CS 161 | HW 5

(1) a. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire) = \alpha$

Note: $\neg p \vee q \equiv p \Rightarrow q$

Smoke	Fire	(Smoke ⇒ Fire)	(¬Smoke ⇒ ¬Fire)	α
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

α is neither valid nor unsatisfiable

b $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire) = \alpha$

Smoke	Fire	Heat	(Smoke ⇒ Fire)	(Smoke ∨ Heat) ⇒ Fire	α
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

α is neither valid nor unsatisfiable

① α ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

$= \alpha$

¬P ∨ q
¬q ∨ P

Smoke	Fire	Heat	(Smoke ∧ Heat) ⇒ Fire	Smoke ⇒ Fire	Heat ⇒ Fire	(Smoke ⇒ Fire) ∨ (Heat ⇒ Fire)	α
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	T	T	T

α is valid

② a. M: mythical
I: immortal
m: mammal
H: horned
μ: magical

KB = $M \Rightarrow I \wedge$
$\neg M \Rightarrow (\neg I \wedge m)$
$(I \vee m) \Rightarrow H \wedge$
$H \Rightarrow \mu$

b. KB = $(\neg M \vee I) \wedge (M \vee m) \wedge (\neg I \vee M) \wedge$
$(\neg I \vee H) \wedge (\neg M \vee H)$
$\wedge (\neg H \vee \mu)$

Scratch
$\overline{(I \vee M)} \vee H$
$(I \vee M) \wedge \overline{H}$
$\overline{(I \wedge H)} \vee (M \wedge H)$
$(I \wedge \overline{H}) \wedge \overline{(M \wedge H)}$
$(\overline{I} \vee H) \wedge \overline{(M \wedge H)}$

② c i. $\alpha = M$ (mythical)

$KB \wedge \neg\alpha =$ 1. $(\neg M \vee I)$, 2. $(M \vee M)$, 3. $\neg I \vee M$, 4. $\neg I \vee H$, 5. $\neg M \vee H$, 6. $\neg H \vee M$, 7. $\neg M$

8. (1,3) $\neg M \vee M$

9. (8,7) M

10 (9,7) $M \wedge \neg M$ ✗ contradiction!

$KB \models \alpha$

ii $\alpha = \mu$ (magical)

$KB \wedge \neg\alpha =$ 1. $\neg M \vee I$, 2. $M \vee M$, 3. $\neg I \vee M$, 4. $\neg I \vee H$, 5. $\neg M \vee H$, 6. $\neg H \vee \mu$, 7. $\neg \mu$

8 (7,6) $\neg H$

9. (8,5) $\neg M$

10. (1,3) $\neg M \vee M$

11. (10,9) M

12 (11,5) H

13 (6,12) μ

14 (13,7) $H \wedge \neg H$ ✗ contra-diction!

$KB \models \mu$

iii $\alpha = H$ (horned)

$KB \wedge \neg H =$ 1. $\neg M \vee I$, 2. $M \vee M$, 3. $\neg I \vee M$, 4. $\neg I \vee H$, 5. $\neg M \vee H$, 6. $\neg H \vee \mu$, 7. $\neg H$

8 (7,5) $\neg M$

9. (1,3) $\neg M \vee M$

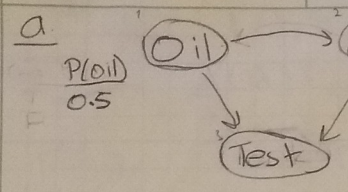
10 (9,8) M

11 (10,5) H

12 (11,7) $H \wedge \neg H$ ✗ contradiction

$KB \models H$

(3) a



P(Oil)
0.5

Oil	P(Gas)
T	0
F	0.4

Oil	Gas	P(Test)
T	T	
T	F	.45
F	T	.06
F	F	

P(a|b) = P(a∧b)/P(b)

b. $P(oil \mid test) = \frac{P(test \mid oil)\,P(oil)}{P(test)}$ ← given

$= \frac{0.9\,(0.5)}{0.54} = 0.833$

$P(test) = P(test \mid oil)\,p(oil) + P(test \mid gas)\,p(gas) + p(test \mid \neg o, \neg g)\,p(\neg o, \neg g) + p_{\emptyset}$

$= 0.91$

Scratch

only oil - .5
only gas - .2 both = 0
neither - .3 P(G=g | O=f)

test: oil pres .9
gas .3
neither .1

P(G|O) = P(G∧O)

1. P(Oil) = P(Oil)

2. $P(Gas \mid Oil) = P(Gas \mid Parents(Gas))$

$\frac{P(Gas \wedge Oil)}{P(Oil)} =$

Gas \ Oil	T	F	prob dist
T	0	0.2	
F	0.5	0.3	

$P(G \mid O=f) = \frac{P(G \wedge O=f)}{P(O=f)} = \frac{0.2}{0.5} = .4$

3. $P(T \mid G, O) = P(T \mid G, O)$

Test	O	¬O	G	¬G
→T		.45	.06	.03

$P(Test=T \mid O=T, GET) = \frac{P(T=T \wedge O=T \wedge GET)}{P(O=T \wedge GET)} = ($

$P(+ \mid o) = .9 = \frac{P(+ \wedge o)}{P(o)}$

$P(+ \mid g) = .3 = \frac{P(+ \wedge g)}{P(g)}$

$P(+ \mid \neg o, \neg g) = .1 = \frac{P(+ \wedge \neg o \wedge \neg g)}{P(\neg o \wedge \neg g)}$

④ a. $P(A,B,C,D,E,F,G,H) =$
$P(A)P(B)P(C|A)P(D|A,B)P(E|B)P(F|C,D)P(G|F)P(H|F,E)$

b. $P(E,F,G,H) = P(E)P(F)P(G|F)P(H|F,E)$

$= \sum_{e \in E} P(e) \sum_{f \in F} P(f) \sum_{g \in G} P(g|F) \sum_{h \in H} P(h|e,f)$

$= \sum_{e} \begin{bmatrix} P(e) \\ P(\neg e) \end{bmatrix} \times \sum_{f} \begin{bmatrix} P(f) \\ P(\neg f) \end{bmatrix} \times \sum_{g} \begin{bmatrix} P(g|F) \\ P(\neg g|F \end{bmatrix} \times \sum_{h} \begin{bmatrix} P(h|e,f) \\ P(\neg h|e,f) \end{bmatrix}$

c. $P(a, \neg b, c, d, \neg e, f, \neg g, h) =$

$0.2 \times 0.3 \times \underset{P(C|A)}{\gamma} \times 0.6 \times 0.1 \times \underset{P(F|CD)}{\beta} \times \underset{P(G|F)}{\alpha} \times \underset{P(H|EF)}{\Delta}$

d. $P(\neg a, b) = P(\neg a) \times P(b) = 0.8 \times 0.7 = \boxed{0.56}$

↳ A, B are independent of each other, & other events

$P(\neg e|a) = P(\neg e) = 0.7 \times 0.9 + 0.3 \times 0.1 = \boxed{0.66}$

↳ $\neg e$ is conditionally indep. of A (as encoded in the b.n.) but is dependent on B, so these are summed out

e • P(A & B) are independent events

• P(C|A) is conditionally independent of all other nodes

• P(D|A,B), P(E|B), P(F|C,D), P(G|F), P(H|F,B) are all also conditionally independent of all other nodes.

④ E. The blanket consists of A, B, C, E, & f for D

g

A, B, D, E	P(D|AB)P(E|B)
1 1 1 1	0.05
1 1 1 0	0.45
1 1 0 1	0.05
1 0 1 1	0.54
0 1 1 1	0.1 × 0.1 = 0.01
0 1 1 0	0.1 × 0.9 = 0.09
0 1 0 1	0.9 × 0.1 = 0.09
0 0 1 1	0.8 × 0.9 = 0.72
0 0 1 0	0.8 × 0.1 = 0.08
0 0 0 0	0.2 × 0.1 = 0.02
1 0 0 1	0.4 × 0.9 = 0.36
1 1 0 0	0.5 × 0.9 = 0.45
1 0 0 0	0.4 × 0.1 = 0.04
0 1 0 0	0.9 × 0.9 = 0.81
0 0 0 1	0.2 × 0.9 = 0.18
1 0 1 0	0.6 × 0.1 = 0.06

h

A	B	E	$\Sigma_D P(D \mid A,B)\, P(E \mid B)$
1	1	1	0.1
1	1	0	0.9
1	0	1	0.9
0	1	1	0.01 + 0.09 = 0.1
0	1	0	0.18
0	0	1	0.18 + 0.72 = 0.9
0	0	0	0.1
1	0	0	0.4