CS 4920/5920 Spring 2021
HW 2

HW 2 is due on 2/25. Explain how you reached your answers. Answers without explanations will receive no to little credit.

Problem 1. Balls in a Bin

Suppose there are three red balls, two yellow balls, and four green balls in a bin. In each *event*, you pick one ball from the bin and observe the color of the ball (the balls are only distinguishable by their colors). After observation, you put the ball back into the bin.

a. What is the information entropy value for an event?
b. What is the information entropy for six events?
c. Now you add two additional yellow balls in the bin and conduct the events. What is the value of the entropy for an event?
d. Suppose there are only three colors: red, yellow, and green. From Part c. (with a total of eleven balls in the bin), you can either add one ball of any color to the bin or subtract one ball of any color from the bin. What would your action be (add or subtract and which color) to *maximize* the entropy of the events?
e. Suppose there are only three colors: red, yellow, and green. From Part c. (with a total of eleven balls in the bin), you can either add one ball of any color to the bin or subtract one ball of any color from the bin. What would your action be (add or subtract and which color) to *minimize* the entropy of the events?
f. Suppose now you can choose any color beyond red, yellow, and green. From Part c. (with a total of eleven balls in the bin), you can either add one ball of any color to the bin or subtract one ball of any color from the bin. What would your action be (add or subtract and which color) to *maximize* the entropy of the events?

Problem 2. Deck of Cards

There is a standard 52-card deck of cards (e.g., https://en.wikipedia.org/wiki/Standard_52-card_deck). There is a total of 52 cards, e.g., no extra joker cards. In each *event*, you shuffle the deck of cards randomly, pick a card, and observe the suit and the rank.

a. What is the information entropy value for one event?
b. What is the entropy value for six events?
c. Now suppose the face cards (the jack's, queen's, and king's) are considered the same as 10's. That is, jack's, queen's, and king's are effectively the same as 10's. What is the information entropy for one event?
d. Now suppose you only consider the suit in the card-picking event (i.e., rank does not matter and gets ignored). What is the information entropy for one event?
e. You can control and change the rank (but not the suit) of all the cards. How would you modify the deck of cards to minimize the information entropy? What is the resulting information entropy for an event after modifying the cards?
f. You can change both the suit and the rank of the cards. How would you modify the cards to minimize the information entropy? What is the resulting information entropy for an event after modifying the cards?

Problem 3.

Prove the followings:

a. a ≡ b (mod n) and b ≡ c (mod n) imply a ≡ c (mod n)

b. [(a mod n) - (b mod n)] mod n = (a - b) mod n
c. For two consecutive integers n and n+1, GCD(n,n+1)=1
d. Given two integers a and b, prove that Euclidean algorithm, described in Section 2.2, yields the greatest common divisor GCD(a,b)

Problem 4.
Use the variables in class for the following problems.

a. State the Euclidean Algorithm
b. Prove the Euclidean Algorithm
c. State the Extended Euclidean Algorithm
d. Prove the Extended Euclidean Algorithm

Problem 5.
Use Euclidean Algorithm to solve the followings.

a. GCD(2105,425)
b. GCD(3555,12075)
c. GCD(2078,9602)
d. GCD(24142,16762)

Note: Provide a table similar to Table 2.1 in the textbook. Alternatively, write a computer program to solve this problem; if you do so, describe your program and include your code (and the compiler output if applicable) in your homework.

Problem 6.
Using the extended Euclidean algorithm, find the multiplicative inverse of

a. 550 mod 1869
b. 835 mod 3321
c. 8144 mod 39902
d. 10003 mod 33241

Note: Provide a table similar to Table 2.4 in the textbook. Explain the reason if the multiplicative inverse does not exist. Alternatively, write a computer program to solve this problem; if you do so, describe your program and include your code (and the compiler output if applicable) in your homework.