P 7

i) Let $\frac{t}{dt} := N$. Then the required probability is

$$\binom{N}{n} p^n (1-p)^{N-n}, \quad p = B\,dt$$

Note that $N \ggg n$ and hence two or more photons getting detected in the same time interval dt is a rare event.

[Exercise: Convince yourself that this is indeed the case. For concreteness consider 10^5 different boxes and 10 identical balls and $p = 0.1$]. As $dt \to 0$, $N \to \infty$

and $\binom{N}{n} \approx \frac{N^n}{n!}$. Also as $p \approx 0$,

$1-p \approx 1 \Rightarrow (1-p)^{N-n} \approx (1-p)^N$.

$\therefore$ In the limit $dt \to 0$, the probability reduces to

$$\lim_{dt \to 0} \frac{\left(\frac{t}{dt}\right)^n}{n!} (Bdt)^n (1-Bdt)^{t/dt}$$

$$= \frac{(Bt)^n}{n!} e^{-Bt}$$

(Poisson Distribution)

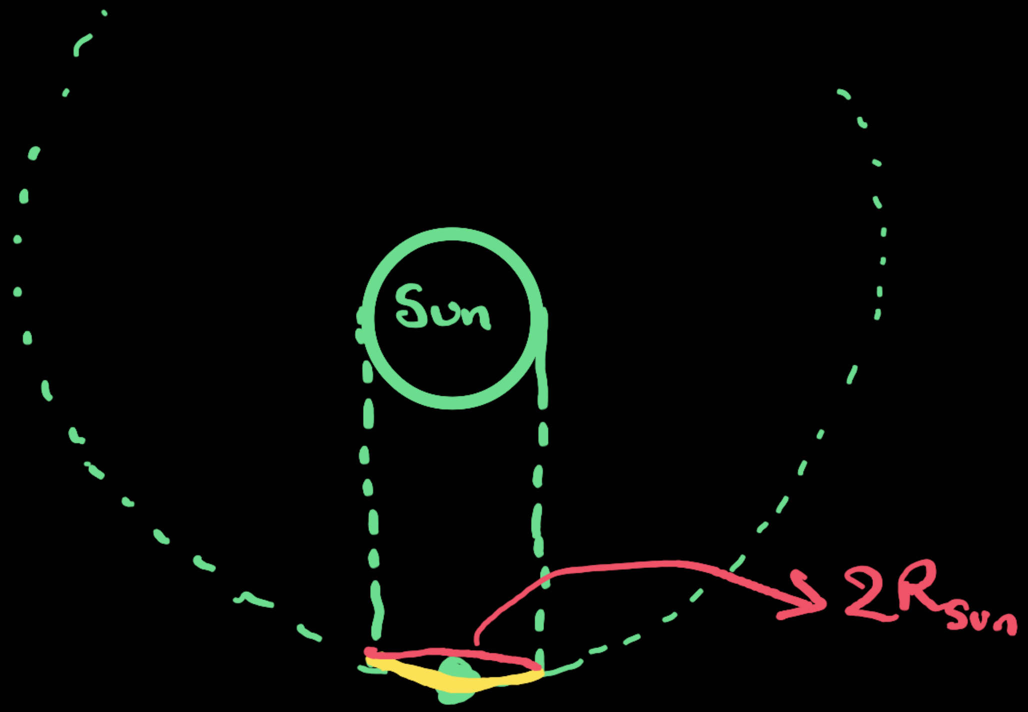
$$\therefore P(\underbrace{X}_{\text{random variable for no. of photons}} = n) = \frac{e^{-Bt}(Bt)^n}{n!}$$

ii) $E[X] = Bt$

$$\text{var}[x] = Bt$$

$$\text{std dev}[x] = \sqrt{Bt}$$

iii)



As $R_{sun} << r_{sun-Earth}$,

$$\text{time for transit} \approx \frac{2R_{sun}}{v_{earth}}$$

v_{earth} → orbital velocity of earth

iv) $f_t = \dfrac{\pi R_{Earth}^2}{\pi R^2} = \left(\dfrac{1}{109}\right)^2 = \dfrac{1}{11881}$

search for an image of a transit to see why this is true!

v) $L_{\odot} = 3.8 \times 10^{26}$ W

$\lambda_{\odot} = 500$ nm

probability of the detector detecting a photon in a time dt $= \frac{L}{4\pi d^2} \times \pi \left(\frac{1}{2}\right)^2 \times \frac{\lambda}{hc} dt$

$\therefore B = \frac{L\lambda}{16 d^2 hc}$ (substitute values)

vi) $f_v \sim \frac{\sqrt{Bt}}{Bt} = \frac{1}{\sqrt{Bt}}$

from v   from iii

$f_t = \left(\frac{1}{\log}\right)^2$

vii) $f_t > f_v$ yields a condition of the form

$$d < C$$ where C is a constant and can be calculated from above.