Part – 1

3.1:

To-Do 1:

Logistic function:

[3]:
```python
def logistic_function(x):
    """
    Computes the logistic function applied to any value of x.
    Arguments:
    x: scalar or numpy array of any size.
    Returns:
    y: logistic function applied to x.
    """
    y = 1 / (1 + np.exp(-x))
    return y
```

Testing the function:

[4]:
```python
import numpy as np
def test_logistic_function():
    """
    Test cases for the logistic_function.
    """
    # Test with scalar input
    x_scalar = 0
    expected_output_scalar = round(1 / (1 + np.exp(0)), 3) # Expected output: 0.5
    assert round(logistic_function(x_scalar), 3) == expected_output_scalar, "Test failed for scalar input"

    # Test with positive scalar input
    x_pos = 2
    expected_output_pos = round(1 / (1 + np.exp(-2)), 3) # Expected output: ~0.881
    assert round(logistic_function(x_pos), 3) == expected_output_pos, "Test failed for positive scalar input"

    # Test with negative scalar input
    x_neg = -3
    expected_output_neg = round(1 / (1 + np.exp(3)), 3) # Expected output: ~0.047
    assert round(logistic_function(x_neg), 3) == expected_output_neg, "Test failed for negative scalar input"

    # Test with numpy array input
    x_array = np.array([0, 2, -3])
    expected_output_array = np.array([0.5, 0.881, 0.047]) # Adjusted expected values rounded to 3 decimals

    # Use np.round to round the array element-wise and compare
    assert np.all(np.round(logistic_function(x_array), 3) == expected_output_array), "Test failed for numpy array input"
    print("All tests passed!")
# Run the test case
test_logistic_function()
```

```
All tests passed!
```

To-Do 2:

Log Loss function:

```python
def log_loss(y_true, y_pred):
    """
    Computes log loss for true target value y ={0 or 1} and predicted target value y' inbetween {0-1}.
    Arguments:
    y_true (scalar): true target value {0 or 1}.
    y_pred (scalar): predicted taget value {0-1}.
    Returns:
    loss (float): loss/error value
    """
    # Ensure y_pred is clipped to avoid log(0)
    y_pred = np.clip(y_pred, 1e-10, 1 - 1e-10)
    loss = -(y_true * np.log(y_pred) + (1-y_true) * np.log(1-y_pred))
    return loss
```

```python
def test_log_loss():
    """
    Test cases for the log_loss function.
    """
    # Test case 1: Perfect prediction (y_true = 1, y_pred = 1)
    y_true = 1
    y_pred = 1
    expected_loss = 0.0 # Log loss is 0 for perfect prediction
    assert np.isclose(log_loss(y_true, y_pred), expected_loss), "Test failed for perfect prediction (y_true=1, y_pred=1)"

    # Test case 2: Perfect prediction (y_true = 0, y_pred = 0)
    y_true = 0
    y_pred = 0
    expected_loss = 0.0 # Log loss is 0 for perfect prediction
    assert np.isclose(log_loss(y_true, y_pred), expected_loss), "Test failed for perfect prediction (y_true=0, y_pred=0)"

    # Test case 3: Incorrect prediction (y_true = 1, y_pred = 0)
    y_true = 1
    y_pred = 0
    try:
        log_loss(y_true, y_pred) # This should raise an error due to log(0)
    except ValueError:
        pass # Test passed if ValueError is raised for log(0)

    # Test case 4: Incorrect prediction (y_true = 0, y_pred = 1)
    y_true = 0
    y_pred = 1
    try:
        log_loss(y_true, y_pred) # This should raise an error due to log(0)
    except ValueError:
        pass # Test passed if ValueError is raised for log(0)

    # Test case 5: Partially correct prediction
    y_true = 1
    y_pred = 0.8
    expected_loss = -(1 * np.log(0.8)) - (0 * np.log(0.2)) # ~0.2231
    assert np.isclose(log_loss(y_true, y_pred), expected_loss, atol=1e-6), "Test failed for partiallycorrect prediction (y_true=1, y_pred=0.8)"
    y_true = 0
    y_pred = 0.2
    expected_loss = -(0 * np.log(0.2)) - (1 * np.log(0.8)) # ~0.2231
    assert np.isclose(log_loss(y_true, y_pred), expected_loss, atol=1e-6), "Test failed for partiallycorrect prediction (y_true=0, y_pred=0.2)"
    print("All tests passed!")

# Run the test case
test_log_loss()
```

```
All tests passed!
```

To-Do 3:

Cost function:

[7]:
```python
def cost_function(y_true, y_pred):
    """
    Computes log loss for inputs true value (0 or 1) and predicted value (between 0 and 1)
    Args:
    y_true (array_like, shape (n,)): array of true values (0 or 1)
    y_pred (array_like, shape (n,)): array of predicted values (probability of y_pred being 1)
    Returns:
    cost (float): nonnegative cost corresponding to y_true and y_pred
    """
    assert len(y_true) == len(y_pred), "Length of true values and length of predicted values do not match"
    n = len(y_true)
    loss_vec = -(y_true * np.log(y_pred) + (1 - y_true) * np.log(1 - y_pred))
    cost = np.sum(loss_vec)/n
    return cost
```

Testing the function:

[8]:
```python
def test_cost_function():
    # Test case 1: Simple example with known expected cost
    y_true = np.array([1, 0, 1])
    y_pred = np.array([0.9, 0.1, 0.8])
    # Expected output: Manually calculate cost for these values
    # log_loss(y_true, y_pred) for each example
    expected_cost = (-(1 * np.log(0.9)) - (1 - 1) * np.log(1 - 0.9) + -(0 * np.log(0.1)) - (1 - 0) * np.log(1 - 0.1) + -(1 * np.log(0.8)) - (1 - 1) * n

    # Call the cost_function to get the result
    result = cost_function(y_true, y_pred)
    # Assert that the result is close to the expected cost with a tolerance of 1e-6
    assert np.isclose(result, expected_cost, atol=1e-6), f"Test failed: {result} != {expected_cost}"
    print("Test passed for simple case!")

# Run the test case
test_cost_function()
```

```
Test passed for simple case!
```

To-Do 4:

Vectorized Cost Function:

[9]:
```python
# Function to compute cost function in terms of model parameters - using vectorization
def costfunction_logreg(X, y, w, b):
    """
    Computes the cost function, given data and model parameters.
    Args:
    X (ndarray, shape (m,n)): data on features, m observations with n features.
    y (array_like, shape (m,)): array of true values of target (0 or 1).
    w (array_like, shape (n,)): weight parameters of the model.
    b (float): bias parameter of the model.
    Returns:
    cost (float): nonnegative cost corresponding to y and y_pred.
    """
    n, d = X.shape
    assert len(y) == n, "Number of feature observations and number of target observations do not match."
    assert len(w) == d, "Number of features and number of weight parameters do not match."
    # Compute z using np.dot
    z = np.dot(X, w) + b # Matrix-vector multiplication and adding bias
    # Compute predictions using logistic function (sigmoid)
    y_pred = logistic_function(z)
    # Compute the cost using the cost function
    cost = cost_function(y, y_pred)
    return cost
```

Testing the function:

[10]:
```python
# Testing the Function:
X, y, w, b = np.array([[10, 20], [-10, 10]]), np.array([1, 0]), np.array([0.5, 1.5]), 1
print(f"cost for logistic regression(X = {X}, y = {y}, w = {w}, b = {b}) = {costfunction_logreg(X, y, w, b)}")
```

```
cost for logistic regression(X = [[ 10  20]
 [-10  10]], y = [1 0], w = [0.5 1.5], b = 1) = 5.500008350784906
```

To-Do 5:

Gradient Function:

[11]:
```python
def compute_gradient(X, y, w, b):
    """
    Computes gradients of the cost function with respect to model parameters.
    Args:
    X (ndarray, shape (n,d)): Input data, n observations with d features
    y (array_like, shape (n,)): True labels (0 or 1)
    w (array_like, shape (d,)): Weight parameters of the model
    b (float): Bias parameter of the model
    Returns:
    grad_w (array_like, shape (d,)): Gradients of the cost function with respect to the weight
    parameters
    grad_b (float): Gradient of the cost function with respect to the bias parameter
    """
    n, d = X.shape # X has shape (n, d)
    assert len(y) == n, f"Expected y to have {n} elements, but got {len(y)}"
    assert len(w) == d, f"Expected w to have {d} elements, but got {len(w)}"
    # Compute predictions using logistic function (sigmoid)
    z = X @ w + b
    y_pred = logistic_function(z)
    # Compute gradients
    grad_w = (1 / n) * (X.T @ (y_pred - y)) # Gradient w.r.t weights, shape (d,)
    grad_b = (1 / n) * np.sum(y_pred - y) # Gradient w.r.t bias, scalar
    return grad_w, grad_b
```

A simple assertion test for the function:

[12]:
```python
# Simple test case
X = np.array([[10, 20], [-10, 10]]) # shape (2, 2)
y = np.array([1, 0]) # shape (2,)
w = np.array([0.5, 1.5]) # shape (2,)
b = 1 # scalar
# Assertion tests
try:
    grad_w, grad_b = compute_gradient(X, y, w, b)
    print("Gradients computed successfully.")
    print(f"grad_w: {grad_w}")
    print(f"grad_b: {grad_b}")
except AssertionError as e:
    print(f"Assertion error: {e}")
```

```
Gradients computed successfully.
grad_w: [-4.99991649  4.99991649]
grad_b: 0.4999916492890759
```

To-Do 6:

[13]:
```python
def gradient_descent(X, y, w, b, alpha, n_iter, show_cost=False, show_params=True):
    """
    Implements batch gradient descent to optimize logistic regression parameters.
    Args:
    X (ndarray, shape (n,d)): Data on features, n observations with d features
    y (array_like, shape (n,)): True values of target (0 or 1)
    w (array_like, shape (d,)): Initial weight parameters
    b (float): Initial bias parameter
    alpha (float): Learning rate
    n_iter (int): Number of iterations
    show_cost (bool): If True, displays cost every 100 iterations
    show_params (bool): If True, displays parameters every 100 iterations
    Returns:
    w (array_like, shape (d,)): Optimized weight parameters
    b (float): Optimized bias parameter
    cost_history (list): List of cost values over iterations
    params_history (list): List of parameters (w, b) over iterations
    """
    n, d = X.shape
    assert len(y) == n, "Number of observations in X and y do not match"
    assert len(w) == d, "Number of features in X and w do not match"
    cost_history = []
    params_history = []
    for i in range(n_iter):
        # Compute gradients
        grad_w, grad_b = compute_gradient(X, y, w, b)

        # Update weights and bias
        w -= alpha * grad_w
        b -= alpha * grad_b

        # Compute predictions
        z = X @ w + b
        y_pred = 1 / (1 + np.exp(-z))

        # Compute cost using your function
        cost = cost_function(y, y_pred)
```

```python
        # Store cost and parameters
        cost_history.append(cost)
        params_history.append((w.copy(), b))

        # # Optionally print cost and parameters
        # if show_cost and (i % 100 == 0 or i == n_iter - 1):
        #     print(f"Iteration {i}: Cost = {cost:.6f}")
        # if show_params and (i % 100 == 0 or i == n_iter - 1):
        #     print(f"Iteration {i}: w = {w}, b = {b:.6f}")

    return w, b, cost_history, params_history
```

Testing the function:

[14]:
```python
# Test the gradient_descent function with sample data
X = np.array([[0.1, 0.2], [-0.1, 0.1]]) # Shape (2, 2)
y = np.array([1, 0]) # Shape (2,)
w = np.zeros(X.shape[1]) # Shape (2,) - same as number of features
b = 0.0 # Scalar
alpha = 0.1 # Learning rate
n_iter = 100000 # Number of iterations
# Perform gradient descent
w_out, b_out, cost_history, params_history = gradient_descent(X, y, w, b, alpha, n_iter, show_cost=True, show_params=False)
# Print final parameters and cost
print("\nFinal parameters:")
print(f"w: {w_out}, b: {b_out}")
print(f"Final cost: {cost_history[-1]:.6f}")
```

```
Final parameters:
w: [38.51304248 18.83386869], b: -2.8176836626325836
Final cost: 0.008254
```

A simple assertion test for the function:

[16]:
```
# Simple assertion test for gradient_descent
def test_gradient_descent():
    X = np.array([[0.1, 0.2], [-0.1, 0.1]]) # Shape (2, 2)
    y = np.array([1, 0]) # Shape (2,)
    w = np.zeros(X.shape[1]) # Shape (2,)
    b = 0.0 # Scalar
    alpha = 0.1 # Learning rate
    n_iter = 100 # Number of iterations
    # Run gradient descent
    w_out, b_out, cost_history, _ = gradient_descent(X, y, w, b, alpha, n_iter, show_cost=False,
    show_params=False)
    # Assertions
    assert len(cost_history) == n_iter, "Cost history length does not match the number of iterations"
    assert w_out.shape == w.shape, "Shape of output weights does not match the initial weights"
    assert isinstance(b_out, float), "Bias output is not a float"
    assert cost_history[-1] < cost_history[0], "Cost did not decrease over iterations"
    print("All tests passed!")

# Run the test
test_gradient_descent()
```

```
All tests passed!
```

To-Do 7:

Decision/Prediction Function:

[17]:
```
def prediction(X, w, b, threshold=0.5):
    """
    Predicts binary outcomes for given input features based on logistic regression parameters.
    Arguments:
    X (ndarray, shape (n,d)): Array of test independent variables (features) with n samples and d
    features.
    w (ndarray, shape (d,)): Array of weights learned via gradient descent.
    b (float): Bias learned via gradient descent.
    threshold (float, optional): Classification threshold for predicting class labels. Default is 0.5.
    Returns:
    y_pred (ndarray, shape (n,)): Array of predicted dependent variable (binary class labels: 0 or 1).
    """
    # Compute the predicted probabilities using the logistic function
    z = X @ w + b
    y_test_prob = logistic_function(z)

    # Classify based on the threshold
    y_pred = (y_test_prob >= threshold).astype(int)
    return y_pred
```

Testing the function:

```
[18]: def test_prediction():
    X_test = np.array([[0.5, 1.0], [1.5, -0.5], [-0.5, -1.0]]) # Shape (3, 2)
    w_test = np.array([1.0, -1.0]) # Shape (2,)
    b_test = 0.0 # Scalar bias
    threshold = 0.5 # Default threshold
    # Updated expected output
    expected_output = np.array([0, 1, 1])
    # Call the prediction function
    y_pred = prediction(X_test, w_test, b_test, threshold)
    # Assert that the output matches the expected output
    assert np.array_equal(y_pred, expected_output), f"Expected {expected_output}, but got {y_pred}"
    print("Test passed!")
test_prediction()
```

```
Test passed!
```

To-Do 8:

```
[19]: def evaluate_classification(y_true, y_pred):
    """
    Computes the confusion matrix, precision, recall, and F1-score for binary classification.
    Arguments:
    y_true (ndarray, shape (n,)): Ground truth binary labels (0 or 1).
    y_pred (ndarray, shape (n,)): Predicted binary labels (0 or 1).
    Returns:
    metrics (dict): A dictionary containing confusion matrix, precision, recall, and F1-score.
    """
    # Initialize confusion matrix components
    TP = np.sum((y_true == 1) & (y_pred == 1)) # True Positives
    TN = np.sum((y_true == 0) & (y_pred == 0)) # True Negatives
    FP = np.sum((y_true == 0) & (y_pred == 1)) # False Positives
    FN = np.sum((y_true == 1) & (y_pred == 0))

    # Confusion matrix
    confusion_matrix = np.array([[TN, FP],
    [FN, TP]])

    # Precision, recall, and F1-score
    precision = TP / (TP + FP) if (TP + FP) > 0.0 else 0.0
    recall = TP / (TP + FN) if (TP + FN) > 0.0 else 0.0
    f1_score = (
        2 * precision * recall / (precision + recall)
        if (precision + recall) > 0.0 else 0.0
    )

    # Metrics dictionary
    metrics = {
    "confusion_matrix": confusion_matrix,
    "precision": precision,
    "recall": recall,
    "f1_score": f1_score
    }

    return metrics
```

3.2:

1:

Importing and Loading of dataset

]:
```python
# Load dataset
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
import matplotlib.pyplot as plt
url = "https://raw.githubusercontent.com/jbrownlee/Datasets/master/pima-indians-diabetes.data.csv"
columns = ['Pregnancies', 'Glucose', 'BloodPressure', 'SkinThickness', 'Insulin', 'BMI', 'DiabetesPedigreeFunction', 'Age', 'Outcome']
data_pima_diabetes = pd.read_csv(url, names=columns)
```

Some Basic Data Cleaning

2]:
```python
# Data cleaning
columns_to_clean = ['Glucose', 'BloodPressure', 'SkinThickness', 'Insulin', 'BMI']
data_pima_diabetes[columns_to_clean] = data_pima_diabetes[columns_to_clean].replace(0, np.nan)
data_pima_diabetes.fillna(data_pima_diabetes.median(), inplace=True)
data_pima_diabetes.info()
```

Summary Statistics

[39]:
```python
data_pima_diabetes.describe()
```

[39]:

	Pregnancies	Glucose	BloodPressure	SkinThickness	Insulin	BMI	DiabetesPedigreeFunction	Age	Outcome
count	768.000000	768.000000	768.000000	768.000000	768.000000	768.000000	768.000000	768.000000	768.000000
mean	3.845052	121.656250	72.386719	29.108073	140.671875	32.455208	0.471876	33.240885	0.348958
std	3.369578	30.438286	12.096642	8.791221	86.383060	6.875177	0.331329	11.760232	0.476951
min	0.000000	44.000000	24.000000	7.000000	14.000000	18.200000	0.078000	21.000000	0.000000
25%	1.000000	99.750000	64.000000	25.000000	121.500000	27.500000	0.243750	24.000000	0.000000
50%	3.000000	117.000000	72.000000	29.000000	125.000000	32.300000	0.372500	29.000000	0.000000
75%	6.000000	140.250000	80.000000	32.000000	127.250000	36.600000	0.626250	41.000000	1.000000
max	17.000000	199.000000	122.000000	99.000000	846.000000	67.100000	2.420000	81.000000	1.000000

2. Training Test Split and Standard Scaling of the Data:

Train - Test Split Followed by Standard Scaling:

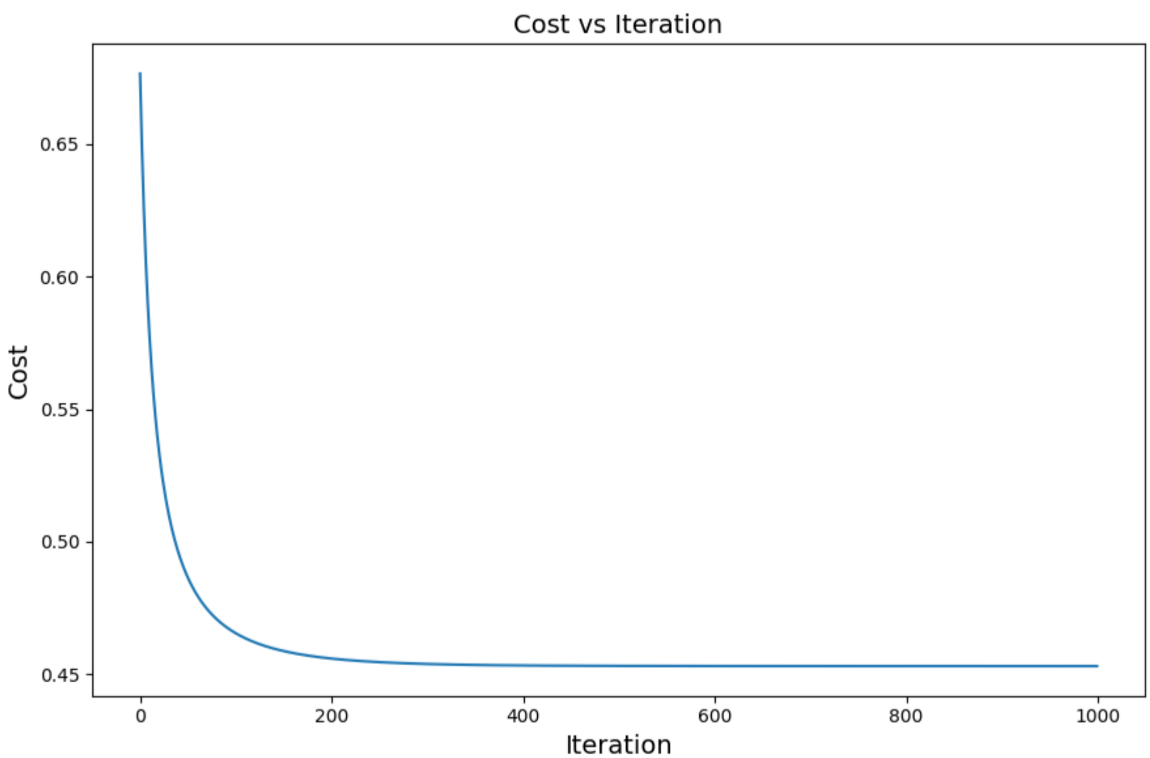
41]:
```python
# Train-test split
X = data_pima_diabetes.drop(columns=['Outcome']).values
y = data_pima_diabetes['Outcome'].values
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_state=42, stratify=y)

# Standardize features
scaler = StandardScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)
```

3. Training of the Sigmoid Regression:

Training the model:

[42]:
```python
# Initialize parameters
w = np.zeros(X_train_scaled.shape[1])
b = 0.0
alpha = 0.1
n_iter = 1000

# Train model
print("\nTraining Logistic Regression Model:")
w, b, cost_history,params_history = gradient_descent(X_train_scaled, y_train, w, b, alpha, n_iter, show_cost=True, show_params=False)
# Plot cost history
plt.figure(figsize=(9, 6))
plt.plot(cost_history)
plt.xlabel("Iteration", fontsize=14)
plt.ylabel("Cost", fontsize=14)
plt.title("Cost vs Iteration", fontsize=14)
plt.tight_layout()
```

```
Training Logistic Regression Model:
```



4. Did the Model Overfitt or Underfitt?

Evaluating Train and Test Performance on Cost Value:

[43]:
```
# Test model
y_train_pred = prediction(X_train_scaled, w, b)
y_test_pred = prediction(X_test_scaled, w, b)

# Evaluate train and test performance
train_cost = costfunction_logreg(X_train_scaled, y_train, w, b)
test_cost = costfunction_logreg(X_test_scaled, y_test, w, b)
print(f"\nTrain Loss (Cost): {train_cost:.4f}")
print(f"Test Loss (Cost): {test_cost:.4f}")
```

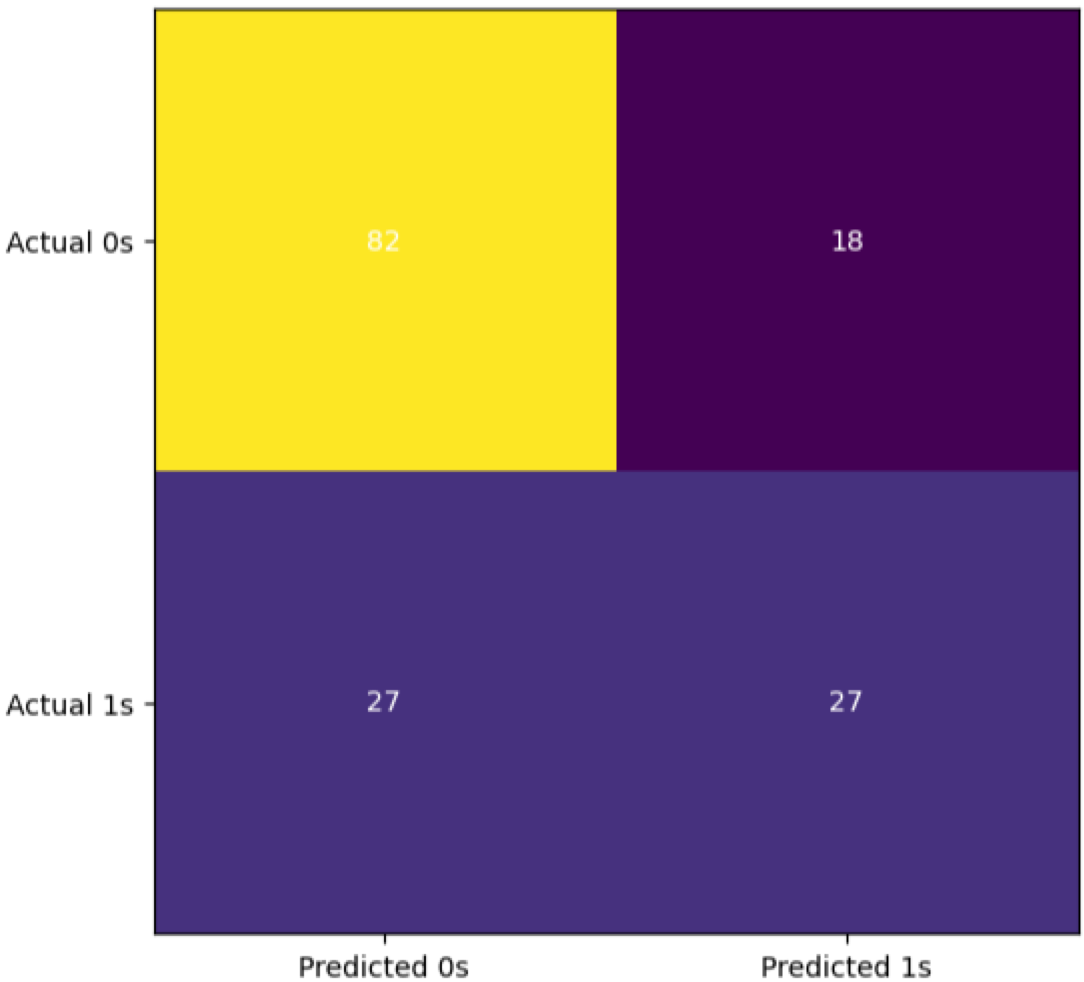
```

Train Loss (Cost): 0.4531
Test Loss (Cost): 0.5146
```

5. How well my model did?

Evalation on various Metrics for Classification:

[49]:
```python
# Accuracy on test data
test_accuracy = np.mean(y_test_pred == y_test) * 100
print(f"\nTest Accuracy: {test_accuracy:.2f}%")

# Evaluation
metrics = evaluate_classification(y_test, y_test_pred)
confusion_matrix = metrics["confusion_matrix"]
precision = float(metrics["precision"])
recall = float(metrics["recall"])
f1_score = float(metrics["f1_score"])
print(f"\nConfusion Matrix:\n{confusion_matrix}")
print(f"Precision: {precision:.2f}")
print(f"Recall: {recall:.2f}")
print(f"F1-Score: {f1_score:.2f}")

#Optional - Visualizing the COnfusion matrix
# Visualizing Confusion Matrix
fig, ax = plt.subplots(figsize=(6, 6))
ax.imshow(confusion_matrix)
ax.grid(False)
ax.xaxis.set(ticks=(0, 1), ticklabels=('Predicted 0s', 'Predicted 1s'))
ax.yaxis.set(ticks=(0, 1), ticklabels=('Actual 0s', 'Actual 1s'))
ax.set_ylim(1.5, -0.5)
for i in range(2):
    for j in range(2):
        ax.text(j, i, confusion_matrix[i, j], ha='center', va='center', color='white')

plt.show()
```

```
Test Accuracy: 70.78%

Confusion Matrix:
[[82 18]
 [27 27]]
Precision: 0.60
Recall: 0.50
F1-Score: 0.55
```



Part – 2

```
•[1]: # Prayan Piya
      # 2462985
      # np03cs4a240098
```

```
[16]: import numpy as np
      import pandas as pd
      import matplotlib.pyplot as plt
```

Implementation of Softmax Regression from Scratch:

To-do 1:

Implementation of Softmax function:

```
[35]: def softmax(z):
          """
          Compute the softmax of a 2D numpy array along the specified axis.
          Parameters:
          z (numpy.ndarray): Input array of shape (m, n) where m is the number of samples and n is the number of classes.

          Returns:
          numpy.ndarray: Softmax probabilities of the same shape as input (m, n), where each row sums to 1 and represents the probability distribution over c

          Notes:
          - Applies a normalization trick to prevent numerical instability by subtracting the max value in each row before exponentiation.
          """

          # Normalize input to prevent numerical instability
          exp_z = np.exp(z - np.max(z, axis=1, keepdims=True))
          return exp_z / np.sum(exp_z, axis=1, keepdims=True)
```

Testing the softmax function:

```
def test_softmax():
    """
    Perform basic assertion tests on the softmax function to validate its correctness.
    Tests:
    - Ensure that the output probabilities sum to 1 for each row.
    - Ensure non-negative values (all probabilities should be >= 0).- Test on edge cases (e.g., all zeros, very large or small values).
    """
    # Test input
    test_cases = [
    (np.array([[0, 0, 0]]), "All zeros"),
    (np.array([[1, 2, 3]]), "Simple case"),
    (np.array([[1000, 1000, 1000]]), "Large identical values"),
    (np.array([[-1000, -1000, -1000]]), "Small identical values"),
    (np.array([[1, 0, -1]]), "Mixed positive and negative")
    ]

    for i, (z, description) in enumerate(test_cases):
        print(f"Test {i + 1}: {description}")
        result = softmax(z)

        # Check that probabilities sum to 1
        assert np.allclose(result.sum(axis=1), 1), f"Failed: Probabilities do not sum to 1 in {description}"

        # Check non-negativity
        assert np.all(result >= 0), f"Failed: Negative probabilities in {description}"

        print("Passed.")

    print("\nAll tests passed for softmax function.")

test_softmax()
```

```
Test 1: All zeros
Passed.
Test 2: Simple case
Passed.
Test 3: Large identical values
Passed.
```

```
Test 4: Small identical values
Passed.
Test 5: Mixed positive and negative
Passed.

All tests passed for softmax function.
```

To-do 2:

Implementation of Categorial Log-Loss function:

```
def loss_softmax(y_true, y_pred):
    """
    Compute the cross-entropy loss for a single observation.

    Parameters:
    y_true (numpy.ndarray): True labels (one-hot encoded) of shape (c,).
    y_pred (numpy.ndarray): Predicted probabilities of shape (c,).

    Returns:
    float: Cross-entropy loss for the observation.
    """
    return -np.sum(y_true * np.log(y_pred + 1e-10)) # Add epsilon to prevent log(0)
```

Testing the Categorial Log-Loss function:

[6]:
```
def test_loss_softmax():
    """
    Test the loss_softmax function using a known input and output.
    """
    # Test Case 1: Perfect prediction
    y_true = np.array([0, 1, 0]) # True label (one-hot encoded)
    y_pred = np.array([0.1, 0.8, 0.1]) # Predicted probabilities
    expected_loss = -np.log(0.8) # Expected loss for perfect prediction
    assert np.isclose(loss_softmax(y_true, y_pred), expected_loss), "Test Case 1 Failed"

    # Test Case 2: Incorrect prediction
    y_true = np.array([1, 0, 0]) # True label (one-hot encoded)
    y_pred = np.array([0.3, 0.4, 0.3]) # Predicted probabilities
    expected_loss = -np.log(0.3) # Expected loss for incorrect prediction
    assert np.isclose(loss_softmax(y_true, y_pred), expected_loss), "Test Case 2 Failed"

    # Test Case 3: Edge case with near-zero probability
    y_true = np.array([0, 1, 0]) # True label (one-hot encoded)
    y_pred = np.array([0.01, 0.98, 0.01]) # Predicted probabilities
    expected_loss = -np.log(0.98) # Expected loss for edge case
    assert np.isclose(loss_softmax(y_true, y_pred), expected_loss), "Test Case 3 Failed"

    print("All test cases passed!")

# Run the test
test_loss_softmax()
```

```
All test cases passed!
```

To-do 3:

Implementation of Cost Function:

[34]:
```
def cost_softmax(X, y, W, b):
    """
    Compute the average cross-entropy cost over all samples.
    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).
    Returns:
    float: Average cross-entropy cost over all samples.
    """
    n, d = X.shape
    z = np.dot(X, W) + b
    y_pred = softmax(z)
    return -np.sum(y * np.log(y_pred + 1e-10)) / n
```

Testing of Cost Function:

```
24]: def test_cost_softmax():
    """
    Test the cost_softmax function using a known input and output.
    """
    # Test Case 1: Small dataset with perfect predictions
    X = np.array([[1, 2], [2, 3], [3, 4]]) # Feature matrix (n=3, d=2)
    y = np.array([[1, 0], [0, 1], [1, 0]]) # True labels (n=3, c=2, one-hot encoded)
    W = np.array([[1, -1], [-1, 1]]) # Weight matrix (d=2, c=2)
    b = np.array([0, 0]) # Bias vector (c=2)
    z = np.dot(X, W) + b
    y_pred = softmax(z) # Predicted probabilities
    expected_cost = -np.sum(y * np.log(y_pred + 1e-10)) / X.shape[0] # Compute expected cost
    assert np.isclose(cost_softmax(X, y, W, b), expected_cost), "Test Case 1 Failed"

    # Test Case 2: All-zero weights and bias
    X = np.array([[1, 0], [0, 1], [1, 1]]) # Feature matrix (n=3, d=2)
    y = np.array([[1, 0], [0, 1], [1, 0]]) # True labels (n=3, c=2, one-hot encoded)
    W = np.zeros((2, 2)) # Zero weight matrix
    b = np.zeros(2) # Zero bias vector
    z = np.dot(X, W) + b
    y_pred = softmax(z) # Predicted probabilities (uniform distribution)
    expected_cost = -np.sum(y * np.log(y_pred + 1e-10)) / X.shape[0] # Compute expected cost
    assert np.isclose(cost_softmax(X, y, W, b), expected_cost), "Test Case 2 Failed"

    print("All test cases passed!")

# Run the test
test_cost_softmax()
```

```
All test cases passed!
```

4. Implementing Gradient Descent for Training Softmax Regression: ¶

4.1 Gradients for Softmax Regression:

Function for computing Gradients for Softmax Regression:

```
]: def compute_gradient_softmax(X, y, W, b):
    """
    Compute the gradients of the cost function with respect to weights and biases.
    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).
    Returns:
    tuple: Gradients with respect to weights (d, c) and biases (c,).
    """
    n, d = X.shape
    z = np.dot(X, W) + b
    y_pred = softmax(z)
    grad_W = np.dot(X.T, (y_pred - y)) / n
    grad_b = np.sum(y_pred - y, axis=0) / n
    return grad_W, grad_b
```

An assertion test for compute_gradient_softmax function:

[11]:
```
# Test function
def test_compute_gradient_softmax():
    # Define simple inputs
    X = np.array([[1, 2], [3, 4]]) # Shape (2, 2)
    y = np.array([[1, 0], [0, 1]]) # Shape (2, 2), one-hot encoded
    W = np.array([[0.1, 0.2], [0.3, 0.4]]) # Shape (2, 2)
    b = np.array([0.01, 0.02]) # Shape (2,)

    # Expected gradients (calculated manually or using a reference implementation)
    z = np.dot(X, W) + b
    y_pred = softmax(z)
    grad_W_expected = np.dot(X.T, (y_pred - y)) / X.shape[0]
    grad_b_expected = np.sum(y_pred - y, axis=0) / X.shape[0]

    # Compute gradients using the function
    grad_W, grad_b = compute_gradient_softmax(X, y, W, b)

    # Assertions
    assert np.allclose(grad_W, grad_W_expected, atol=1e-6), "Gradient W does not match expected values"
    assert np.allclose(grad_b, grad_b_expected, atol=1e-6), "Gradient b does not match expected values"
    print("All tests passed for compute_gradient_softmax!")

# Run the test
test_compute_gradient_softmax()
```

```
All tests passed for compute_gradient_softmax!
```

4.2 Gradient Descent for Softmax Regression:

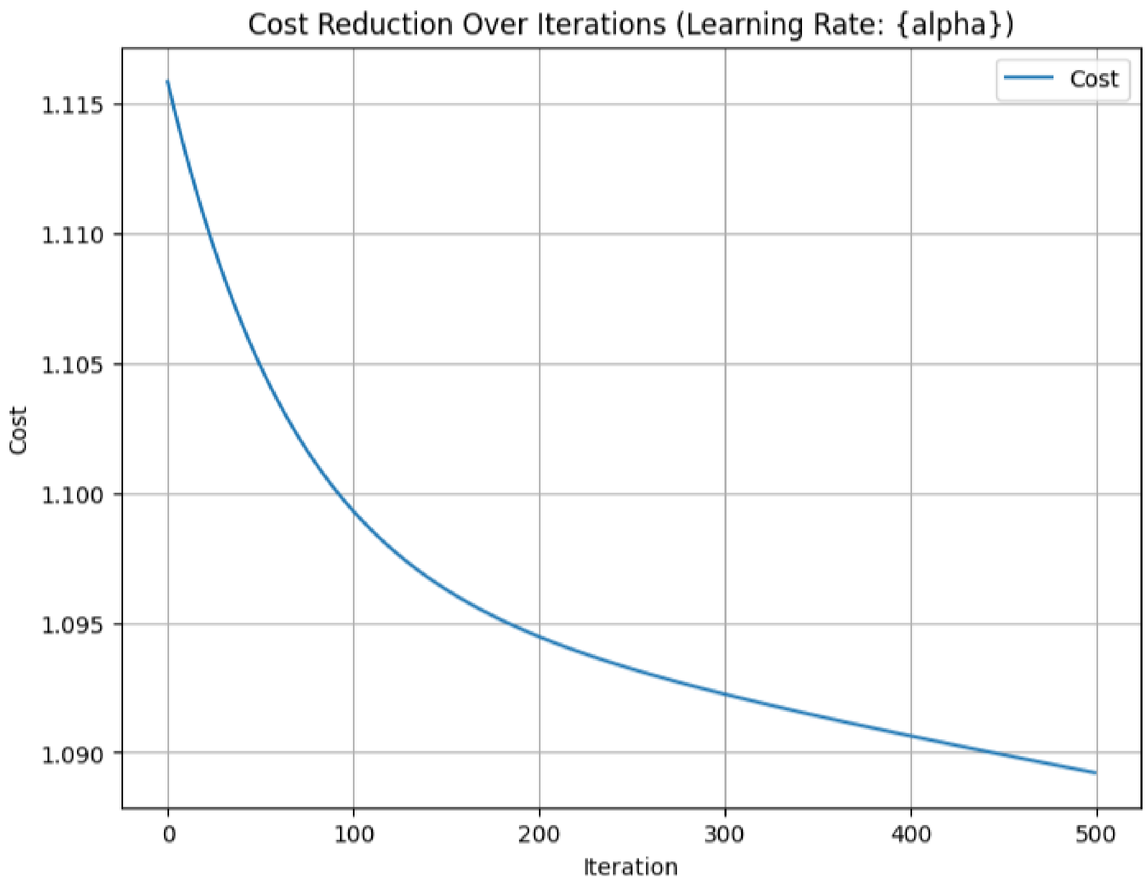
Function for Gradient Descent for Softmax Regression:

[14]:

```
# Gradient Descent
def gradient_descent_softmax(X, y, W, b, alpha, n_iter, show_cost=False):
    """
    Perform gradient descent to optimize the weights and biases.
    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    y (numpy.ndarray): True labels (one-hot encoded) of shape (n, c).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).
    alpha (float): Learning rate.
    n_iter (int): Number of iterations.
    show_cost (bool): Whether to display the cost at intervals.
    Returns:
    tuple: Optimized weights, biases, and cost history.
    """
    cost_history = []
    for i in range(n_iter):
        grad_W, grad_b = compute_gradient_softmax(X, y, W, b)
        W -= alpha * grad_W
        b -= alpha * grad_b
        cost = cost_softmax(X, y, W, b)
        cost_history.append(cost)
        if show_cost and (i % 100 == 0 or i == n_iter - 1):
            print(f"Iteration {i}: Cost = {cost:.6f}")
    return W, b, cost_history
```

A simple test for gradient_descent_softmax Function:

18]:
```
# To test a Gradient Descent we plot the Cost vs. Iterations and observe the behaviour and flow of the plot.
def test_gradient_descent_softmax_with_plot():
    # Generate synthetic data for testing
    np.random.seed(0)
    n, d, c = 100, 5, 3 # 100 samples, 5 features, 3 classes
    X = np.random.rand(n, d)
    y_indices = np.random.randint(0, c, size=n)
    y = np.zeros((n, c))
    y[np.arange(n), y_indices] = 1 # One-hot encoding

    # Initialize weights and biases
    W = np.random.rand(d, c)
    b = np.random.rand(c)

    # Parameters for gradient descent
    alpha = 0.01 # Learning rate
    n_iter = 500 # Number of iterations

    # Run gradient descent
    W_opt, b_opt, cost_history = gradient_descent_softmax(X, y, W, b, alpha, n_iter, show_cost=False)

    # Plot the cost history
    plt.figure(figsize=(8, 6))
    plt.plot(range(n_iter), cost_history, label="Cost")
    plt.xlabel("Iteration")
    plt.ylabel("Cost")
    plt.title("Cost Reduction Over Iterations (Learning Rate: {alpha})")
    plt.legend()
    plt.grid()
    plt.show()

    # Final cost should ideally be less than initial cost
    print(f"Initial Cost: {cost_history[0]}")
    print(f"Final Cost: {cost_history[-1]}")

test_gradient_descent_softmax_with_plot()
```




```
Initial Cost: 1.1158608479596404
Final Cost: 1.0892233754862581
```

2. Constructing One Hot Encoding for Label Vector:

One Hot Encoding for Target:

[30]:
```python
from sklearn.preprocessing import OneHotEncoder

X = iris.data # Creates a Feature Matrix
y = iris.target # Creates a Target Vector
encoder = OneHotEncoder(sparse_output=False)
y_onehot = encoder.fit_transform(y.reshape(-1, 1))
y_onehot
```

Testing of Prediction function:

[37]:
```
def test_predict_softmax():
    # Generate synthetic data for testing
    np.random.seed(0)

    n, d, c = 10, 5, 3 # 10 samples, 5 features, 3 classes
    X = np.random.rand(n, d)
    W = np.random.rand(d, c)
    b = np.random.rand(c)

    # Compute the predictions using the function
    predictions = predict_softmax(X, W, b)

    # Check the shape of the output
    assert predictions.shape == (n,), f"Shape mismatch: expected {(n,)}, got {predictions.shape}"

    # Verify that all predicted labels are within the range of class indices
    assert np.all(predictions >= 0) and np.all(predictions < c), (f"Predictions out of range: expected 0 to {c-1}, got {predictions}")

    # Check that the predicted labels are integers
    assert np.issubdtype(predictions.dtype, np.integer), f"Predictions are not integers: {predictions.
    dtype}"
    print("All tests passed for predict_softmax!")

# Run the test
test_predict_softmax()
```

```
All tests passed for predict_softmax!
```

19]:
```
# Evaluation Function
def evaluate_classification(y_true, y_pred):
    """
    Evaluate the classification performance using confusion matrix, precision, recall, and F1-score.
    Parameters:
    y_true (numpy.ndarray): True class labels of shape (n,).
    y_pred (numpy.ndarray): Predicted class labels of shape (n,).
    Returns:
    tuple: Confusion matrix, precision, recall, and F1-score.
    """
    from sklearn.metrics import confusion_matrix, precision_score, recall_score, f1_score
    cm = confusion_matrix(y_true, y_pred)
    precision = precision_score(y_true, y_pred, average="weighted", zero_division=0)
    recall = recall_score(y_true, y_pred, average="weighted", zero_division=0)
    f1 = f1_score(y_true, y_pred, average="weighted", zero_division=0)
    return cm, precision, recall, f1
```

Summary Statistics:

[22]:
```python
print("\nDataset Description Before Cleaning:")
X.describe()
```

```
Dataset Description Before Cleaning:
```

[22]:

	sepal length (cm)	sepal width (cm)	petal length (cm)	petal width (cm)
count	150.000000	150.000000	150.000000	150.000000
mean	5.843333	3.057333	3.758000	1.199333
std	0.828066	0.435866	1.765298	0.762238
min	4.300000	2.000000	1.000000	0.100000
25%	5.100000	2.800000	1.600000	0.300000
50%	5.800000	3.000000	4.350000	1.300000
75%	6.400000	3.300000	5.100000	1.800000
max	7.900000	4.400000	6.900000	2.500000

4.2 Putting Helper Function to Action - Softmax Regression for the dataset:

1. Some Basic Data Operation, Loading, Analysis and Cleaning:

Necessary Import and Checking the Data:

[ ]:
```python
import numpy as np
import pandas as pd
from sklearn.datasets import load_iris

# Load the Iris dataset
iris = load_iris()
X = pd.DataFrame(iris.data, columns=iris.feature_names)
y = pd.Series(iris.target, name="target")

# Display information about the dataset before cleaning
print("Dataset Info:")
X.info()
```

```
Dataset Info:
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 150 entries, 0 to 149
Data columns (total 4 columns):
 #   Column             Non-Null Count  Dtype  
---  ------             --------------  -----  
 0   sepal length (cm)  150 non-null    float64
 1   sepal width (cm)   150 non-null    float64
 2   petal length (cm)  150 non-null    float64
 3   petal width (cm)   150 non-null    float64
dtypes: float64(4)
memory usage: 4.8 KB
```

5. Decision/Prediction Function for Softmax Regression for Multi-class Classification:

Decision/Precision Function:

[36]:
```python
# Prediction Function
def predict_softmax(X, W, b):
    """
    Predict the class labels for input data.
    Parameters:
    X (numpy.ndarray): Feature matrix of shape (n, d).
    W (numpy.ndarray): Weight matrix of shape (d, c).
    b (numpy.ndarray): Bias vector of shape (c,).
    Returns:
    numpy.ndarray: Predicted class labels of shape (n,).
    """
    z = np.dot(X, W) + b
    y_pred = softmax(z)
    return np.argmax(y_pred, axis=1)
```

Testing of Prediction function:

[37]:
```python
def test_predict_softmax():
    # Generate synthetic data for testing
    np.random.seed(0)

    n, d, c = 10, 5, 3 # 10 samples, 5 features, 3 classes
    X = np.random.rand(n, d)
    W = np.random.rand(d, c)
    b = np.random.rand(c)

    # Compute the predictions using the function
    predictions = predict_softmax(X, W, b)

    # Check the shape of the output
    assert predictions.shape == (n,), f"Shape mismatch: expected {(n,)}, got {predictions.shape}"

    # Verify that all predicted labels are within the range of class indices
    assert np.all(predictions >= 0) and np.all(predictions < c), (f"Predictions out of range: expected 0 to {c-1}, got {predictions}")

    # Check that the predicted labels are integers
    assert np.issubdtype(predictions.dtype, np.integer), f"Predictions are not integers: {predictions.
    dtype}"
    print("All tests passed for predict_softmax!")

# Run the test
test_predict_softmax()
```

```
All tests passed for predict_softmax!
```

[19]:
```python
# Evaluation Function
def evaluate_classification(y_true, y_pred):
    """
    Evaluate the classification performance using confusion matrix, precision, recall, and F1-score.
    Parameters:
    y_true (numpy.ndarray): True class labels of shape (n,).
    y_pred (numpy.ndarray): Predicted class labels of shape (n,).
    Returns:
    tuple: Confusion matrix, precision, recall, and F1-score.
    """
    from sklearn.metrics import confusion_matrix, precision_score, recall_score, f1_score
    cm = confusion_matrix(y_true, y_pred)
    precision = precision_score(y_true, y_pred, average="weighted", zero_division=0)
    recall = recall_score(y_true, y_pred, average="weighted", zero_division=0)
    f1 = f1_score(y_true, y_pred, average="weighted", zero_division=0)
    return cm, precision, recall, f1
```

3. Train Test Split and Standard Scaling of the Data:

Train - Test Split Followed by Standard Scaling:

```
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler

# Split data into train and test sets
X_train, X_test, y_train, y_test = train_test_split(X, y_onehot, test_size=0.2, random_state=42, stratify=y)

# Initialize the scaler and scale the data
scaler = StandardScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)

# Assertions to check the shape of X_train_scaled and X_test_scaled
assert X_train_scaled.shape == (X_train.shape[0], X_train.shape[1]), f"X_train_scaled shape mismatch: {X_train_scaled.shape}"
assert X_test_scaled.shape == (X_test.shape[0], X_test.shape[1]), f"X_test_scaled shape mismatch: {X_test_scaled.shape}"

print("Shape assertions passed!")
```

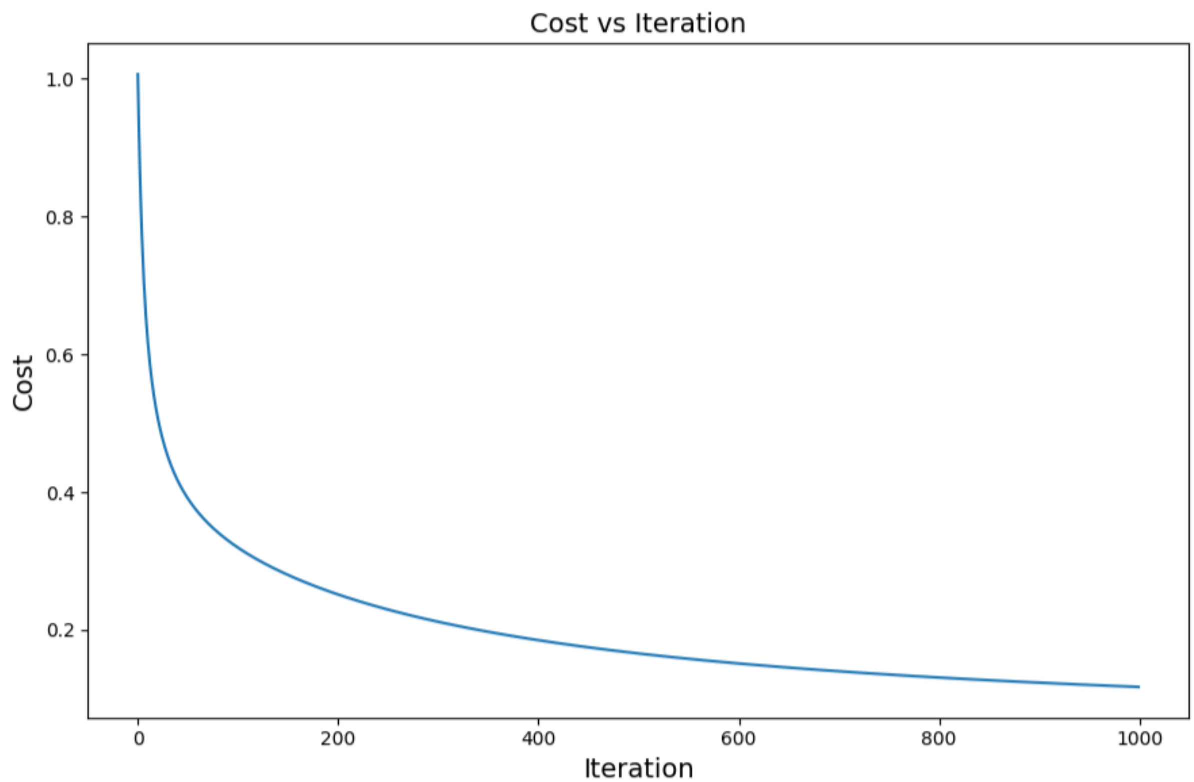
```
Shape assertions passed!
```

4. Training of the Softmax Regression:

Training the Model:

```
# Intitalizations:
num_classes = y_train.shape[1]
num_features = X_train_scaled.shape[1]
W = np.zeros((num_features, num_classes))
b = np.zeros(num_classes)
alpha = 0.1
n_iter = 1000
print("\nTraining Softmax Regression Model:")
W, b, cost_history = gradient_descent_softmax(X_train_scaled, y_train, W, b, alpha, n_iter, show_cost=True)

#Plot Cost Vs. Iterations:
plt.figure(figsize=(9, 6))
plt.plot(cost_history)
plt.xlabel("Iteration", fontsize=14)
plt.ylabel("Cost", fontsize=14)
plt.title("Cost vs Iteration", fontsize=14)
plt.tight_layout()
plt.show()
plt.show()
```

```
Training Softmax Regression Model:
Iteration 0: Cost = 1.006823
Iteration 100: Cost = 0.319428
Iteration 200: Cost = 0.251376
Iteration 300: Cost = 0.211741
Iteration 400: Cost = 0.185004
Iteration 500: Cost = 0.165744
Iteration 600: Cost = 0.151226
Iteration 700: Cost = 0.139898
Iteration 800: Cost = 0.130812
Iteration 900: Cost = 0.123360
Iteration 999: Cost = 0.117192
```



5. Did the Model Overfitt or Underfitt?

Evaluating Train and Test Performance on Cost Value:

]:
```
# Test model
y_train_pred = predict_softmax(X_train_scaled, W, b)
y_test_pred = predict_softmax(X_test_scaled, W, b)

# Evaluate train and test performance
train_cost = cost_softmax(X_train_scaled, y_train, W, b)
test_cost = cost_softmax(X_test_scaled, y_test, W, b)
print(f"\nTrain Loss (Cost): {train_cost:.4f}")
print(f"Test Loss (Cost): {test_cost:.4f}")
```

```
Train Loss (Cost): 0.1172
Test Loss (Cost): 0.1575
```

5. How well my model did?

Evalation on various Metrics for Classification:

[41]:

```
# Accuracy on test data
y_test_true = np.argmax(y_test, axis=1)
test_accuracy = np.mean(y_test_pred == y_test_true) * 100

print(f"\nTest Accuracy: {test_accuracy:.2f}%")

# Evaluation
y_test_true = np.argmax(y_test, axis=1)
cm, precision, recall, f1 = evaluate_classification(y_test_true, y_test_pred)
print("\nConfusion Matrix:")
print(cm)
print(f"Precision: {precision:.2f}")
print(f"Recall: {recall:.2f}")
print(f"F1-Score: {f1:.2f}")

#Optional - Visualizing the COnfusion matrix
# Visualizing Confusion Matrix
# Visualization
fig, ax = plt.subplots(figsize=(6, 6))
ax.imshow(cm, cmap='Blues') # Use a color map for better visualization
# Set tick labels for the axes
ax.set_xticks(range(3))
ax.set_yticks(range(3))
ax.set_xticklabels(['Predicted 0', 'Predicted 1', 'Predicted 2'])
ax.set_yticklabels(['Actual 0', 'Actual 1', 'Actual 2'])
# Add labels to each cell in the confusion matrix
for i in range(cm.shape[0]):
    for j in range(cm.shape[1]):
        ax.text(j, i, cm[i, j], ha='center', va='center', color='white' if cm[i, j] > np.max(cm) / 2 else 'black')

# Add grid lines and axis labels
ax.grid(False)
plt.title('Confusion Matrix', fontsize=14)
plt.xlabel('Predicted Label', fontsize=12)
plt.ylabel('Actual Label', fontsize=12)
plt.tight_layout()
plt.show()
```

```
Test Accuracy: 93.33%

Confusion Matrix:
[[10  0  0]
 [ 0  9  1]
 [ 0  1  9]]
Precision: 0.93
Recall: 0.93
F1-Score: 0.93
```