

# 1 Introduction

A table of frequently used symbols:

Symbol	Representation
$g$	the input graph
$v$	the set of nodes in $g$
$e$	the set of edges in $g$
$n$	a node in $v$
$G$	the constructed metagraph
$V$	the set of metanodes in $G$
$E$	the set of metaedges in $G$
$N$	a metanode in $V$ of the form $\{n_i : x_i\}$ where $n_i \in v$ and $x_i$ is the number of atoms at $n_i$

Each  $N \in V$  is composed of  $i$  many nodes from  $v$ . A metanode takes the form  $\{n_i : x_i\}$  where  $n_i \in v$  and  $x_i$  is the number of atoms at  $n_i$ . The  $\sum_0^i x_i = k$  at any given  $N$  ensuring that all atoms are accounted for at each metanode

# 2 Algorithms

---

## Algorithm 1: ConstructMetaGraph

---

**Input:**  $g = (v, e)$ : the input graph,  $\{s, t\} \in v$ : the start and target compounds,  $k$ : flow/number of atoms to conserve

**Output:**  $G = (V, E)$ : the meta-graph

```

1  $MG \leftarrow (MV = \emptyset, ME = \emptyset)$  /* initialize new metagraph */
2  $stack \leftarrow$  initialize empty stack;
3  $start \leftarrow \{s : k\}$  /* Create a metanode with all  $k$  atoms at the start compound */
4  $V \leftarrow V \cup start$  /* Add the start state to the set of metanodes */
5  $stack.push(start)$  /* Add the start state to the stack to find its neighbors */
6  $G \leftarrow \text{PopulateMetaGraph}(G, stack, target)$  /* Find neighboring metanodes to build  $G$  */
7 return  $G$ 
```

---



---

## Algorithm 2: PopulateMetaGraph

---

**Input:**  $G = (V, E)$ : the metagraph,  $stack$ : stack of metanodes that need to be explored,  $target$ : the metanode state with all  $k$  atoms at node  $t$

**Output:**  $G = (V, E)$ : metagraph with any newly found metanodes added in

```

1 while  $|stack| > 0$  do
2    $current \leftarrow stack.pop()$  /* Pop off a metanode to explore */
3   if  $current = target$  then
4     continue /* If we've reached the target, no need to find nbrs */
5   else
6      $nbrCount \leftarrow \text{IterativeFindMetaNbrs}(current)$ ;
7     if  $nbrCount = 0$  and  $current \neq target$  then
8        $G \leftarrow \text{Prune}(current)$  /* This metanode is a terminus, so remove it from the metagraph */
9 return  $G$ 
```

---

---

**Algorithm 3:** IterativeFindMetaNbrs

---

**Input:** *parent*: the metanode to find nbrs for

**Output:**  $G = (V, E)$ : metagraph with any newly found metanodes added in

```
1 for  $n \in N$  /* Iterate for each node involved in the metanode state */
2 do
3   for each inner node  $n$  in  $N$ , for inner nbr  $m$ , move 0 to  $x_i$  flow. For each move, attempt to move the
     remaining flow at  $n$  to the remaining inner nbrs  $m$ . Then repeat for each  $n$  in  $N$ . Check if any of the
     newly generated states are valid and add them to MG and stack
```

---