SPATIAL SOUNDFIELD RECORDING OVER A LARGE AREA USING DISTRIBUTED HIGHER ORDER MICROPHONES

*Prasanga N. Samarasinghe, Thushara D. Abhayapala**

Applied Signal Processing Group
Research School of Engineering, CECS,
The Australian National University,
ACT 0200, Canberra, Australia.

M.A. Poletti

Industrial Research Limited,
PO Box 31-310, Lower Hutt,
New Zealand.

ABSTRACT

Recording and reproduction of spatial sound fields over a large area is an unresolved problem in acoustic signal processing. This is due to the the inherent restriction in recording higher order harmonic components using practically realizable microphone arrays. As the frequency increases and as the region of interest becomes large the number of microphones needed in effective recording increases beyond practicality. In this paper, we show how to use higher order microphones, distributed in a large area, to record and accurately reconstruct 2D spatial sound fields. We use sound field coefficient translation between origins to combine distributed field recording to a single sound field over the entire region. We use simulation examples in (i) interior and (ii) exterior fields to corroborate our design.

1. INTRODUCTION

Consider the designing of a microphone array system for recording sound over a large area for spatial sound reproduction. Whilst there has been progress in microphone array design in beam forming and recording higher order harmonics, to capture enough higher order modes to reproduce a sound field over a large region needs a microphone array with an impractical aperture size. In this paper, we will present a systematic way of using a limited number of lower order microphones distributed over a large area to capture the spatial sound field. In particular, we will explicitly show how to combine the distributed sound field recordings to construct the entire spatial sound field.

A system for recording height invariant 2D sound fields using circular microphone arrays is given in [1] where the sound field is mainly described in terms of plane waves. 3D recording systems based on spherical harmonics are given in [2, 3, 4], where a spherical array of $Q = (M+1)^2$ omni directional microphones is required to design a higher order microphone of order M. Although an M^{th} order 2D microphone is capable of extracting $2M+1$ harmonic coefficients with respect to the microphone's position, recording a large region like a room environment remains a challenge as the design and implementation become too complex and impractical as the order increases.

Our approach is to distribute a limited number of realizable higher order microphones over the large region of interest and use the local sound field coefficients recorded by them to determine the global sound field. For example, the higher order microphone arrays given in [2, 5] can be used in implementing this system. This paper is restricted to the 2D space assuming a height invariant sound field and since our design is applicable in both interior and exterior sound fields it can be employed in 2D surround sound systems as well as in horizontal holography.

*This work was supported by the Australian Research Council Discovery Grant DP 110103369.

2. PROBLEM FORMULATION

We initially consider the representation of a sound field with respect to the global origin O. In our first case of interest, we consider a circular region η within a radius of R_0 where the existing sound field is caused by one or more sources lying outside the area of concern (Interior field). Then the sound pressure at any point $\boldsymbol{x} = (R, \Omega)$ inside of η (See Fig.1(a)) is given in terms of cylindrical harmonics [6] as

$$S(\boldsymbol{x}, k) = \sum_{n=-\infty}^{\infty} \alpha_n^{(0)}(k) J_n(kR) e^{in\Omega} \tag{1}$$

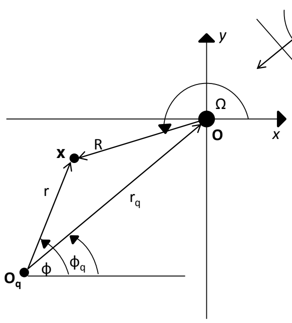
where $k = 2\pi f/c$ is the wave number, f is the frequency, c is the speed of sound propagation, $J_n(\cdot)$ represents the cylindrical Bessel function of order n and $\alpha_n^{(0)}(k)$ are the interior sound field coefficients with respect to O.

In the exterior case, we consider a region ζ, where the existing sound field is caused by one or more sources lying inside the area of concern as in Fig.1(b) (sources are within the circle of radius R_s). The corresponding cylindrical harmonic expansion for sound pressure at any point $\boldsymbol{x} = (R, \Omega)$ for $R \geq R_s$, is of the form [6]

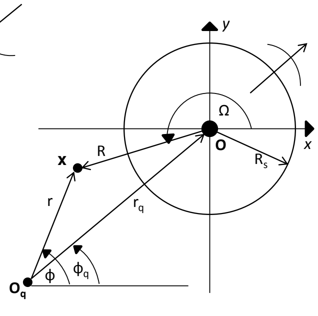
$$S(\boldsymbol{x}, k) = \sum_{n=-\infty}^{\infty} \beta_n^{(0)}(k) H_n(kR) e^{in\Omega} \tag{2}$$

where $H_n(\cdot)$ represents the n^{th} order cylindrical Hankel function of the first kind and $\beta_n^{(0)}(k)$ are the exterior sound field coefficients with respect to O.

The representations (1) and (2) are valid for sound fields caused by both near field (spherical waves) and far field (plane waves) sources. It is important to note that the exterior and interior sound field coefficients $\alpha_n^{(0)}(k)$ and $\beta_n^{(0)}(k)$ are independent of the position of interest and are with respect to the global origin. If we record these coefficients accurately enough, the entire sound field within the desired region can be predicted up to a certain order.



(a) Interior sound field



(b) Exterior sound field

Figure 1: Two dimensional geometry of the existing sound field

3. SOUND FIELD RECORDING

In this paper we aim to determine the global sound field using a set of distributed recordings over the region of interest. Consider Q number of M^{th} order microphones distributed over the region of interest where the q^{th} microphone is at O_q as shown in Fig.1(a) and Fig.1(b). Each of these higher order microphones will capture local sound field coefficients $\alpha_m^{(q)}(k)$ with respect to O_q, for $m = -M....M$. Our intention is to use this information in determining the global sound field coefficients in (1) and (2).

3.1. Interior sound field

The sound pressure at any point $\boldsymbol{x}$ inside the region of interest (see Fig.1(a)) is given by (1) and our aim is to determine the set of coefficients $\alpha_n^{(0)}(k)$. We first express the sound field at $\boldsymbol{x}$ with respect to a local origin O_q, using Graf's cylindrical addition theorem for Bessel functions [7] given by

$$J_n(kR)e^{in\Omega} = \sum_{m=-\infty}^{\infty} J_{m-n}(kr^{(q)})J_m(kr)e^{im\phi}e^{i(n-m)\phi^{(q)}} \tag{3}$$

The sound field at $\boldsymbol{x}$ with respect to O_q becomes,

$$S(r,\phi,k) = \sum_{m=-\infty}^{\infty} \alpha_m^{(q)} J_m(kr)e^{im\phi} \tag{4}$$

where

$$\alpha_m^{(q)} = \sum_{n=-\infty}^{\infty} \alpha_n^{(0)}(k)J_{m-n}(kr^{(q)})e^{i(n-m)\phi^{(q)}}. \tag{5}$$

The relationship (5) obtained between the local and global sound field coefficients can be written in matrix form

$$\mathbf{A} = \boldsymbol{T}^{(I)} \times \mathbf{B} \tag{6}$$

where $\mathbf{A} = [\alpha_{-M}^{(1)}, ..\alpha_{M}^{(1)}, ...\alpha_{-M}^{(Q)}, ...\alpha_{M}^{(Q)}]^T$ is a $(2M+1)Q \times 1$ matrix representing the captured local sound field coefficients (5), $\mathbf{B} = [\alpha_{-N}^{\prime(0)}, ...\alpha_{N}^{\prime(0)}]^T$ is a $(2N+1) \times 1$ matrix representing the global interior sound field coefficients, $N = keR_0/2$ is the truncation limit of the infinite summation given in (1) (the explanation for this is given in Section 4) and

$$\boldsymbol{T}^{(I)} = \begin{bmatrix} T_{(-M+N)}^{(1)} & \cdots & \cdots & \cdots & T_{(-M-N)}^{(1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ T_{(M+N)}^{(Q)} & \cdots & \cdots & \cdots & T_{(M-N)}^{(Q)} \end{bmatrix}$$

is a $[(2M+1)Q] \times (2N+1)$ matrix where $T_l^{(q)} = J_l(kr^{(q)})e^{-il\phi^{(q)}}$.

A solution to (6) exists only when the system is under determined (i.e $(2M+1)Q \geq (2N+1)$) which limits the maximum sound field order recordable by an array of Q number of M^{th} order microphones to $[(2M+1)Q-1]/2$. Corresponding maximum frequency recordable is the Nyquist frequency for order M

$$f_{N(I)} = \frac{c[(2M+1)Q-1]}{2\pi eR_0}. \tag{7}$$

3.2. Exterior sound field

In exterior sound field recording , the sound pressure at any point $\boldsymbol{x}$ within the region $R \geq R_s$ (see Fig.1(b)), is given in the form (2). We express this with respect to O_q using Graf's cylindrical addition theorem for Hankel functions [7] given by

$$H_n(kR)e^{in\Omega} = \sum_{m=-\infty}^{\infty} H_{m-n}(kr^{(q)})J_m(kr)e^{im\phi}e^{i(n-m)\phi^{(q)}} \tag{8}$$

and obtain (4) where $\alpha_m^{(q)}$ is now

$$\alpha_m^{(q)} = \sum_{n=-\infty}^{\infty} \beta_n^{(0)}(k)H_{m-n}(kr^{(q)})e^{i(n-m)\phi^{(q)}}. \tag{9}$$

This relationship (9) can be written in Matrix form as

$$\mathbf{A} = \boldsymbol{T}^{(E)} \times \mathbf{D} \tag{10}$$

where $\mathbf{D} = [\beta_{-N_s}^{\prime(0)}, ...\beta_{N_s}^{\prime(0)}]^T$ is a $(2N_s+1) \times 1$ matrix representing the global sound field coefficients, $N_s = keR_s/2$ is the truncation limit for the infinite summation given in (2), and

$$\boldsymbol{T}^{(E)} = \begin{bmatrix} T_{(-M+N)}^{\prime(1)} & \cdots & \cdots & \cdots & Te'(1)_{(-M-N)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ T_{(M+N)}^{\prime(Q)} & \cdots & \cdots & \cdots & T_{(M-N)}^{\prime(Q)} \end{bmatrix}$$

is a $[(2M+1)Q] \times (2Ns+1)$ matrix where $T_l^{\prime(q)} = H_l(kr^{(q)})e^{-il\phi^{(q)}}$.

As described in Section 3.1, a solution to (10) exists only when $(2M+1)Q \geq (2Ns+1)$ which limits the maximum sound field order recordable by the array to $[(2M+1)Q-1]/2$. Thus the maximum frequency recordable by the array or the Nyquist frequency for microphone order M results in

$$f_{N(E)} = \frac{c[(2M+1)Q-1]}{2\pi eRs} \tag{11}$$

Note that the difference between (7) and (11) is due to the different truncation limits in (1) and (2).

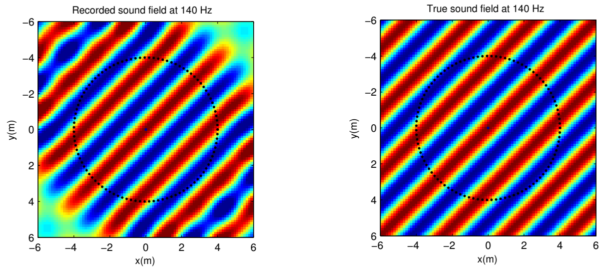


Figure 2: Interior field: Recorded sound field for $M = 1$ and the existing sound field of $f = 140$Hz.

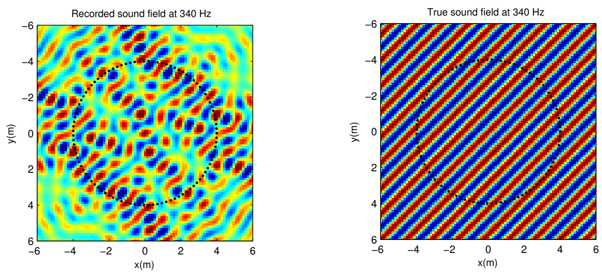


Figure 3: Interior field: Recorded sound field for $M = 1$ and the existing sound field of $f = 340$Hz.

Error involved with the recorded sound field relative to the actual field can be calculated over the region of interest by

$$\varepsilon(k) = \frac{\int_{R_1}^{R_2} \int_0^{2\pi} \|S_E(\boldsymbol{x},k) - S_R(\boldsymbol{x},k)\|^2 \, d\Omega dR}{\int_{R_1}^{R_2} \int_0^{2\pi} \|S_E(\boldsymbol{x},k)\|^2 \, d\Omega dR} \tag{12}$$

where $S_E(\boldsymbol{x},k)$ is the existing sound field within the region of concern and $S_R(\boldsymbol{x},k)$ represents the recorded sound field

$$S_R(\boldsymbol{x},k) = \begin{cases} \sum_{n=-\infty}^{\infty} \alpha_n'^{(0)}(k) J_n(kR) e^{in\Omega} & \text{Interior} \\ \sum_{n=-\infty}^{\infty} \beta_n'^{(0)}(k) H_n(kR) e^{in\Omega} & \text{Exterior} \end{cases} \tag{13}$$

4. SIMULATION RESULTS

Matlab simulations were performed for an array of $Q = 10$ equally spaced microphone units in a circle[1] of radius $r_q = R_0 = 4$m. This produces a first order array Nyquist frequency of 145 Hz (7) and 580 Hz (11) for interior and exterior cases respectively. We investigated the array's behaviour with increasing microphone order $M = 2$ and $M = 3$ keeping $Q = 10$ a constant. For interior field recording we assumed the existing field to be a plane wave arriving from a direction $\pi/4$ and the sound field of interest was within the region $R \leq R_0$. For exterior field recording the existing sound field was assumed to be produced by $L = 4$ line sources equally spaced on $R_s = 1$ m and the region of interest was within $R \geq R_s$. The existing field $S_E(\boldsymbol{x},k)$ can be given by

$$S_E(\boldsymbol{x},k) = \begin{cases} e^{ikR\cos(\Omega-\pi/4)} & \text{Existing interior field} \\ \sum_{l=1}^{4} H_0(k \left\| \boldsymbol{x} - \boldsymbol{x}_s^{(l)} \right\|) & \text{Existing exterior field} \end{cases} \tag{14}$$

where $\boldsymbol{x}_s^{(l)}$ are the source locations. These can be represented by cylindrical harmonic expansions of forms (1) and (2) as

$$S_E(\boldsymbol{x},k) = \begin{cases} \sum_{n=-\infty}^{\infty} \underbrace{i^n e^{-in\pi/4}}_{\alpha_n^{(0)}} J_n(kR) e^{in\Omega} \\ \sum_{n=-\infty}^{\infty} \underbrace{\sum_{l=1}^{L} J_n(kR_s) e^{-in\phi_s^{(l)}}}_{\delta_n^{(0)}} H_n(kR) e^{in\Omega} \end{cases} \tag{15}$$

Due to inherent properties of Bessel functions, the representation (15) given by infinite summations of orthogonal modes, can be truncated to $N = [ekR_0/2]$ and $N_s = [ekR_s/2]$ terms [8] for interior and exterior cases, respectively.

Fig. 2 shows a recorded plane wave field (interior) of frequency 140 Hz by a first order($M = 1$) array. The circle represents the array of radius 4 m inside of which our region of interest lies. For the same array Fig. 3 shows a recorded plane wave field of frequency 340 Hz much higher than the Nyquist frequency and the results are incorrect. This sound field can be successfully recorded by either increasing the number of microphones Q or by increasing the microphone order M. Fig. 4 is the result of the second approach where the array is now of $M = 3$ higher order microphones for which the

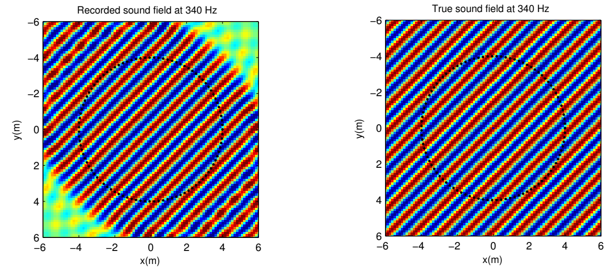


Figure 4: Interior field: Recorded sound field for $M = 3$ and the existing sound field of $f = 340$Hz.

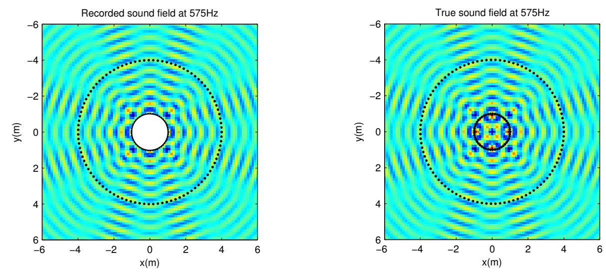


Figure 5: Exterior field: Recorded sound field for $M = 1$ and the existing sound field of $f = 575$Hz.

[1] It is not necessary to choose a circle other than for convenience.

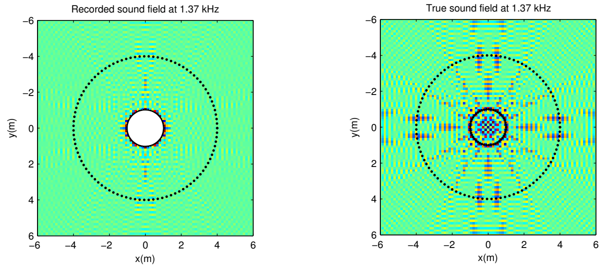


Figure 6: Exterior field: Recorded sound field for $M = 1$ and the existing sound field of $f = 1.37$kHz.

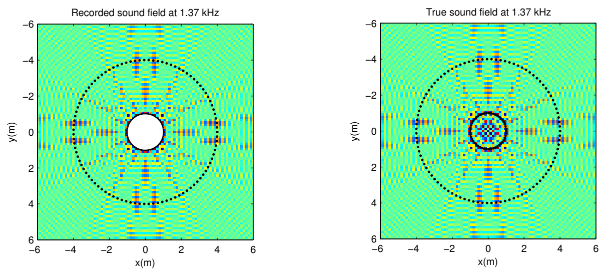


Figure 7: Exterior field: Recorded sound field for $M = 3$ and the existing sound field of $f = 1.37$kHz.

Nyquist frequency is 345 Hz. For a better understanding of the array's performance in recording an interior plane wave field, we plot the error curves (12) over $R_1 = 0$ $R_2 = 4$ m for $M = 1, 2$ and 3 as shown in Fig.8. We can observe that the error is zero until the Nyquist frequency for each M and starts building beyond that.

We use the same circular array in recording an example exterior field caused by 4 line sources. Our region of interest lies beyond the outer most source $Rs = 1$ m. Fig.5 depicts a recorded sound field of frequency 575 Hz by a first order($M = 1$) array. The outer circle depicts the microphone array and the inner circle depicts the source array. Similar to interior field recording, for frequencies higher than the Nyquist frequency, the array's performance degrade. In proving this we plot Fig.6 which is the recorded sound field of frequency 1.37 kHz. But if it is desirable to record a sound field at this frequency, we can increase the microphone order to $M = 3$ for which the Nyquist frequency is 1.38 kHz and the result is shown in Fig.7. It is important to note that the array is capable of accurately recording the entire sound field beyond the outer most source, up to any required radius. The error involved with recording an exterior field (12) over $R_1 = 1.5 : R_2 = 4$ m, is plotted in Fig.9. We omit the low frequencies $f < 500$ Hz as the condition number of the matrix $\boldsymbol{T}^{(E)}$ in (10) become significantly large with increasing M producing incorrect solutions for $\alpha_n'^{(0)}$ in (13). Again we can observe that the error only builds up beyond the Nyquist frequency for each M.

5. CONCLUSION

We have formulated the theory for designing a 2D recording system with the use of distributed higher order microphones of order M. It has been showed in simulations that the system is capable of accurately recording interior and exterior sound fields within a large region occupying only a limited number of microphone units.

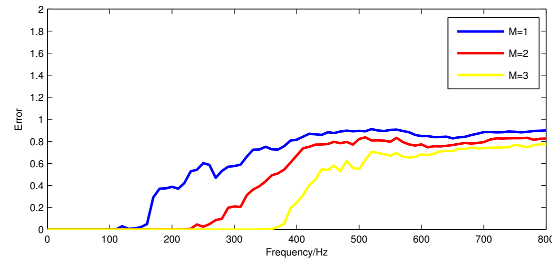


Figure 8: Error in interior field recording for $M = 1, 2, 3$.

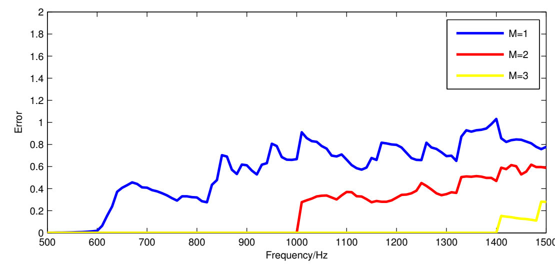


Figure 9: Error in exterior field recording for $M = 1, 2, 3$.

We intend to extend the theory and simulation to a 3D recording system employing addition theorems in 3D and investigate the system's broadband pertinence in a future publication.

6. REFERENCES

[1] M. Poletti, "A unified theory of horizontal holographic sound systems," *Audio Engineering Society*, vol. 48, no. 12, pp. 1155–1182, December 2000.

[2] T. Abhayapala and D. Ward, "Theory and design of high order sound field microphones using spherical microphone array," in *Acoustics, Speech, and Signal Processing (ICASSP), IEEE International Conference on*, vol. II, 2002, pp. II–1949.

[3] Z. Li, R. Duraiswami, and N. Gumerov, "Capture and recreation of higher order 3d sound fields via reciprocity," in *ICAD'04*, 2004, pp. 6–9.

[4] B. Rafaely, "The spherical shell microphone array," *IEEE Trans. Acoust. Speech Sig. Proc.*, vol. 16, no. 4, pp. 740–747, 2008.

[5] G. Elko and J. Meyer, "Second-order differential adaptive microphone array," in *IEEE International Conference on Acoustics, Speech and Signal Processing, ICASSP*. IEEE, 2009, pp. 73–76.

[6] E. Williams, *Fourier Acoustics: Sound Radiation and Nearfield Acoustic Holography*. London, UK: Academic Press, 1999, pp. 115–125.

[7] G. Watson, *A treatise on the theory of Bessel functions*. London, UK: Cambridge University Press, 1995, p. 361.

[8] Y. Wu and T. Abhayapala, "Theory and design of soundfield reproduction using continuous loudspeaker concept," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 17, no. 1, pp. 107–116, January 2009.