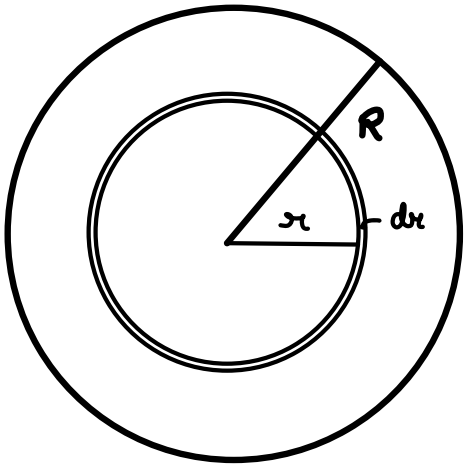
Q) Given a star made of ideal gas with molecular mass M_0. Assume that the density of the star is constant. The mass of the star is M and radius R. ($T = mp$ for ideal gas, m is constant)

Find pressure as function of distance from centre (hydrostatic equilibrium)

Given constant density ρ

$$dM_r = (4\pi r^2 dr)\rho$$

$$\therefore \frac{dM}{dr} = 4\pi r^2 \rho$$



For equilibrium,

$$dP\,dA = \left(-\frac{GM_r}{r^2}\right)\rho\, dr\, dA$$

$$= -\left(\frac{GMr}{R^3}\right)\left(\frac{M}{\frac{4}{3}\pi R^3}\right) dr\, dA \qquad \left(\because \text{constant density}\right)$$

$$\therefore \int_{P_r}^{0} dP = \frac{-3GM^2}{4\pi R^6} \int_{r}^{R} r\, dr$$

$$\therefore P_r = \frac{3GM^2}{8\pi R^6}\left(R^2 - r^2\right)$$