Pravega AstroWiz Solutions

Q5- First option

a) Let us say we project the satellite with speed V_0 w.r.t ~~earth~~ ~~Eridani~~ Rustum
We want to ~~eas~~ interpret the expression

$$\sqrt{2\left(\frac{GM_s}{R}+\frac{GM_p}{r}\right)}$$

Let us find Gravitational potential energy due to interaction of satellite with planet and Star

$$PE = -\frac{GM_s}{R} - \frac{GM_p}{r}$$

If we naively equate this with initial KE we get ~~the~~ concerned expression for speed

This expression is obviously wrong because it does not take into account that planet we are projecting the satellite w.r.t ~~[illegible]~~ which itself is moving. Another subtle point is that it does not take into account change in Kinetic energy of ~~[illegible]~~ planet
(This has been discussed precisely in next question)

b) Let us consider the expression we arrived at in a). Now the speed obtained is expected to be wrt Sun. If we want this speed to be minimum in ~~Earth's~~ planet's frame we must project it tangentially to ~~Earth's~~ planet's orbit along it's velocity (of ~~Earth~~ planet)

Then we obtain the expression

$$\sqrt{2\left(\frac{GM_s}{R}+\frac{GM_p}{r}\right)} - v$$

The flaw here is that in elastic two body collision (rather interaction) the energy (KE) exchanged is zero only in com frame. If say one particle has extremely large mass (when compared to other), then ~~the~~ energy (KE) change of large mass is zero in frame moving with large mass initially.

In frame of star, the ~~earth~~ planet picks a non negligible amount of kinetic energy from the KE initially supplied to ~~Ear~~ satellite. Thus we also need to take into account the increase in planet ~~Earth's~~ KE (which is small compared to initial KE of ~~earth~~ planet but comparable to KE of satellite)

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c) As argued in Q b) we must work in frame of massive bodies to obtain correct result without accounting change in KE of massive body. It can be seen that Gravitational field due to star near ~~Ea~~ planet's surface is way less than due to planet.
Thus we make an assumption that satellite first escapes planet's field and then the star's field.

We must project the satellite tangentially to planet's orbit along its ~~speed~~ velocity.

The benefit of this is when we change our frame to star we add the planet's ~~Earth's~~ speed to remnant speed of of satellite (wrt planet). Had we projected satellite in some other direction we would have got the two vectors at some angle, thus leading to non maximum resultant velocity

d) We assume that satellite first escapes planet's field and then escapes star's field.
Say we project the satellite w.r.t. ~~Earth~~ planet with speed v along its tangent of planet's orbit.

Using Energy conservation (in frame of planet) the speed of ~~g~~ satellite after it escapes the planet's field is

$$\sqrt{v^2 - \frac{2GM_p}{r}}$$

Now to get the speed wrt star we add the orbital speed of planet.
Reason for this is that the two would be almost along same line since we projected the satellite tangentially
wrt star speed is

$$\sqrt{v^2 - \frac{2GM_p}{r}} + v_0$$

Now wrt star, this satellite must escape the Gravitational field of star it must posses KE ~~also~~ $\geq$ the PE of satellite with star.
Now distance of satellite is almost R from star. This is because gravitational field of planet is significant in much smaller region than that of star. ∴ By energy conservation

$$\frac{1}{2} m \left(\sqrt{v^2 - \frac{2GM_p}{r}} + v_0\right)^2 = \frac{GM_s m}{R}$$

$$\Rightarrow v = \left(\left[\sqrt{\frac{2GM_s}{R}} - \sqrt{\frac{GM_s}{R}}\right]^2 + \frac{2GM_p}{r}\right)^{1/2}$$

$$= 16.49 \text{ km/s}$$