UNIT-VI – BACKTRACKING

Backtracking: General method, Applications- N-QUEEN Problem, Sum of Sub Sets problem, Graph Coloring, Hamiltonian Cycles.

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Introduction

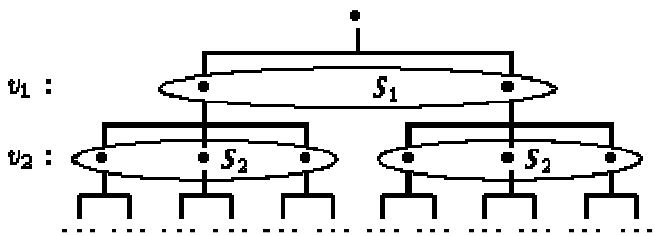
Backtracking is a refinement of the brute force approach, which systematically searches for a solution to a problem among all available options. It does so by assuming that the solutions are represented by vectors (v_1, ..., v_m) of values and by traversing, in a depth first manner, the domains of the vectors until the solutions are found.

When invoked, the algorithm starts with an empty vector. At each stage it extends the partial vector with a new value. Upon reaching a partial vector (v_1, ..., v_i) which can't represent a partial solution, the algorithm backtracks by removing the trailing value from the vector, and then proceeds by trying to extend the vector with alternative values.

```
ALGORITHM try(v1,...,vi)
{
   IF (v1,...,vi) is a solution THEN RETURN (v1,...,vi)
   FOR each v DO
      IF (v1,...,vi,v) is acceptable vector  THEN
        sol = try(v1,...,vi,v)
        IF sol != () THEN RETURN sol
      END
   END
   RETURN ()
}
```

If S_i is the **domain** of v_i, then $S_1 \times ... \times S_m$ is the **solution space** of the problem. The **validity criteria** used in checking for acceptable vectors determines what portion of that space needs to be searched, and so it also determines the resources required by the algorithm.

The traversal of the solution space can be represented by a depth-first traversal of a tree. The tree itself is rarely entirely stored by the algorithm in discourse; instead just a path toward a root is stored, to enable the backtracking.



In case of greedy and dynamic programming techniques, we will use Brute force approach. It means, we will evaluate all possible solutions, among which, we select one solution as optimal solution. In backtracking technique, we will get same optimal solution with less number of steps. So we use backtracking technique. We can solve problems in an efficient way when compared to other methods like greedy method and dynamic programming. In this we will use bounding functions (criterion functions), implicit and explicit conditions. While explaining the general method of backtracking technique, there we will see implicit and explicit constraints. The major advantage of backtracking method is, if a partial solution $(x_1,x_2,x_3\ldots..,x_i)$ can't lead to optimal solution then $(x_{i+1}\ldots x_n)$ solution may be ignored entirely.

Explicit constraints: These are rules which restrict each xi to take on values only from a given set.

Example

1)Knapsack problem, the explicit constraints are,

i) $x_i=0$ or 1

ii) $0<=x_i<=1$

2)4-queens problem : in 4 queens problem, the 4 queens can be placed in 4x4 chess board in 4^4 ways.

Implicit constraints: These are rules which determine which of the tuples in the solution space satisfy criterion function.

Example: In 4 queens problem, the implicit constraints are no 2 queens can be on the same row, same column and same diagonal.

Let us see some terminology which is being used in this method.

1)**Criterion Function**: it is a function p(x1,x2,x3,…xn) which needs to be maximized or minimized for a given problem.

2)**Solution Space** : All tuples that satisfy the explicit constraints define a possible solution space for a particular instance 'i' of the problem.

For example consider the following tree. ABD, ABE, AC are the tuples in solution space.

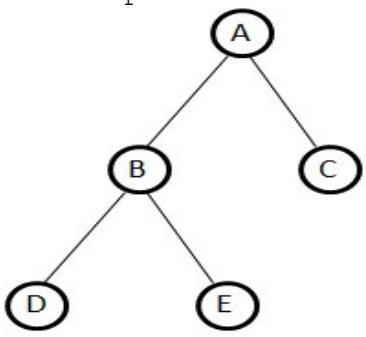


Fig) The organization of a solution space

3)**Problem state:** each node in the tree organization defines a problem state. So, A,B ,C are problem states.

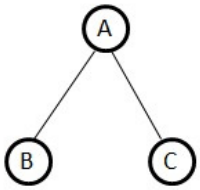


Fig) Tree (Problem State)

4)**Solution states:** These are those problem states S for which the path from the root to S define a tuple in the solution space.

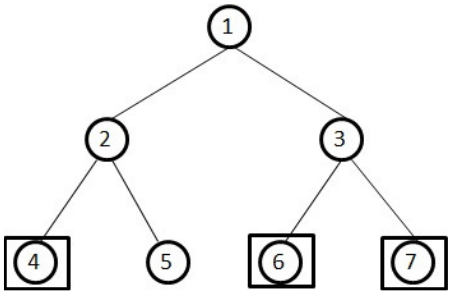


Fig) Tree (Solution state)

Here square nodes indicate solution. For the above solution space, there exists 3 solution states. These solution states represented in the form of tuples i.e. (1,2,4), (1,3,6) and (1,3,7) are the solution states.

5)**state space tree:** if we represent solution space in the form of a tree then the tree is referred as the state space tree.

For example given is the state space tree of 4-queen problem. Initially x1=1 or 2 or 3 or 4. It means we can place first queen in either of 1/2/3/4 column. If x1=1 then x2 can be paced in either 2nd,3rd , or 4th column. If x2=2 then x3 can be placed either in 3rd or 4th column. If x3=3 then x4=4. So nodes 1-2-3-4-5 is one solution in solution space. It may or may not be feasible solution. Similarly we can observe the remaining solutions in the figure.

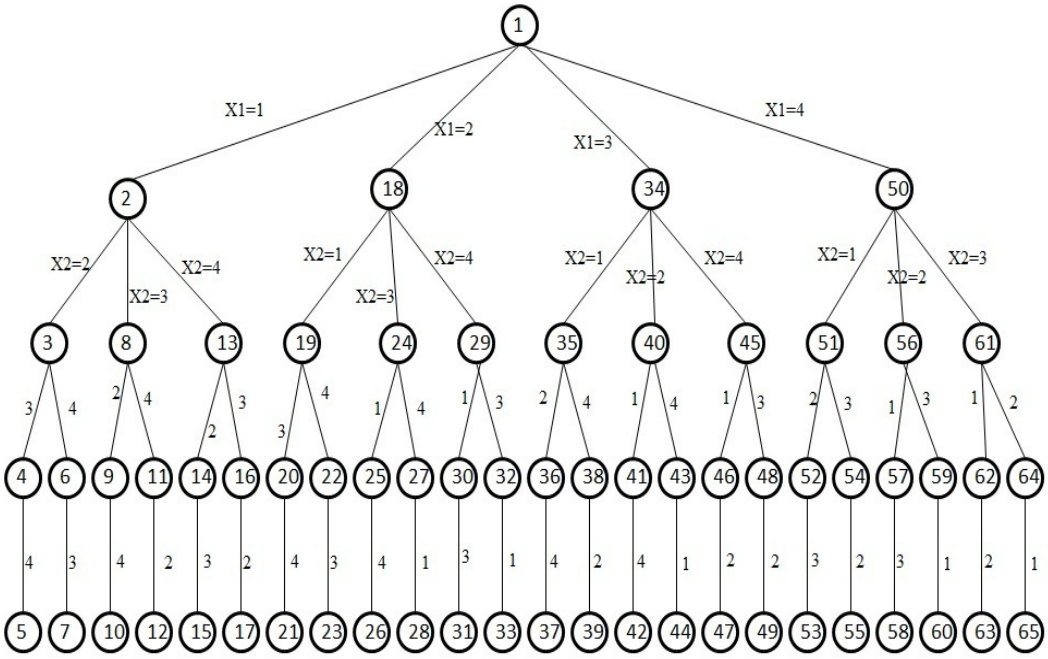


Fig) Tree organization of the 4 queen solution space

6)**Answer states :** These solution states s for which the path from the root to S defines a tuple which is a member of the set of solutions.(i.e. it satisfies the implicit constraints) of the problem. Here 3,4, are answer states. (1,3) and (1,2,4) are solution states.

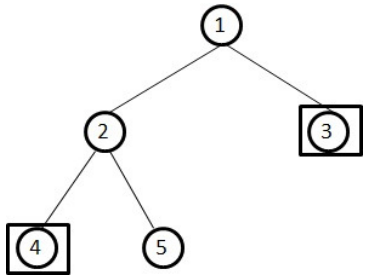


Fig) Tree (answer states)

7)**Live node:** A node which has been generated and all of whose children have not yet been generated is live node. In the fig (a) node A is called live node since the children of node A have not yet been generated.

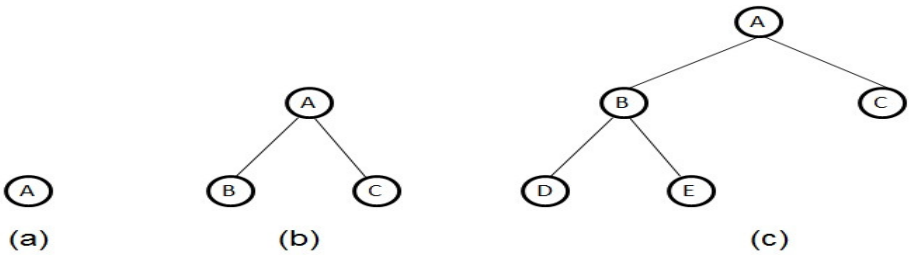


Figure) Live nodes

In fig (b) node A is not a live node but B,C are live nodes.

In fig(c) nodes A,B are not live and D,E C are live nodes.

8)**E-node** : The live node whose children are currently being generated is called E-node.(node being expanded).

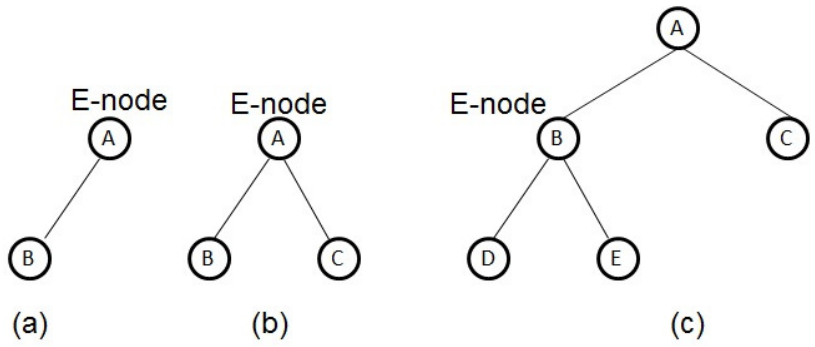


Figure) E-nodes

9)**Dead node:** it is a generated node that is either not to be expanded further or one for which all of its children have been generated.

Ex)In figure(a) nodes A,B,C are dead nodes since node A's children already generated and Nodes B,C are not expanded.

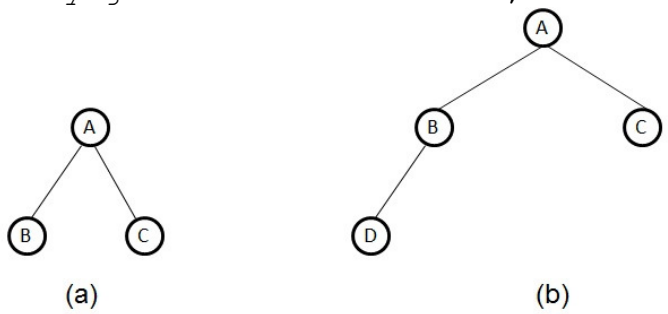


Figure) Dead nodes

In figure (b) assumed that node B can generate one more node so nodes A,D,C are dead nodes.

Applications:

1)n-Queens Problem (4-Queens and 8-Queens Problem)

Consider an nxn chess board. Let there are n Queens. These n Queens are to be placed on the nxn chess board so that no two queens are on the same column, same row or same diagonal.

n-queens Problem: The n-queens problem is a generalization of the 8-queens problem. Now n-queens are to be placed on an nxn cross board so that no two attack; that is no two queens are on the same row, column, or diagonal. The solution space consists of all n! permutations of n-tuple (1,2,3,..n).

The following figure shows a possible tree organization for the case n = 4. A tree such as this is called a permutation tree. The edges are labeled by possible values of x_i.

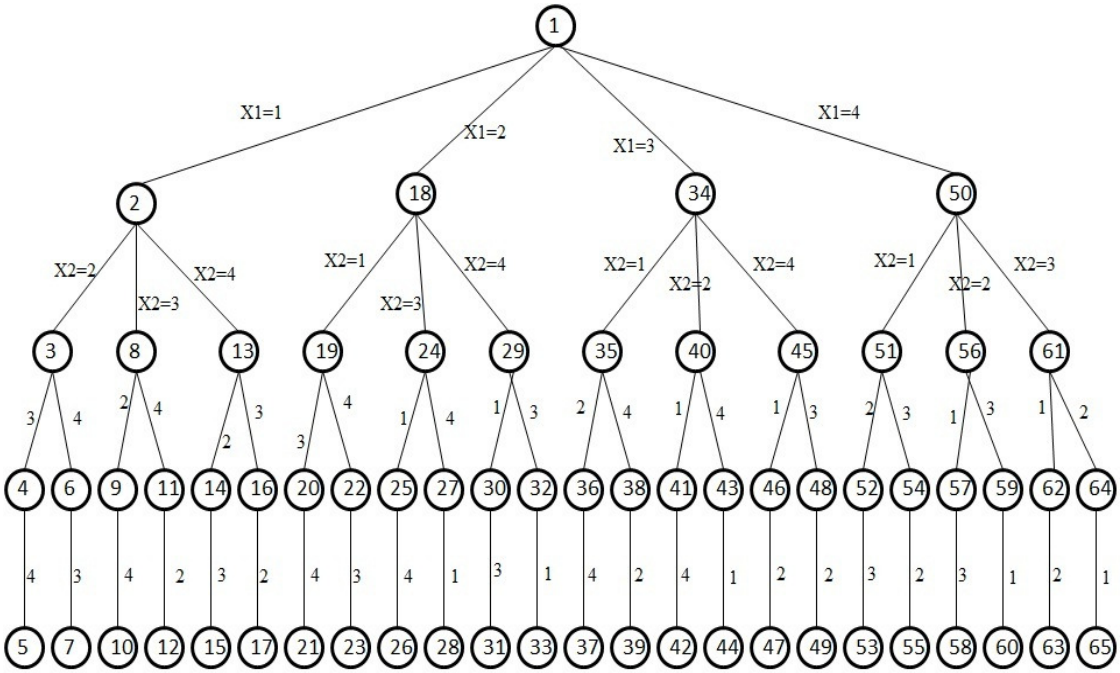


Figure)The organization of 4-queens solution space

Edges from level 1 to level 2 nodes specify the values for x_1. Thus the left most sub-tree contains all solutions with $x_1=1$.

Edges from level i to level i+1 are labeled with the values of x_i. The solution space is defined by all paths from the root node to a leaf node. There are 4!=24 leaf nodes in the permutation tree.

If we imagine the chess board squares being numbered as the indices of the two dimensional array a[1..n,1..n] then we observe that every element on the same diagonal that runs from upper left to lower right has the same row-col value.

	1	2	3	4	5	6	7	8
1				Q				
2						Q		
3								Q
4		Q						
5							Q	
6	Q							
7			Q					
8					Q			

Consider the queen at a[4,2]. The squares that are diagonal to this queen (running from upper left to lower right) are a[3,1],a[5,3], a[6,4], a[7,5],a[8,6]. All these squares have a (row - column) value of 2. Also every element on the same diagonal that goes from the upper right to the lower left has the same (row + column) value.

Suppose two queens are placed at positions (i,j) and (k,l)then by the above we can say they are on the same diagonal if

i-j=k-l which is primary diagonal
or
i+j=k+l which is secondary diagonal

Equation for primary diagonal	Equation for Secondary diagonal
i-j = k-l	i+j = k+l
this can be written as follows	this can be written as follows
j-l = i-k	j-l = k-i

Therefore two queens lie on the same diagonal if and only if |j-l| = |i-k|.

The algorithm place(k,i) returns a Boolean value that is true if kth queen can be placed in column 'i'. it tests both whether 'i' is distinct from all previous values x[i]..x[k-1] and whether there is no other queen on the same diagonal.

Its computing time is O(k-1).

The array x[1..n] is a global array. Let (x1,x2,x3,…xn) be the solution vector where xi is the column number on which the ith queen is placed. (i may be row number).

Using the algorithm place() a queen is placed in k^{th} row , i^{th} column and return true otherwise false.

```
Algorithm place(k,i)
{
  for j:=1 to k-1 do
    if ((x[j]=i)  or  (abs(x[j]-i) = abs(j-k)) then
       return false;

  return true;
}
```

This algorithm is invoked by nqueens(1,n).

The algorithm for obtaining solution to n-queens problem is given below.

```
Algorithm nqueens(k,n)
{
    for i:=1 to n do
      {
         if (place(k,i) then
          {
             X[k]:=i;
             if (k=n) then
               write(x[i:n);
             else
               nqueens(k+1,n);
           }
      }
}
```

For an 8x8 chess board there are $^{64}C_8$ possible ways to place 8 Queens using brute force approach. However by allowing only placements of queens on distinct rows and columns, we require the examination of at most 8! Tuples.

For a 4x4 chess board there are $^{16}C_4$ possible ways to place 4 Queens using brute force approach. However by allowing only placements of queens on distinct rows and columns, we require the examination of at most 4! Tuples.

Place first queen in the first row in the first column.
As it is the first queen it is not under attack.
X[1]=1 (column value is assigned)

	1	2	3	4
1	1			
2				
3				
4				

X[1]=1

To place second queen in second row
Start with first column.
It is under attack
Second column also Under attack
Third column not under attack by other queens.
So we place queen in 3rd column.
X[2]=3

	1	2	3	4
1	1			
2	-	-	2	
3				
4				

X[2]=3

To place third queen in third row
first col under attack
second column under attack
third column under attack
fourth column under attack
not possible to place queen in third row
because placement of previous queens is
not correct.So backtrack to previous row
and move the queen to another possible place
and continue.

	1	2	3	4
1	1			
2			2	
3	-	-	-	-
4				

Go to second row
Move the queen to another col.
Another possibility is column 4.
Move to col 4.
Now X[2]=4

	1	2	3	4
1	1			
2	-	-	-	2
3				
4				

X[2]=4

Go to third row to place 3rd queen
First col under attack
Second column not under attack by other
queens
So place the queen in 2nd col.
X[3]=2

	1	2	3	4
1	1			
2				2
3	-	3		
4				

X[3]=2

Now to place 4th queen in 4th row
First col under attack by other queen
Second col under attack by other queen
Third col under attack by other queen
Fourth col under attack by other queen
Not possible to place the queen in 4th row
as there is a problem in the placement of
previous queens
Back track to previous placements
Goto 3rd row and try to move the queen to
another place.
The other places are under attack go to 2nd row
Already we checked all possibilities in 2nd row
we backtrack to first row.

	1	2	3	4
1	1			
2				2
3		3		
4	-	-	-	-

First queen is moved to 2nd column
X[1]=2

	1	2	3	4
1		1		
2				
3				
4				

X[1]=2

Second queen in second row
First col under attack
Second column under attack
Third col under attack
4th col not under attck
So place queen in 4th col
X[2]=4

	1	2	3	4
1		1		
2	-	-	-	2
3				
4				

X[2]=4

To place third queen in third row
First col not under attack
So place the queen in first col
X[3]=1

	1	2	3	4
1		1		
2				2
3	3			
4				

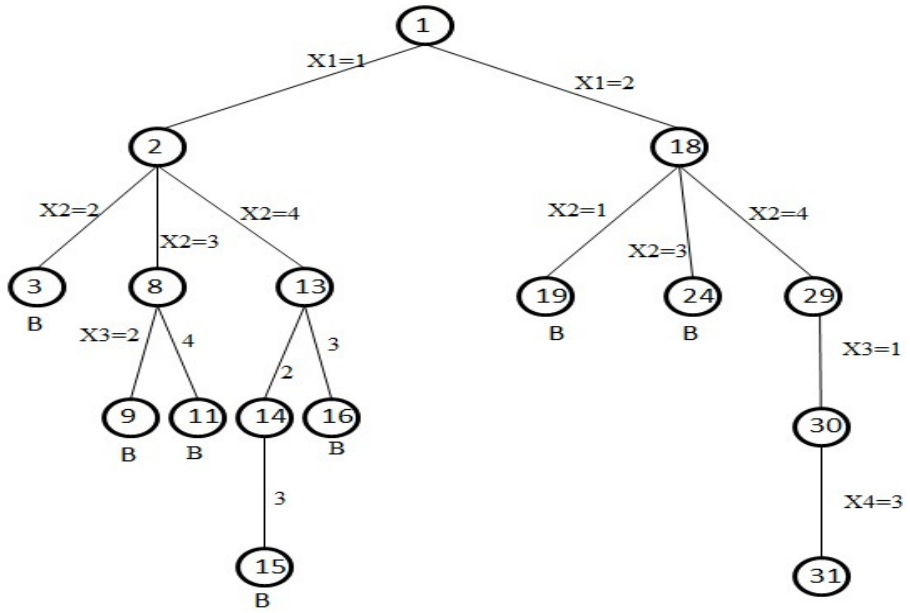
X[3]=1

To place 4th queen in 4th row
first col under attack
second col under attack
third col not under attack
so place the 4th queen in 3rd col
x[4]=3

	1	2	3	4
1		1		
2				2
3	3			
4	-	-	4	

X[4]=3

All four queens are placed in the
4x4 chess board without attacking each other.
In the same way it is possible to place all 8 queens in
an 8x8 chess board without attacking each other.



Portion of the tree that is generated during backtracking

Figure shows the part of the solution space tree that is generated. The tree generated as per the above processing. Nodes are numbered in the order in which they are generated. A node that gets killed as a result of backtracking has a B under it.

Tracing of the algorithm to place 4 queens on a 4x4 cross board such that no two queens attack each other.

```
nqueens(k,n)    place(k,i)

nqueens(1,4)    place(1,1) returns True so x[1]=1

nqueens(2,4)    place(2,1) returns False
                place(2,2) returns False
                place(2,3) returns True so X[2]=3

nqueens(3,4)    place(3,1) returns False
                place(3,1) returns False
                place(3,1) returns False
                place(3,1) returns False
                                      Backtracking

nqueens(2,4)    place(2,4) returns True so X[2]=4

nqueens(3,4)    place(3,1) returns False
                place(3,2) returns True so X[3]=2
nqueens(4,4)    place(4,1) returns False
                place(4,2) returns False
                place(4,3) returns False
                place(4,4) returns False
                                   Backtracking
nqueens(1,4)    place(1,2) returns True so x[1]=2

nqueens(2,4)    place(2,1) returns False
                place(2,2) returns False
                place(2,3) returns False
                place(2,4) returns True so X[2]=4

nqueens(3,4)    place(3,1) returns True so X[3]=1

nqueens(4,4)    place(4,1) returns False
                place(2,2) returns False
                place(2,3) returns True so X[4]=3
```

The solution vector for a 4x4 cross board to place 4 non attacking queens is

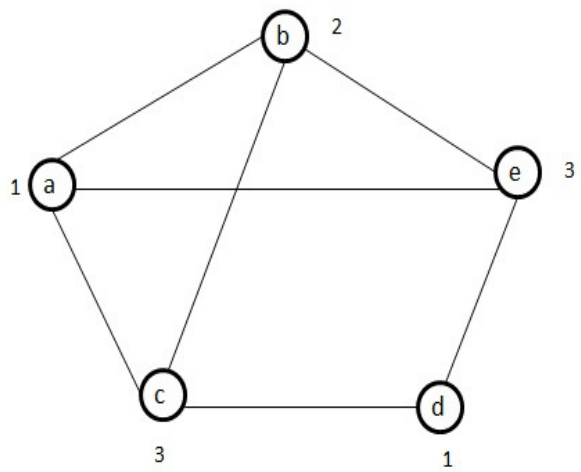
```
x[1]=2
x[2]=4
x[3]=1
x[4]=3
```

2)GRAPH COLORING

Let G be a graph and m be a given positive integer. We want to discover whether the node of G can be colored in such a way that no two adjacent nodes have the same color yet only m colors are used.

This is termed the m-colorability decision problem. Note that if d is the degree of the given graph, then it can be colored with d+1 colors. The m-colorability optimization problem asks for the smallest integer m for which the graph G can be colored. The integer is referred to as the chromatic number of the graph.

For example the following graph can be colored with three colors 1, 2 and 3. The color of each node is indicated next to it. It can also be seen that thee colors are needed to color this graph's chromatic number is 3.



An example graph and its coloring

State space tree for coloring a graph containing 3 nodes using 3 colors

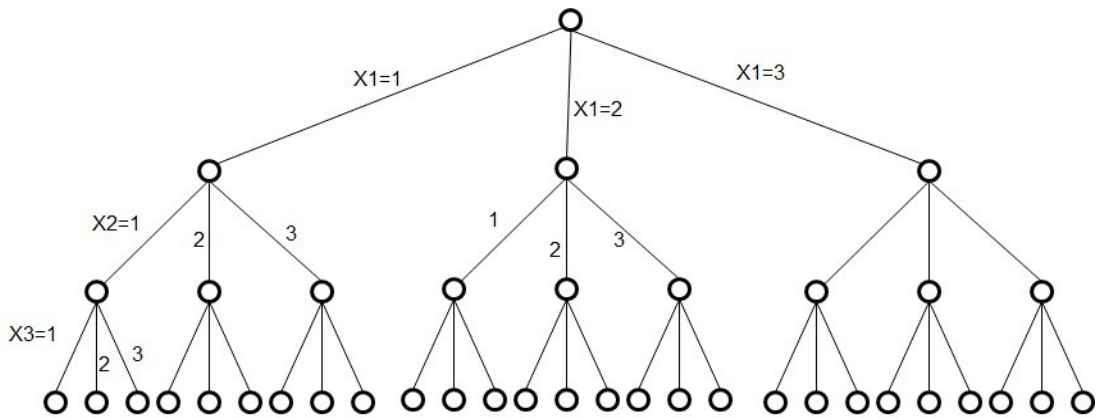


Fig)State space tree for mColoring when n=3 and m=3

The algorithm mcoloring was formed using the recursive backtracking schema. The graph is represented by its Boolean adjacency matrix G[1:n,1:n]. All assignments of 1,2,…m to the vertices of the graph such that adjacent vertices are assigned distinct integers are printed. K is the index of the next vertex to color.

```
Algorithm mcoloring(k)
{
  repeat
   {
      nextvalue(k);
      if (x[k] = 0) then return;
      if (k=n) then
         write(x[1:n]);
      else
         mcoloring(k+1);
   }until(false);
}
```

No of vertices= n
No of colors= m
Solution vector = X[1], X[2], X[3]……..X[n]
The values of solution vector may belongs to {0,1,2,3..m}

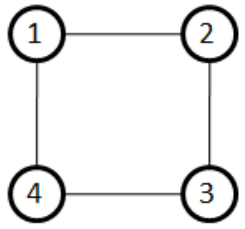
The following Algorithm is used to generate next color.

Assume that X[1],..x[k-1] have been assigned integer values in the range [1,m] such that adjacent vertices have distinct integers.

A value for x[k] is determined in the range [0,m].

X[k] is assigned the next highest numbered color while maintaining distinctness from the adjacent vertices of vertex k. if no such color exists, the x[k]=0.

```
Algorithm nextvalue(k)
{
  Repeat
  {
     X[k]=(x[k]+1)mod(m+1);   // next highest color
     if (x[k]=0) then
          return;         //all colors have been used
     for j:=1 to n do
      {
       if ((G[k,j]!=0) and (x[k]=x[j])) then break;
                      //g[k,j] an edge and
                      //vertices k and j have same color
      }
      if (j=n+1) then return;
    }until (false);
 }
```

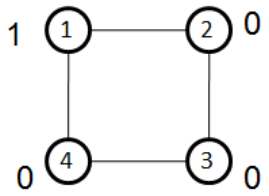


Graph

Adjacency Matrix G

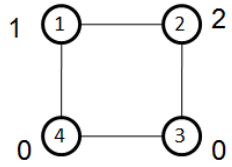
	1	2	3	4
1	0	1	0	1
2	1	0	1	0
3	0	1	0	1
4	1	0	1	0

```
 Assume that n=4 and m=3
 X[1]=0,x[2]=0,x[3]=0,x[4]=0
 If we call the algorithm mcoloring(k)

mcoloring(1)    i.e. k=1
nextvalue(1)
k=1
x[1]=(x[1]+1) mod (m+1)
x[1]= 0+1 mod 4
x[1]=1

             G[k,j]!=0 and x[k]=x[j]
j=1 G[1,1]     false   and   true   = false
j=2 G[1,2]     true    and   false  = false
j=3 G[1,3]     false   and   false  = false
j=4 G[1,4]     true    and   false  = false

x[1]=1 ,x[2]=0,x[3]=0,x[4]=0
```

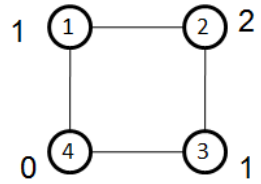



```
mcoloring(2)    i.e. k=2
nextvalue(2)
k=2
x[2]=(x[2]+1) mod (m+1)
x[2]= 0+1 mod 4
x[2]=1
             G[k,j]!=0 and x[k]=x[j]
j=1 G[2,1]     True    and     True  =  True     break
             G[2,1] is an edge and
             adjacent vertices have same color

x[2]=(x[2]+1) mod (m+1)
x[2]=(1+1) mod 4=2 mod 4
x[2]=2

             G[k,j]!=0 and x[k]=x[j]
j=1 G[2,1]     True    and   False   = False
j=2 G[2,2]     False   and   True    = False
j=3 G[2,3]     True    and   False   = False
j=4 G[2,4]     False   and   False   = False

x[1]=1 ,x[2]=2,x[3]=0,x[4]=0
```

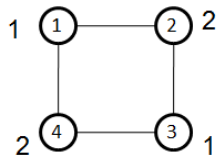
assume that the number mentioned outside the node belongs to color

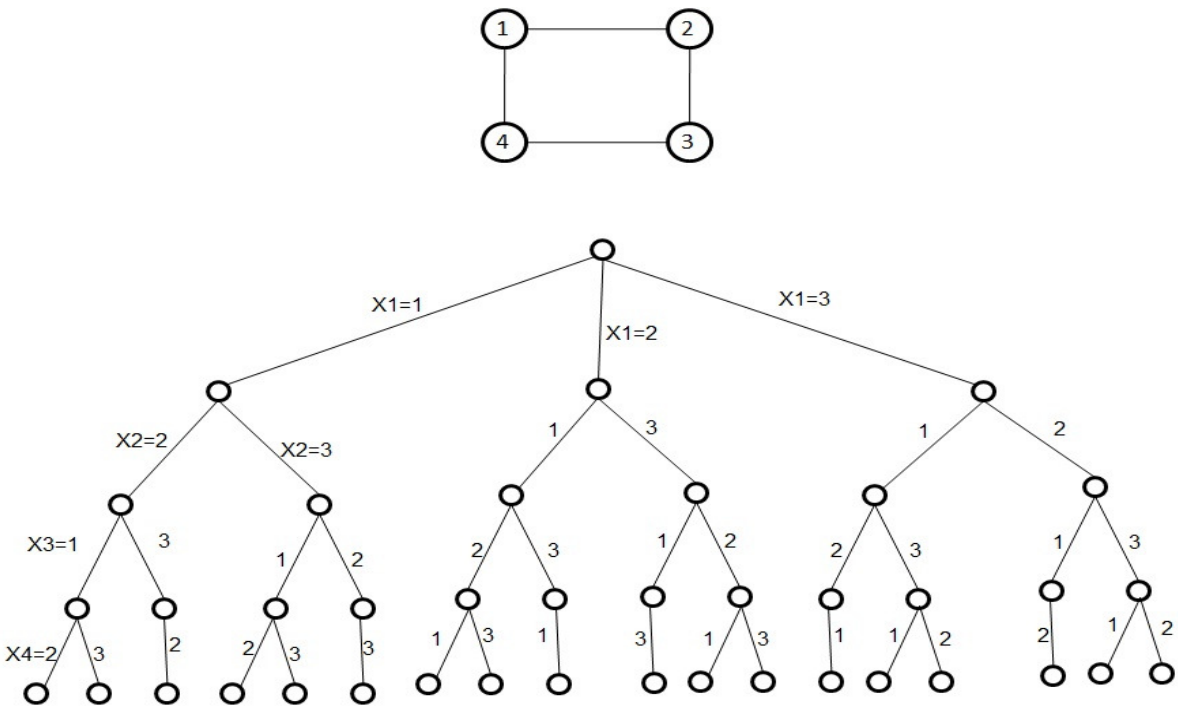



```
mcoloring(3)    i.e. k=3
nextvalue(3)
k=3
x[3]=(x[3]+1) mod (m+1)
x[3]= 0+1 mod 4
x[3]=1

             G[k,j]!=0 and x[k]=x[j]
j=1 G[3,1]     False   and   True  =  False
j=2 G[3,2]     True    and   False =  False
j=3 G[3,3]     False   and   True  =  False
j=4 G[3,4]     True    and   False =  False

x[1]=1 ,x[2]=2,x[3]=1,x[4]=0.
```




```
mcoloring(4)    i.e. k=4
nextvalue(4)
k=4
x[4]=(x[4]+1) mod (m+1)
x[4]= 0+1 mod 4
x[4]=1
             G[k,j]!=0 and x[k]=x[j]
j=1 G[4,1]     true    and   true  =  True   so break
                    adjacent vertices have same color

x[4]=(x[4]+1) mod (m+1)
x[4]= 1+1 mod 4
x[4]=2
             G[k,j]!=0 and x[k]=x[j]
j=1 G[4,1]     True    and   False  =  False
j=2 G[4,2]     False   and   True   =  False
j=3 G[4,3]     True    and   False  =  False
j=4 G[4,4]     False   and   True   =  False

x[1]=1 ,x[2]=2,x[3]=1,x[4]=2
```

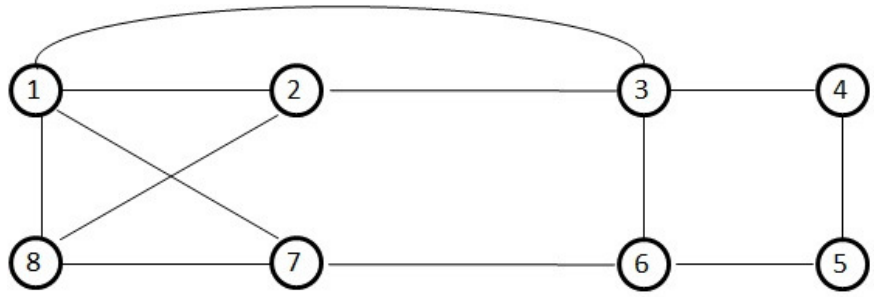




A 4-node graph and all possible 3-colorings

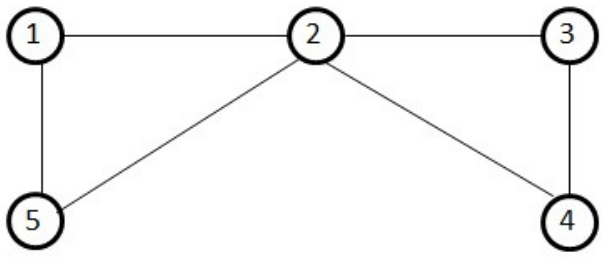
HAMILTONIAN CYCLES

Let G=(V,E) be a connected graph with n vertices. A Hamiltonian cycle is a round trip path along n edges of G that visits every vertex once and returns to its starting position. In other words if a Hamiltonian cycle begins at some vertex v1 ∈ G and the vertices of G are visited in the order v_1, v_2,…v_{n+1} then the edges (v_i,v_{i+1}) are in E, 1<=i<=n, and the v_i are distinct except for v_1 and v_{n+1}, which are equal.



Graph containing a Hamiltonian Cycle

The above graph contains the Hamiltonian cycles
1,2,3,4,5,6,7,8,1
1,3,4,5,6,7,8,2,1
1,2,8,7,6,5,4,3,1



Graph Does not containing a Hamiltonian Cycle

The graph contains no Hamiltonian cycle.

To check whether there is a Hamiltonian cycle or not we may use backtracking method. The graph may be directed or undirected. Only distinct cycles are output.

The backtracking solution vector (X_1,X_2,X_3,…X_n) is defined so that xi represents the ith visited vertex of the proposed cycle.

Now all we need to do is determine how to compute the set of possible vertices for x_k if x_1,..x_{k-1} have already been chosen. If k=1 then x1 can be any of the n vertices.

The algorithm nextvalue(k) which determines a possible next vertex for the proposed cycle.

Using nextvalue we can particularize the recursive backtracking schema to find all Hamiltonian cycles. This algorithm is started by first initializing the adjacency matrix G[1:n,1:n], then setting x[2:n] to 0 and x[1] to 1 and then executing Hamiltonian(2).

```
// x[1:k-1] is a path of k-1 distinct vertices
// if x[k]=0 then no vertex has yet been assigned to x[k]
// after execution x[k] is assigned to the next highest
// numbered vertex which does not already appear in
// x[1:k-1]. Otherwise  x[k]=0.
// if k=n then in addition x[k] is connected to x[1].

Algorithm nextvalue(k)
{
  Repeat
  {
    X[k]:=(x[k]+1)mod(n+1);
    if (x[k]=0) then
          return;
    if (G[x[k-1],x[k]]]!=0) then
      {
        For j:=1 to k-1 do
            if (x[j]=x[k]) then
                break;
        if (j=k) then
          if((k<n) or ((k=n)and G[x[n],x[1]]!=0))then
                return;
      }
   }until (false);
}
```

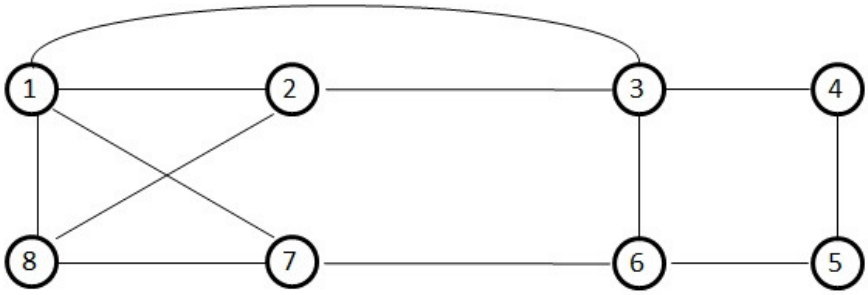
Algorithm to generate next vertex.

The algorithm Hamiltonian() uses the recursive formulation of backtracking to find all the Hamiltonian cycles of a graph. The graph is stored as an adjacency matrix G[1:n,1:n]. All cycles begin at node 1.

```
Algorithm Hamiltonian(k)
{
   Repeat
   {
     nextvalue(k);
     if (x[k]=0) then return;
     if (k=n) then write (x[1:n]);
     else
       Hamiltonian(k+1);
   }until(false);
}
```

Algorithm to find all Hamiltonian cycles.

Example)



Graph containing a Hamiltonian Cycle

No of vertices n=8

Adjacency matrix G

	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	1	1
2	1	0	1	0	0	0	0	1
3	1	1	0	1	0	1	0	0
4	0	0	1	0	1	0	0	0
5	0	0	0	1	0	1	0	0
6	0	0	1	0	1	0	1	0
7	1	0	0	0	0	1	0	1
8	1	1	0	0	0	0	1	0

Solution vertex

X[1]	1
X[2]	0
X[3]	0
X[4]	0
X[5]	0
X[6]	0
X[7]	0
X[8]	0

Algorithm starts with vertex 1 as initial vertex.
Solution vertex must contain a series of vertices in the cycle.
X[1:n] i.e. x[1:8]
X[1]=1
and x[2:8]=0
we will add one by one vertices to the solution vector.

```
Hamiltonian(2)     k=2
Nextvalue(2)       k=2
X[2]=(x[2]+1) mod (8+1)=(0+1)mod 9 = 1
Is there an edge between k and k-1
G[x[k-1],x[k]]!=0
G[1,1] no edge False
X[2]=(x[2]+1) mod (8+1)
    = (1+1) mod 9
    = 2

If (G[x[k-1],x[k]]!=0)
      G[1,2] edge True
```

Solution vector

X[1]	1
X[2]	2
X[3]	0
X[4]	0
X[5]	0
X[6]	0
X[7]	0
X[8]	0

```
Are there duplicate vertices in the path
   J=1  is x[j]=x[k]
        is x[1]=x[2] no false

         k < n return  still we need to add vertices
```

```
Hamiltonian(3)    k=3
Nextvalue(3)      k=3
X[3]=(x[3]+1) mod (8+1)=(0+1)mod 9 = 1
X[3]=1
Is there an edge between k and k-1

If (G[x[k-1],x[k]]!=0)
      G[2,1] edge True

Are there duplicate vertices in the path
   J=1  is x[j]=x[k]
        is x[1]=x[3]
              1=1   True   break

X[3]=(x[3]+1) mod (8+1)
X[3]=(1+1)mod 9
    = 2

If (G[x[k-1],x[k]]!=0)
      G[2,2] edge False

X[3]=(x[3]+1) mod (8+1)
X[3]=(2+1)mod 9
    = 3

If (G[x[k-1],x[k]]!=0)
      G[2,3] edge True

J=1   is x[j]=x[k]
      Is 1=2 no false
J=2   is 2=3 no false

      as  k < n return  still we need to add vertices


Hamiltonian(4)  k=4
Nextvalue(4)    k=4

X[4]=(x[4]+1) mod (8+1)
    = (0+1)mod 9 = 1
X[4]=1
Is there an edge between k and k-1

If (G[x[k-1],x[k]]!=0)
      G[3,1] edge True

Are there duplicate vertices in the path
   J=1  is x[j]=x[k]
        is x[1]=x[4]
              1=1   True   break
```

Solution vector

X[1]	1
X[2]	2
X[3]	3
X[4]	0
X[5]	0
X[6]	0
X[7]	0
X[8]	0

```
X[4]=(x[4]+1) mod (8+1)
X[4]= 2

If (G[x[k-1],x[k]]!=0)
      G[3,2] edge True

 Are there duplicate vertices in the path
   J=1  is x[j]=x[k]
        is x[1]=x[4]
              1=2   False

    J=2  is x[j]=x[k]
         is x[2]=x[4]
              2=2   True Break



X[4]=(x[4]+1) mod (8+1)
X[4]=3


If (G[x[k-1],x[k]]!=0)
      G[3,3] edge False

X[4]=(x[4]+1) mod (8+1)
X[4]=4

If (G[x[k-1],x[k]]!=0)
      G[3,4] edge True


J=1   is x[j]=x[k]
      Is 1=4 no false
J=2   is 2=4 no false
J=3   is 3=4 no false
      as  k < n return  still we need to add vertices

Hamiltonian(5)  k=5
Nextvalue(5)    k=5

X[5]=(x[5]+1) mod (8+1)
    = (0+1)mod 9 = 1
X[5]=1
Is there an edge between k and k-1

If (G[x[k-1],x[k]]!=0)
      G[4,1] no edge False

X[5]=(x[5]+1) mod (8+1)
    = (1+1)mod 9 = 2

If (G[x[k-1],x[k]]!=0)
      G[4,2] no edge False

X[5]=(x[5]+1) mod (8+1)
```

Solution vector

X[1]	1
X[2]	2
X[3]	3
X[4]	4
X[5]	0
X[6]	0
X[7]	0
X[8]	0

```
    = (3+1)mod 9 = 3
If (G[x[k-1],x[k]]!=0)
      G[4,3]  edge True

Are there duplicate vertices in the path
   J=1  is x[j]=x[k]
        is x[1]=x[5]
              1=3   False
J=2  is x[j]=x[k]
        is x[2]=x[5]
              2=3   False
J=3  is x[j]=x[k]
        is x[3]=x[5]
              3=3   True  duplicate found break

X[5]=(x[5]+1) mod (8+1)
X[5]= 5

If (G[x[k-1],x[k]]!=0)
      G[4,5] edge True

 Are there duplicate vertices in the path
   J=1  is x[j]=x[k]
        is x[1]=x[5]
              1=5   False

    J=2  is x[j]=x[k]
         is x[2]=x[5]
              2=5   False

   J=3  is x[j]=x[k]
        is x[3]=x[5]
              3=5   False

    J=4  is x[j]=x[k]
         is x[4]=x[5]
              4=5   False
```

Solution vector

X[1]	1
X[2]	2
X[3]	3
X[4]	4
X[5]	5
X[6]	0
X[7]	0
X[8]	0

```
      as  k < n return  still we need to add vertices


Hamiltonian(6)  k=6
Nextvalue(6)    k=6



The solution vector for hamiltonian cycles
1,2,3,4,5,6,7,8,1
1,8,2,3,4,5,6,7,1
1,3,4,5,6,7,8,2,1
```

SUM OF SUBSETS

Suppose we are given n distinct positive numbers(usually called weights) and we desire to find all combinations of these numbers whose sum are m.

This is called the sum of subsets problem.

Ex1) given positive numbers W_i, 1<=i<=n, and m, this problem calls for finding all subsets of w_i whose sums are m. For example, if n=4, $(w_1,w_2,w_3,w_4)=(7,11,13,24)$ and m=31, then the desired subsets are (7,11,13) and (7,24).

Rather than representing the solution vector by w_i which sum to m, we could represent the solution vector by giving the indices of these w_i.

Now the two solutions are described by the vectors (1,2,3) and (1,4).

In general all solution subset is represented by n-tuple $(X_1,X_2,X_3,\ldots X_n)$ such that Xi∈ {0,1},1<=i<=n. The X_i is 0 if w_i is not chosen and xi=1 if w_i is chosen. The solutions to the above instances are (1,1,1,0) and (1,0,0,1). This formulation expresses all solutions using a fixed sized tuple.

The sum of sub set is based on fixed size tuple. Let us draw a tree structure for fixed tuple size formulation.

All paths from root to a leaf node define a solution space. The left subtree of the root defines all subsets containing W_1 and the right subtree defines all subsets not containing W_1 and so on.

Step 1)Start with an empty set

Step 2)Add next element in the list to the sub set

Step 3)If the subset is having sum = m then stop with that sub set as solution.

Step 4)If the sub set is not feasible or if we have reached the end of the set then backtrack through the subset until we find the most suitable value.

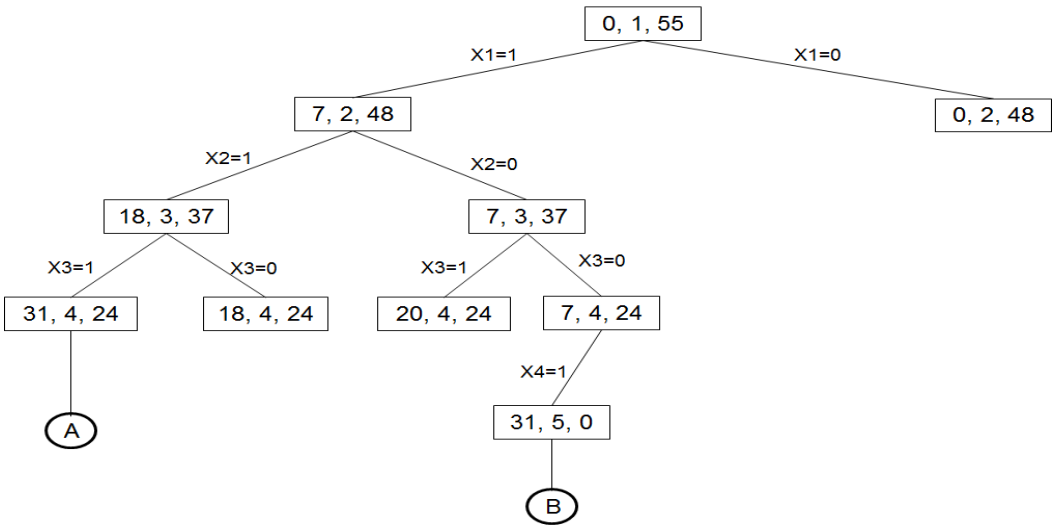
Step 5)if the subset is feasible then repeat step 2

Step 6)if we have visited all elements without finding a suitable subset and if no backtracking is possible, then stop with no solution.

s - sum of all selected elements
k - denotes the index of chosen element
r - initially sum of all elements. After selection of some element from the set subtract the chosen value from r each time.
W(1:n) - represents set containing n elements.
X[i]-solution vector 1<=i<=k

```
Algorithm sumofsubsets(s,k,r)
{
  X[k]:=1;
  if (s+w[k]=m) then write (x[1:k]); // subset found
  else
    if (s+w[k]+w[k+1]<=m) then
       sumofsubsets(s+w[k],k+1,r-w[k]);

   //generate right child and evaluate Bk.

   if ((s+r-w[k]>=m) and (s+w[k+1]<=m)) then
    {
      X[k]:=0;
      sumofsubsets(s,k+1,r-w[k]);
    }
}
```

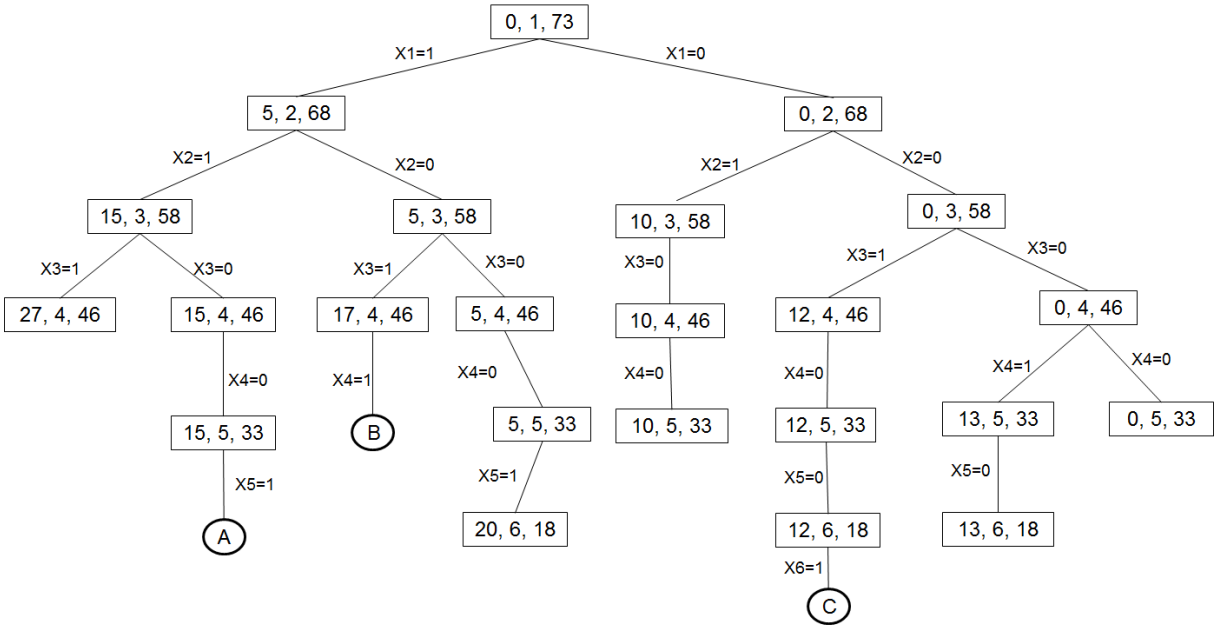
Ex) n=4, $(w_1,w_2,w_3,w_4)=(7,11,13,24)$ and m=31
Solution Vector=(x[1],x[2],x[3],x[4])



Portion of state space Tree

Solution A = {1,1,1,0}
Solution B = {1,0,0,1}

Ex2) n=6, m=30 and w[1:6]={5,10,12,13,15,18}.
Portion of the state space tree generated by sum of subsets



State space tree with solution

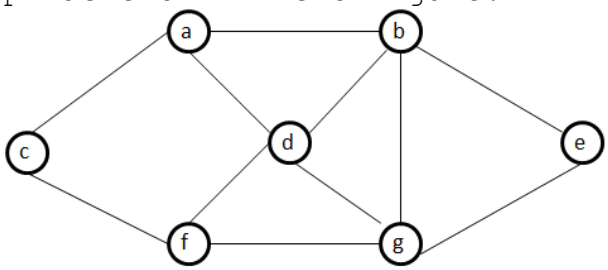
The rectangular nodes list the values of s,k and r.
Circular nodes represent points at which subsets with sums m are printed out.

Solution A = (1,1,0,0,1)
Solution B = (1,0,1,1)
Solution C = (0,0,1,0,0,1)

Note that the tree contains only 23 rectangular nodes. The full space tree for n=6 contains $2^6-1=63$ nodes from which calls could be made.

Important questions

1)Describe problem state, solution state and answer state with examples.
2) Write the control abstraction of backtracking
3)Explain the applications of backtracking
4)Describe 4-queen problem using backtracking
5)Write an algorithm of finding all m-colorings of a graph
6)Draw the state space tree for m-coloring graph using suitable graph
7)Apply backtracking to find Hamiltonian cycle in the following graph as shown in the figure.



8)Write backtracking algorithm for 8-queens problem