Mechanics Of Composites

Term Project

Hand Written Solution

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Given Fibre Properties :-

AS4 :- $E_1 = 225$ GPa $\quad E_2 = E_3 = 15$ GPa $\quad G_{12} = 15$ GPa

$\nu_{12} = 0.2$ $\quad \nu_{23} = 0.25$

Epoxy 2 :- $E_1 = 5.35$ GPa $\quad \nu_{12} = 0.35$

V_f (fibre volume fraction) $= 0.512$

Layup Sequence $= [0, 39, -39, 90]_s$

$\Rightarrow$ ① Effective ply properties.

$$E_{1\,ply} = V_f E_{1f} + V_m E_{1m}$$

$$= (0.512)(225) + (0.488)(5.35)$$

$$= 117.81 \text{ GPa}$$

$$E_{2\,ply} = \frac{E_{2f} \times E_{2m}}{E_{2f} V_m + E_{2m} V_f} = 7.98 \text{ GPa}$$

$$\nu_{12\,ply} = \nu_f V_f + \nu_m V_m = 0.2732$$

$$G_m = \frac{E_m}{2(1+nu)} = \frac{5.35}{2(1+0.35)} = 1.981\ GPa$$

$$G_{12\,ply} = \frac{G_m\, G_{12f}}{G_{12f} V_m + G_m V_f} = 3.57\ GPa$$

Calculating Q matrix (According to CLT assumption)

$$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

$$Q_{11} = \frac{E_{1ply}}{\left(1 - \nu_{12ply}^2 \frac{E_{2ply}}{E_{1ply}}\right)} = 118\ GPa$$

$$Q_{22} = \frac{E_{2ply}}{\left(1 - \nu_{12ply}^2 \frac{E_{2ply}}{E_{1ply}}\right)} = 8.02\ GPa$$

$$Q_{12} = \frac{\nu_{12ply} E_{2ply}}{\left(1 - \nu_{12ply}^2 \frac{E_{2ply}}{E_{1ply}}\right)} = 2.19\ GPa$$

$$Q_{66} = G_{12} = 3.57\ GPa$$

$$\therefore [Q]_0 = \begin{bmatrix} 118 & 2.19 & 0 \\ 2.19 & 8.02 & 0 \\ 0 & 0 & 3.57 \end{bmatrix} GPa.$$

$$[Q]_{90} = \begin{bmatrix} 8.02 & 2.19 & 0 \\ 2.19 & 118 & 0 \\ 0 & 0 & 3.57 \end{bmatrix} GPa$$

$$[Q]_{39} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix}^{-1} [Q]_0 \begin{bmatrix} m^2 & n^2 & mn \\ n^2 & m^2 & -mn \\ -2mn & 2mn & m^2-n^2 \end{bmatrix}$$

where $m = \cos 39$ & $n = \sin 39$.

$$Q_{xx} = m^4 Q_{11} + 2m^2n^2 (Q_{12} + 2Q_{66}) + n^4 Q_{22}$$

$$Q_{xx} = 48.76 \text{ GPa}$$

$$Q_{yy} = n^4 Q_{11} + 2m^2n^2 (Q_{12} + 2Q_{66}) + m^4 Q_{22}$$

$$Q_{yy} = 25.90 \text{ GPa}$$

$$Q_{xy} = m^2n^2 (Q_{11} + Q_{22} - 4Q_{66}) + Q_{12} (m^4 + n^4)$$

$$Q_{xy} = 27.87 \text{ GPa}$$

$$Q_{xs} = m^3 n (Q_{11} - Q_{12}) + mn^3 (Q_{12} - Q_{22}) - 2mn(m^2 - n^2) Q_{66}$$

$$Q_{xs} = 32.35 \text{ GPa}$$

$$Q_{ys} = mn^3 (Q_{11} - Q_{12}) + m^3 n (Q_{12} - Q_{22}) + 2mn(m^2 - n^2) Q_{66}$$

$$Q_{ys} = 21.43 \text{ GPa}$$

$$Q_{ss} = m^2n^2 (Q_{11} + Q_{22} - 2Q_{12}) + (m^2 - n^2)^2 Q_{66}$$

$$Q_{ss} = 29.84 \text{ GPa}$$

$$\therefore \; [Q]_{39} = \begin{bmatrix} 48.76 & 27.87 & 32.35 \\ 27.87 & 25.90 & 21.43 \\ 32.35 & 21.43 & 29.84 \end{bmatrix} \text{GPa}$$

$$[Q]_{-39} = \begin{bmatrix} 48.76 & 27.87 & -32.35 \\ 27.87 & 25.90 & -21.43 \\ -32.35 & -21.43 & 29.84 \end{bmatrix} \text{GPa}$$

② Calculation of ABD matrix.

Total thickness of laminate = 0.001 m

Thickness of each ply = 0.000125 m = t_K

	0°	39°	-39°	90°	90°	-39°	39°	0°
z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8
$-4t_K$	$-3t_K$	$-2t_K$	$-t_K$	0	t_K	$2t_K$	$3t_K$	$4t_K$.

$$A_{ij} = \sum_{K=1}^{N} [Q_{ij}]_K (z_K - z_{K-1})$$

$$= \Big([Q]_0 + [Q]_{39} + [Q]_{-39} + [Q_{90}] \Big) t_K \times 2$$

$$= \begin{bmatrix} 223.54 & 60.12 & 0 \\ 60.12 & 178 & 0 \\ 0 & 0 & 66.82 \end{bmatrix} \text{GPa} \times 0.25 \times 10^{-3}$$

$$= \begin{bmatrix} 55.885 & 15.03 & 0 \\ 15.03 & 44.8 & 0 \\ 0 & 0 & 16.705 \end{bmatrix} \text{MPa}$$

$$B_{ij} = \frac{1}{2} \sum_{K=1}^{N} (Q_{ij})_K \left[z_K^2 - z_{K-1}^2 \right]$$

Since the composite is symmetric

$$[B] = 0$$

$$D_{ij} = \frac{1}{3} \sum_{K=1}^{N} [Q_{ij}]_K \left[z_K^3 - z_{K-1}^3 \right]$$

$$= \frac{2}{3} \left[[Q]_0 (4^3 - 3^3) + [Q]_{39} (3^3 - 2^3) + [Q]_{-39} (2^3 - 1) + [Q]_{90} (1^3 - 0^3) \right]$$

$$\times t_K^3$$

$$= \begin{bmatrix} 7.37 & 1.055 & 0.51 \\ 1.055 & 1.42 & 0.33 \\ 0.51 & 0.33 & 1.17 \end{bmatrix}$$

$$\therefore ABD = \begin{bmatrix} 55.885 \times 10^6 & 15.03 \times 10^6 & 0 & 0 & 0 & 0 \\ 15.03 \times 10^6 & 44.8 \times 10^6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16.705 \times 10^6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.37 & 1.055 & 0.51 \\ 0 & 0 & 0 & 1.055 & 1.42 & 0.33 \\ 0 & 0 & 0 & 0.51 & 0.33 & 1.17 \end{bmatrix}$$

$$\text{Load} = \begin{bmatrix} 100000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 100000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} & & \\ & ABD & \\ & & \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_s^0 \\ K_x \\ K_y \\ K_s \end{bmatrix}$$

$$\Rightarrow \{\varepsilon_P\} = \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_s^0 \\ K_x \\ K_y \\ K_s \end{bmatrix} = [ABD]^{-1} \begin{bmatrix} 100000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \{\varepsilon_P\} = \begin{bmatrix} 0.002 \\ -6.6 \times 10^{-4} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\{\varepsilon_P\}_K = \{\varepsilon^0\} + z\{K_x\}$$

$\Rightarrow$ $\{\varepsilon_P\}$ for every ply $= \begin{bmatrix} 0.0019 \\ -0.00064 \\ 0 \end{bmatrix}$

$$[\sigma]_K = [Q]_K [\varepsilon]_K$$

$$[\sigma]_0 = [Q]_0 [\varepsilon]$$

$$= \begin{bmatrix} 118 & 2.19 & 0 \\ 2.19 & 8.02 & 0 \\ 0 & 0 & 3.57 \end{bmatrix} \begin{bmatrix} 0.0019 \\ -0.00064 \\ 0 \end{bmatrix}$$

$$[\sigma]_0 = \begin{bmatrix} 230.9 \\ -1.02 \\ 0 \end{bmatrix} \text{MPa}$$

similarly

$$[\sigma]_{39} = \begin{bmatrix} 77.4 \\ 37.7 \\ 49.5 \end{bmatrix} \text{MPa} \qquad [\sigma]_{-39} = \begin{bmatrix} 77.4 \\ 37.7 \\ 49.5 \end{bmatrix} \text{MPa}$$

$$[\sigma]_{90} = \begin{bmatrix} 14.2 \\ -74.3 \\ 0 \end{bmatrix} \text{MPa}$$

Now to apply hashin failure criteria

we need to rotate the stress is ply coordinate axis.

$$\therefore [\sigma]'_{\theta} = \begin{pmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{pmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}$$

$m = \cos\theta \qquad n = \sin\theta$

$$\therefore [\sigma]'_{0} = \begin{bmatrix} 230.9 \\ -1.02 \\ 0 \end{bmatrix} \text{MPa}$$

$$[\sigma]'_{39} = \begin{bmatrix} 110.03 \\ 5.03 \\ -9.16 \end{bmatrix} \text{MPa}$$

$$[\sigma]'_{-39} = \begin{bmatrix} 110.03 \\ 5.03 \\ -9.16 \end{bmatrix} \text{MPa}$$

$$[\sigma']_{90} = \begin{bmatrix} -74.3 \\ 14.2 \\ 0 \end{bmatrix} \text{MPa}$$

$X = 3200\,\text{MPa}$, $X_c = 150\,\text{MPa}$, $Y = 150\,\text{MPa}$, $Y' = 80$, $S_{12} = S_{23} = 80$

For 0° ply :- $\sigma_{11} = 230.9\,\text{MPa}$, $\sigma_{22} = -1.02\,\text{MPa}$, $\sigma_{12} = 0$

Since $\sigma_{22} < 0$

Matrix compressive failure mode.

$$\left[\left(\frac{80'}{2\times 80'}\right)^2 - 1\right] R\left(\frac{-1.02}{80}\right) + \left(\frac{1.02}{4\times 80}\right)^2 R^2 + 0 = 1$$

$$\Rightarrow 1.016\times 10^{-5} R^2 + 0.009 R = 0$$

$\Rightarrow$ $R =$ negative (not consider since $R<0$)

Tensile fibre failure mode :-

$$R^2\left(\frac{230.9}{3200}\right)^2 = 1 \qquad \left\{ \left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 = 1 \right\}$$

$\therefore \Rightarrow R = 13.85$

$\therefore$ for 0° ply $\boxed{R = 13.85}$

For 90° ply :- $\sigma_{11} = -74.3$, $\sigma_{22} = 14.2$, $\sigma_{12} = 0$

Matrix tensile failure mode.

$$\left(\frac{14.2}{150}\right)^2 R^2 = 1 \qquad \left\{ \left(\frac{\sigma_2}{Y^2}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 = 1 \right\}$$

$\Rightarrow R = 10.56$

~~Tensile fibre failure mode~~

Compressive fibre failure mode

$$\left(\frac{74.3}{1500}\right)^2 R^2 = 1 \qquad \left\{ \left(\frac{\sigma_1}{X'}\right)^2 = 1 \right\}$$

$\Rightarrow$ ~~$R = 20.18$~~.

$\boxed{R = 20.2}$

$\therefore$ ~~for 90° ply~~ ~~$\boxed{R = 20.189}$~~