3D Reconstruction of Cameras and Structure

- Chapter 10 (continue …) -

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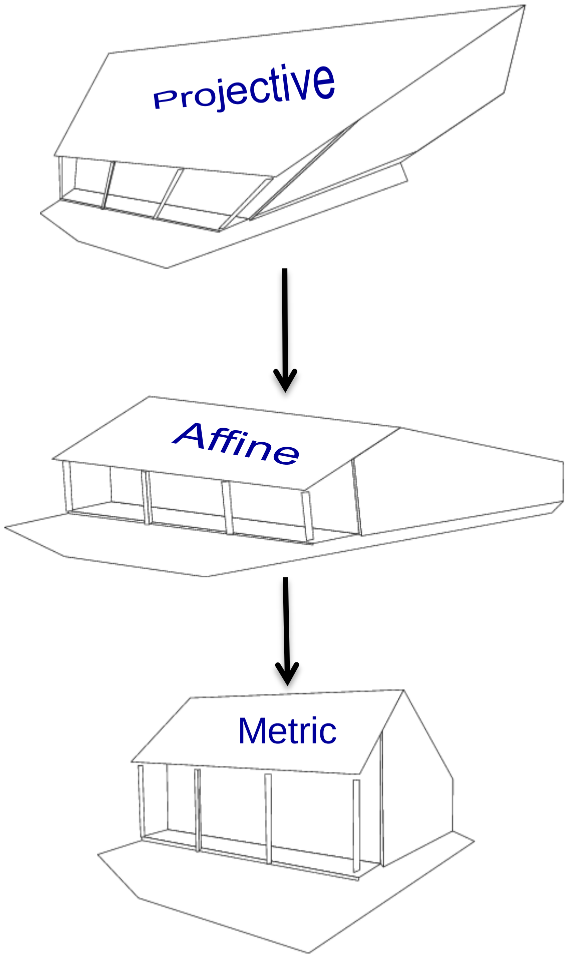
Based on Marc Pollefeys' presentations (www.cs.unc.edu/~marc/mvg/slides.html)

Content

- Chapter 10 continue …
  - Stratified scene reconstruction
  - Direct scene reconstruction

Stratified Scene Reconstruction

- Begin with a Projective reconstruction
- Refine it to an Affine reconstruction
  - Reconstructed scene is than an affine Projective transformation of the actual scene
- Then refine it to a metric reconstruction
  - Reconstructed scene is then a scaled version of actual scene
- Need further information either about the scene, the motion, or the camera calibration



From Projective to Affine Reconstruction

- The essence of affine reconstruction is to locate the plane at infinity
- We have determined a projective reconstruction of a scene: $(\mathrm{P}, \mathrm{P}', \{\mathrm{X}_i\})$
- Find a projective transformation that maps

$$\pi = (A, B, C, D)^{\mathsf{T}} \mapsto \pi_{\infty} = (0,0,0,1)^{\mathsf{T}}$$

$$\mathrm{H}^{-\mathsf{T}}\pi = (0,0,0,1)^{\mathsf{T}}$$

- Such a transformation is $\mathrm{H} = \begin{bmatrix} \mathrm{I} \mid 0 \\ \pi^{\mathrm{T}} \end{bmatrix}$
- H is applied to all points and the two cameras
- Can be sufficient depending on application, e.g.,
  - Computation of the mid-point
  - Computation of the centroid of a set of points

Translational Motion

Case: Suppose the motion of the cameras is a pure translation with no rotation and no change in the internal parameters

- Points at infinity are fixed for a pure translation and will map to the same points in two images related by translation
- Given a projective reconstruction, reconstruct a point X_i lie on the plane of infinity by corresponding match $x_i \leftrightarrow x_i$
- **3 points needed** → uniquely define plane at infinity
- Fundamental matrix:

$$F = [e]_\times = [e']_\times$$

- Choose the two cameras as:

$$P = [I \mid 0]$$

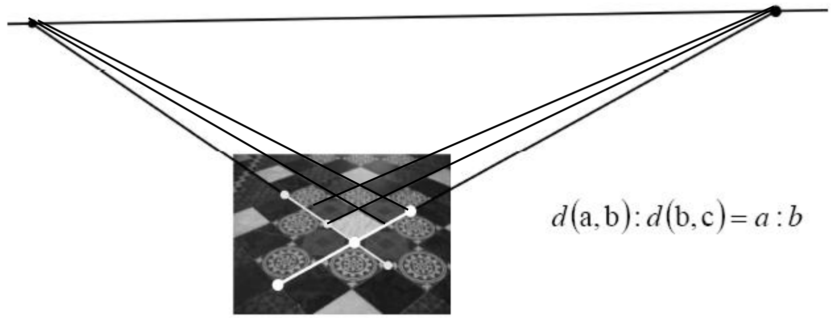
$$P' = [I \mid e']$$

Scene Constraints

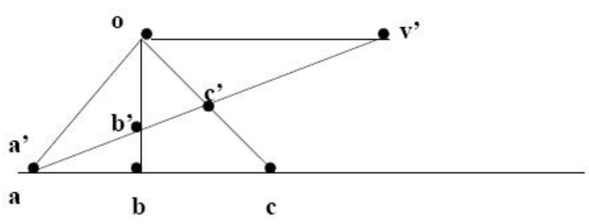
- **Parallel lines**
  - Parallel lines intersect at infinity
  - Reconstruction of corresponding vanishing point yields point on π_∞
  - 3 sets of parallel lines allow to uniquely determine plane at infinity

Remark: Obtaining vanishing point in one image can be sufficient

- **Distance ratios on a line**
  - Known distance ratio along a line allow to determine point at infinity



$$d(\mathrm{a},\mathrm{b}):d(\mathrm{b},\mathrm{c}) = a:b$$



Infinity Homography

- Image to image mapping via plane at infinity

$$x' = H_\infty x$$

- For given two cameras $P = [M \mid m]$ and $P' = [M' \mid m']$, let a point $X = (\tilde{X}^T, 0)^T$ on the plan at infinity

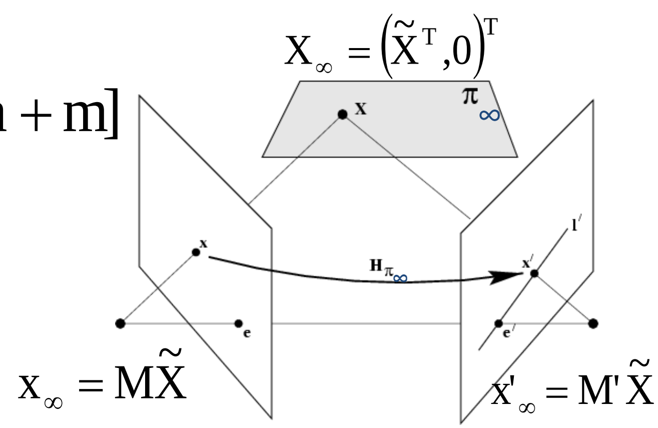
$$\Rightarrow x = M\tilde{X} \text{ and } x' = M'\tilde{X} \Rightarrow x' = M'M^{-1}x \text{ for } \pi_\infty$$

$$\Rightarrow H_\infty = M'M^{-1}$$

- Unchanged under affine transformations:

$$P = [M \mid m]\begin{bmatrix} A & a \\ 0 & 1 \end{bmatrix} = [MA \mid Ma + m]$$

$$H_\infty = M'AA^{-1}M^{-1}$$



- Cameras of affine reconstruction:

$$P = [I \mid 0] \quad P' = [H_\infty \mid e']$$

- H_∞ computed from correspondences of 3 vanishing points or correspondences of a vanishing lines and vanishing point together with F

One of the Cameras is Affine

- The principal plane of an affine camera is at infinity
- For affine reconstruction
  - It is sufficient to find the principal plane of the camera supposed to be affine and map it to the plane $(0,0,0,1)^T$.
- Compute **H** that maps third row of **P** to $(0,0,0,1)^T$ and apply to cameras and reconstruction
- **Example**: e.g. if **P**=[**I**|0], swap 3rd and 4th column, i.e., to obtain

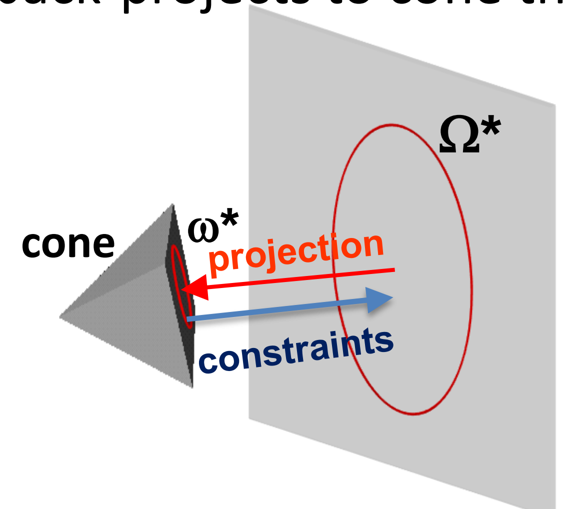
$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

From Affine to Metric Reconstruction

- The essence of metric reconstruction is to identify absolute conic in the plane at infinity
- Transform so that $\Omega_\infty : X^2 + Y^2 + Z^2 = 0, \text{ on } \pi_\infty$

- Then projective transformation relating original and reconstruction is a similarity transformation

- In practice, find image of Ω_∞
- Image ω_∞ back-projects to cone that intersects π_∞ in Ω_∞



note that image is independent of particular reconstruction

From Affine to Metric Reconstruction

- Given $\mathrm{P} = [\mathrm{M} \mid \mathrm{m}]$ and image of absolute conic ω
- Possible transformation from affine to metric is

$$\mathrm{H} = \begin{bmatrix} \mathrm{A}^{-1} & 0 \\ 0 & 1 \end{bmatrix} \qquad \mathrm{AA}^{\mathrm{T}} = \left(\mathrm{M}^{\mathrm{T}} \omega \mathrm{M}\right)^{-1}$$

(Cholesky Factorization)

Proof: $\mathrm{P}_{\mathrm{M}} = \mathrm{PH}^{-1} = [\mathrm{MA} \mid \mathrm{m}]$

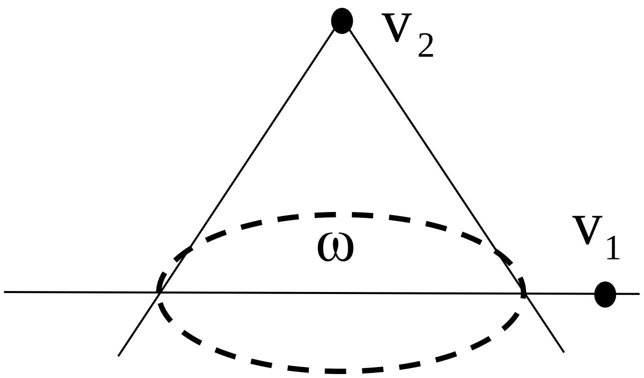
$$\omega^{*} = \mathrm{M}_{\mathrm{M}} \mathrm{M}_{\mathrm{M}}^{\mathrm{T}} = \mathrm{MAA}^{\mathrm{T}} \mathrm{M}^{\mathrm{T}} \qquad \left(\Omega^{*} = \begin{bmatrix} \mathrm{I} & 0 \\ 0 & 0 \end{bmatrix} \right)$$

$$\mathrm{M}^{-1} \omega^{-1} \mathrm{M}^{-\mathrm{T}} = \mathrm{AA}^{\mathrm{T}}$$

Constraints from Scene Orthogonality

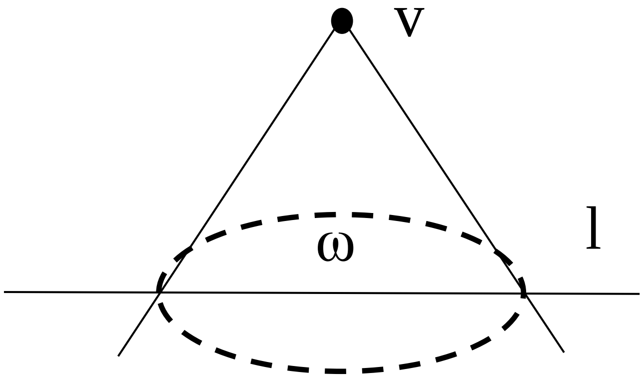
- Pairs of vanishing points arising from orthogonal scene line place constraints on the absolute conic:

$$v_1^T \omega v_2 = 0$$



- A vanishing point and a vanishing line arising from a direction and plane which are orthogonal place constraints on the absolute conic:

$$l = \omega v$$



Constraints from Known Internal Parameters

- If the calibration matrix of a camera is equal to K, the image of the absolute conic is

$$\omega = K^{-T}K^{-1}$$

K used to constrain or determine the elements of ω

Constraints from Same Camera for All Images

- Cameras P and P' have the same calibration matrix

  $\Rightarrow$ the image of the absolute conic is the same in both images e.g. moving cameras

- Given sufficient images there is in general only one conic that projects to the same image in all images, i.e. Ω_∞

- Image of the absolute conic lies on the plane at infinity may be transferred from one view to the other by the infinite homography

$$\omega' = H_\infty^{-T} \omega H_\infty^{-1}$$

  where ω and ω' are images of Ω_∞ in the two views

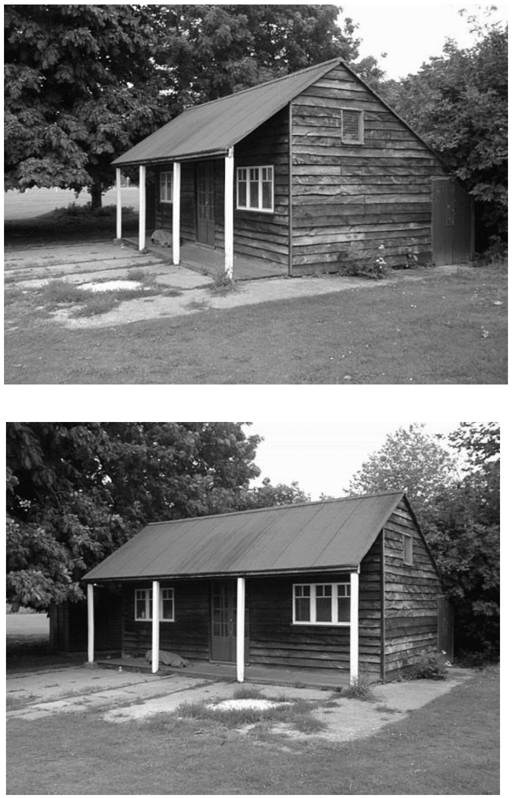
Direct Metric Reconstruction using Image of the Absolute Conic

- Approach I
  - Image of absolute conic is known in each image
  - Compute K from image of the absolute conic using

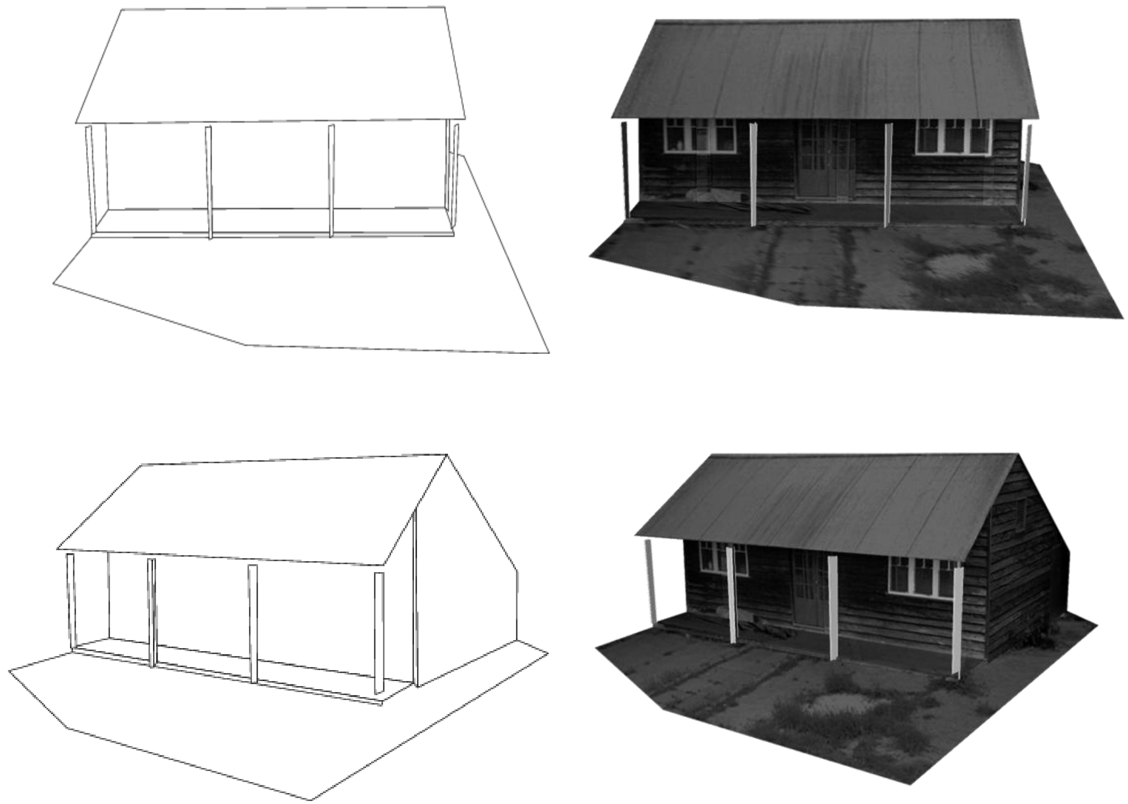
$$\omega = K^{-T}K^{-1} \Rightarrow K$$

  - Reconstruct the scene using essential matrix
- Approach II
  - Compute projective reconstruction
  - Back-project image of the absolute conic from both images to obtain cone
  - Intersection defines Ω_∞ and its support plane at infinity

Example of Metric Reconstruction

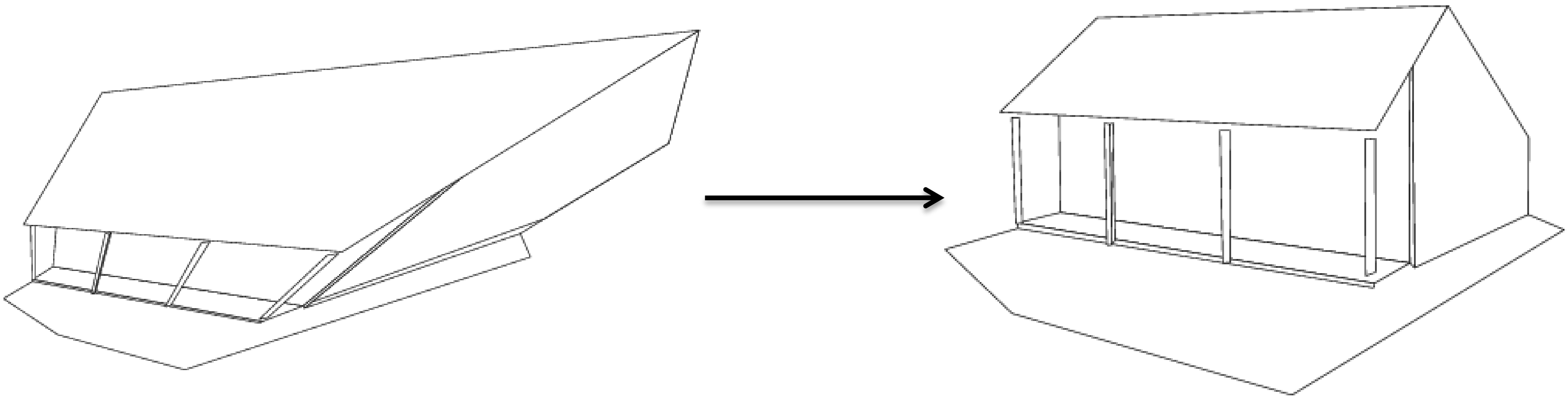


Original



Metric Reconstruction

Direct Scene Reconstruction



Projective Metric

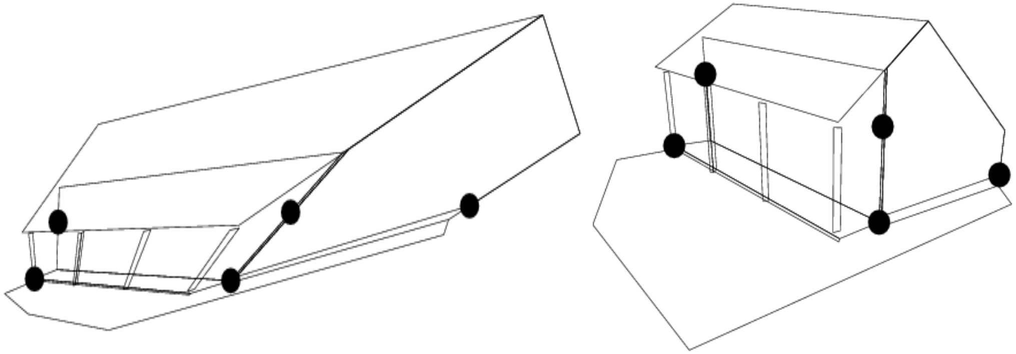
- Ground control points
  - Points with know 3D locations in a Euclidean world frame.
  - Its 3D location $\{X_i\}$ in the projective reconstruction computed from their image correspondences $x_i \leftrightarrow x'_i$.

Direct Scene Reconstruction using Ground Truth

- Use control points X_{Ei} with know coordinates to go from projective to metric

$$X_{Ei} = HX_i$$
$$x_i = PH^{-1}X_{Ei}$$

- How many points?
  - Each point correspondence provides 3 linearly independent equations on the elements of H, and since H has 15 DOF, a linear solution is obtained provided n > 4 and no four of the control points are coplanar.



<u>Objective</u>: Given two uncalibrated images compute a metric reconstruction $(P_M, P'_M, \{X_{Mi}\})$ of the cameras and scene structure.

<u>Algorithm :</u>

(i) Compute a projective reconstruction $(P, P', \{X_i\})$:

- (a) Compute F from point correspondences $x_i \leftrightarrow x'_i$ between the images.
- (b) Compute the camera matrices P,P' from F.
- (c) Triangulate X_i from point correspondence $x_i \leftrightarrow x'_i$

(ii) Rectify the projective reconstruction to metric:

- **Direct method**: Compute H from control points $X_{Ei} = HX_i$

$$P_M = PH^{-1}, P'_M = P'H^{-1}, X_{Mi} = HX_i$$

- **Stratified method**:

(a) Affine reconstruction: Compute plane at infinity

$$H = \begin{bmatrix} I \mid 0 \\ \pi^T{}_\infty \end{bmatrix}$$

(b) Metric reconstruction: Compute the image of the absolute conic and the upgrade the affine to metric reconstruction with the

$$H = \begin{bmatrix} A^{-1} & 0 \\ 0 & 1 \end{bmatrix} \qquad AA^T = \left(M^T \omega M\right)^{-1}$$