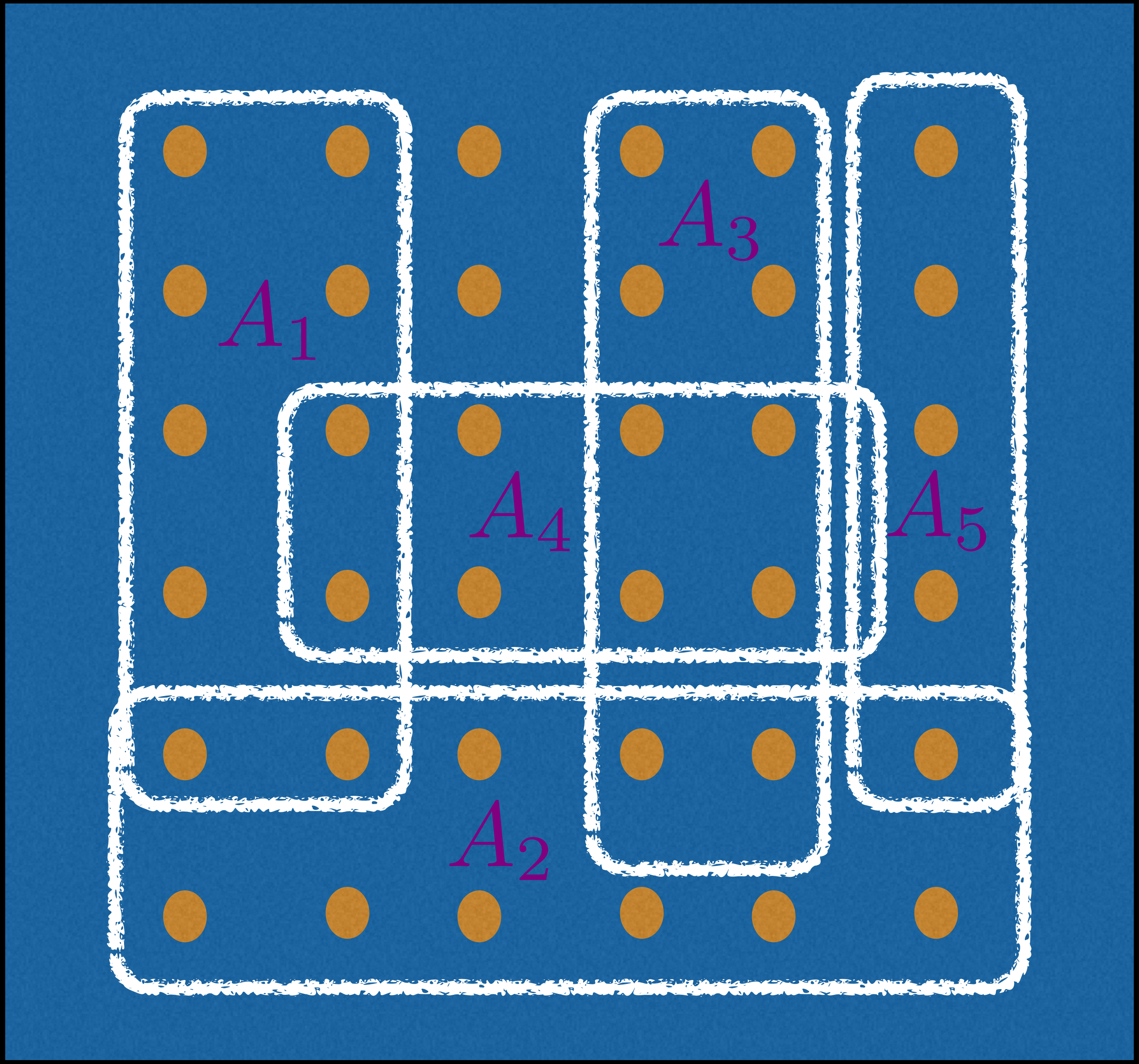
Learning Coverage Functions and Private Release of Marginals

Vitaly Feldman
IBM Research

Pravesh Kothari
UT Austin

Coverage Functions

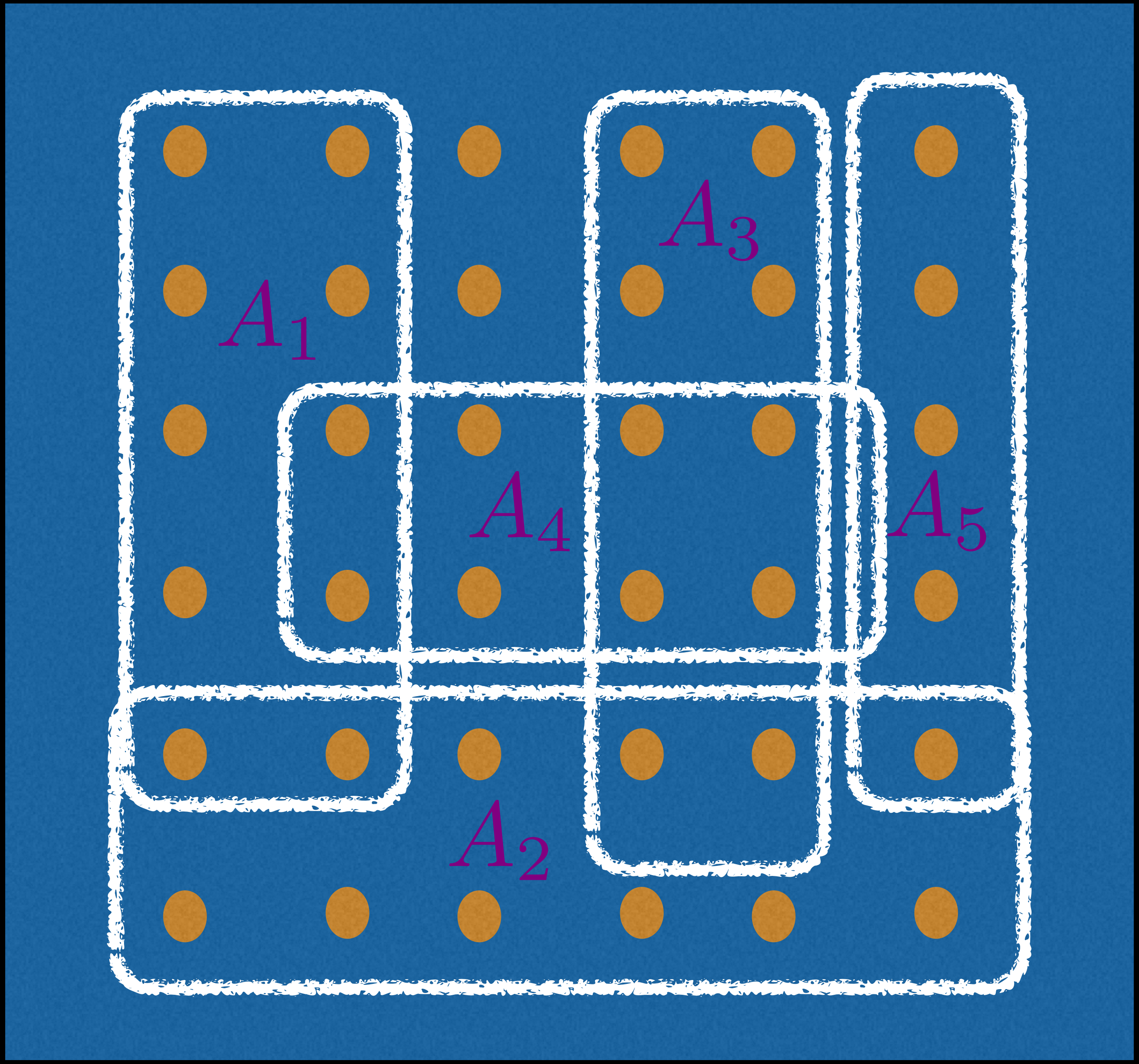
A function $f : 2^{[n]} \to \mathbb{R}^+$ is a *coverage function* if:



Coverage Functions

A function $f : 2^{[n]} \to \mathbb{R}^+$ is a *coverage function* if:

Ground Set U

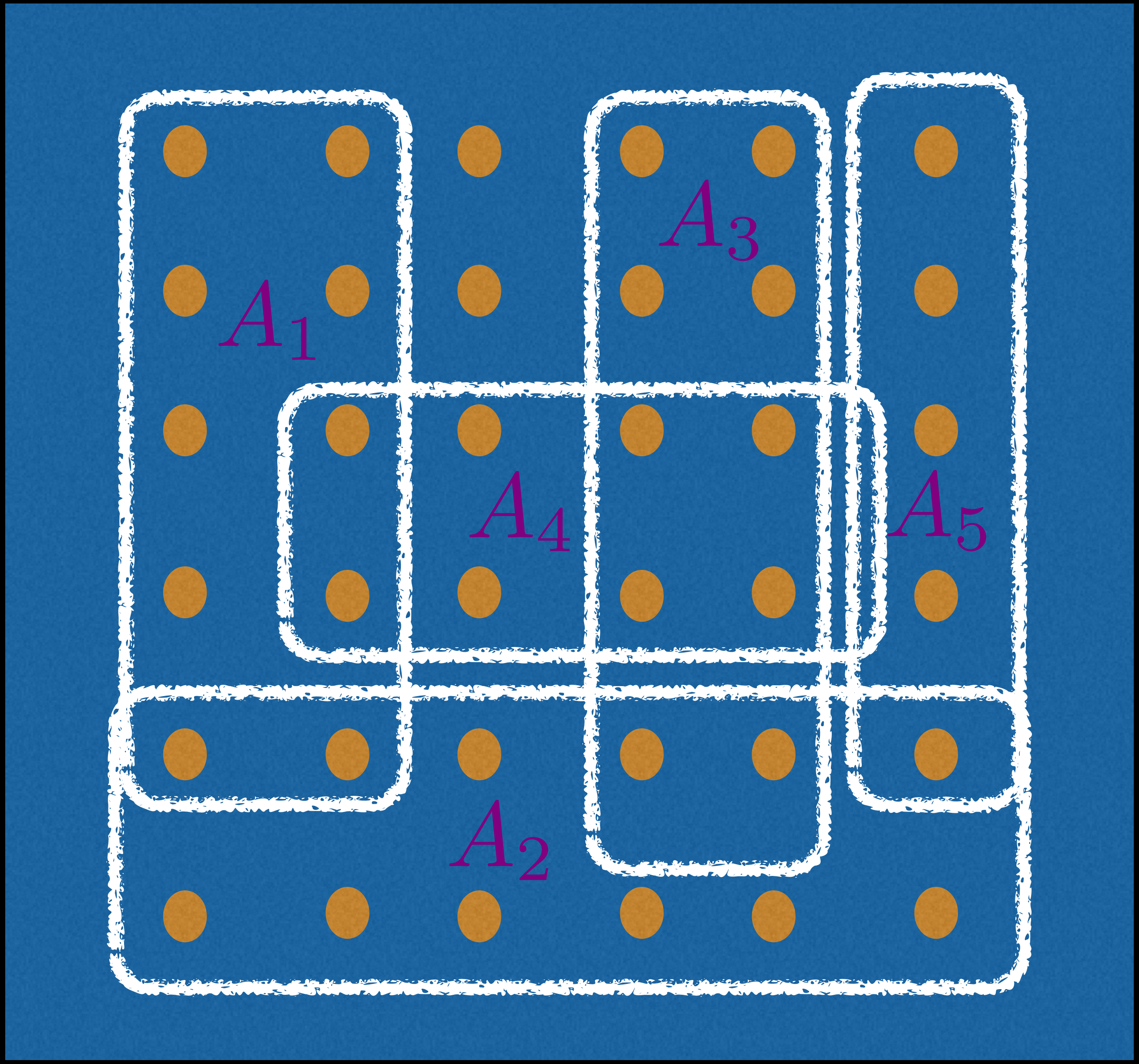


Coverage Functions

A function $f : 2^{[n]} \to \mathbb{R}^+$ is a *coverage function* if:

Ground Set U

$A_1, A_2, \ldots, A_n \subseteq U$



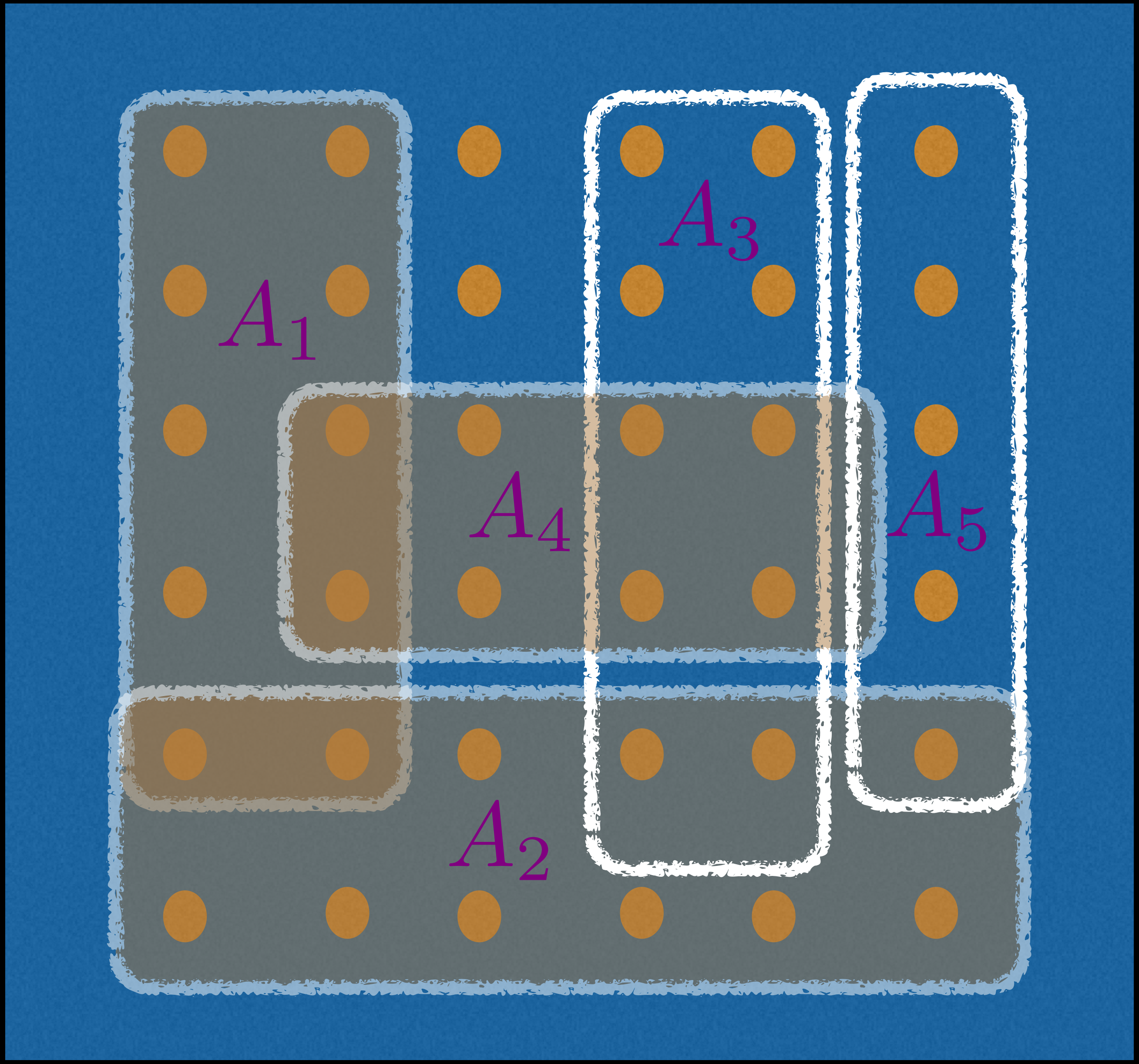
Coverage Functions

A function $f : 2^{[n]} \to \mathbb{R}^+$ is a *coverage function* if:

Ground Set U

$A_1, A_2, \ldots, A_n \subseteq U$

For any $S \subseteq [n]$



Coverage Functions

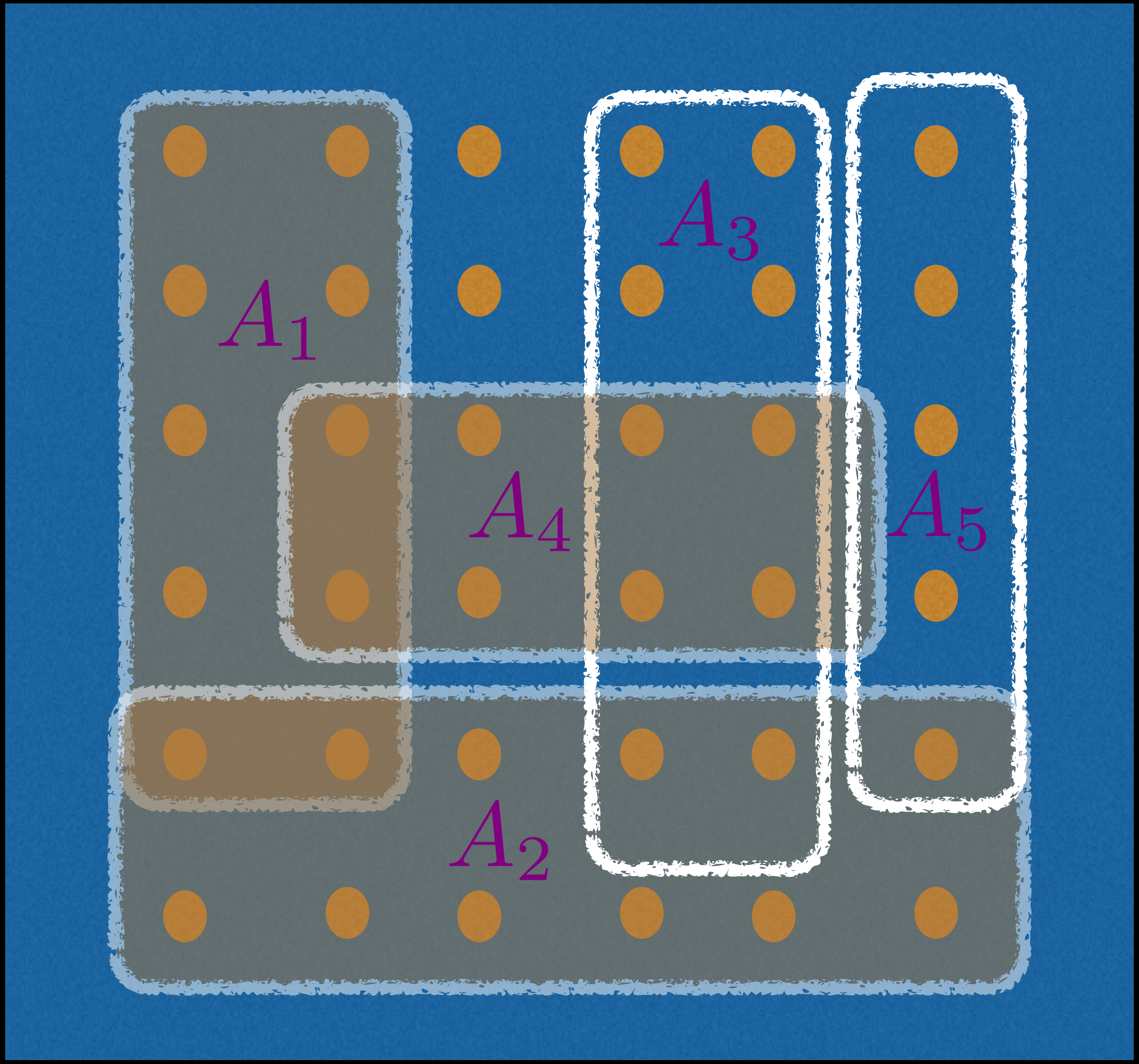
A function $f : 2^{[n]} \to \mathbb{R}^+$ is a *coverage function* if:

Ground Set U

$A_1, A_2, \ldots, A_n \subseteq U$

For any $S \subseteq [n]$

$$\boxed{f(S) = |\cup_{i \in S} A_i|}$$



Coverage Functions

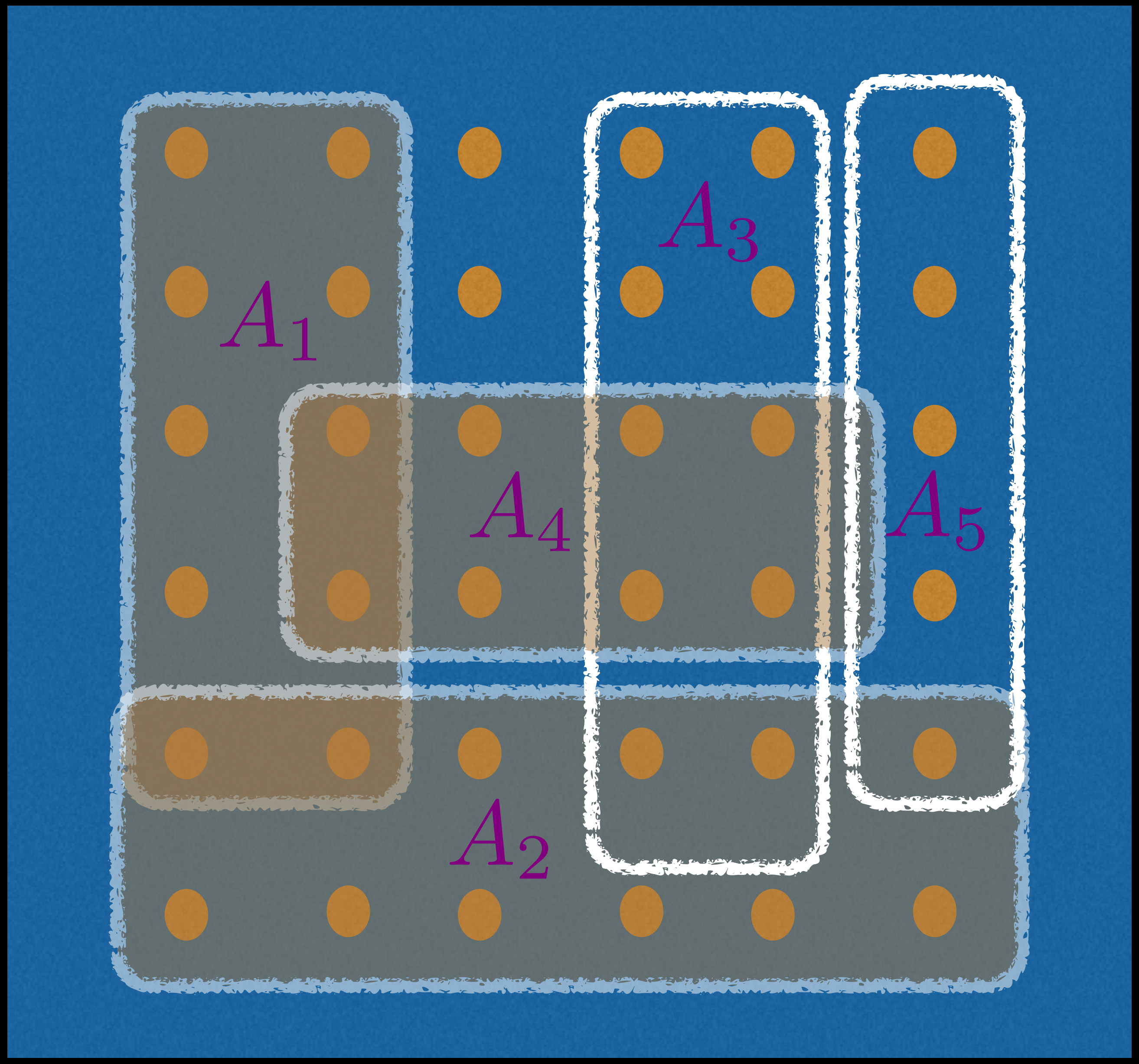
A function $f : \cancel{2^{[n]}} \{-1,1\}^n \to \mathbb{R}^+$ is a *coverage function* if:

Ground Set U

$A_1, A_2, \ldots, A_n \subseteq U$

For any $S \subseteq [n]$

$$\boxed{f(S) = |\cup_{i \in S} A_i|}$$



Coverage Functions

normalization

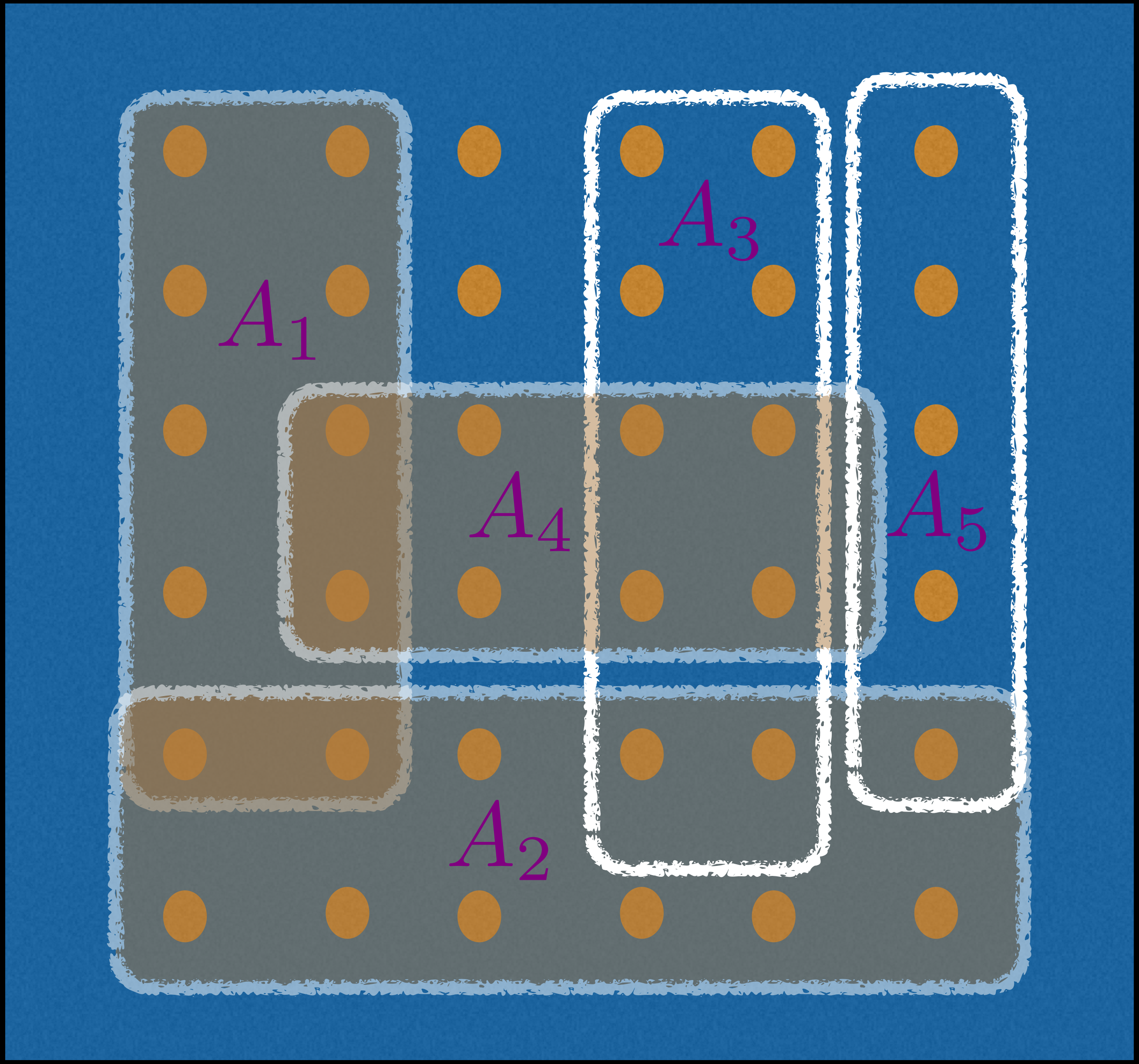
A function $f : \{-1,1\}^n \to [0,1]$ is a *coverage function* if:

Ground Set U

$A_1, A_2, \dots, A_n \subseteq U$

For any $S \subseteq [n]$

$$\boxed{f(S) = |\cup_{i \in S} A_i|}$$



Coverage Functions

$f : \{-1,1\}^n \to [0,1]$ *coverage function:*

$$f(x) = \sum_{S \subseteq [n]} \alpha_S \cdot \mathsf{OR}_S(x)$$

Coverage Functions

$f : \{-1,1\}^n \to [0,1]$ *coverage function:*

$$f(x) = \sum_{S \subseteq [n]} \alpha_S \cdot \mathsf{OR}_S(x)$$

non-negative

Coverage Functions

$f : \{-1,1\}^n \to [0,1]$ *coverage function:*

$$f(x) = \sum_{S \subseteq [n]} \alpha_S \cdot \mathsf{OR}_S(x)$$

monotone
disjunction on S

Coverage Functions

$f : \{-1,1\}^n \to [0,1]$ *coverage function:*

$$f(x) = \sum_{S \subseteq [n]} \alpha_S \cdot \mathsf{OR}_S(x)$$

$$\mathsf{OR}_S(x) = 0 \Leftrightarrow x_i = 1 \text{ for every } i \in S$$

“non-negative linear combination of monotone disjunctions”

Coverage Functions

$f : \{-1,1\}^n \to [0,1]$ *coverage function:*

$$f(x) = \sum_{S \subseteq [n]} \alpha_S \cdot \mathsf{OR}_S(x)$$

$$\sum_{S \subseteq [n]} \alpha_S \leq 1$$

Coverage Functions

- ✦Important subclass of submodular functions.
  - Topic of intense research in machine learning.
- ✦Algorithmic Game Theory\Economics
  - Valuation Functions - utilities, prices.

    Lehmann-Nisan-06, Dobzinski et. al.-05, Vondrák-08, Papadimitriou et. al.-08, Dughmi et. al. 11
- ✦Differential Privacy: in this talk

Our Results

1. Learnability of Coverage Functions.
2. Applications to Private Data Release

Learning Coverage Functions

"PAC Learning with ℓ_1 error"

Learner sees random examples labeled the target coverage function f.

Must return a hypothesis h such that

$$\mathbb{E}[|h(x) - f(x)|] \leq \epsilon$$

Learning Coverage Functions

"PAC Learning with ℓ_1 error"

Coverage functions are learnable on the uniform distribution in time $\tilde{O}(n)\text{poly}(1/\epsilon)$ *and samples* $\log(n)\text{poly}(1/\epsilon)$.

Previous Work: Submodular Functions

[Cheraghchi,Klivans,K,Lee 2012],[Gupta,Hardt,Roth,Ullman 2012],[Feldman-K-Vondrak 2013],[Feldman and Vondrak 2013]

Best: $2^{O(1/\epsilon^2)} \cdot \text{poly}(n)$

exponential dependence *necessary*.[Feldman-K-Vondrak 2013]

Learning Coverage Functions

PMAC Model [Balcan and Harvey2011]

"Probably *Mostly* Approximately Correct"

Learner sees random examples from the target coverage function $f : \{-1, 1\}^n \to \mathbb{R}^+$.

Must return a hypothesis h such that

$$\Pr[h(x) \leq f(x) \leq (1+\gamma)h(x)] \geq 1 - \delta$$

More challenging model to learn than PAC.

Learning Coverage Functions

PMAC Model [Balcan and Harvey2011]

"Probably *Mostly* Approximately Correct"

Coverage functions are PMAC learnable on the uniform distribution in time $\tilde{O}(n)\text{poly}(\frac{1}{\delta \cdot \gamma})$ *and samples* $\log(n)\text{poly}(\frac{1}{\delta \cdot \gamma})$.

Previous Work: Submodular Functions:

[Balcan and Harvey 2011] [Feldman and Vondrak 2013]

Best: $2^{O(1/(\delta\gamma)^2)}\text{poly}(n)$

First *fully* polynomial time *PMAC* learning algorithm for an interesting class.

Learning Coverage Functions

"Agnostic Learning with ℓ_1 error"

learner gets random examples from an *arbitrary* f.

Must return h:

$$\mathbb{E}[|h(x) - f(x)|] \leq opt + \epsilon$$

Learning Coverage Functions

"Agnostic Learning with ℓ_1 error"

learner gets random

$$opt = \min_{coverage\ c} \mathbb{E}[|c(x) - f(x)|]$$

Must retur

$$\mathbb{E}[|h(x) - f(x)|] \leq opt + \epsilon$$

Learning Coverage Functions

Coverage functions are agnostically learnable over all product and symmetric distributions in time and samples $n^{O(\log(1/\epsilon))}$.

Learning Coverage Functions

PDF at x
depends only on weight of
x

Coverage functions are agnostically learnable over all product and symmetric distributions in time and samples $n^{O(\log(1/\epsilon))}$.

Hardness assumption: *Sparse LPN*

Learning Coverage Functions

distribution independent setting:

*reduction from Disjoint-DNFs**

$\mathrm{poly}(n, 1/\epsilon)$ *PAC algorithm for coverage functions*

$$\Downarrow$$

$\mathrm{poly}(n, 1/\epsilon)$ PAC algorithm for Disjoint-DNFs.

*DNFs where any assignment satisfies at most one term.

Learning Coverage Functions

distribution independent setting:

reduction from DISJOINT-DNFs

Best: $n^{O(n^{1/3})}$

[Klivans-Servedio 2001]

$\text{poly}(n, 1/\epsilon)$ *PAC algorithm for coverage functions*

$\Downarrow$

$\text{poly}(n, 1/\epsilon)$ PAC algorithm for DISJOINT-DNFs.

Application: Private Data Release

Database $D \subseteq \{-1, 1\}^n$

Application: Private Data Release

Database $D \subseteq \{-1,1\}^n$

Counting Query $q : \{-1,1\}^n \to \{0,1\}$

Application: Private Data Release

Database $D \subseteq \{-1, 1\}^n$

Counting Query $q : \{-1, 1\}^n \rightarrow \{0, 1\}$

Reply $\dfrac{1}{|D|} \sum_{r \in D} q(r)$

Application: Private Data Release

Database $D \subseteq \{-1,1\}^n$

Counting Query $q : \{-1,1\}^n \to \{0,1\}$

Reply $\dfrac{1}{|D|} \sum_{r \in D} q(r)$

goal: answer queries *accurately* while ensuring "*privacy*" of participants of the database

Application: Private Data Release

Differential Privacy

[Dwork,McSherry,Nissim,Smith 2006]

Adjacent Databases: $|D \Delta D'| = 1$

Algorithm $A : 2^{\{-1,1\}^n} \to R$ is κ-differentially private if for any adjacent databases D and D', if for every $T \subseteq R$

$$\Pr[A(D) \subseteq T] \leq e^{\kappa} \Pr[A(D') \subseteq T]$$

"algorithm behavior doesn't change much on adjacent databases."

Application: Private Data Release

Marginal Queries

q: monotone conjunction

Application: Private Data Release

Releasing Marginals

- simple and natural query class.

Application: Private Data Release

Releasing Marginals

- simple and natural query class.
- official stats of US census, Dept. of Labor and IRS.

Application: Private Data Release

Releasing Marginals

- simple and natural query class.
- official stats of US census, Dept. of Labor and IRS.
- well studied.

Barak et. al.07, UllmanVadhan11, Cheraghchi et. al. 12,
Hardt. et. al.12, Thaler et. al. 12, Bun et. al.13,
Chadrasekharan et. al.14, Dwork et. al. 13

Application: Private Data Release

Releasing Marginals

Counting queries with monotone conjunction predicate.

Goal: A differentially private algorithm that, given D, publishes a summary, that, for any monotone conjunction c, can be used to compute

$$\frac{1}{|D|}\sum_{r \in D} c(r) \pm \beta$$

Application: Private Data Release

Releasing Marginals

Counting queries with conjunction predicate.

Goal: A differentially private algorithm that, given D, publishes a summary, that, for any monotone conjunction c, can be used ... te

error

$$\frac{1}{|D|}\sum_{r\in D} c(r) \pm \beta$$

Application: Private Data Release

Releasing Marginals

Counting queries with conjunction predicate.

Goal: A differentially private algorithm that, given D, publishes a summary, that, for any monotone conjunction c, can be used to compute

$$\frac{1}{|D|}\sum_{r \in D} c(r) \pm \beta$$

Worst Case Error: Best $n^{O(\sqrt{n})}$

[Thaler,Ullman,Vadhan 2012][Dwork,Nikolov,Talwar 2013]

Application: Private Data Release

Releasing Marginals with low average error

Application: Private Dat

w.r.t. some distribution over queries.

Releasing Marginals with low average error

Application: Private Data Release

Releasing Marginals with low average error

Introduced by [Gupta,Hardt,Roth,Ullman 2011]

[Cheraghchi,Klivans,K,Lee 2012],[Hardt,Rothblum,Servedio 2012],
[Dwork,Nikolov,Talwar 2013]

Application: Private Data Release

Releasing Marginals with low average error

Introduced by [Gupta,Hardt,Roth,Ullman 2011]

[Cheraghchi,Klivans,K,Lee 2012],[Hardt,Rothblum,Servedio 2012],
[Dwork,Nikolov,Talwar 2013]

Releasing Marginals: $n^{O(1/\bar{\alpha}^2)}$

$\bar{\alpha}$:average error

Application: Private Data Release

PAC Learning Algorithm

$$\Downarrow$$

$\text{poly}(n, 1/\epsilon)/\kappa$ *algorithm for releasing conjunction counting queries privately with low average error over all conjunctions.*

Application: Private Data Release

Releasing Marginals with low average error

average error over *all* monotone conjunctions?

Most applications require releasing only *short* marginals.

Application: Private Data Release

Releasing Short Marginals

"k-way marginals"

Counting queries on monotone conjunctions *of length k.*

Worst Case Error: $n^{O(\sqrt{k})}$

[Thaler,Ullman,Vadha 2012] [Dwork,Nikolov,Talwar 2013]

Application: Private Data Release

Releasing Marginals with low average error

"k-way marginals"

average error over all monotone conjunctions *of length k*?

previous results not applicable.

Application: Private Data Release

Releasing k-way marginals

Agnostic learning algorithm
on *symmetric* distributions.

$\Downarrow$

k-way marginals can be released with average error $\bar{\alpha}$ *in time* $n^{O(\log 1/\bar{\alpha})}$.

Application: Private Data Release

Counting Query Function

$$c_D : \mathcal{C} \rightarrow [0, 1]$$

"maps queries to their correct answers on a database D".

Application: Private Data Release

Counting Query Function

$$c_D : \mathcal{C} \to [0,1]$$

Counting Query Class

$$\mathbb{Q}(\mathcal{C}) = \{c_D \mid D \subseteq \{-1,1\}^n\}$$

Application: Private Data Release

Counting Query Class

$$\mathbb{Q}(\mathcal{C}) = \{c_D \mid D \subseteq \{-1,1\}^n\}$$

For $\mathcal{C}$= monotone conjunctions,

$\mathbb{Q}(\mathcal{C})$ = coverage functions.

Learning on a distribution

$$\Downarrow$$

private release of marginals with low average error.

used in previous works on PDR.

PAC and *PMAC* Learning coverage functions.

PAC Learning

On the Uniform Distribution*

Simple properties of the Fourier spectrum

f: coverage function.

PAC Learning

On the Uniform Distribution

Simple properties of the Fourier spectrum

f: coverage function.

Monotonicity of Fourier coeffs:

For $T \subseteq V$, $\quad |\hat{f}(T)| \geq |\hat{f}(V)|$

PAC Learning

On the Uniform Distribution

Simple properties of the Fourier spectrum

f: coverage function.

Monotonicity of Fourier coeffs:

For $T \subseteq V$, $\quad |\hat{f}(T)| \geq |\hat{f}(V)|$

"find large Fourier coeffs efficiently"

PAC Learning

On the Uniform Distribution

Simple properties of the Fourier spectrum

f: coverage function.

Monotonicity of Fourier coeffs:

For $T \subseteq V$, $\quad |\hat{f}(T)| \geq |\hat{f}(V)|$

L1 spectral norm of f is bounded above by 2.

PAC Learning

On the Uniform Distribution

Simple properties of the Fourier spectrum

f: coverage function.

Monotonicity of Fourier coeffs:

sum of absolute values of Fourier coefficients.

$$|\hat{f}(T)| \geq |\hat{f}(V)|$$

L1 spectral norm of f is bounded above by 2.

PAC Learning

On the Uniform Distribution

Simple properties of the Fourier spectrum

f: coverage function.

Monotonicity of Fourier coeffs:

For $T \subseteq V$, $\quad |\hat{f}(T)| \geq |\hat{f}(V)|$

L1 spectral norm of f is bounded above by 2.

“not too many large Fourier coeffs”

PAC Learning

On the Uniform Distribution

Simple properties of the Fourier spectrum

f: coverage function.

Monotonicity of Fourier coeffs:

For $T \subseteq V$, $\quad |\hat{f}(T)| \geq |\hat{f}(V)|$

L1 spectral norm of f is bounded above by 2.

“return linear combination of large Fourier coefficients”

PAC Learning
On the Uniform Distribution

Junta Approximation

For a coverage function c, there is a coverage function c', that depends on $O(1/\epsilon^2)$ variables and L1 approximates c within ϵ on the uniform distribution.

PAC Learning
On the Uniform Distribution

Junta Approximation

For a coverage function c, there is a coverage function c', that depends on $O(1/\epsilon^2)$ variables and L1 approximates c within ϵ on the uniform distribution.

"reduce sample complexity - attribute efficiency"

PAC Learning

On the Uniform Distribution

Coverage functions are PAC learnable over the uniform distribution in time $\tilde{O}(n) \cdot \text{poly}(1/\epsilon)$ *and samples* $\log(n) \cdot \text{poly}(1/\epsilon)$.

PMAC Learning

On the Uniform Distribution

if values of target coverage functions are "*large*", then additive error $\Rightarrow$ multiplicative error.

PMAC Learning

On the Uniform Distribution

if values of target coverage functions are "*large*", then additive error $\Rightarrow$ multiplicative error.

- Decompose target coverage function into "regions".

PMAC Learning

On the Uniform Distribution

if values of target coverage functions are "*large*", then additive error $\Rightarrow$ multiplicative error.

- Decompose target coverage function into "regions".
- In each region, value of coverage function large compared to the max *in the region.*

PMAC Learning

On the Uniform Distribution

For any coverage* function with maximum M, either

$$\Pr[c(x) \geq M/4] \geq 1 - \delta/2$$

or

$$\exists i \in [n] \quad f(x) \geq \frac{M}{16 \ln(2/\delta)}$$

for every x: $x_i = -1$

Recurse on x: $x_i = 1$

PMAC Learning
on the Uniform Distribution

- Concentration of Lipschitz submodular functions.

[Boucheron,Lugosi and Massart 2000]

- for monotone submodular f $\mathbb{E}[f] \geq M/4$

[Feige 2006]

PMAC Learning

On the Uniform Distribution

Coverage functions are PMAC learnable over the uniform distribution in time $\tilde{O}(n) \cdot \mathsf{poly}(\frac{1}{\gamma\delta})$ *and samples* $\log(n) \cdot \mathsf{poly}(\frac{1}{\gamma\delta})$.

Summary

Learning Coverage Functions

- Fully polynomial PAC and PMAC learning on uniform distribution.
- Agnostic,Product\Symmetric distributions: $n^{O(\log(1/\epsilon))}$
- Arbitrary Distributions: Reduction from Disjoint-DNFs

Applications

- Faster private data release algorithms for marginals.

Future Work

Learning on more general distributions?


Questions?

Application: Private Data Release

Proper PAC Learning Algorithm

$\Downarrow$

Fully polynomial time algorithm for releasing *sanitized database* to answer conjunction counting queries privately with low average error over all conjunctions.

PAC Learning
On the Uniform Distribution

Algorithm: Key Idea

- Find variable set I of the junta approx.: estimate singleton Fourier coeffs. monotonicity+ small spectral norm

PAC Learning
On the Uniform Distribution

Algorithm: Key Idea

- Find variable set I of the junta approx.: estimate singleton Fourier coeffs. monotonicity+ small spectral norm
- Estimate Large Fourier coeffs with indices subsets of I.

$$|\hat{f}(T)| \leq \epsilon \Rightarrow |\hat{f}(V)| \leq \epsilon \text{ for every } V \supseteq T$$

PAC Learning

On the Uniform Distribution

Algorithm: Key Idea

- Find variable set I of the junta approx.: estimate singleton Fourier coeffs. monotonicity+ small spectral norm
- Estimate Large Fourier coeffs with indices subsets of I.

$$|\hat{f}(T)| \leq \epsilon \Rightarrow |\hat{f}(V)| \leq \epsilon \text{ for every } V \supseteq T$$

- Return linear combination of large Fourier coeffs.