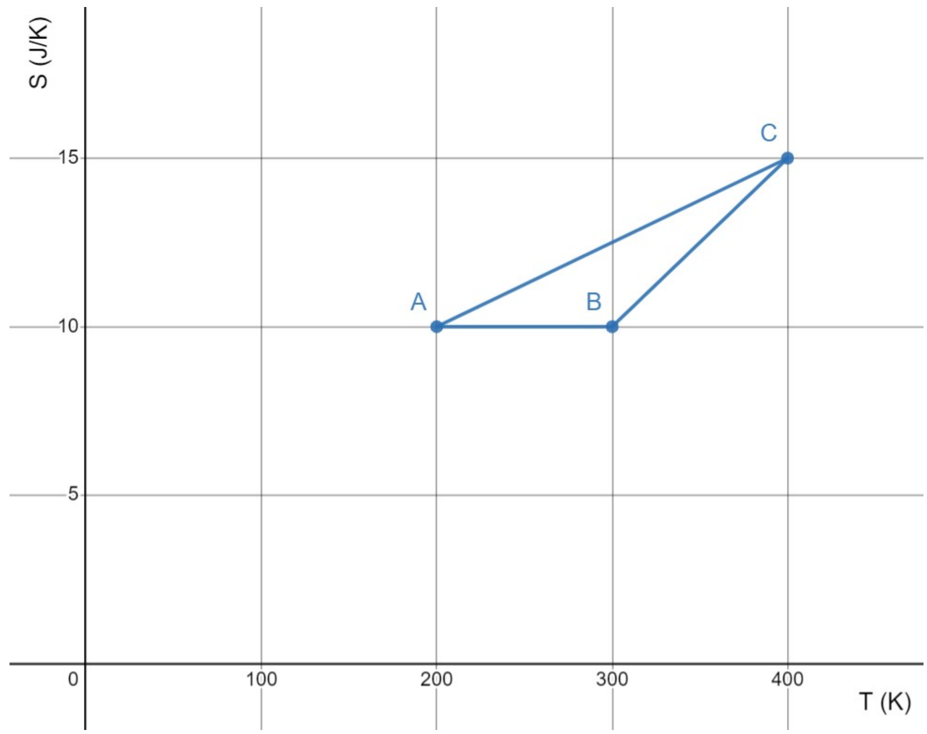

Figure 5: T-S diagram

a. 500 J
b. 250 J
c. -250 J
d. -500 J

Q.13. An open hemispherical metal bowl of radius R with negligible thickness is filled with equal volumes of two immiscible liquids of density ρ and 2ρ. It is now held up against a wall as shown in figure 6. Supposing the wall is frictionless, what is the magnitude of the least force required to ensure that the bowl stays static?

a. $\sqrt{2\pi^2 + \frac{4\pi}{3} + \frac{4}{9}} \times \rho R^3 g$

b. $\sqrt{\dfrac{5\pi^2}{4}+\dfrac{4\pi}{3}+\dfrac{9}{16}}\times\rho R^3 g$
c. $\sqrt{2\pi^2-\dfrac{2\pi}{3}+\dfrac{1}{9}}\times\rho R^3 g$
d. $\sqrt{\dfrac{5\pi^2}{4}-\dfrac{2\pi}{3}+\dfrac{1}{9}}\times\rho R^3 g$

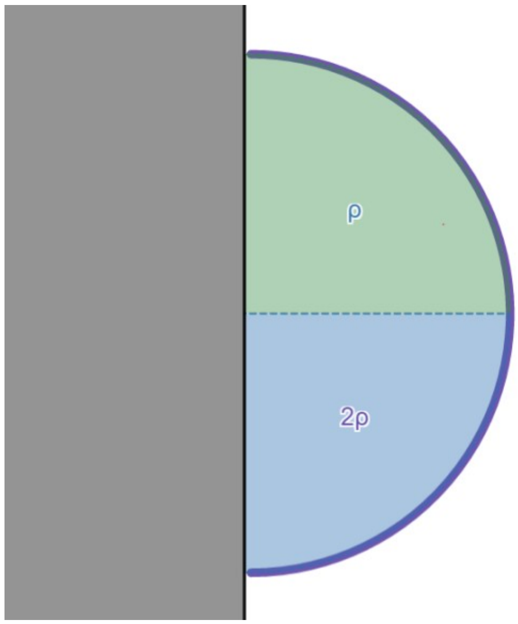

Figure 6: Bowl held against wall

Q.14. A siphon tube is placed in a large tank of water as shown in figure 7. To get the siphoning started, a suction pump is connected at A. What should be the minimum pressure difference creation by the suction pump to get the siphoning started?
Density of water $= 1000 kg/m^3$
Acceleration due to gravity $= 10 m/s^2$

a. 1500 Pa
b. 1000 Pa
c. 800 Pa
d. 500 Pa

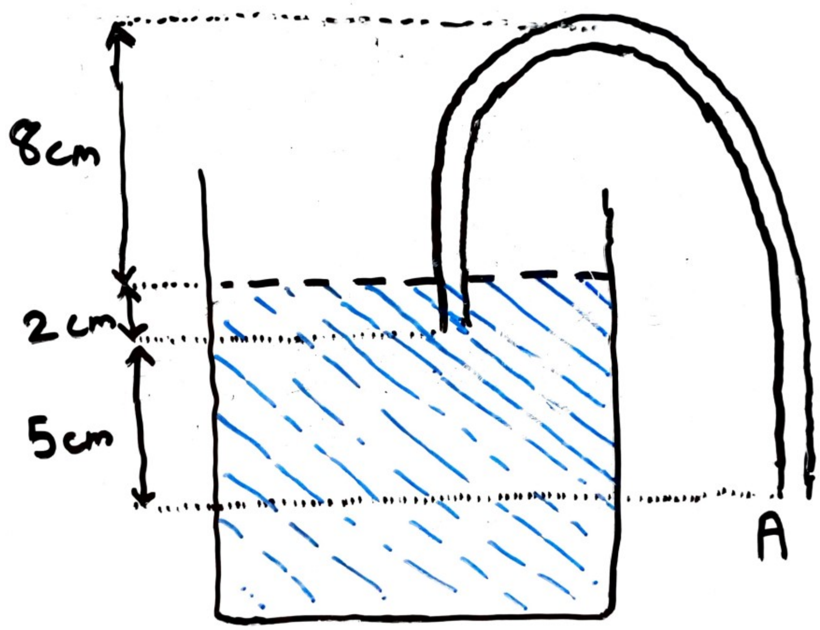

Figure 7: Siphon

Q.15.* Scientists discover a very heavy particle A of mass M and a very light particle B of mass m. When A and B are at a distance r, the potential energy of the interaction is given by:

$$U = -\frac{E}{\frac{r}{\rho} + \left(\frac{r}{\rho}\right)^2}$$

with E and ρ being constants with dimensions of energy and length respectively. Now, A and B can form a bound system only when the angular momentum of B around A is less than $a \times \sqrt{mE\rho^2}$. Then a is:

a. 1
b. $\sqrt{3}$
c. 2
d. $\sqrt{2}$

P.T.O.

Multiple options correct

Q.16.* Suppose the electric potential from a point charge Q varies as

$$\phi = -\alpha Q ln(r)$$

in an alternate universe. In this universe, there is a ring in the x-y plane, with charge q uniformly distributed along its circumference, radius 3 units, with centre at the origin. Which of the following statements is true about the potential due to the ring?
a. At (3,4,0), $\phi = -1.61\alpha q$
b. At (1,1,0), $\phi = -0.69\alpha q$
c. At (1,2,0), ϕ is half that at (3,4,0)
d. At (2,2,0), $\phi = -1.10\alpha q$

Q.17. Which of the following is/are responsible for us seeing the green colour in leaves?
a. Absorption of blue colour wavelengths by chlorophyll.
b. Thermal emission from the leaves.
c. Resonance of chlorophyll molecule at frequency corresponding to red light.
d. Mobility of π electrons around the chlorophyll molecule.

Q.18. Two masses m and $2m$ are placed at rest connected as shown in figure 8 by an inextensible string of length l. At a given instant, the mass m is imparted velocity v in the upward direction. Which of the following is/are true?
a. The path of mass m is a cycloid.
b. The velocity of mass m varies periodically with period $2\pi l/v$.
c. The acceleration of mass m varies periodically with mean zero.
d. The path of mass $2m$ is a cycloid.

Figure 8: Masses and string

Q.19. A boy rides a bicycle and accelerates it from rest to some finite velocity $\vec{v}$ in a straight line. If the mass of the bike is m and that of the boy is M, which of

the following is/are true for the process?
a. The work done on the bike is $mv^2/2$.
b. The net force on the boy-bike system is only due to friction.
c. The net torque on the boy-bike system about the centre of mass is zero.
d. The total impulse of the frictional force on the front tyre is $(M+m)\vec{v}/2$.

Q.20. 100 round frictionless particles of mass m are placed in a box and initially imparted a speed v_0 each, in random directions. Given the fact that all collisions are elastic, what can you say about the speeds of the balls (v_i) after a long time.
a. Average value of $v_i < v_0$.
b. Average value of $v_i^2 > v_0^2$.
c. Average value of v_i^2 is $3\pi/4$ times the square of the average value of v_i.
d. The most likely value of v_i is $\sqrt{\pi}/2$ times the average value of v_i.